

Laboratory physical modelling of block toppling instability by means of tilt tests

Original

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Laboratory physical modelling of block toppling instability by means of tilt tests

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Abstract:	<p>In this paper we present a physical modelling approach where the stability of rock blocks against toppling in the field can be estimated using a tilt table, engineered rock models and 3D-printed small-scale versions of a natural rock boulder. To achieve this goal, first, simple geometry rock elements are tilted and results interpreted according to analytical formulations. Then, more complex geometry engineered rock blocks, including some whose centers of gravity do not project on the center of the base element, are tested and results properly interpreted. Eventually, the 3D-printed version of the rock boulder is produced from 3D point clouds recovered in the field by means of a combination of photogrammetry and laser scanner techniques. Analytical formulations and numerical calculations have been used in order to validate the proposed approach, to explain the physical phenomena involved, and to allow for possible extension of the physical modelling results to different scenarios, such as those considering the influence of water or seismic loading on stability.</p>
Suggested Reviewers:	Juan Ramon Vidal Romani juan.vidal.romani@udc.es Resat Ulusay resatulusay1987@gmail.com Andrea Segalini andrea.segalini@unipr.it
Response to Reviewers:	<p>Dear Editor and Reviewers:</p> <p>On behalf of my co-authors, we kindly acknowledge all the illustrative and enriching comments on the original version of our manuscript. We have tried to implement all the corrections by following the comments and suggestions, as well as to polish the English usage of the entire manuscript.</p> <p>Please, find all the changes and comments in the 'Response Letter to Editor and Reviewers' attached to the re-submission of our manuscript.</p> <p>Sincerely, Dr. Ignacio Pérez-Rey (corresponding author) ignacio.perez@cedex.es</p>

Response letter to Editor and Reviewers

Laboratory physical modelling of block toppling instability by means of tilt tests

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NOTE: In the following, we present the ‘letter of reply’ to the questions raised by the Editor and Reviewers regarding our article *Laboratory physical modelling of block toppling instability by means of tilt tests*. The original text of the Editor and Reviewers has been kept in blue and our responses are presented in black. (Modifications to the original manuscript are presented in green in the corresponding updated version).

Editor and Reviewer comments:

Two reviewers found the manuscript to be of interest and already in a rather good shape. I concur.

Specifics

1) Language - please check again and polish. I noticed some problematic sentences (see on example below).

20 Then, more complex geometry engineered rock blocks, including some where its center of gravity does not project on the center of the base element, are tested and results properly interpreted." - please improve English; for example, "some where its center of gravity does not" should probably read: some whose centers of gravity do not

2) Figures - 23 is a lot and I wonder if you really need all of them; perhaps some could be combined?

3) Highlights - please add (see Journal's Guide for Authors)

Authors

Dear Editor:

First, we would like to acknowledge the positive general assessment of the manuscript. Additionally, we thank you and both reviewers for all the comments on our manuscript, since they will ultimately contribute to the improvement of our work.

Regarding your specific comments and suggestions:

- 1) English usage has been polished again and some sentences were re-written by considering also those language corrections proposed by **Reviewer #2**. These sentences are indicated in the new version of the manuscript in green color.
- 2) Some Figures were combined into one so as to reduce its total number (now 20 instead of 23), as proposed. Figures combined were: (Fig. 12 + Fig. 13) now **Fig. 13**, (Fig. 16 + Fig. 17) now **Fig. 16**, (Fig. 18 + Fig. 19) now **Fig. 17**, (Fig. 21 + Fig. 22) now **Fig. 19**. A new figure was included following a reviewer's suggestion and Fig. 19 has been slightly modified for better understanding.
- 3) The ‘Highlights’ have been included according to the Journal's Guide of Authors in the new Submission process in a separate document.

Reviewer #1:

The paper presents an interesting study and is clearly presented, well written and easy to read. On my personal opinion it can be published as it is, without further revisions.

Authors

Dear Reviewer #1:

We kindly acknowledge these positive comments and the opinion regarding our paper. Thanks.

Reviewer #2 (R#2):

Corrections, comments, suggestions in the attached file.

Authors

Dear Reviewer #2:

We kindly acknowledge all the comments and suggestions. We thank you for the enriching comments and we tend to agree with all the suggested modifications, which have been implemented in the new version of the manuscript and indicated in green.

R#2 (comment made by R#2 referred to Eq. 3 in the original version of our manuscript)

It is a "correction", but an extension. Fig 3 is derived from static equations only, and it is not incorrect. Sagaseta extended the graph considering the dynamic equations. His graph should be included here, after equation (3) with a suitable explanation.

Authors

Thanks for this suggestion. In order to clarify this issue, a new Figure (**Fig. 4**) containing the boundary conditions for toppling, as published by Sagaseta (1986) has been embedded in the new version of the manuscript. The chart has been remade based on the equations established by Sagaseta (1986) for dynamic conditions, as presented in our paper, and it is included with a suitable explanation, as requested by R#2, after Eq. 3 in the way shown below:

If Eq. 3 is considered, the boundary conditions for the possible modes of failure for a slab-like straight block with slenderness b/h , placed on an inclined plane dipping α degrees and with a friction angle $\phi = 30^\circ$ can be correctly plotted (**Fig. 4**).

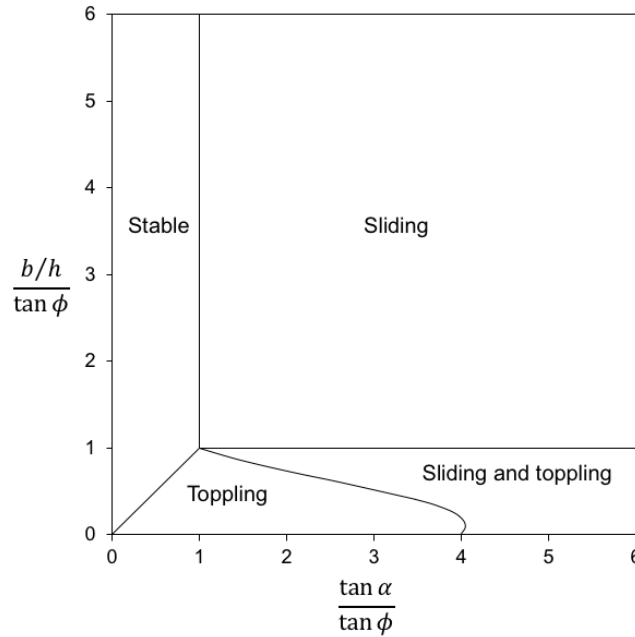


Fig. 4 Stability chart for dynamic conditions, considering a block of dimensions $b \times h$ placed on a inclined plane dipping α degrees (as modified from Sagasetta (1986)). The line dividing toppling and sliding + toppling failure regions corresponds to a friction angle $\phi = 30^\circ$.

For a block such as the already referred, in Fig. 4 the stability region is determined by the vertical line ($\tan \alpha = \tan \phi$) and the 1:1 line ($\tan \alpha = b/h$). The sliding failure will take place for the region defined by the same vertical line and the condition $\tan \phi = b/h$ (horizontal line). The curved line, as derived from Eq. 3, divides the toppling and toppling + sliding failure regions. Note that the position of this line will depend on the friction angle, and will always intercept the horizontal axis at a value $\tan \alpha = 4 \cdot \tan \phi$.

R#2 (comment made by R#2 referred to Eq. 22 in the original version of our manuscript)

Why not display the equation for α_{crit} as a function of a and b , as in PM-2?

Authors

Thanks for this comment. We find this suggestion appropriate, and we have added a short paragraph with a new equation relating the stabilizing and overturning (toppling) moments as a function of a and b (as in PM-2). The new text and Equation are provided below, as included in the new (corrected) version of the manuscript:

In a similar way as presented in Eq. 8, the angle of critical toppling can also be estimated as a function of a and b dimensions of the specimen (Eq. 23):

$$\alpha_{cr} = \operatorname{atan} \left[\frac{(W_1 + W_2) \frac{b}{2}}{W_1 \cdot a + W_2 \cdot \frac{3a}{2}} \right] \quad (23)$$

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13

14

15 **ABSTRACT**

16 In this paper we **present** a physical modelling approach where the stability of rock blocks
17 against toppling in the field can be estimated using a tilt table, engineered rock models
18 and 3D-printed small-scale versions of a natural rock boulder. To achieve this goal, first,
19 simple geometry rock elements are tilted and results interpreted according to analytical
20 formulations. Then, more complex geometry engineered rock blocks, including some
21 **whose centers of gravity do** not project on the center of the base element, are tested and
22 results properly interpreted. Eventually, the 3D-printed version of the rock boulder is
23 produced from 3D point clouds recovered in the field by means of a combination of
24 photogrammetry and laser scanner techniques. Analytical formulations and numerical
25 calculations have been used in order to validate the proposed approach, to explain the
26 physical phenomena involved, and to allow for possible extension of the physical
27 modelling **results** to different scenarios, such as those considering the influence of water
28 or seismic loading on stability.

29

30 **List of symbols**

31	α	base plane inclination angle
32	α_{cr}	critical angle of toppling
33	b	width of the block
34	cog	center of gravity
35	ϕ	friction angle
36	FS	factor of safety
37	g	acceleration of gravity (9.81 m/s ²)
38	h	height of the block or model
39	P	rotation pivot of a given block or model and origin of the (x_i, y_i) coordinates
40	r	radius
41	r_c	radius of the block corners (expressed as the radius of curvature)
42	x_i	distance from P to the cog of the i -th sub-section along the x-axis for a given
43		block or model
44	y_i	distance from P to the cog of the i -th sub-section along the y-axis for a given
45		block or model
46	w	width of the block (out of plane)
47	W	weight of the block
48	W_i	weight of the i -th section

49 **1. Introduction**

50 Analysing the stability of natural or man-made slopes in rock masses is a complex task.
51 The principles of the discipline addressing this problem (typically known as rock slope
52 engineering) were established in the seminal works developed during the 1960s and 1970s
53 at Imperial College in London and by Richard Goodman in Berkeley, and they are
54 summarized in the book entitled “Rock Slope Engineering” (Hoek and Bray, 1974). In
55 this book, it was clearly stated that, unlike in the case of soils, where failure mechanisms
56 typically involve sliding along a rotational or planar sliding surface, for rocks, a variety
57 of failure mechanisms could occur according to the number, continuity, spacing,
58 orientation and geomechanical characteristics of existing discontinuity sets.

59

60 Four typical types of failure mechanisms were identified at the time as the most common
61 possibilities for rock slope instabilities: planar, wedge, (block) toppling and circular. This
62 classification combined with the simple mathematical tools provided to analyse their
63 stability, represented a significant step forward in the rock engineering field, leading to
64 improved rock slope stability analyses and designs. However, as wisely noted in the
65 preface of Hoek and Bray’s (1974) book, ‘Because the rock mass behind every slope is
66 unique, there are no standard recipes or routine solutions which are guaranteed to produce
67 the right answer each time they are applied’. This is why it is important to bear in mind
68 that rock slope instability phenomena do not necessarily occur according to simple failure
69 mechanisms as those described in these early works.

70

71 Whereas three of these basic mechanisms refer to sliding phenomena, only one of them
72 involves toppling. Toppling corresponds to rotation of relatively slender rock columns or
73 blocks about a fixed base. There are two distinct types of toppling failure mechanisms
74 named block and flexural toppling (Goodman and Bray, 1976). In the first case, the
75 toppling block is already fully detached from the rest of the rock mass. The second implies
76 flexural or tensile failure, where the block is not completely detached of the rock mass
77 such that new tensile failure cracks need to occur to fully detach the block from the
78 surrounding ones for toppling instability to occur. In this study we focus on the analysis
79 of block toppling.

80

81 The simplest toppling case considered in this study is that of a single toppling block. The
82 first analyses of the stability against toppling of a single block were performed by Ashby
83 (1971) and then extended and formalized by other authors (Hoek and Bray, 1974;
84 Sagaseta, 1986). All these analyses start with strict geometrical assumptions considering
85 perfectly rectangular blocks with sharp corners resting on a dipping plane striking in the
86 same direction as the block face in contact with the plane.

87

88 Some studies have recently begun to address the influence of more realistic shapes of
89 blocks on block stability against toppling. Alejano et al. (2015) studied the influence of
90 the rounding of block corners on the stability of a block. This geometric condition
91 observed in some rock masses and associated with spheroidal weathering erosive
92 processes was shown to significantly affect block stability. The authors proposed a
93 formulation to compute the stability of a round-cornered block to account for this effect.
94 This formulation relies on the position of the potential rotation axis and was verified using
95 physical models and field observations. While such a contribution is interesting and
96 extends the type of toppling instability that can be analyzed, when one observes certain
97 natural rock environments, more complex irregular block geometries exist for which no
98 analysis methods can be observed.

99

100 As an example, Fig. 1 illustrates three zones in two different mountain environments,
101 where a number of toppled and stable blocks are observed. In most of the cases and based
102 on our current capabilities to analyze stability of blocks, it would be indeed difficult to
103 rigorously perform a back analysis of the instability or stability of these blocks to explain
104 why are they stable or unstable. For the stable blocks, it would also be difficult to assess
105 their stability after a potential change in loading conditions, such as in the case of a
106 seismic event. Accordingly, the main goal of this study is to develop a methodology
107 through which the stability of complex blocks (such as those illustrated in Fig. 1) can be
108 analyzed in a more or less reliable manner.

