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*Original*

Mining SpatioTemporally Invariant Patterns / Colomba, L., Cagliero, L., Garza, P.. - ELETTRONICO. - (2022).  
(International Conference on Advances in Geographic Information Systems (SIGSPATIAL) 2022 Seattle, Washington  
(USA) 01/11/2022 - 04/11/2022) [10.1145/3557915.3560998].

*Availability:*

This version is available at: 11583/2971092 since: 2022-12-02T12:39:51Z

*Publisher:*

ACM

*Published*

DOI:10.1145/3557915.3560998

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# Mining SpatioTemporally Invariant Patterns

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## ABSTRACT

Discovering patterns that represent key spatial or temporal dependencies among data is a well-known exploratory data mining task. However, prior works either separately analyze spatial and temporal dependencies or discover joint spatiotemporal properties of specific trajectories observed over a region of interest. With the goal of generalizing the information provided by spatiotemporal patterns, in this paper we extract sequences of discrete events showing spatiotemporally invariant properties. We seek patterns whose corresponding instances in the source data differ only due to an invariant spatiotemporal transformation. We denote such a new type of patterns as SpatioTemporally Invariant. We also propose an efficient algorithm to mine STInvs and validate its efficiency and effectiveness on real data.

## CCS CONCEPTS

• **Information systems** → *Spatial-temporal systems; Data mining.*

## KEYWORDS

Spatiotemporal Data, Pattern Mining, Data Mining

### ACM Reference Format:

Luca Colomba, Luca Cagliero, and Paolo Garza. 2022. Mining SpatioTemporally Invariant Patterns. In *The 30th International Conference on Advances in Geographic Information Systems (SIGSPATIAL '22)*, November 1–4, 2022, Seattle, WA, USA. ACM, New York, NY, USA, 4 pages. <https://doi.org/10.1145/3557915.3560998>

## 1 INTRODUCTION

Designing and applying data mining algorithms tailored to spatiotemporal data has become relevant to several application domains (e.g., [4, 7]). Sequential pattern mining is an unsupervised technique aimed at discovering recurrent patterns from sequential data, which provide actionable insights. Traditional sequential pattern mining approaches (e.g., [3]) discover sequences of discrete events. They allow end-users to enforce domain-specific temporal constraints [1, 10] while neglecting the spatial dimension. Conversely, arbitrary spatial relationships can be modelled as spatial trajectories [2] or co-location patterns [9] while ignoring the temporal aspects.

More recently, the research community has addressed the joint analysis of spatiotemporal correlations among data. Prior works

focus on (1) Characterizing specific routes by means of trajectory and STAR patterns [2, 8], or (2) Finding frequent co-located and co-occurring contiguous events in a sequence by extracting propagation/influential patterns [6]. However, the discovered patterns are tailored to specific locations or regions and not easy to abstract from specific data instances.

This paper aims at generalizing geospatial event relations by introducing the notion of *spatiotemporal invariance*. Given a trigger event of interest, we look for associated sequences of events whose instances frequently show similar spatial and temporal relative gaps. For instance, we seek sequences of multiple events taking place at relatively similar distances both in space and time. To address this issue, we propose a new type of patterns, namely the SpatioTemporally Invariant Patterns (STInvs). They describe a trigger event followed by a sequence of spatiotemporally related ones. The related events are characterized by a (constrained) spatial and temporal gap with respect to the trigger event. Thus, STInvs provide a high-level description of the spatiotemporally invariant relations holding between a trigger event and those occurring in its neighborhood regardless of the specific data instance.

The proposed pattern differs from existing ones because events are not necessarily located in close spatial proximity (unlike the events described by co-location patterns [9]), are not constrained to specific trajectories (unlike trajectory patterns [2, 8]), and can represent non-contiguous sequences of co-occurring and co-located events (unlike propagation patterns [6]).

The contributions of this paper are as follows.

- **Formalization.** Introduction of the notion of spatiotemporal invariance.
- **Pattern.** Definition of a new pattern (namely the STInv patterns).
- **Algorithm.** Design and development of a new mining algorithm (namely STInvMiner).
- **Experiments.** Empirical evaluation on a real case study related to bicycle sharing system management.

