Vector Fitting of Noisy Frequency Responses via Smoothing Regularization

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***Abstract*—We present a simple and effective strategy to compute reduced-order rational macromodels from noisy frequency responses. The reference macromodeling engine is the basic Vector Fitting (VF) scheme, which is well known to be sensitive to noise in the training data. This problem is here avoided by augmenting the VF cost function with a penalization term related to the second derivative of the model, which effectively acts as a regularizer. The results obtained on a set of noisy measurements of a Surface Acoustic Wave (SAW) filter demonstrate the effectiveness of proposed approach in rejecting noise and producing smooth models.**

I. Introduction

Vector Fitting (VF) is currently the most common algorithm for the identification of behavioral models of Linear and Time-Invariant (LTI) systems for electrical and electronic applications. Since its original formulation [1], several improvements have been documented, including concurrent fitting of multiple responses [2] and related implementation to multicore computing hardware [3], [4]. A comprehensive overview of the VF algorithm and its applications is available in [5], including post-processing algorithms for passivity enforcement and SPICE equivalent network synthesis.

It has been shown in [6] that noise in the training data may have the detrimental effect of impairing VF convergence, which usually takes place for "clean" data in few iterations. Various methods have been presented for handling noise in VF application, including hard pole relocation [7], mixed VF-Newton iterations [6], and instrumental variable approaches [8], [9]. All these approaches provide viable and effective solutions, which however require significant modifications to the main VF algorithm and code.

In this paper, we present an alternative, simple and effective approach based on a dedicated penalization term added to the VF cost function. Due to this structure, a minimal modification is required to any basic VF code, including the public available implementation [10]. This penalization is here derived based on an estimate of the second derivative of the model frequency responses. Second-order derivative minimization is in fact a well-known data processing technique for data smoothing in presence of significant noise [11]. The algorithm that we propose uses smoothed data during the pole relocation phase, in order to train a minimal number of basis poles. A second step for residue identification employs second-derivative penalization to make overall model behavior smooth and insensitive to noise. The proposed formulation is discussed in Sec. II.

We tested the proposed algorithm on several examples, confirming its effectiveness in noise rejection. We present in Sec. III some results obtained from a set of measured responses of a discrete filter component. The proposed penalization leads to models of significantly lower order than plain VF application, which for this case dramatically fails unless the number of poles is set to an unrealistically large number.

II. Formulation

A. Smoothing regularization

The main issue with approximating data that is corrupted with noise is that the resulting model might represent not only the main features of the underlying system, but also spurious contributions induced by noise. A well-known solution to this problem, denoted as *overfitting*, is regularization. In this particular work, we follow the standard idea of smoothing regularization [11] with the purpose of obtaining a new set of samples that are close to the original ones, with an additional condition on their smoothness.

Let us assume that K samples of some transfer function $H(j\omega)$ measured at uniformly spaced frequencies $\omega = \omega_1, ..., \omega_K$ are collected in the vector $\tilde{\mathbf{H}} \in \mathbb{C}^K$. In order to measure the smoothness of $\tilde{\mathbf{H}}$, we resort to a discretized second derivative matrix operator $\mathbf{T}$, as in [11], such that $[\mathbf{TH}]_k \approx H''(j\omega_k)$ (i.e., the k-th element of the the resulting vector $\mathbf{TH} \in \mathbb{C}^{K-2}$). If the frequency samples are uniformly spaced in frequency, this operator takes the following form

$$\mathbf{T} = \begin{bmatrix} 1 & -2 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 & 0 & 0 \\ \dots & \dots & 0 & 0 & 0 & 1 & -2 & 1 \end{bmatrix} \quad (1)$$

up to an irrelevant scaling factor $\Delta\omega^2$, where $\Delta\omega$ is the frequency spacing.

Consider an unknown vector $\mathbf{H} \in \mathbb{C}^K$ that solves the following problem

$$\begin{aligned} &\text{minimize} \quad \|\mathbf{TH}\|_2^2 \\ &\text{subject to} \begin{cases} |\,\mathrm{Re}\{H_k - \tilde{H}_k\}| \le \varepsilon \\ |\,\mathrm{Im}\{H_k - \tilde{H}_k\}| \le \varepsilon, \end{cases} \end{aligned} \quad (2)$$

for $k = 1, \dots, K$. Here, ε is the maximum allowed deviation of the reconstructed and smoothed samples H_k from the original ones. This hyperparameter can often be estimated based on a noise characterization of the source that produced the dataset [e.g. the noise floor of a Vector Network Analyzer

(VNA)]. The solution vector $\mathbf{H}$ of (2) is thus a smooth approximation of $\tilde{\mathbf{H}}$ and can be constructed to be almost free from the artifacts induced by noise by tuning ε.