109

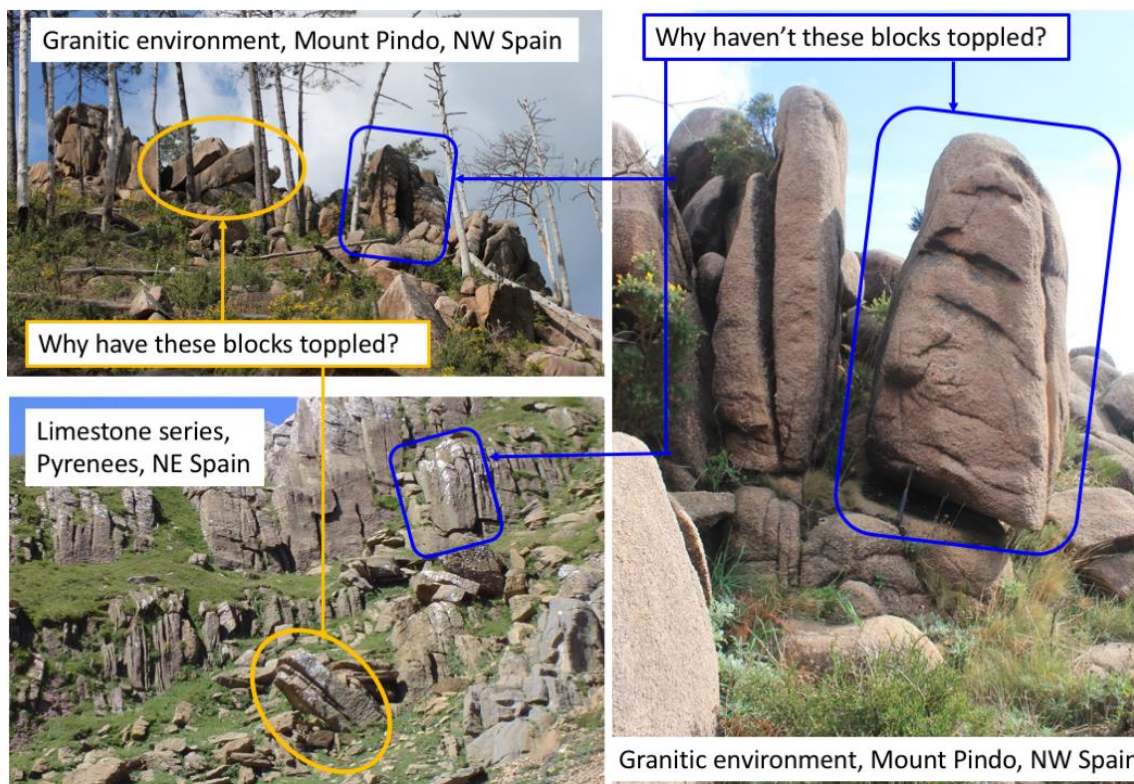
110 Since their early studies, the authors have considered cases involving increasingly
111 complex estimates of the toppling stability of individual rock blocks taking into account
112 the occurrence of rounded corners on the toppling blocks (Alejano et al., 2017) or rock
113 boulders (Alejano et al., 2010b; Pérez-Rey et al., 2019). They also considered other
114 studies available from literature focusing on the potential destabilizing effects of

115 earthquakes on rock blocks or groups of rock blocks (Christianson et al., 1995; Shi et al.,
116 1996; Vann et al., 2019), where it **has been necessary** to extend available methodologies
117 to **model** the nature of these structures in a more realistic manner.

118

119 In the process of developing the above-mentioned studies, **tilting small-scale rock**
120 **elements with simple geometries** has revealed as an interesting technique that, in
121 combination with analytical formulations can help to understand a number of issues
122 associated to toppling phenomena. Therefore, a number of simple and a little bit more
123 complex geometry engineered rock blocks have been tilted and results explained in this
124 document to illustrate the potential interest of extending this technique to more realistic
125 **block shapes**.

126



127

128 Fig. 1. Different geological environments where toppled and stable rock blocks are observed.

129

130 The study is an extension of basic toppling equilibrium calculations and was developed
131 in parallel with particular rock slope stability studies. The initial concepts associated with
132 the proposed methodology developed while studying the stability of **footwall** slopes
133 (Alejano et al., 2011) and masonry retaining walls (Alejano et al., 2012). In both cases it
134 was possible to resort to physical models subjected to tilt tests in order to carry out simple

135 analyses with the aim of confirming particular failure mechanisms and the validity of
136 some formulations (Fig. 2). This previously developed approach is valid if only friction
137 is involved, and block's geometries can be easily reproduced using common rock cutting
138 approaches (typically with saw-blades).

139



140

141 Fig. 2. Physical models of dry masonry retaining walls used to understand the failure mechanism
142 of these structures and check limit equilibrium calculations.

143

144 Since the turn of this century, photogrammetric and laser scanning techniques for point
145 cloud acquisition have become widely available, such that a very good representation of
146 a given rock slope geometry can be achieved (Alejano et al., 2013; Armesto et al., 2009;
147 Ferrero et al., 2011, 2009; Riquelme et al., 2014). Most recently, relatively large and
148 accurate 3D point clouds have become available at very reasonable costs (Girardeau-
149 Montaut, 2018). It is therefore possible to obtain 3D point clouds representing the
150 complex block geometries observed in nature (such as in Fig. 1). Based on such geometric
151 data, the center of gravity (*cog*), which is critical for toppling stability analyses, can be
152 precisely located. Other relevant geometrical aspects of the rock blocks or boulders such
153 as the position of the contact base between the block and basal plane can be also
154 rigorously defined. This geometrical information largely facilitates computing the factor
155 of safety of these boulders against toppling, as the ratio of stabilizing moments to
156 overturning moments.

157

158 Additionally, in the last five years, 3D printing has improved so rapidly that it is relatively
159 easy to produce a scaled 3D printed version of any rock block based on the point cloud
160 representing its outer surface (Bader et al., 2018; Virtanen et al., 2014). Therefore, it
161 allows for the possibility of using tilt testing as a methodology to analyze the stability of
162 blocks with complex geometry against toppling.

163 **Consequently**, in this paper we introduce a new physical modelling approach, where a tilt
164 table, classic limit equilibrium computations, and rock physical models using a 3D-
165 printed version of a real boulder are used to estimate the factor of safety against toppling
166 of rock blocks in *Nature*. **The use of analytical formulations and numerical models in**
167 **parallel with this approach is strongly recommended. This would allow a deeper**
168 **understanding of the phenomena at stake and will help to extend physical modelling**
169 **observations to different scenarios (water pressure or dynamic loadings).**

170

171

172 **2. The mechanics of toppling**

173

174 Despite the fact that some authors consider the occurrence of pure toppling as an
175 uncommon failure mechanism, and that toppling is typically associated with larger
176 failures or as a consequence of other mechanisms like slope undercutting or weakening
177 at the toe (Hencher, 2015), toppling has been extensively reported as the origin of several
178 rock-mass failures experienced in different fields such as open-pit mining (Al Mandalawi
179 et al., 2019; Alejano et al., 2010a; Amini and Ardestani, 2019), civil engineering
180 (Akbarpour et al., 2012; Cai et al., 2019; Tu et al., 2007) and natural rock slopes (Guo et
181 al., 2017).

182

183 The mechanism of toppling had **long** been partially identified in the field by some authors
184 (Müller, 1968; Terzaghi, 1962), though it was not until the late 1960's that it began to be
185 considered as a mode of failure unto itself (Bray, 1969). A few years later, some authors
186 started the study of toppling in a more rigorous way, through both laboratory models
187 (Ashby, 1971; Barton, 1971) and early applications of numerical methods (Cundall, 1971;
188 St. John, 1972).

189

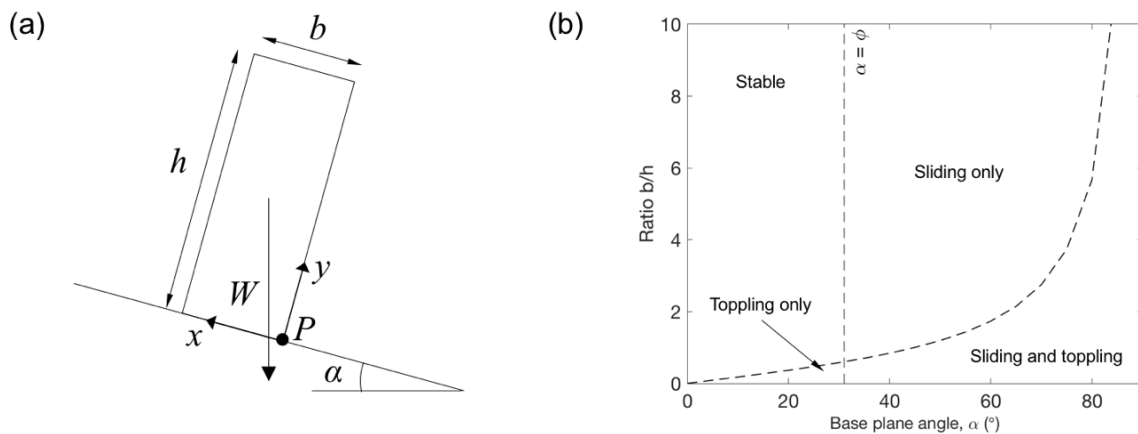
190 Ashby (1971) developed a seminal study on different sliding and toppling modes of
191 failure based on laboratory physical models and **observations on** rock slopes. This author
192 derived a simplified analytical 2-D toppling model for an isolated straight block of height
193 h and width b , resting on a plane dipping α degrees with a friction angle between contacts,
194 ϕ (Fig. 3).

195 Considering the base plane angle, $\alpha < \phi$ and the x and y components of the weight (W)
 196 of the block, a factor of safety against toppling can be estimated by relating the stabilizing
 197 and overturning moments with respect to the rotation pivot, P (Fig. 3a), as presented in a
 198 general form in Eq. 1.

$$FS = \frac{\sum M_{stabilizing}}{\sum M_{overturning}} \quad (1)$$

199

200



201

202 Fig. 3. (a) 2D sketch of a single block placed on an inclined plane for toppling stability analysis;
 203 (b) conditions for sliding and toppling according to Ashby (1971).

204

205 It is therefore easy to derive that the toppling condition (represented as $FS \leq 1$ in Eq. 1)
 206 solely depends on the slenderness of the block and the plane dip (α) (Ashby, 1971) and is
 207 defined by the geometrical relationship presented in Eq. 2:

208

$$\frac{b}{h} \leq \tan \alpha \quad (2)$$

209

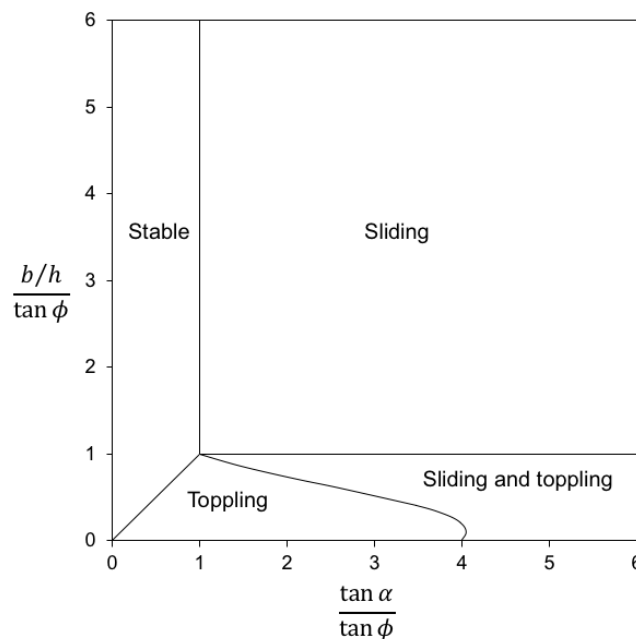
210 According to Fig. 3b and depending on the considered kinematic conditions established by
 211 Eq. 1 and the condition $\alpha = \phi$, that refers to sliding stability, four zones can be identified:
 212 stability, sliding, toppling and sliding and toppling simultaneously occurring. Even
 213 though this model laid the groundwork for subsequently reproduced studies on toppling
 214 (Hoek and Bray, 1974), some errors were detected in the conditions presented in Fig. 3b,
 215 as corrected by other authors (Bray and Goodman, 1981; Sagaseta, 1986). The

216 “corrected” solution for a single block toppling condition, as proposed by Sagaseta
 217 (1986), is given by Eq. 3:

$$\frac{4 \cdot \tan \alpha \cdot \left[1 + \left(\frac{b}{h} \right)^2 \right] - 3 \cdot \left(\tan \alpha - \frac{b}{h} \right)}{4 \cdot \left[1 + \left(\frac{b}{h} \right)^2 \right] + 3 \cdot \frac{b}{h} \cdot \left(\tan \alpha - \frac{b}{h} \right)} \leq \tan \phi \quad (3)$$

218

219 If Eq. 3 is considered, the boundary conditions for the possible modes of failure for a
 220 slab-like straight block with slenderness b/h , placed on an inclined plane dipping α
 221 degrees and with a friction angle $\phi = 30^\circ$ can be correctly plotted (Fig. 4).



222

223 Fig. 4 Stability chart for dynamic conditions, considering a block of dimensions $b \times h$ placed on
 224 an inclined plane dipping α degrees (as modified from Sagaseta (1986)). The line dividing toppling
 225 and sliding + toppling failure regions corresponds to a friction angle $\phi = 30^\circ$.

226 For a block such as the already referred, in Fig. 4 the stability region is determined by the
 227 vertical line ($\tan \alpha = \tan \phi$) and the 1:1 line ($\tan \alpha = b/h$). The sliding failure will take
 228 place for the region defined by the same vertical line and the condition $\tan \phi = b/h$
 229 (horizontal line). The curved line, as derived from Eq. 3, divides the toppling and toppling
 230 + sliding failure regions. Note that the position of this line will depend on the friction
 231 angle, and will always intercept the horizontal axis at a value $\tan \alpha = 4 \cdot \tan \phi$.

232

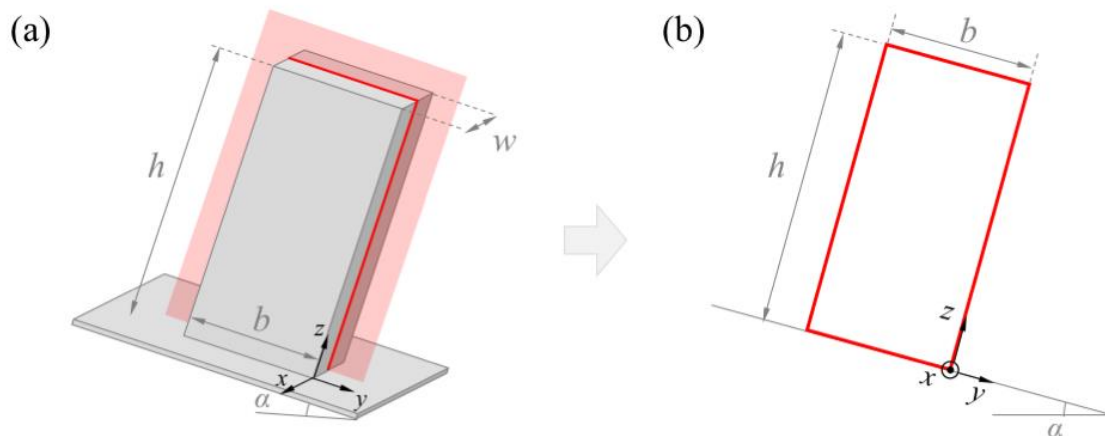
233

234 To consider the case of rounding on the block edges caused by weathering, the single 2D
 235 straight block model was reevaluated several years later (Alejano et al., 2015). Starting
 236 from Eq. 2, the authors proposed a new equation by considering a radius of curvature (r_c)
 237 representative of the edge rounding in such a way that toppling condition is given by the
 238 geometrical relationship presented in Eq. 4:

$$\frac{b - 2r_c}{h} < \tan \alpha \quad (4)$$

240
 241 It must be noted that the approaches presented above should only be applied to 2D
 242 problems or to simplifications of 3D bodies presenting a constant cross-section, as
 243 illustrated in Fig. 5. Additionally, the rotating pivot is already known, and defined by an
 244 identifiable edge.