## 2 THE SPATIOTEMPORALLY INVARIANT PATTERN

We describe the occurrences of a set of discrete events of interest according to their spatiotemporal properties. For example, the manager of a bike sharing system can be interested in recording and analyzing critical occupancy levels of the stations in terms of percentage of available docks.

Domain experts are asked to (i) define a set of possible event types  $E$  and (2) mark a subset of them, hereafter denoted by *trigger events*, as relevant to effectively support decision-making. For example, when the occupancy level of a specific station is too high a trigger event can be recorded.

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SIGSPATIAL '22, November 1–4, 2022, Seattle, WA, USA  
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ACM ISBN 978-1-4503-9529-8/22/11.  
<https://doi.org/10.1145/3557915.3560998>

**Table 1: Summary of the notation used.**

Symbol	Description
$A$	the geographical area under analysis (e.g., the urban area where the bike sharing system is active)
$l$	a geographical location within $A$ (e.g., the location of a docking station)
$r$	spatial resolution, i.e., the radius of the circular neighborhood of a geographical location $l$
$td$	temporal resolution, i.e., duration of the considered time slots
$length$	max sequence length, i.e., the maximum number of time slots that appear in an input sequence
$\delta s$	spatial gap (multiple of $r$ )
$\delta t$	temporal gap (multiple of $td$ )
$E$	set of event types under analysis (e.g., car accident, very low level of occupancy of a bike sharing station)
$E^{tr}$	trigger event types ( $E^{tr} \subseteq E$ )
$\langle e, t, l \rangle$	occurrence of event type $e$ at timestamp $t$ in location $l$

When an arbitrary event is observed, we keep track of the corresponding location  $l$  and time  $t$  (see Table 1 for a detailed description of the notation used). The location indicates the geographical position where the event of type  $e$  took place. Similar to co-location patterns [9], locations are also characterized by a local neighborhood, which indicates the potential area of influence of an event occurrence. In our context, the local neighborhood of a location  $l$  is defined as a circular area of radius  $r$  centered in  $l$ . To enable a multi-resolution spatial analysis, we define various amplitudes of neighborhood represented by concentric circles of radius  $k \cdot r$  centered in  $l$  ( $k \in \mathbb{Z}^+$ ). Hereafter,  $r$  will be also denoted as *spatial resolution*.

The timestamp of occurrence of a discrete event  $t$  is discretized into discrete time slots of equal duration  $td$ .  $td$  indicates the temporal resolution at which we consider the occurrences of discrete events.

**Example.** In Figure 1 the temporal resolution is 15 minutes. All the events occurring in the same time slot (e.g.,  $e_1$  and  $e_2$ ) are assumed to be concurrent.

A spatiotemporally annotated occurrence of event  $e$  at timestamp  $t$  in location  $l$  is denoted as a triplet  $\langle e, t, l \rangle$ . For the sake of brevity, we will also denote the event occurrence  $\langle e_i, t_j, l_z \rangle$  where  $e_i \in E$ ,  $t_j \in T$ , and  $l_z \in A$ , as  $occ_{e_i t_j l_z}$  whenever clear from the context.

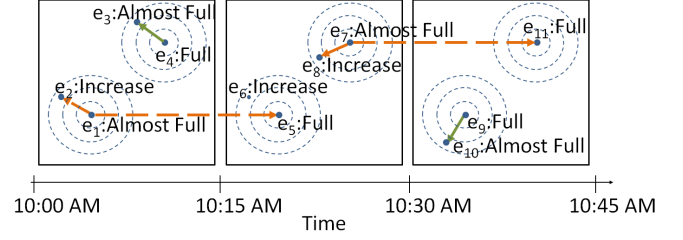
## 2.1 The event sequence mining task

We explore temporal sequences of discrete events [3].