B. Regularized Vector Fitting

In this work, we combine the above regularization strategy with the Relaxed Vector Fitting (RVF) formulation [12]. The RVF algorithm can be broken down into two phases: the pole identification step, during which an estimate of the N dominant system poles $\{p_i\}_{i=1,\ldots,N}$ is iteratively refined, and a final step, during which the residues are calculated by solving a least-squares approximation problem that fits the given data to a linear combination of the elements of a partial fraction basis corresponding to p_i.

1) Pole identification: In this phase, we solve the ordinary least squares problem described in [12], constructed based on the smoothed samples H_k, which are here considered as the "true" data to be fitted. This allows the pole relocation to quickly converge to a good estimate of the basis poles that are strictly required to model the important features in the transfer function, disregarding the effects of noise. The result of this phase is a pole set $\{p_i\}$, whose number depends on the initial choice of model order N.

2) Final phase: In this second phase, we fit the original (noisy) data $\tilde{H}_k$ using the partial fractions centered at p_i as basis functions for the model. The cost function to be minimized in this step is

$$f(\mathbf{r}) = ||\mathbf{W}\,(\mathbf{\Phi}\,\mathbf{r} - \tilde{\mathbf{H}})||_2 + \gamma\,||\mathbf{U}\,\mathbf{T}\,\mathbf{\Phi}\,\mathbf{r}||_2 \tag{3}$$

where $\mathbf{\Phi} \in \mathbb{C}^{K\times(N+1)}$ with $\Phi_{ki} = (j\omega_k - p_i)^{-1}$, $\mathbf{r} \in \mathbb{C}^{N+1}$ is the unknown vector containing the model coefficients (the residues), $\mathbf{W}$, $\mathbf{U}$ are real diagonal matrices of weights, and γ is a hyperparameter. The first term in (3) is the weighted approximation error and the second term is a measure of the model smoothness based on its curvature, which is numerically estimated through $\mathbf{T}$ and whose relative importance is controlled by γ. In our numerical experiments, the last term is based on the ratio between the second derivative and the actual value of the function, so that, if inverse magnitude weights are used, $\mathbf{W} = \mathrm{diag}\{|\tilde{H}_k|^{-1}\}$ and $\mathbf{U} = \mathrm{diag}\{|\tilde{H}_k|^{-2}\}$.

III. Results

We carried out a validation of the proposed strategy on laboratory measurements of a narrow bandpass filter (Crystek SAW 902.5 MHZ, CBPFS-0902). Measurements were performed with a constant incident power of 0 dBm by using the Agilent E5071B VNA for various configurations of the intermediate frequency bandwidth (IFBW) of the receiver [13], leading to different amounts of noise captured by the measurement process. Such different noise levels are clearly visible in Fig. 1 and Fig. 2, especially at low frequency where the filter response has a small magnitude that is completely shaded by the measurement noise. For this example, we have two conflicting requirements: fitting data using a relative error control in order to represent the full dynamic range and the small magnitude portion of the frequency responses; and making the model insensitive to noise, which hides the true filter response in the low and high frequency rejection bands.

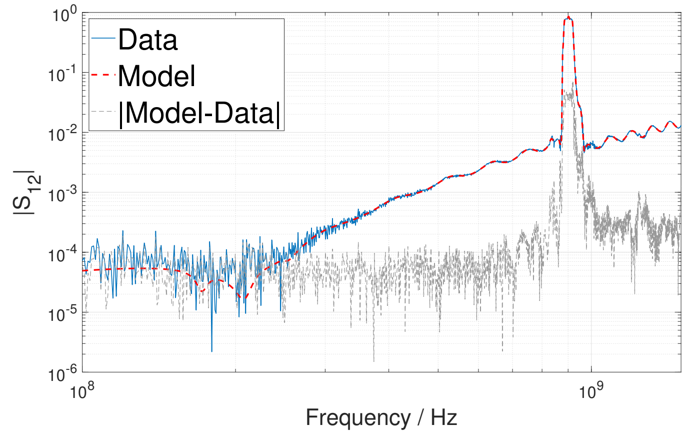

Fig. 1. Noisy data compared with the final model built with $\gamma = 10^{-3}$. These measurements were taken with IFBW =500 Hz

Figures 1 and 2 show the noisy data collected with an IF bandwidth equal to 500 Hz and 70 kHz, respectively, and the model responses resulting from the proposed algorithm. The noise floor is clearly visible in both cases. In particular, the noise floor is used to estimate an appropriate smoothing parameter, which is set to $\epsilon = 10^{-3}$. Both figures report the model responses (red) and the model-data error (grey) in addition to the original noisy data samples (blue). We see that the data behavior in the filter passband is captured correctly, as well as the out-of-band ripples both beyond the upper cut-off frequency and at low frequency. In this experiment, the model order was set to $N = 50$ and $\gamma = 10^{-3}$.