245



246
 247 Fig. 5. (a) 3D body resting on an inclined plane with constant cross-section along the x-direction;
 248 (b) 2D simplification of the problem on the z - y plane.

249

250 Nevertheless, this situation is seldom found in natural blocks (particularly, in rock
 251 boulders), that usually present irregular shapes and poorly defined contact planes, such
 252 that 2D analyses are unrealistic. In these cases, the third dimension cannot be disregarded
 253 and other approaches are needed for a correct assessment of the toppling mechanism
 254 (Domokos et al., 2012; Yeung and Wong, 2007; Zábranová et al., 2020).

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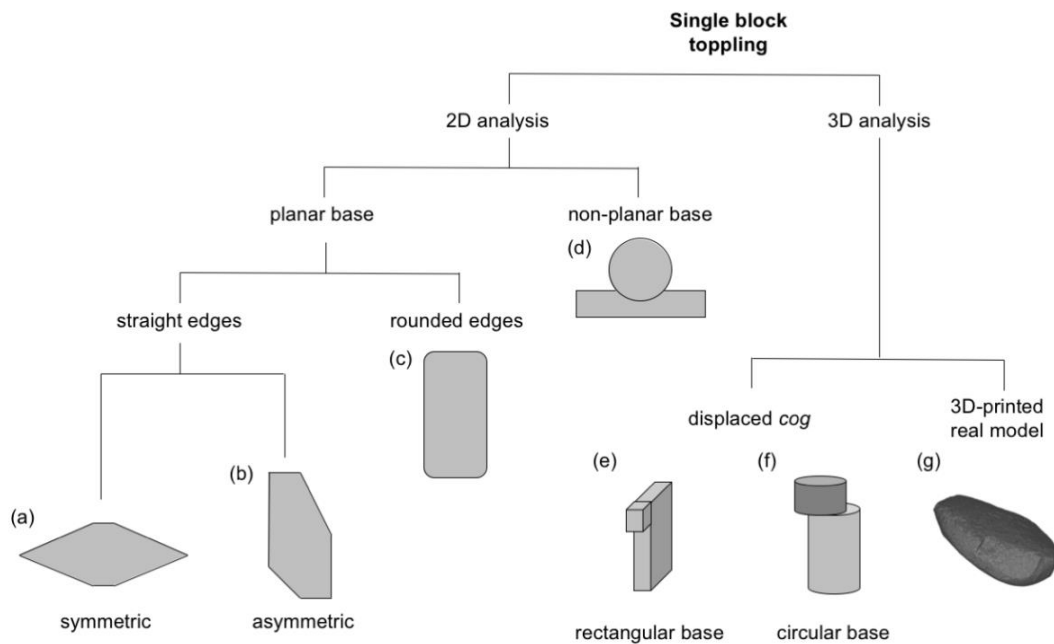
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257

258 **3. Analytical assessment and laboratory physical modelling**

259 An experimental program was designed in order to study the critical angle of toppling of
260 engineered models subjected to tilt tests under laboratory conditions. The idea was to test
261 different blocks or assemblies presenting features that determine their kinematic behavior
262 with respect to toppling: the symmetry of the block section, the edge rounding, the
263 concavity of the contact base and the position of the center of gravity with respect to a
264 central cross-section of the block. A sketch showing these features and the models used
265 is presented in Fig. 6.

266



267

268 Fig. 6. Sketch showing the specimens used for 2D and 3D toppling analyses according to the type
269 of contact base (planar or non-planar), edge rounding (straight or rounded), symmetry of the
270 cross-section and position of the cog.

271

272

273

274

275 **3.1. Physical models and laboratory testing**

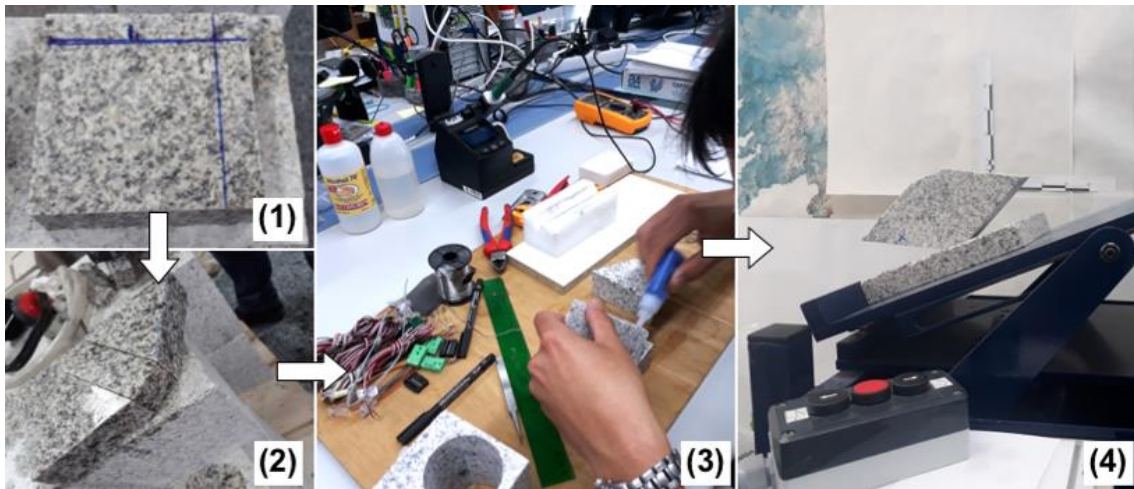
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277 The models presented in Fig. 6 were selected to perform the laboratory tilt tests. The
278 majority of them (Fig. 6 a–e) consisted of some saw-cut pieces of two igneous rocks
279 (granite and orthogneiss) assembled by gluing them together to form the physical model
280 (Fig. 7). A photo of each model is presented in Fig. 8. In mode case PM-6 (Fig. 6f and
281 Fig. 8f), steel (with a density of 7900 kg/m³) was used to fabricate the upper cylinder; as
282 can be appreciated, this part of the model was intended to displace the center of gravity

283 of the corresponding assembly. The replica of a real boulder was 3D-printed with PLA
284 (polylactide) plastic based on a 3D point cloud recovered from the original geological
285 structure (Pérez-Rey et al., 2019).

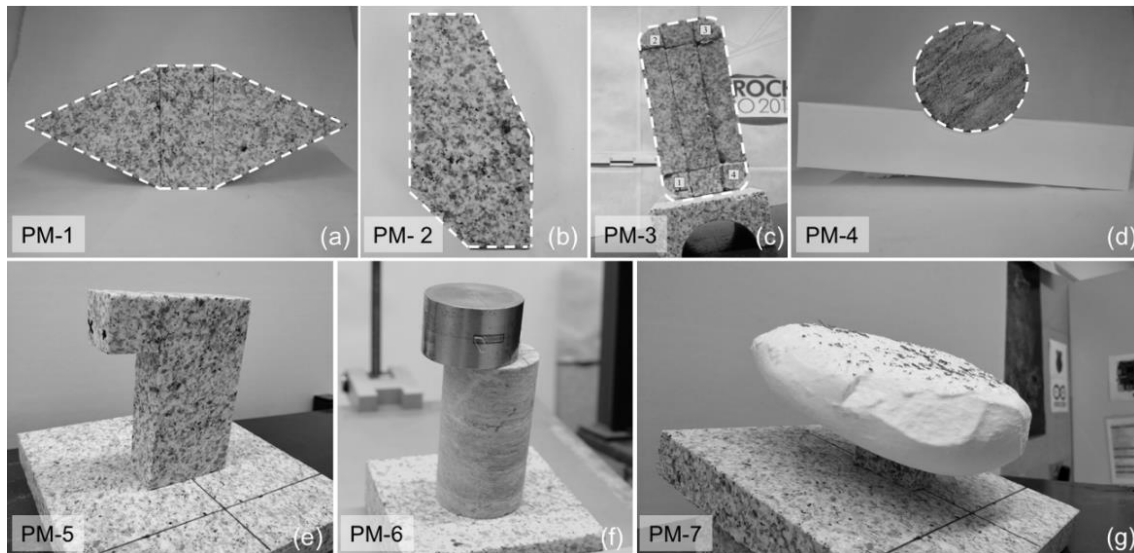
286

287 The density of the plastic is lower than that of the rock. Additionally, the printed version
288 presents an internal plastic pattern with regular hollow zones. Since toppling stability
289 depends in both the stabilizing and overturning moments and they are both proportional
290 to the block weight, density does not influence stability in uniform bodies. The friction
291 force in the base is also proportional to the weight in case the contact is considered planar,
292 though it may affect results for rough joints, as Barton's formula suggests. Since the
293 contacts of the studied elements are typically planar, the weight does not significantly
294 affect the frictional response. If one wants to avoid the sliding mechanism, in cases where
295 it can take place at lower tilt angles than toppling, a piece of sand paper can be glued to
296 the base of the element to increase friction strength. In conclusion, density, when constant,
297 and friction angle are considered to have a negligible impact on the presented results.



298

299 Fig. 7. Process of fabrication of the rock models (1, 2) dimensioning and parts of a model; (3)
300 gluing of parts to create the complete model and (4) model being tested.



301

302 **Fig. 8.** Photos of the physical models used in this study (a. symmetric section; b. asymmetric
 303 section; c. rounded edges; d. concave base; e. displaced *cog* (rectangular base); f. displaced *cog*
 304 (circular base); g. 3D printed replica of a real boulder). The name assigned to each model is also
 305 provided.

306

307 Regarding the tilt tests, they were carried out with a testing frame designed at the
 308 University of Vigo that consists of a metallic platform able to rotate about a fixed axis
 309 and driven by an asynchronous motor that assures constant angular lifting velocities (from
 310 $0.1^\circ/\text{min}$ to $26^\circ/\text{min}$) are maintained. The ‘start’ and ‘stop’ orders can be given to the
 311 machine by a manual control. A constant record of the tilting angle is kept using an
 312 inclinometer (*Leica DISTO D5*) with an accuracy of 0.1° attached to the platform (**Fig.**
 313 **9**). The critical angle of toppling for each tested model could be registered by simply
 314 stopping the rotation of the platform as the onset of instability was observed. For that
 315 purpose, the tilting velocity was set at $12^\circ/\text{min}$ for all tests in order to balance precision
 316 in determination of the critical angle with testing time. Each model was tested three times
 317 until toppling (or sliding, if it was the case) occurred, and results were collected for
 318 comparison with analytical and numerical predictions.

319



320

321 **Fig. 9.** Tilting platform used for carrying out tilt tests with the physical models (1. rotating
322 platform; 2. connection box and velocity control; 3. 'start and stop' control; 4. digital
323 inclinometer).

324

325

326 **3.2. Analytical assessment of toppling**

327

328 With the models PM-1 to PM-4 presented in **Fig. 6**, it is possible to develop simple 2D
329 analytical predictions of the critical angle of toppling. To do that, some cross-sections
330 were divided into simpler shapes (like squares, rectangles and triangles). The driving
331 forces were located at the centroids of each subsection and the stabilizing and overturning
332 moments were calculated by considering the rotation axis as the lower corner of the model
333 in contact with the tilting table, which was also set as the origin of the coordinate system
334 for calculations. The critical angle of toppling, α_{cr} , can easily be estimated by imposing a
335 factor of safety, $FS = 1$ in Eq. 1.

336

337 Other scenarios, like those presented for models PM-5 to PM-7, require a more detailed
338 analysis on the positioning of the *cog* as well as the rotation pivot.

339

340 It is relevant to note that the relative stability of any of the blocks under scrutiny for any
341 position can be both computed in terms of factor of safety and in terms of critical stability
342 angle. Obviously, these two approaches are related to one another, so they can be

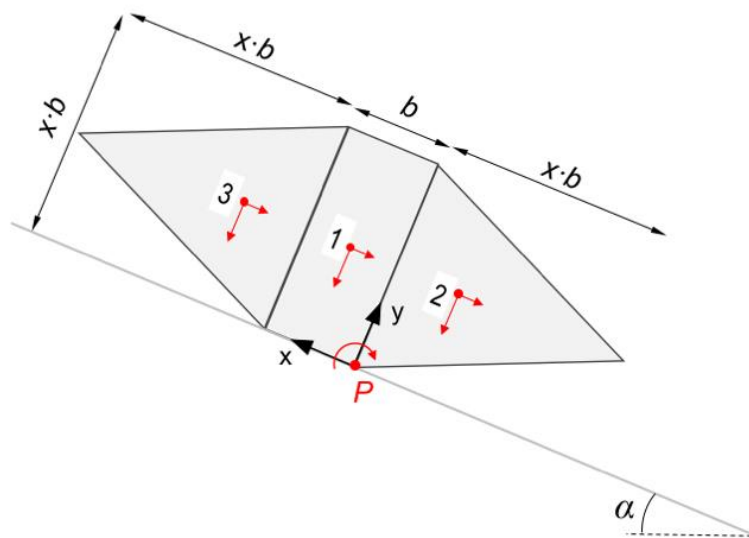
343 computed for different scenarios. For test interpretation purposes where no additional
344 actions exist (water pressures, earthquakes), the authors have selected the critical angle
345 approach, thinking it could be more illustrative for output comparison purposes.
346 However, a factor of safety approach could also be developed.

347

348 3.2.1. Symmetric model with straight edges and planar base

349 The simplest geometry studied (PM-1), consists of a symmetric block with a constant
350 cross-section that can be divided in two triangles and a rectangle. The model is defined
351 in terms of the central block breadth, b . A sketch of this model is presented in Fig. 10,
352 where the relevant force components and the rotation axis for a given inclination of the
353 base (α) are also shown.