**Definition 2.1.** An *event sequence* of length  $n$  (also denoted as event  $n$ -sequence) is an ordered list of items, each one consisting of an event occurrence:  $[occ_{e_1 t_1 l_1}, occ_{e_2 t_2 l_2}, \dots, occ_{e_n t_n l_n}]$ , where  $t_1 \leq t_2 \leq \dots \leq t_n$ .  $\square$

An arbitrary sequence may include events of different types and contemporary events happening in different locations.

For our purposes, all the timestamps associated with the event occurrences are discretized into time slots (e.g., [10am, 10.15am]) with fixed duration  $td$ . We denote  $td$  as the *temporal resolution*.

**Figure 1: Examples of SpatioTemporally Invariant patterns.**

We apply a time windowing approach [5] to collect a database consisting of a set of event sequences. .

**Example.** Let us consider the time slot  $TS_1$  including timestamps  $t_1$  and  $t_2$ , time slot  $TS_2$  including timestamps  $t_3$ , and time slot  $TS_3$  including  $t_4$ . Let  $occ_{e_1 t_1 l_1}$ ,  $occ_{e_2 t_1 l_2}$ ,  $occ_{e_2 t_2 l_1}$ ,  $occ_{e_4 t_2 l_3}$ ,  $occ_{e_1 t_3 l_1}$ , and  $occ_{e_5 t_4 l_4}$  be the observed events.

Let us define a window of size  $length \cdot td$ , being  $length = 2$ , sliding over the time span and including pairs of consecutive time slots. The corresponding sequence database contains the following sequences  $S_1, \dots, S_3$  respectively corresponding to  $TS_1, \dots, TS_3$ :

$$\begin{aligned} S_1 &: [occ_{e_1 t_1 l_1}, occ_{e_2 t_1 l_2}, occ_{e_2 t_2 l_1}, occ_{e_4 t_2 l_3}, occ_{e_1 t_3 l_1}] \\ S_2 &: [occ_{e_1 t_3 l_1}, occ_{e_5 t_4 l_4}] \\ S_3 &: [occ_{e_5 t_4 l_4}] \end{aligned}$$

The sequence database  $D = \bigcup_{i=1}^3 S_i$  collects all the sequences generated by the sliding process.

## 2.2 The spatiotemporal invariance property

We look for the sequences of discrete events showing invariant spatiotemporal properties. To this end, we first introduce the concepts of time and spatial gaps, which indicate the sequential variation of the recorded timestamps and locations, respectively.

**Definition 2.2.** Let  $S = [occ_{e_1 t_1 l_1}, occ_{e_2 t_2 l_2}, \dots, occ_{e_n t_n l_n}]$  be an event  $n$ -sequence. The *temporal gap sequence* relative to  $S$ ,  $\delta t(S)$  in short, is the  $(n-1)$ -sequence  $[t_2 - t_1, t_3 - t_1, \dots, t_n - t_1]$ .  $\square$

The temporal gaps  $t_j - t_1$  are multiples of  $td$ .

**Definition 2.3.** Let  $S = [occ_{e_1 t_1 l_1}, occ_{e_2 t_2 l_2}, \dots, occ_{e_n t_n l_n}]$  be an event  $n$ -sequence. The *spatial gap sequence* relative to  $S$ ,  $\delta s(S)$  in short, is the  $(n-1)$ -sequence  $[dist(l_2, l_1), dist(l_3, l_1), \dots, dist(l_n, l_1)]$ , where  $dist(l_x, l_y) = \left\lceil \frac{\text{haversine\_distance}(l_x, l_y)}{r} \right\rceil$ .

The spatial distance among two event occurrences depends on  $r$ .

**Example.** In the left-hand-side picture of the example in Figure 1, the (discretized) spatial gap between events  $e_1$  and  $e_2$  is equal to  $3 \cdot r$  because event  $e_2$  lies in the third circle centered in  $e_1$ .

Two  $n$ -sequences are spatially/temporally invariant if their corresponding spatial/temporal gap sequences are coincident.