As reference, we report in Fig. 3 the results of a model obtained through the standard RVF scheme with inverse magnitude weights and same dynamic order $N = 50$. This figure shows that the pole relocation of RVF fails in identifying proper dominant pole locations. Most of the poles are driven by the algorithm to fit noise spikes at low frequency, with the effect of draining poles from the locations required to represent the actual features of the underlying system.

IV. Discussion

The presented test case can be regarded as a proof of concept for proposed methodology. Availability of the hardware and the possibility to tune the amount of noise in the measurements allowed us to investigate the performance of smoothing regularization and penalization, specifically in the rejection band of the filter where the response magnitude is small. Depending on the use of the model, the requirement for an aggressive accuracy in this rejection band may be questionable. However, there are several scenarios in which the measured responses are inherently small, and yet they need to be accurately represented by a model even in presence of small signal to noise ratios. A notable example is crosstalk characterization in shielded and/or twisted differential cables and more generally in multiconductor high-speed signal links.

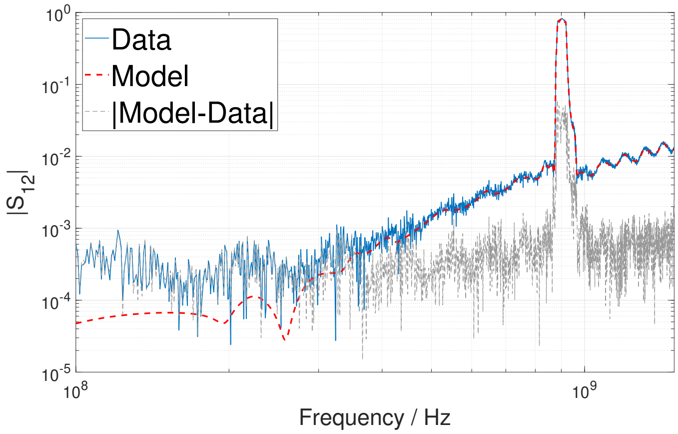

Fig. 2. Noisy data compared with the final model where $\gamma = 10^{-3}$. These measurements were taken with IFBW = 70 kHz

Application to such test cases will be the subject of a forthcoming extended report.

V. Conclusions

This paper presented a simple algorithm based on smoothing regularization combined with Vector Fitting, with the objective of computing rational macromodels from noisy frequency responses. An operator that provides an estimate of the second derivative along frequency is used both to smooth data samples used for VF pole relocation, and as a penalization term in the VF cost function for estimating model residues. The result is a robust and noise-insensitive rational fitting scheme, whose effectiveness is here demonstrated on a set of noisy measured responses of a SAW filter. Future investigations will be devoted to a more complete validation campaign and to an automated estimation of the hyperparameters that control algorithm performance and noise rejection.

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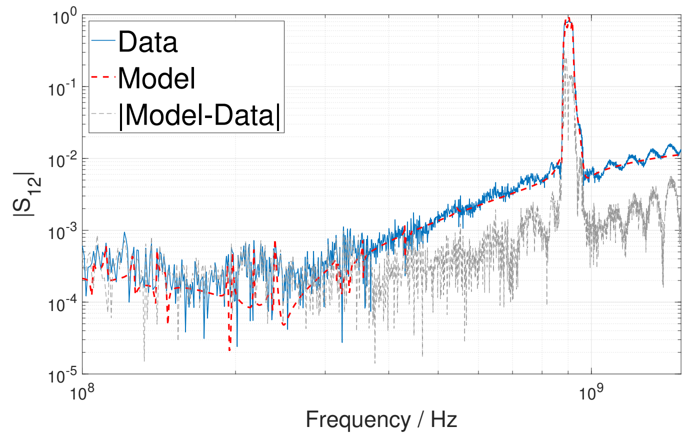

Fig. 3. Noisy data compared with the final model obtained from the standard VF algorithm with inverse magnitude weighting. These measurements were taken with IFBW = 70 kHz

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