354



355

356 Fig. 10. Sketch of the symmetric model with constant cross-section, where the centroid of each
357 sub-element is shown as well as the rotation pivot and origin of x-y coordinates.

358

359 For demonstrative purposes, we present the derivation of the critical angle formulation,
360 based on the computation of the factor of safety for the rock block illustrated in Fig. 10.

361 If one considers the rotation pivot (P) as the origin, the expression for estimating the factor
362 of safety (FS) dividing the block in its three basic elements is as follows:

363

$$FS = \frac{\sum M_{stabilizing}}{\sum M_{overturning}} = \frac{W_1 \cos \alpha \frac{b}{2} - W_2 \cos \alpha \frac{xb}{3} + W_3 \cos \alpha \frac{(3+x)b}{3}}{W_1 \sin \alpha \frac{xb}{2} + W_2 \sin \alpha \frac{xb}{2} + W_3 \sin \alpha \frac{xb}{2}} \quad (5)$$

364

365 In this case, all forces acting parallel to the x-axis correspond strictly to destabilizing
 366 moments, while those acting parallel to the y-axis contribute to both the stabilization (sub-
 367 sections 1 and 3) and destabilization (sub-section 2) of the block shown in Fig. 10.

368

369 Accounting for the fact that $W_2 = W_3$, and that $W = W_1 + W_2 + W_3$, Eq. 5 can be simplified to
 370 Eq. 6:

$$FS = \frac{\sum M_{stabilizing}}{\sum M_{overturning}} = \frac{W \cos \alpha \frac{b}{2}}{W \sin \alpha \frac{xb}{2}} = \frac{1}{x \cdot \tan \alpha} \quad (6)$$

371

372 Equating the FS to 1, the point at which instability initiates, the critical angle for toppling
 373 in Fig. 10 can be derived as Eq. 7:

$$\alpha_{cr} = \text{atan} \left(\frac{1}{x} \right) \quad (7)$$

374

375 This implies that, as mentioned above, the size of the sample does not influence results,
 376 so the physical model represents the behavior of any smaller or larger homothetic block.
 377 For the analyzed case and considering that $x = 2.7$, the critical angle in this case will be
 378 $\alpha_{cr} = 20.32^\circ$ as derived from Eq. 7. The influence of the height of the central rectangular
 379 block could be easily computed by testing different values of x in Eq. 7. This kind of
 380 analytical solution is therefore quite suitable for evaluation of the influence of some
 381 geometrical parameters of the blocks under scrutiny.

382

383 Alternatively, the FS can be computed for the case the cog is known from the beginning
 384 accounting for the stabilizing and overturning moments of the weight as presented in Eq.
 385 1. For blocks with more complex geometry, it therefore tends to be most convenient to
 386 compute the position of the cog to simplify subsequent computations.

387

388

389

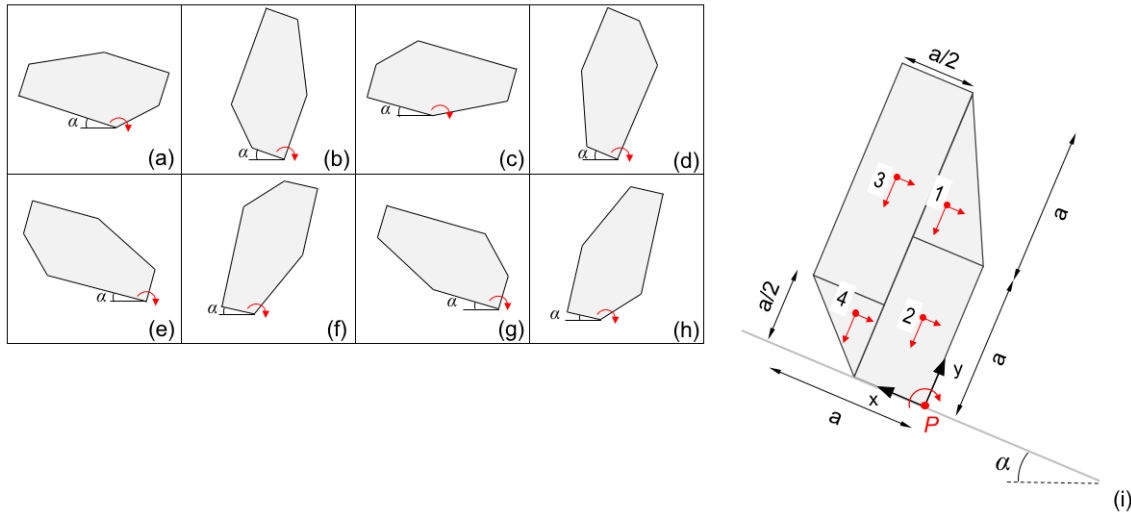
390

391 **3.2.2. Asymmetric model with straight edges and planar base**

392

393 This model (named as PM-2) presents an asymmetric cross-section that can be divided
 394 into simpler shapes (i.e. two rectangles and two triangles) to simplify calculations, as
 395 presented in Fig. 11.

396



397

398 **Fig. 11.** Sketch of an asymmetric model with constant cross section and straight edges. All the
 399 possible testing positions for toppling analyses are presented (a-h). The sub-sections (1-4)
 400 indicated in the enlarged sketch (i) on the left are kept for all models.

401

402

403 For the model sketched in Fig. 11i (equivalent to that in Fig. 11b), the angle of critical
 404 toppling can be estimated with Eq. 8, in which the cross section was divided in four
 405 simpler subsections.

$$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{a}{3} + W_2 \cdot \frac{a}{4} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{2a}{3}}{W_1 \cdot \frac{4a}{3} + W_2 \cdot \frac{a}{2} + W_3 \cdot \frac{5a}{4} + W_4 \cdot \frac{a}{3}} \right) \quad (8)$$

406

407 By modifying the position of the PM-2 block as presented in Fig. 11i, it is possible to get
 408 different geometries that will change the cross section to be analyzed, as shown in Fig.
 409 11a-h and, consequently, the critical angle of toppling. This procedure is valuable when
 410 performing the experimental part of the present work, since it will allow testing a single
 411 model in eight different positions.

412

413 To illustrate how the stability against toppling and, ultimately, the critical angle of this
 414 relatively complex geometry block could be computed, the complete set of equations (Eq.

415 9-16) for calculating the analytical critical angle of toppling for each position of the block
 416 is presented in Table 1. Again, in this case, as in any other, the stability against toppling
 417 will be completely independent of the block size.

418

419 Table 1. Equations for estimating the FS against toppling and angles of critical toppling for all
 420 the positions presented in Fig. 10 (a-h).

Block position	Equation
(a)	$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{5a}{6} + W_3 \cdot \frac{3a}{4} - W_4 \cdot \frac{a}{6}}{W_1 \cdot \frac{2a}{3} + W_2 \cdot \frac{3a}{4} + W_3 \cdot \frac{a}{4} + W_4 \cdot \frac{a}{3}} \right) \quad (9)$
(b)	$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{a}{3} + W_2 \cdot \frac{a}{4} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{2a}{3}}{W_1 \cdot \frac{4a}{3} + W_2 \cdot \frac{a}{2} + W_3 \cdot \frac{5a}{4} + W_4 \cdot \frac{a}{3}} \right) \quad (10)$
(c)	$\alpha_{cr} = \arctan \left(\frac{-W_1 \cdot \frac{a}{3} + W_2 \cdot \frac{a}{2} - W_3 \cdot \frac{a}{4} + W_4 \cdot \frac{2a}{3}}{W_1 \cdot \frac{a}{3} + W_2 \cdot \frac{a}{4} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{2a}{3}} \right) \quad (11)$
(d)	$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{2a}{3} + W_2 \cdot \frac{3a}{4} + W_3 \cdot \frac{a}{4} + W_4 \cdot \frac{a}{3}}{W_1 \cdot \frac{2a}{3} + W_2 \cdot \frac{3a}{2} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{5a}{3}} \right) \quad (12)$
(e)	$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{2a}{3} + W_2 \cdot \frac{3a}{2} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{5a}{3}}{W_1 \cdot \frac{2a}{3} + W_2 \cdot \frac{3a}{4} + W_3 \cdot \frac{a}{4} + W_4 \cdot \frac{a}{3}} \right) \quad (13)$
(f)	$\alpha_{cr} = \arctan \left(\frac{-W_1 \cdot \frac{a}{6} - W_2 \cdot \frac{a}{4} + W_3 \cdot \frac{a}{4} + W_4 \cdot \frac{a}{6}}{W_1 \cdot \frac{2a}{3} + W_2 \cdot \frac{3a}{2} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{5a}{3}} \right) \quad (14)$
(g)	$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{4a}{3} + W_2 \cdot \frac{a}{2} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{a}{3}}{W_1 \cdot \frac{a}{3} + W_2 \cdot \frac{a}{4} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{2a}{3}} \right) \quad (15)$
(h)	$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{a}{6} + W_2 \cdot \frac{a}{4} - W_3 \cdot \frac{a}{4} - W_4 \cdot \frac{a}{6}}{W_1 \cdot \frac{4a}{3} + W_2 \cdot \frac{a}{2} + W_3 \cdot \frac{5a}{4} + W_4 \cdot \frac{a}{3}} \right) \quad (16)$

421

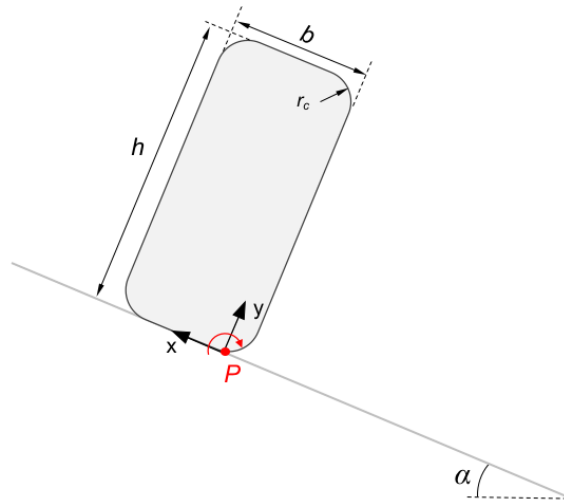
422

423 3.2.3. Symmetric model with rounded edges and planar base

424 Another model presenting a symmetric cross-section is that corresponding to a
 425 rectangular prism with rounded corners (PM-3), as originally analyzed by Alejano et al.
 426 (2015). The cross-section of the model is shown in Fig. 12 and, due to the simplicity of
 427 this section, only the *cog* of the entire model was considered.

428 If the moment equilibrium calculation is performed for the model presented in Fig. 12,
 429 the angle of critical toppling can be estimated (Eq. 17) as rearranged from Eq. 4. Note
 430 that the addition of a non-zero radius of curvature (r) reduces the critical angle of toppling,
 431 since it decreases the actual contact width by $2r_c$.

$$\alpha_{cr} = \arctan\left(\frac{b - 2r_c}{h}\right) \quad (17)$$



434
 435 Fig. 12. Sketch of the symmetric model with rounded edges (for the studied model: $h/b = 2.125$
 436 and $r_c/h = 0.147$).

437
 438
 439 **3.2.4. Model with non-planar (concave) base**

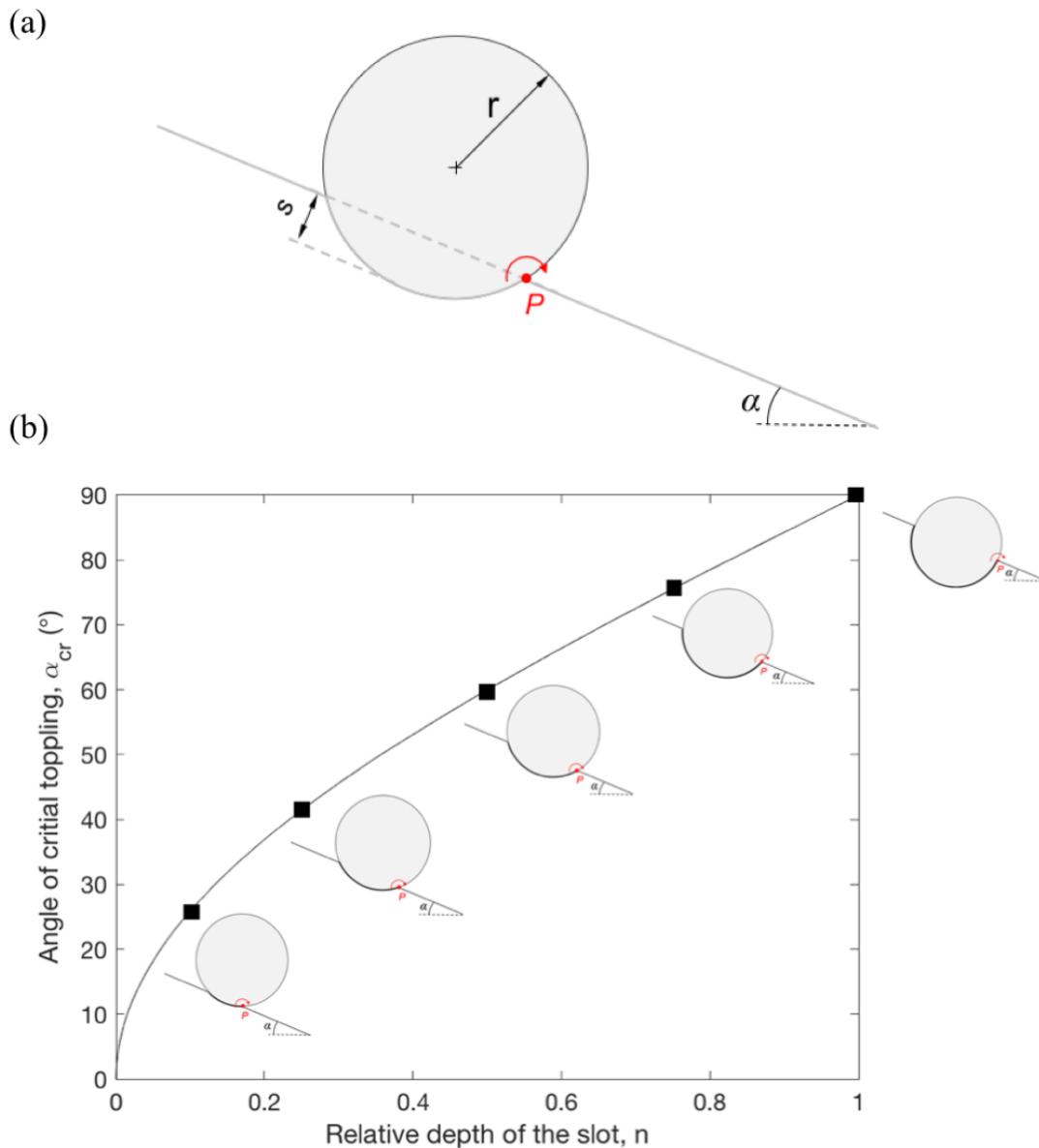
440 The present model is intended to illustrate the effect of a non-planar (concave) base, as a
 441 potentially stabilizing factor. The idea was to study a circular cross-section in which part
 442 of it is embedded in the inclined plane, resting on a concave base that coincides with the
 443 curvature of the cross section (see Fig. 7b and Fig. 13a).