**Definition 2.4.** Let  $S_1$  and  $S_2$  be two event  $n$ -sequences and let  $\delta t(S_1)$  and  $\delta t(S_2)$  be the corresponding temporal gap sequences.  $S_1$  and  $S_2$  are *temporally invariant* if and only if  $\delta t(S_1) = \delta t(S_2)$ .  $\square$

**Definition 2.5.** Let  $S_1$  and  $S_2$  be two event  $n$ -sequences and let  $\delta s(S_1)$  and  $\delta s(S_2)$  be the corresponding spatial gap sequences.  $S_1$  and  $S_2$  are *spatially invariant* if and only if  $\delta s(S_1) = \delta s(S_2)$ .  $\square$

**Example.** Let us consider the following sequences:  $S_1$ : a car accident occurred at 9pm in location  $l_1$ , then a car accident occurred at 9:30pm in  $l_1$ .  $S_2$ : a car accident occurred at 7:00am in location  $l_2$ , then a car accident occurred at 7:30am in  $l_2$ .  $S_1$  and  $S_2$  are spatiotemporal invariant sequences because the corresponding pairs of events occurred within the same position (spatial gap=0) with a temporal gap of 30 minutes.

### 2.3 The SpatioTemporally Invariant pattern

The main purpose of STInv is to characterize relevant spatiotemporally invariant trends in sequence databases. An STInv summarizes a set of event sequences  $S_1, S_2, \dots, S_q$  such as all the pairs of summarized sequences  $(S_i, S_j)$   $1 \leq i, j \leq q$  are spatiotemporally invariant.

*Definition 2.6.* Let  $S_1, S_2, \dots, S_q$  be a set of  $q$  spatiotemporally invariant sequences sharing the same trigger event as first event type ( $e_1 \in E^{tr}$ ). The STInv is a sequence of triplets  $[ \langle e_1, \delta s_1, \delta t_1 \rangle, \langle e_2, \delta s_2, \delta t_2 \rangle, \dots, \langle e_n, \delta s_n, \delta t_n \rangle ]$  where  $e_x$  is  $x$ -th event type in  $S_1, S_2, \dots, S_q$ ,  $\delta s_x$  and  $\delta t_x$  are the  $x$ -th spatial and temporal gap values in the corresponding sequences.  $\square$

**Example.** Figure 1 shows an example of event sequences related to the monitoring of the occupancy of the bike sharing stations in an urban environment. Specifically, the monitored events, i.e., *fully occupied, almost fully occupied, and increasing occupancy level* indicate the levels of occupancy of the stations of a bike sharing system, expressed in terms of percentage of available docks. We consider as temporal resolution the 15-minute time slots depicted as consecutive time frames. To model the neighborhood of trigger events  $e_1$  and  $e_4$ , we set the spatial resolution to  $r=100m$  and plot surrounding circles indicating the spatial distances  $1 \cdot r=100m$ ,  $2 \cdot r=200m$ , and  $3 \cdot r=300m$ , respectively. Two STInvs in Figure 1 are depicted as oriented splines connecting different occurrences of spatiotemporal events. Specifically, they are represented as a dashed line and a continuous line, respectively.

*Pattern example 1.* Let us consider the STInv with trigger event (*Almost full*). It is observed in the time slot [10.00am, 10.15am]. It shows a spatial correlation between events  $e_1$  (*Almost full*) and  $e_2$  (*Increase*). More specifically,  $e_2$  occurs in the neighborhood of the trigger event at a distance ranging between  $2 \cdot r=200m$  and  $3 \cdot r=300m$ , i.e., (200m - 300m). Next, 15 minutes after the occurrence of the trigger event  $e_1$ , we can also observe in the same location the occurrence of event  $e_5$  (*Full*). Such a combination of spatiotemporal events ( $e_1$ - $e_2$  and then  $e_5$ ) is worth considering because another spatiotemporally invariant occurrence can be observed at the upper-right corner of the spatial areas in time slots [10.15am, 10.30am] and [10.30am, 10.45am] (events  $e_7$ - $e_8$  and then  $e_{11}$ ). The STInv can be formulated as follows:  $[ \langle \text{Almost Full}, \delta s=0m, \delta t=0 \rangle, \langle \text{Increase}, \delta s=200m-300m, \delta t=0 \rangle, \langle \text{Full}, \delta s=0m, \delta t=15mins \rangle ]$