444
 445 It can be demonstrated that the angle of critical toppling, α_{cr} , of the model presented in
 446 Fig. 13a solely depends on the depth of the slot (s) relative to the radius of the circular
 447 cross-section (r), n , where $n = s/r$ for $n \in (0, 1)$. The angle of critical toppling can
 448 therefore be calculated from Eq. 18.

$$\alpha_{cr} = \text{atan}\left\{\frac{\sin[\text{acos}(1 - n)]}{1 - n}\right\} \quad (18)$$

449

450 With the aim of illustrating the effect of a concave contact on the angle of critical toppling
 451 of such a circular section of radius r , the critical angle is plotted against the depth of the
 452 slot expressed as a proportion of the radius (n) in Fig. 13b.



453
 454 Fig. 13. (a) Sketch of the model with non-planar (concave) base; (b) Dependence of the angle of
 455 critical toppling (y-axis) with the relative depth of the slot, n (x-axis), for the model depicted in
 456 (a).
 457

458 As is shown in Fig. 13b, the effect of a concave contact clearly influences the angle of
 459 critical toppling of a block with a circular section. This effect is most significant for low
 460 values of n , where the derivative $d\alpha_{cr}/dn$ is larger

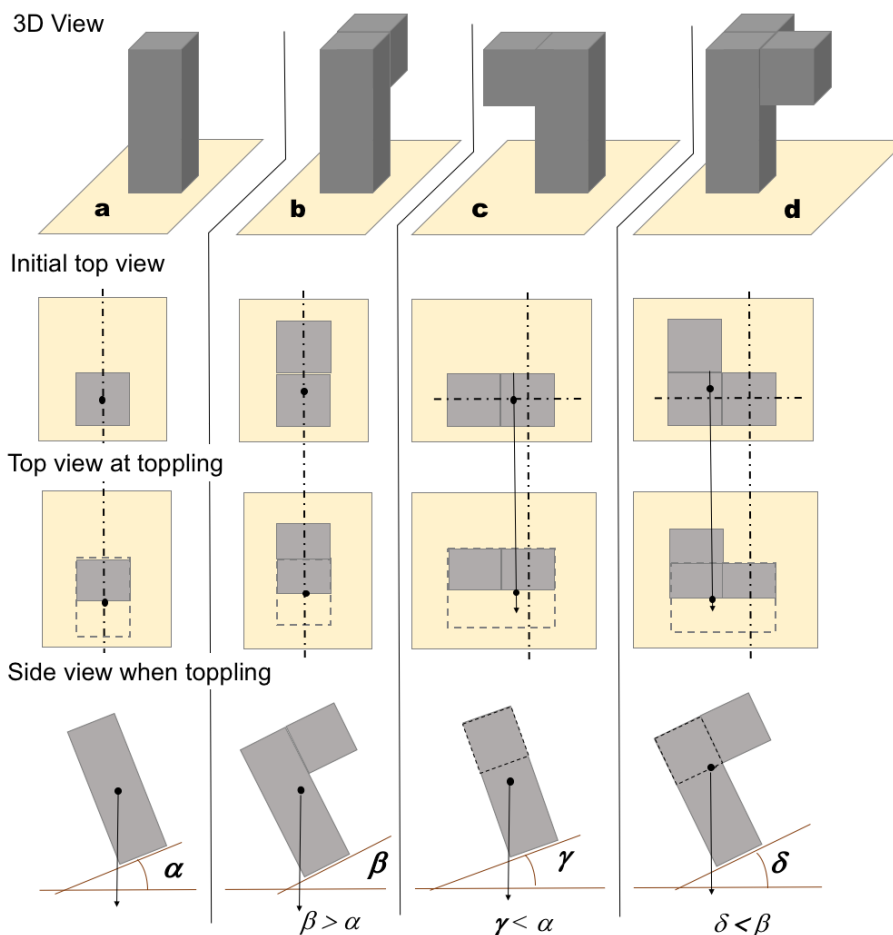
461 Though almost no blocks are expected to match the geometry shown in Fig. 13a perfectly,
 462 in practice, these results mean that even relatively small concavities of the basal plane of

463 a block of more irregular shape have the potential to increase the angle of critical toppling
 464 by a few degrees.

465

466 **3.2.5. Model with the *cog* not contained in a symmetry plane: squared/rectangular**
 467 **base**

468 The position of the center of gravity (*cog*) clearly influences the toppling behavior of an
 469 element as in the case of the blocks already analyzed in this study. If a block is positioned
 470 onto a flat surface and the vertical projection of its *cog* falls inside the contact area, the
 471 block will remain stable against toppling. Consider the situation illustrated in Fig. 14a for
 472 a square-based, symmetric and homogeneous block. In this case, if the *cog* is projected
 473 onto the base, that projection will fall on the center of the square section. If the block is
 474 progressively tilted along the dash-dot plane, it will topple once the projection of the *cog*
 475 falls out the base.



476

477 **Fig. 14.** Different 3D elements (a, b, c and d) to be subjected to a tilt test to illustrate the role of
 478 geometry on toppling. On the upper row, 3D view of the elements resting on a horizontal base to
 479 be tilted. On the second row, initial top view with the projection including the *cog*. On the third
 480 row, top view of the surface after tilting and in the moment of toppling and, on the last row, side
 481 view of platform and element when toppling.

482 Fig. 14b shows a more complex geometry with similar prism and cube attached to its
483 upper back face. The *cog* of this model, when placed on a flat surface, will not project on
484 the center of the base but will instead project somewhere behind the center of the base
485 due to the added mass. Because of this, when tilting the plane where this element stands,
486 it will topple at a higher angle than the previous case; in other words, the angle of critical
487 toppling in this case (α) will be higher than the one observed for the element shown in
488 Fig.14a (β).

489

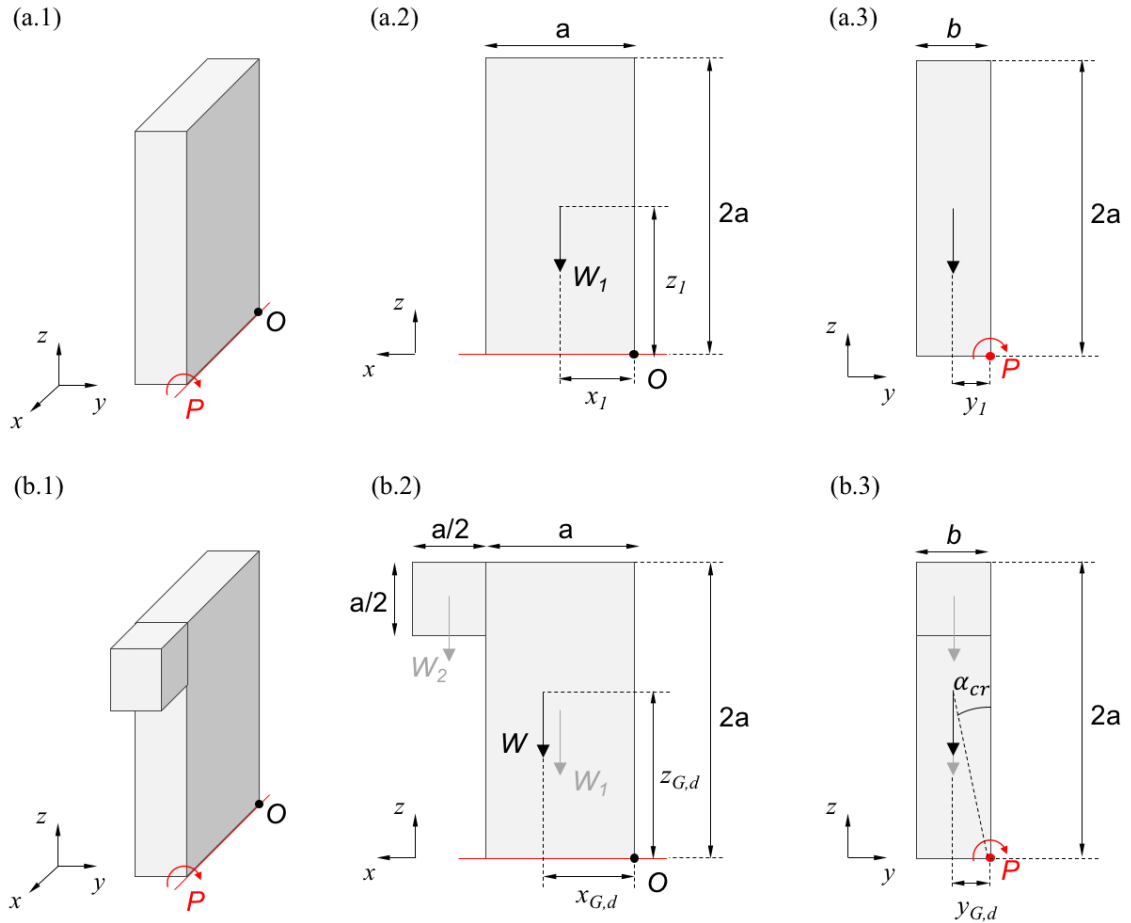
490 The third element (Fig. 14c) is similar to element 'b', but the added cube is now attached
491 on the upper part of a lateral face. Its *cog* will be at the same height as for element b (since
492 it is the same element in the same position), but its projection onto a horizontal plane will
493 be moved to the left in relation to element 'a' (Fig. 14c, second row). When tilting the
494 platform where element 'c' rests, it will topple at a lower angle than α , because its *cog* is
495 located higher than in case 'a', meaning its projection will fall outside its base at a lower
496 angle γ , which will be also less steep than β .

497

498 Element 'd' is a rectangular prism with a square cross-section where two cubes are
499 attached on the upper part of its lateral backward and leftward faces. In this case, the *cog*
500 of the element will be even more displaced upwards than in the case of elements 'b' and
501 'c' and the *cog* projection on its base will be slightly moved backwards and a little bit to
502 the left in relation to the case of element 'a' in Fig 14. This will clearly be less stable than
503 'b' (since the side-attached cubes displace the *cog* upwards) but more stable than 'c'
504 (since the back-attached cube will increase its stability by moving the projection of its
505 *cog* backwards). According to the previous analyses, a hierarchy on the angle of critical
506 toppling can be established for the studied models shown in Fig. 14 as: $\beta > \delta > \alpha > \gamma$.

507

508 With the goal of assessing the effect of a displaced *cog* on the toppling behavior in a more
509 detailed way and following the ideas described in the previous paragraphs, the model
510 presented in Fig. 15 has been considered. It consists of a rectangular-based block with a
511 small prismatic slab attached on the upper part of a lateral face (see Fig. 14b.1).



512 **Fig. 15.** (a) 3D view of the model with the axis of rotation and origin of coordinates for positioning
 513 the cog marked; (b) levelled front view of the assembly and (c) levelled lateral view of the
 514 assembly.
 515
 516

517 As previously mentioned, the effect of adding such a piece to a rectangular-based block
 518 as shown in Fig. 15 implies a displacement of the cog, which will move towards the added
 519 mass. This positioning of the cog can be easily determined by resorting to Eqs. 19, 20 and
 520 21 for each coordinate, respectively, when considering $m_i = W_i / g$.

521

$$x_{G,d} = \frac{\sum(m_i \cdot x_i)}{\sum m_i} \quad (19)$$

$$y_{G,d} = \frac{\sum(m_i \cdot y_i)}{\sum m_i} \quad (20)$$

$$z_{G,d} = \frac{\sum(m_i \cdot z_i)}{\sum m_i} \quad (21)$$

522

523

524 Once the 3D coordinates of the displaced *cog* are set ($x_{G,d}$, $y_{G,d}$, $z_{G,d}$), the angle of critical
525 toppling for the model sketched in Fig. 15b.1 can be estimated using Eq. 22:

526

$$\alpha_{cr} = \text{atan}\left(\frac{y_{G,d}}{z_{G,d}}\right) \quad (22)$$

527

528 In a similar way as presented in Eq. 8, the angle of critical toppling can also be estimated
529 as a function of dimensions a and b of the specimen (Eq. 23):

530

$$\alpha_{cr} = \text{atan}\left[\frac{(W_1 + W_2) \frac{b}{2}}{W_1 \cdot a + W_2 \cdot \frac{3a}{2}}\right] \quad (23)$$

531

532 It has to be highlighted that all the models with a rectangular or squared-base such as
533 those shown in the previous sections and particularly the last shown in Fig. 15 will always
534 have a pre-defined axis of rotation, which coincides with an edge of the block in contact
535 with the base. This is not the case for circular or irregular-based specimens, as considered
536 in Section 3.2.6.

537

538

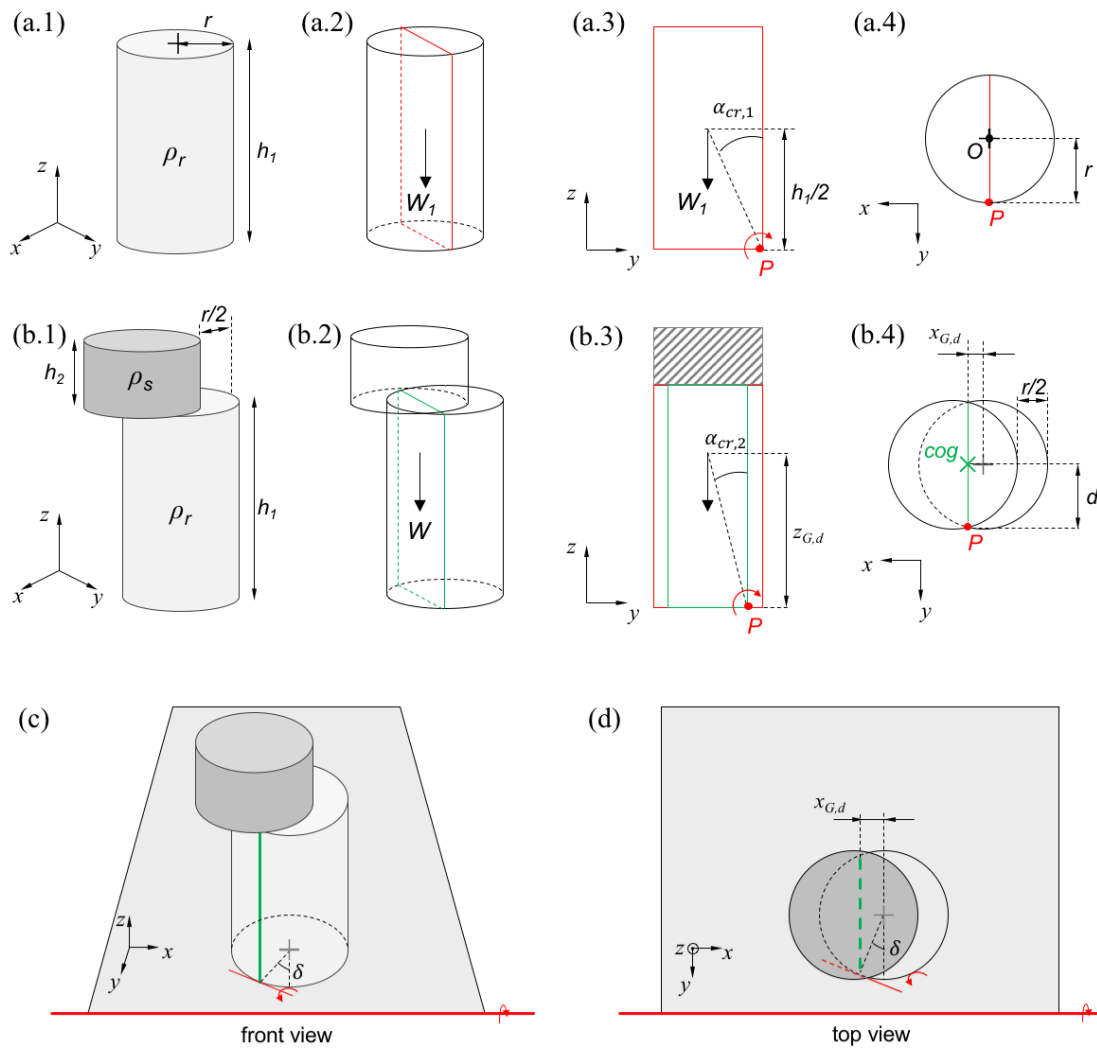
539 **3.2.6. Model with the *cog* not contained in a symmetry plane: circular base**

540 Real rock blocks and boulders as found in Nature rarely have *cogs* that project in the
541 center of their bases when resting on horizontal surfaces, nor do they have well-defined
542 axes of rotation against toppling mechanism due to their typically irregular shapes. As
543 already noted, for a precise assessment of their stability against toppling, it will be
544 necessary to correctly position the *cog* as well as the pivot or axis around which the
545 toppling mechanism will take place.

546

547 The study of these features has been carried out using a laboratory physical model (as
548 shown in Fig. 6f) consisting of two cylinders composed of different materials (rock and
549 steel with densities of $\rho_r = 2700 \text{ kg/m}^3$ and $\rho_s = 7900 \text{ kg/m}^3$, respectively). Both cylinders
550 have the same radius ($r = 27 \text{ mm}$) but different heights, with the rock cylinder measuring
551 100 mm in height and the steel cylinder measuring 35 mm in height. The assembly is
552 sketched in Fig. 16b.1.

553 This test consisted of placing the two cylinders as shown in Fig. 16b.1, with the top piece
 554 displaced a distance of $r/2$ along the x direction. This position of the top steel cylinder
 555 moves the center of gravity out of the center of the base, as can be seen in the front view
 556 shown in Fig. 16c. Then, the specimen was progressively tilted (being the platform
 557 rotation around the x -axis) until toppling of the entire set occurred, when the tilting angle
 558 (angle of critical toppling) was reached.



559
 560 Fig. 16. (a.1-4), (b.1-4) Different views of the model containing the *cog* out of a symmetry axis
 561 (model PM-6); (c), (d) Different views of the model PM-6 at the equilibrium limit state, where
 562 the axis of rotation does not coincide with that of the rotating platform.
 563

564 By considering the origin of the coordinate system at the center of the base of the lower
 565 cylinder (Fig. 16a.4), and regarding that $z_{g,d}$ can be easily obtained as function of the
 566 densities and block geometries, the angle of critical toppling for the model presented in
 567 Fig. 16b.1 when tilted around the x -axis can be calculated using Eq. 24.

568

$$\alpha_{cr} = \text{atan}\left(\frac{d}{z_{G,d}}\right) \quad (24)$$

569

570 Unlike the other examples considered in this study, this model will behave differently
571 once the limit equilibrium for toppling has been reached. Specifically, although the
572 critical angle of toppling can be calculated in a similar way to that of the other models
573 (Eq. 12), the rotation of the cross-section containing the *cog* of the assembly (shown in
574 green in Fig. 16b.3 and Fig. 16b.4) has to be considered, as it influences the rotation axis.

575

576 As shown in Fig. 16c, d, the axis (pivot) of rotation will not coincide with that of the
577 rotating platform and will move laterally due to the displacement of the *cog* and the
578 circular base in such a way that the new axis of rotation corresponds to the tangent at the
579 intersection point between the plane containing the new *cog* (shown in green color in Fig.
580 16c, d) and the base of the model. The displacement angle, δ , as shown in Fig. 16d can
581 be calculated using Eq. 25.

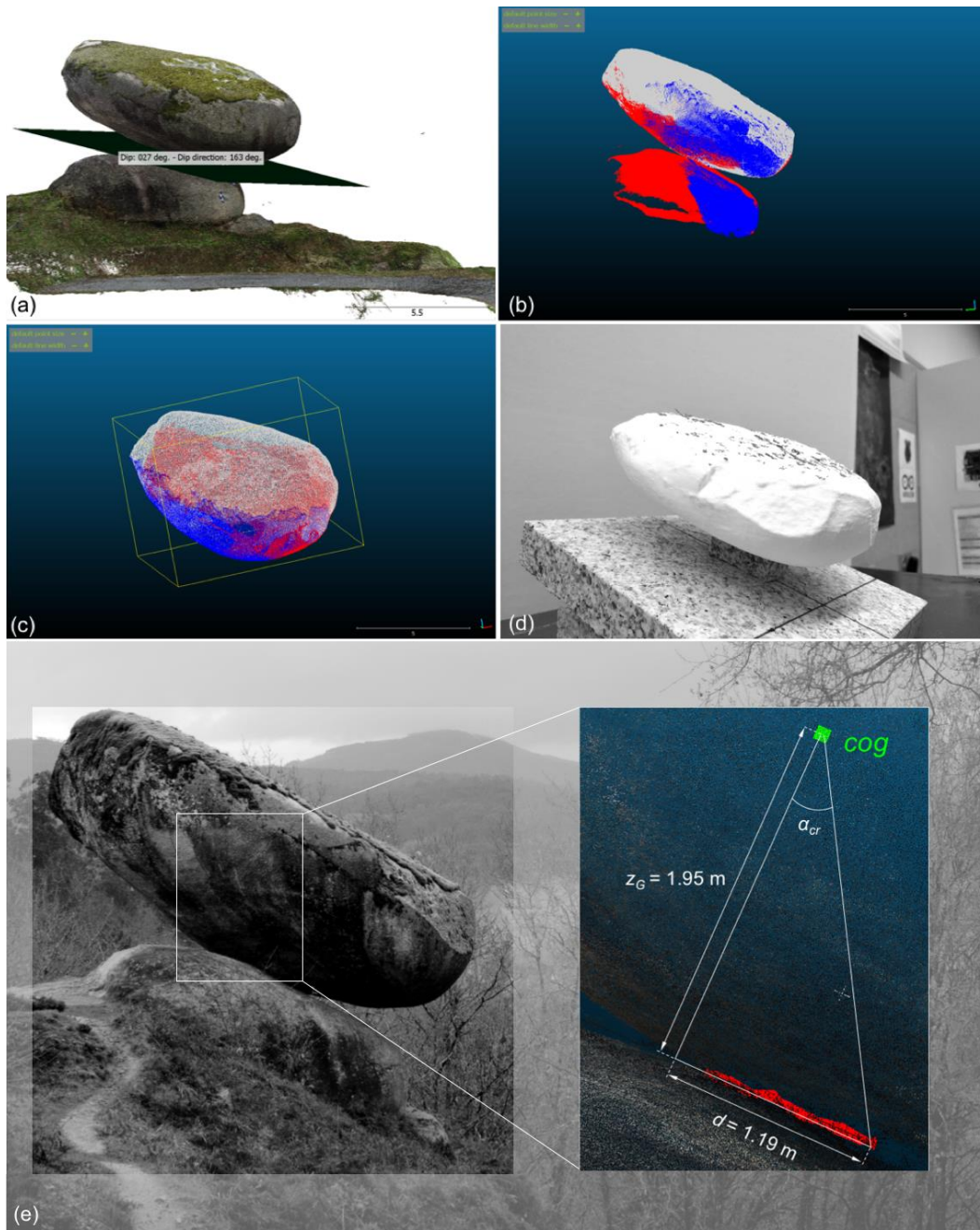
582

$$\delta = \text{asin}\left(\frac{x_{G,d}}{r}\right) \quad (25)$$

583

584 **3.2.7. 3D-printed model representative of a real boulder**

585 The last model (PM-7) considered in this study corresponds to a plastic (PLA) replica of
586 a real granitic boulder located in the NW of Spain, as studied by Pérez-Rey et al. (2019).
587 The replica, made at a scale of approximately 1:50, was created from an 3D point cloud
588 of the real boulder collected in the field, which was afterwards processed with the
589 software *CloudCompare* (Girardeau-Montaut, 2018) and *Meshlab* (Cignoni et al., 2008)
590 in order to develop the 3D printing stage with a *BCN Sigma 3D* printer (see Fig. 17).



591

592 **Fig. 17** (a, b) Two views of the 3D point cloud of the studied boulder; (c) Isolation of the boulder
 593 from the rest of the structure; (d) 3D-printed replica on the testing platform and (e) Detailed view
 594 of the position of the *cog* projected onto the contact plane, showing the distance *d* required
 595 for estimating the critical angle of toppling.

596

597 By taking advantage of such a precise 3D point cloud and with the assistance of
 598 *CloudCompare* software, it is possible to approximate, in a reasonable manner, the
 599 contact area between the boulder and the base and to position the *cog*.

600 Using the relationship presented in Eq. 23 and the geometrical parameters presented in
 601 **Fig. 17e**, it is possible to estimate the approximate angle of critical toppling of the boulder
 602 to be $\alpha_{cr} = 31.4^\circ$.

603 **4. Results**

604 **4.1. Comparison of analytical and experimental results**

605 After carrying out all calculations of the angle of critical toppling of each of the models
 606 considered for this study, as presented in Section 3, all analytical results are shown in
 607 Table 1. Together with these results, the experimental angles of toppling obtained for
 608 each series of three tests carried out with the seven **physical** models are also provided
 609 with an averaged result. As it can be observed in this table, the discrepancy of the analytical
 610 and average laboratory results is always less than 1.3°, and the median error is 0.66°.

611 It must be noted that some models did not achieve toppling failure in the laboratory tests
 612 (in particular, the PM-2 model in some positions). This occurs when the theoretical
 613 toppling angle is greater than the basic friction angle of the base contact surface, **and** the
 614 block slides before reaching its toppling angle. These results are indicated in Table 2 with
 615 an ‘s’. It has also been observed that PM-2 model in position (h) was not self-stable in a
 616 horizontal position ($\alpha_{cr} < 0$).

617 Table 2. Analytical and experimental results for the angle of critical toppling (**in degrees**) as
 618 obtained for the seven studied models.

Model	Position	Analytical	Experimental (s = slide)			
		α_{cr}	α_1	α_2	α_3	α_{mean}
PM-1	—	20.32	20.2	20.3	20.5	20.3
PM-2	(a)	44.22	30.4 (s)	27.2 (s)	30.1 (s)	29.2 (s)
	(b)	28.66	29.1	29.0	29.2	29.1
	(c)	4.18	3.2	3.2	3.1	3.2
	(d)	24.55	25.4	25.3	25.6	25.4
	(e)	65.45	33.5 (s)	30.9 (s)	27.0 (s)	30.5 (s)
	(f)	1.41	2.6	2.7	2.7	2.7
	(g)	61.34	29.0 (s)	25.6 (s)	26.5 (s)	27.0 (s)
	(h)	< 0	—	—	—	—
PM-3	—	10.01	11.0	11.0	9.67	10.6
PM-4	(a) (n =1/6)	33.56	33.3	32.5	32.8	32.9
	(b) (n=1/3)	48.19	47.3	48.3	48.4	48.0
PM-5	—	11.88	11.6	12.3	12.4	12.1
PM-6	—	17.29	16.5	15.9	16.2	16.2
PM-7	—	31.39	30.8	30.4	30.7	30.6