To simplify the representation and improve readability, we can omit the temporal gap indication at the event occurrences while reporting the relative temporal gap  $\delta t$  observed between the event sets occurring in different time slots:

$$\{ \text{Almost Full}(\delta s=0m), \text{Increase}(\delta s=200m-300m) \} \xrightarrow{15 \text{ mins}} \{ \text{Full}(\delta s=0m) \}$$

This STInv can be interpreted as follows: *If a bike sharing station is almost full and the occupancy level of at least one of its nearby stations (between 200m and 300m) is increasing then within 15 minutes*

*the occupancy of the same station will get full.*

**The pattern mining task.** Let  $minsup$  be a minimum support threshold and  $D$  be a sequence database. A *frequent* STInv in  $D$  is an STInv whose number of corresponding event sequences in  $D$  is above  $minsup$ .

Given a sequence database  $D$ , our purpose is to automatically extract all the frequent STInvs in  $D$ .

## 3 THE STINV-MINER ALGORITHM

We present a new algorithm, namely STInv-Miner, to address the STInv pattern mining task. STInv-Miner leverages a prefix-projected-like pattern growth strategy [3]. To enable its adoption for STInv pattern mining, we first need to tailor the concept of sequence projection to STInvs.

The key idea is to transform the original event sequences, annotated with absolute temporal and spatial information, into their corresponding projected versions including relative time and spatial gaps with respect to each trigger event.

For each trigger event in the sequence we generate a projected sequence from the original one that contains the sequence of non-trigger events annotated with their relative spatial and temporal distance, i.e., the spatial and temporal gaps. The projected sequences corresponding to all event sequences in  $D$  are stored in a projected sequence database  $D^p$ .

*Definition 3.1.* Let be  $S_i$  be an event  $n$ -sequence and let  $E^{tr}(S_i)$  be the set of trigger events in  $S_i$  (which are a subset of the whole event set  $E(S_i)$  occurring in  $S_i$ ). The projection function maps each  $S_i$  to a set of  $q$  distinct sequences  $S_1^p, S_2^p, \dots, S_q^p$ , namely the *projected sequences*. Each projected sequence corresponds to a distinct trigger event in  $E^{tr}(S_i)$  belonging to the first time slot of the sequence  $S_i$ . The  $j$ -th projected sequence  $S_j^p$  is defined as follows

$[ \langle e^{tr}, \delta s = 0, \delta t = 0 \rangle, \langle e_2, \delta s_2, \delta t_2 \rangle, \dots, \langle e_m, \delta s_m, \delta t_m \rangle ]$   
where  $e^{tr}$  is the  $j$ -th trigger event,  $e_2, \dots, e_m$  are the events in  $S_i$  that are spatially correlated with  $e^{tr}$ , and the corresponding  $\delta s$  and  $\delta t$  are the spatial and temporal gaps relative to the trigger event.  $\square$

The STInv-Miner algorithm leverages a projected version of the sequence database by tightly integrating the projection and mining phase into a scalable, distributed mining process executed using Apache Spark. Event projections are computed on top of a discretized version of the spatiotemporal event occurrences fulfilling the desired temporal and spatial resolutions.

## 4 EXPERIMENTS

### 4.1 Datasets and configurations

The *Bike Sharing Dataset* contains data related to bike sharing stations' occupancy rates of 5 different cities in the San Francisco Bay Area, sampled every minute in a 2-year period. For simplicity, all the reported results are related to San Francisco, Palo Alto and the global view (35, 5, and 70 stations).

To characterize the occupancy levels of the stations, we divided the temporal axis into timeslots of fixed size (i.e., temporal resolution) and generated an event dataset, defining the following events: *Increase* in station's occupancy level, *Decrease* in station's occupancy level, *Almost full* (at most 2 free slots available) and *Full*.