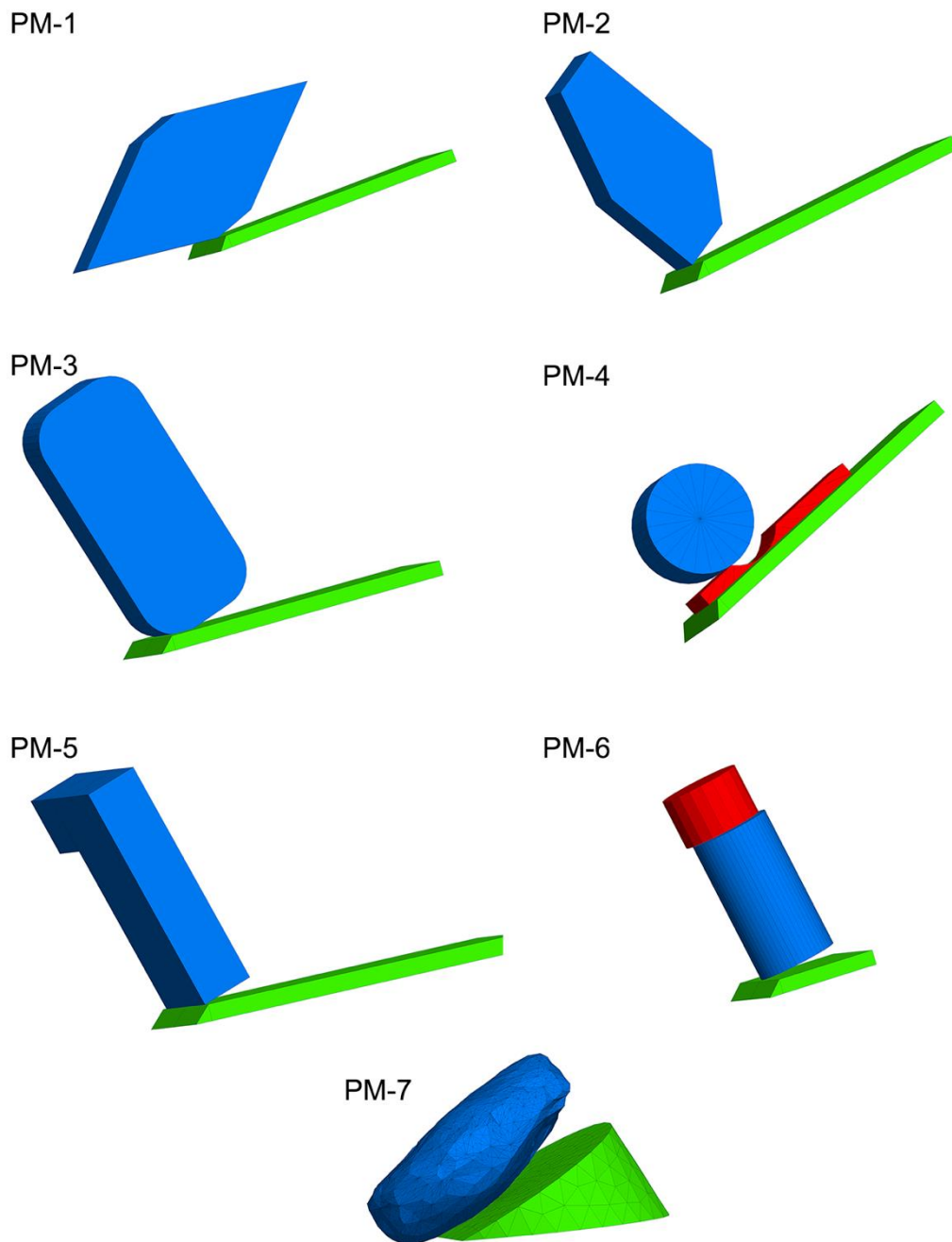
619

620

621 **4.2. 3D discrete numerical modelling**

622 Another way to validate the results of the physical models is by comparing these results
623 with those obtained by numerical analysis. For this part of the study, we have utilized the
624 Distinct Element Method (DEM), which applies an explicit finite difference method for
625 modelling large displacements and rotations of block systems (Cundall, 1971). This
626 method has been used in numerous studies of toppling (Brideau and Stead, 2010; Lanaro
627 et al., 1997; Pritchard and Savigny, 1990). In this case, we used the DEM as implemented
628 in the software 3DEC v5.20 (Itasca, 2016).

629 Discrete Element Methods can deal with geological structures of any size and shape, and
630 with a great variety of constitutive models for both the intact rock and the discontinuities.
631 They also allow for simulation of complex hydrogeological environments or time-
632 dependant phenomena like rock-dynamics or creep. Another advantage of this study is
633 that it does not require the prior definition of a displacement direction (as required in the
634 analytical calculation), meaning the results of the other methods can be confirmed in cases
635 where there is any doubt about the displacement direction, such as for models with a
636 displaced *cog* (PM-5 and PM-6), or the model with the irregular, complex shape (PM-7).
637 The same tilt tests performed with the physical models were simulated in 3DEC (Fig. 18).



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Fig. 18. Results for the numerical simulation of the tilt tests performed.

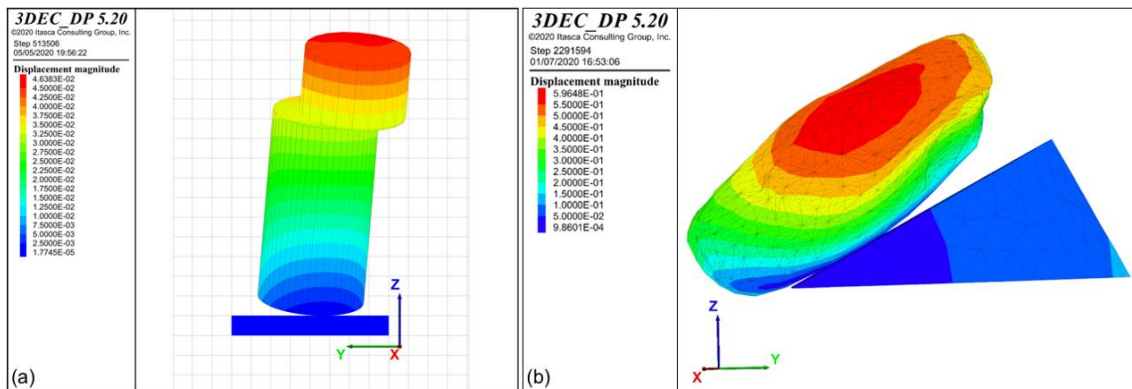
In these models, both the tilt table and the specimens were modelled as rigid blocks. The contact stiffnesses were set to k_n 650 GPa/m and k_s 150 GPa/m. The tilting rate was set slow enough to ensure that no inertial effect was produced so the test could be considered quasi-static. The results of these models are presented in Table 3, where the maximum difference observed between results obtained using various methods is displayed.

647 Table 3. Angles of critical toppling calculated by different methods and absolute maximum
 648 difference between results. Involved methods indicated in brackets.

Model	Angle of critical toppling (°)			Absolute max. difference (°)
	Experimental (E)	Analytical (A)	DEM (D)	
PM-1	20.3	20.32	20.1	0.22 (E-A)
PM-2	29.1	28.66	28.6	0.5 (E-N)
PM-3	10.6	10.01	10.0	0.6 (E-N)
PM-4	48.0	48.19	48.1	0.19 (E-A)
PM-5	12.1	11.88	11.9	0.22 (E-A)
PM-6	16.2	17.29	17.3	1.1 (E-N)
PM-7	30.6	31.39	31.3	0.79 (E-A)

649

650 The results obtained using the DEM models agree with those obtained by both the
 651 physical models and the analytical method, even in the cases where the centre of gravity
 652 is not located in the plane of symmetry, or the toppling involves complex movement not
 653 parallel to the tilting direction, as in the case of PM-6 (Fig. 19a).
 654



655

656 Fig. 19. (a) 3DEC results of the tilt simulation on a rock + steel set with and upper cylinder
 657 uncentered $r/2$ to the left (PM-6). After the block starts to topple, its movement does not follow
 658 the tilting direction because the *cog* is not located in the symmetry plane as assumed in section
 659 3.2.6. (b) Displacement magnitude after tilt-test of PM-7 calculated by 3DEC.
 660

661 It is relevant to note that the DEM and the analytical approach match so closely here (and
 662 in fact in general). This suggests that the errors observed in the experimental results are
 663 largely a function of limitations in the “manufacturing” processes used to build the
 664 various physical models.

665 The critical angle in models with complex geometries (PM-7) measured by the three
 666 methods were also similar, confirming the validity of both the experimental and the
 667 analytical approaches. This model (Fig. 19b) presents both an asymmetrical geometry and
 668 an irregular base shape, resulting in complex movement after destabilization.

669

670

671 **5. Discussion**

672 It is not difficult to find **in Nature** rock blocks or groups of blocks that could potentially
673 become unstable due to toppling. In some case, these blocks are irregular enough so as to
674 be considered heritage or part of natural parks, so they are **preserved** (Fig. 20). On the
675 other hand, the instability of some other less aesthetically appealing rock blocks may
676 jeopardize infrastructure or even people, lives and properties. In any of these cases, it is
677 important to be able to analyze the stability of these blocks under different conditions
678 such that appropriate protective measures can be defined.



679 **Fig. 20.** Balanced stones in Natural Parks. (a) ‘The Three Sisters’ balancing rocks, Matopos
680 National Park, Matabeleland, Zimbabwe (b) *Roque de García*, basaltic horn at Teide National
681 Park, Tenerife, Canary Islands, Spain (c) 3,500 t balanced rock, Arches National Park, Utah, USA
682 and (d) 700 t balanced rock, ‘The Garden of Gods’, Colorado Springs, USA. Photos by the
683 authors.
684
685

687 Although some approaches were developed in the past to compute the stability of blocks
688 against toppling, in many cases, and specifically those corresponding to complex
689 geometry blocks, it was indeed difficult to accurately compute stability against toppling.
690 Recent advances in theoretical stability analysis based on idealized geometries (rounded
691 corners, concave or convex surfaces) have contributed to a better understanding of
692 toppling phenomena. The approaches presented here based on modern block geometry
693 reconstruction methods, 3D printing of a block replica and testing of this replica using a
694 tilt table, help to reproduce the potential instability phenomena of these blocks and to
695 assess their ‘degree of stability’ or instability, i.e., how far from toppling they are.

696

697 One notable limitation of the approach demonstrated in this study is the lack of knowledge
698 of the geometry (concavity or concaveness and roughness) of the contact between the
699 block and the surface where it rests, as well as its actual frictional behavior. However, the
700 proposed approach, in combination with detailed in-situ characterization and the
701 application of analytical and numerical calculation techniques as illustrated in this
702 document, has the potential to contribute to improved assessments of the stability of
703 irregular rock blocks or boulders.

704

705 **6. Concluding remarks**

706

707 All over the world, and particularly in mountainous terrain in hot and temperate regions,
708 rock blocks or boulders occur, and may exist in a state of marginal stability. In most of
709 these cases, the potential instability of these blocks does not represent a hazard to human
710 life or property. In some cases, however, it may be important to quantify block stability
711 either due to an associated hazard, or due to its significance to the community or its natural
712 landscape value.

713

714 Analyzing the stability of these blocks is not an easy task, primarily due to their complex
715 geometry and because it is also difficult to characterize in sufficient detail all the features
716 actually affecting their stability, including block geometry, geometry of the contact with
717 the base surface, strength and deformability characteristics of this contact —of particular
718 relevance when considering rough joints and infill material with non-negligible tensile
719 strength—, and potential triggers such as water pressure or earthquake loading.

720

721 Recently developed remote-sensing tools, such as photogrammetry or LiDAR can be used
722 in order to recover a rather accurate geometry of a block of interest as well as an
723 approximate representation of the contact area (typically hidden). Based on the recovered
724 3D point cloud, a scaled **physical model** of the rock block or boulder can be 3D-printed,
725 and its toppling behavior physically **analyzed** using a tilting platform, since toppling is
726 exclusively dependent on the geometry of the potentially overturning object and the
727 concavity of the base. This approach can be applied in combination with analytical or
728 numerical techniques to study the mechanisms involved and to check physical testing
729 results.

730

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866

Laboratory physical modelling of block toppling instability by means of tilt tests

HIGHLIGHTS

- Physical modelling represents an effective method for toppling-stability assessment
- Tilt-testing of small scale rock elements reproduce mechanisms at larger scales
- Different parameters influencing toppling behaviour have been analysed
- 3D point clouds of rock structures can be obtained from TLR and UAV photos
- 3D Distinct Element methods allow stability calculations of odd-shaped rocks

Laboratory physical modelling of block toppling instability by means of tilt tests

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ABSTRACT

In this paper we introduce a physical modelling approach where the stability of rock blocks against toppling in the field can be estimated using a tilt table, engineered rock models and 3D-printed small-scale versions of a natural rock boulder. To achieve this goal, first, simple geometry rock elements are tilted and results interpreted according to analytical formulations. Then, more complex geometry engineered rock blocks, including some where its center of gravity does not project on the center of the base element, are tested and results properly interpreted. Eventually, the 3D-printed version of the rock boulder is produced from 3D point clouds recovered in the field by means of a combination of photogrammetry and laser scanner techniques. Analytical formulations and numerical calculations have been used in order to validate the proposed approach, to explain the physical phenomena involved, and to allow for possible extension of the physical modelling observations to different scenarios, such as those considering the influence of water or seismic loading on stability.

30 **List of symbols**

31	α	base plane inclination angle
32	α_{cr}	critical angle of toppling
33	b	width of the block
34	cog	center of gravity
35	ϕ	friction angle
36	FS	factor of safety
37	g	acceleration of gravity (9.81 m/s ²)
38	h	height of the block or model
39	P	rotation pivot of a given block or model and origin of the (x_i, y_i) coordinates
40	r	radius
41	r_c	roundness of the block corners (expressed as the radius of curvature)
42	x_i	distance from P to the cog of the i -th sub-section along the x-axis for a given
43		block or model
44	y_i	distance from P to the cog of the i -th sub-section along the y-axis for a given
45		block or model
46	w	width of the block (out of plane)
47	W	weight
48	W_i	weight of the i -th section

49 **1. Introduction**

50 Analysing the stability of natural or man-made slopes in rock masses is a complex task.
51 The principles of the discipline addressing this problem (typically known as rock slope
52 engineering) were established in the seminal works developed during the 1960s and 1970s
53 at Imperial College in London and by Richard Goodman in Berkeley and summarized in
54 the book entitled “Rock Slope Engineering” (Hoek and Bray, 1974; Wyllie and Mah,
55 2004). In this book, it was clearly stated that, unlike in the case of soils, where all the
56 failure mechanisms involve sliding along a rotational or planar sliding surface, for rocks,
57 a variety of failure mechanisms could occur according to the number, continuity, spacing,
58 orientation and geomechanical characteristics of existing discontinuity sets.

59
60 Four typical types of failure mechanisms were identified at the time as the most common
61 possibilities in rock slopes: planar, wedge, (block) toppling and circular failure. This
62 classification combined with the simple mathematical tools provided to analyse their
63 stability, represented a significant step forward in the rock engineering field, leading to
64 improved rock slope stability analyses and designs. However, as wisely noted in the
65 preface of Hoek and Bray’s (1974) book, ‘Because the rock mass behind every slope is
66 unique, there are no standard recipes or routine solutions which are guaranteed to produce
67 the right answer each time they are applied’. This is why it is important to bear in mind
68 that rock slope instability phenomena do not necessarily occur according to simple failure
69 mechanisms as those described in these early works.