**Table 2: Configuration settings.**

	Config 1	Config 2
<i>Dataset</i>	Bike Sharing	Bike Sharing
<i>Temporal Resolution (td)</i>	10 min	10 min
<i>length (nr. of time slots)</i>	6	6
<i>Spatial resolution (r)</i>	100m	100m
<i>Geographical area</i>	Global, City	Global, City
<i>Max pattern length</i>	6	6
<i>Event types</i>	Full, Almost Full, Increase	Full, Almost Full, Increase, Decrease
<i>Trigger event types</i>	Full, Almost Full, Increase	Full, Almost Full, Increase

**Table 3: Number of generated events for the bike sharing dataset. Values associated to events are expressed in %.**

	Global		San Francisco		Palo Alto	
	Conf 1	Conf 2	Conf 1	Conf 2	Config 1	Config 2
<i>Full</i>	3.94	2.26	5.03	2.92	2.39	1.42
<i>Almost Full</i>	22.12	12.67	23.45	13.60	30.26	18.05
<i>Increase</i>	73.94	42.35	71.52	41.47	67.35	40.18
<i>Decrease</i>	N/A	42.72	N/A	42.01	N/A	40.35
<i>Total events</i>	1.08M	1.88M	708K	1.22M	74K	124K

We performed 2 different experiments (Conf 1 and 2 in Table 2), with the sole difference that Conf 2 also considers the *Decrease* events and analyses events taking place in the top-N departure stations instead of considering the station’s neighborhood.

## 4.2 Pattern extraction and analysis

We performed several pattern extractions on geographical areas of different size. We adopted two configurations: (i) Conf 1 to mine local patterns and understand interactions between neighboring stations, and (ii) Conf 2 to identify long-term spatial departure-arrival patterns. *minsup* is set to 1 in both cases.

*Statistics on events and STInv patterns.* Table 3 reports the event type distribution, while Table 4 summarizes the characteristics of the mined STInvs when considering the global view (i.e., all cities together). Several complex and expressive patterns are mined. They are characterized by an average number of event occurrences per STInv close to 6. Since many of them show different spatial/temporal gaps, they represent non-trivial invariant patterns that human experts are unlikely to detect during a manual data exploration.

*Comparison with traditional event sequences and non-spatially invariant sequences.* Analyzing the percentage of STInvs with  $\delta s > 0$  and  $\delta t > 0$ , we exclude all the patterns that disregard variations in the spatial and temporal dimension. Both the values are above 99.9% (Table 4), thus confirming the added value of exploring STInvs rather than simpler event sequences. Finally, we compare the number of STInvs with the number of traditional sequences mined by considering the absolute location of the events (Table 5), relaxing the spatial invariance constraint. The results demonstrate the summarization capability of the proposed approach.

**Table 4: Statistics on the bicycle sharing dataset. Global area.**

	Config 1	Config 2
<i>Total num. of STInvs</i>	45.3M	11.0M
<i>Mean #triplets per sequence</i>	5.90	5.77
<i>Mean discrete spatial distance</i>	2.40	8.31
<i>Mean discrete temporal distance</i>	2.13	2.17
<i>#sequences with at least one <math>\delta s &gt; 0</math> (%)</i>	99.95	99.89
<i>#sequences with at least one <math>\delta t &gt; 0</math> (%)</i>	99.99	99.99
<i>#sequences with at least one <math>\delta s &gt; 0</math> and at least one <math>\delta t &gt; 0</math> (%)</i>	99.29	98.36

**Table 5: Compression Ratio (CR) of STInv compared to absolute non-spatially invariant patterns (Abs).**

	Conf 1			Conf 2		
	#STInv	#Abs	CR	#STInv	#Abs	CR
<i>San Francisco</i>	45.3M	2350M	51.90	6.24M	28.2M	4.53
<i>Palo Alto</i>	111K	143K	1.29	344K	592K	1.72

## 5 CONCLUSIONS

We present a new type of patterns denoting spatiotemporally invariant relations among sequences of discrete events. The key contribution is the generalization of the joint spatial and temporal relations over multiple data instances. The empirical results, achieved on a real-world dataset, show that (i) STInvs averagely summarize the temporal correlations described by tens of traditional sequences and (ii) STInv patterns show non-zero spatial and temporal gaps in most cases.

## ACKNOWLEDGMENTS

This work is partially funded by the SmartData@PoliTO center.

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