70
71 Whereas three of these basic mechanisms refer to sliding phenomena, only one of them
72 involves toppling. Toppling corresponds to rotation of relatively slender rock columns or
73 blocks about a fixed base. There are two distinct types of toppling failure mechanisms
74 named block and flexural toppling (Goodman and Bray, 1976). In the first case, the
75 toppling block is already fully detached from the rest of the rock mass. The second implies
76 flexural or tensile failure, where the block is not completely detached of the rock mass
77 such that new tensile failure cracks need to occur to fully detach the block from the
78 surrounding ones for toppling instability to occur. In this study we focus on the analysis
79 of block toppling.

80

81 The simplest toppling case as considered by this study is that of a single toppling block.
82 The first analyses of the stability against toppling of a single block were performed by
83 Ashby (1971) and then extended and formalized by other authors (Hoek and Bray, 1974;
84 Sagaseta, 1986). All these analyses start with stringent geometrical assumptions
85 considering perfectly rectangular blocks with sharp corners resting on a tilted plane
86 striking in the same direction as the block face in contact with the plane.

87

88 Some studies have recently begun to address the influence of more realistic shapes of
89 blocks on block stability against toppling. Alejano et al. (2015) studied the influence of
90 the rounding of block corners on the stability of a block. This geometric condition
91 observed in some rock masses and associated with spheroidal weathering erosive
92 processes was shown to significantly affect stability of blocks. The authors proposed a
93 formulation to compute the stability of a round-cornered block to account for this effect.
94 This formulation relies on the position of the potential rotation axis and was verified using
95 physical models and field observations. While such a contribution is interesting and
96 extends the type of toppling instability that can be analyzed, when one visits certain
97 natural rock environments, it can be observed that many more complex irregular block
98 geometries exist for which no analysis methods exist.

99

100 As an example, Fig. 1 illustrates three zones in two different mountain environments,
101 where a number of toppled and stable blocks are observed. In most of the cases and based
102 on our current capabilities to analyze stability of blocks, it would be indeed difficult to
103 rigorously perform a back analysis of the instability or stability of these blocks to explain
104 why are they stable or unstable. For the stable blocks, it would also be difficult to assess
105 their stability after a potential change in loading conditions, such as in the case of a
106 seismic event. Accordingly, the main goal of this study is to develop a methodology
107 through which the stability of complex blocks (such as those illustrated in Fig. 1) can be
108 rigorously analyzed.

109

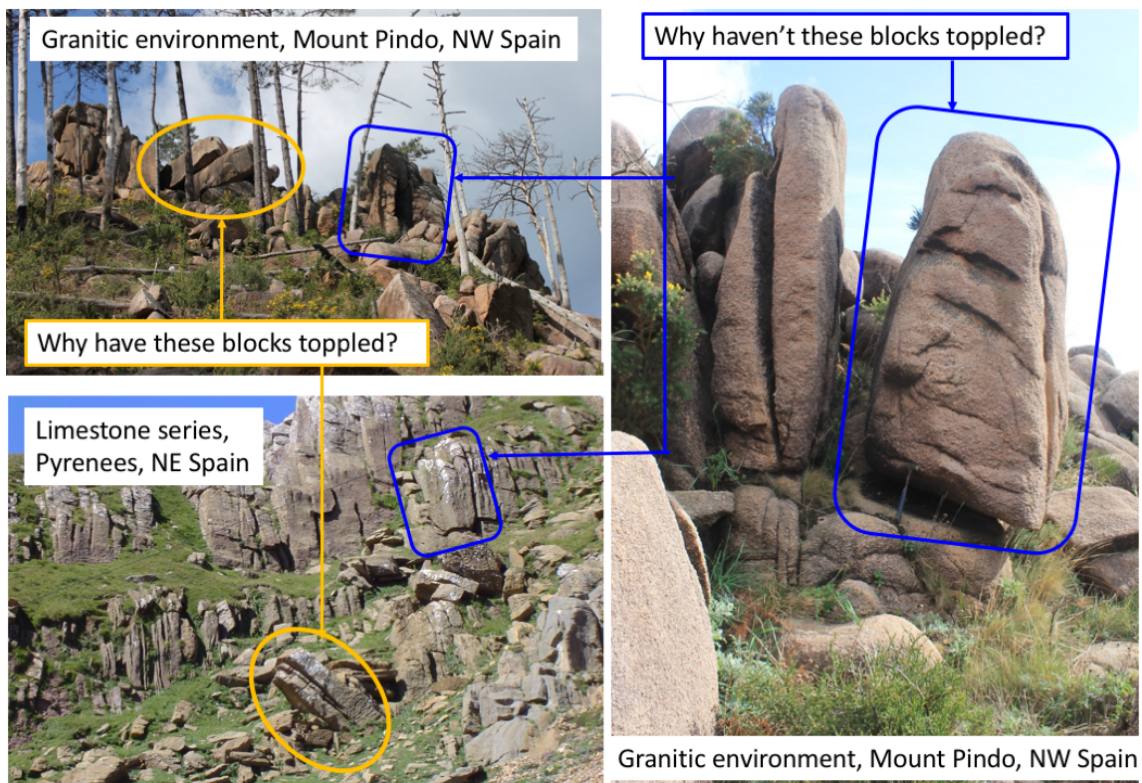
110 Since the early studies, the authors have considered cases involving increasingly complex
111 estimates of the toppling stability of individual rock blocks taking into account the
112 occurrence of rounded corners on the toppling blocks (Alejano et al., 2017) or rock
113 boulders (Alejano et al., 2010b; Pérez-Rey et al., 2019) and considered other studies
114 available from literature focusing on the potential destabilizing effects of earthquakes on

115 rock blocks or groups of rock blocks (Christianson et al., 1995; Shi et al., 1996; Vann et
116 al., 2019) where it was necessary to extend available methodologies to reflect the nature
117 of these structures in a more realistic manner.

118

119 In the process of developing the above-mentioned studies, tilting simple geometry rock
120 elements has revealed as an interesting technique that, in combination with analytical
121 formulations can help to understand a number of issues associated to toppling
122 phenomena. Therefore, a number of simple and a little bit more complex geometry
123 engineered rock blocks have been tilted and results explained in this document to illustrate
124 the potential interest of extending this technique to more realistic forms.

125



126

127 Fig. 1. Different geological environments where toppled and stable rock blocks are observed.

128

129 The study is an extension of basic toppling equilibrium calculations and was developed
130 in parallel with particular rock slope stability studies. The initial concepts associated with
131 the proposed methodology developed while studying the stability of wall slopes (Alejano
132 et al., 2011) and of masonry retaining walls (Alejano et al., 2012). In both these cases it
133 was possible to resort to physical models subjected to tilt tests in order to carry out simple
134 analyses with the aim of confirming particular failure mechanisms and the validity of

135 some formulations (Fig. 2). This previously developed approach is valid if the only type
136 of strength involved is friction and if geometry of the blocks can be easily reproduced
137 using common rock cutting approaches (typically with saw-blades).

138



139

140 Fig. 2. Physical models of dry masonry retaining walls used to understand the failure mechanism
141 of these structures and check limit equilibrium calculations.

142

143 Since the turn of the century, photogrammetric and laser scanning techniques for point
144 cloud acquisition have become widely available, such that a very good representation of
145 a given rock slope geometry can be achieved (Alejano et al., 2013; Armesto et al., 2009;
146 Ferrero et al., 2011, 2009; Riquelme et al., 2014). Most recently, relatively large and
147 accurate 3D point clouds have become available at a very reasonable cost (Girardeau-
148 Montaut, 2018). It is therefore possible to obtain 3D point clouds representing the
149 complex block geometries observed in nature (such as in Fig. 1). Based on such geometric
150 data, the center of gravity (*cog*), which is critical for toppling stability analyses, can be
151 precisely located. Other relevant geometrical aspects of the rock blocks or boulders such
152 as the position of the contact base between the block and basal plane can be also computed
153 rigorously. This geometrical information largely facilitates computing the factor of safety
154 of these boulders against toppling, as the ratio of stabilizing moments to overturning
155 moments.

156

157 Additionally, in the last five years, 3D printing has been improved so rapidly that it is
158 relatively easy to produce a scaled 3D printed version of any rock block based on the
159 point cloud representing its outer surface (Bader et al., 2018; Virtanen et al., 2014). This
160 therefore allows for the possibility of using tilt testing as a methodology to analyze the
161 stability of blocks with complex geometry against toppling.

162

163 Accordingly, in this paper we introduce a new physical modelling approach, where a tilt
164 table, classic limit equilibrium computations, and rock physical models using a 3D-
165 printed version of a real boulder are used to estimate the factor of safety against toppling
166 of rock blocks in the field. It is recommended that analytical formulations or numerical
167 models should be used in parallel with this approach to allow for a complete
168 understanding of the relevant phenomena to be developed and to allow for possible
169 extension of the physical modeling results to different scenarios, just as those considering
170 the influence of water pressure or seismic loading.

171

172

173 **2. The mechanics of toppling**

174

175 Despite the fact that some authors consider the occurrence of pure toppling as an
176 uncommon failure mechanism, and that toppling is typically associated with larger
177 failures occurs or as a consequence of other mechanisms like slope undercutting or
178 weakening at the toe (Hencher, 2015), toppling has been extensively reported as the origin
179 of several rock-mass failures experienced in different fields such as open-pit mining
180 (Alejano et al., 2010a; Al Mandalawi et al., 2019; Amini and Ardestani, 2019), civil
181 engineering (Tu et al., 2007; Akbarpour et al., 2012; Cai et al., 2019) and natural rock
182 slopes (Guo et al., 2017).

183

184 The mechanism of toppling had been partially identified in the field by some authors
185 (Müller, 1968; Terzaghi, 1962), though it was not until the late 1960's (Bray, 1969) that
186 it began to be considered as a mode of failure unto itself. A few years later, some authors
187 started the study of toppling in a more rigorous way, through both laboratory models
188 (Ashby, 1971; Barton, 1971) and early applications of numerical methods (Cundall, 1971;
189 St. John, 1972).

190

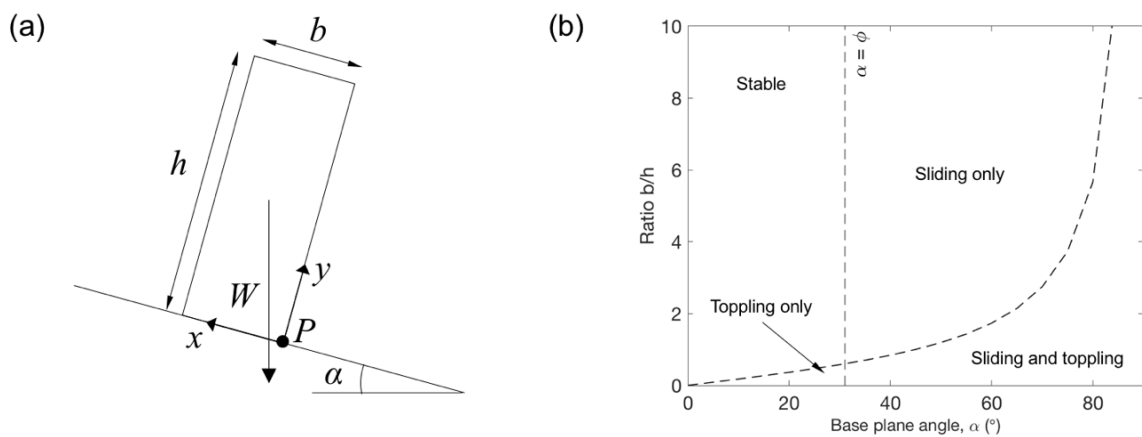
191 Ashby (1971) developed a seminal study on different sliding and toppling modes of
192 failure based on laboratory physical models and rock slopes. This author derived a
193 simplified analytical 2-D toppling model for an isolated straight block of height h and
194 width b , resting on a plane dipping α degrees with a friction angle between contacts, ϕ
195 (Fig. 3).

196 Considering the base plane angle, $\alpha < \phi$ and the x and y components of the weight (W)
 197 of the block, a factor of safety against toppling can be estimated by relating the stabilizing
 198 and overturning moments with respect to the rotation pivot, P (Fig. 3a), as presented in a
 199 general form in Eq. 1.

$$FS = \frac{\sum M_{stabilizing}}{\sum M_{overturning}} \quad (1)$$

200

201



202

203 Fig. 3. (a) 2D sketch of a single block placed on an inclined plane for toppling stability analysis;
 204 (b) conditions for sliding and toppling according to Ashby (1971).

205

206 It is therefore easy to derive that the toppling condition (represented as $FS \leq 1$ in Eq. 1)
 207 solely depends on the slenderness of the block and the plane dip (α) (Ashby, 1971) and is
 208 driven by the geometrical relationship presented in Eq. 2:

209

$$\frac{b}{h} \leq \tan \alpha \quad (2)$$

210

211 According to Fig. 3b and depending on the considered kinematic conditions established by
 212 Eq. 1 and the line $\alpha = \phi$, four potential conditions can be identified: stability, sliding,
 213 toppling and sliding and toppling simultaneously occurring. Even though this model laid
 214 the groundwork for subsequently reproduced studies on toppling (Hoek and Bray, 1974),
 215 some errors were detected in the conditions presented in Fig. 3b, as corrected by other

216 authors (Bray and Goodman, 1981; Sagaseta, 1986). The “corrected” solution for a single
 217 block toppling condition, as proposed by Sagaseta (1986), is given by Eq. 3:

$$\frac{4 \cdot \tan \alpha \cdot \left[1 + \left(\frac{b}{h} \right)^2 \right] - 3 \cdot \left(\tan \alpha - \frac{b}{h} \right)}{4 \cdot \left[1 + \left(\frac{b}{h} \right)^2 \right] + 3 \cdot \frac{b}{h} \cdot \left(\tan \alpha - \frac{b}{h} \right)} \leq \tan \phi \quad (3)$$

218

219 To consider the commonly observed case of rounding on the block edges caused by
 220 weathering, the single 2D straight block model was reevaluated several years later
 221 (Alejano et al., 2015). Starting from Eq. 2, the authors proposed a new equation by
 222 considering a radius of curvature (r_c) representative of the edge rounding in such a way
 223 that toppling condition is given by the geometrical relationship presented in Eq. 4:

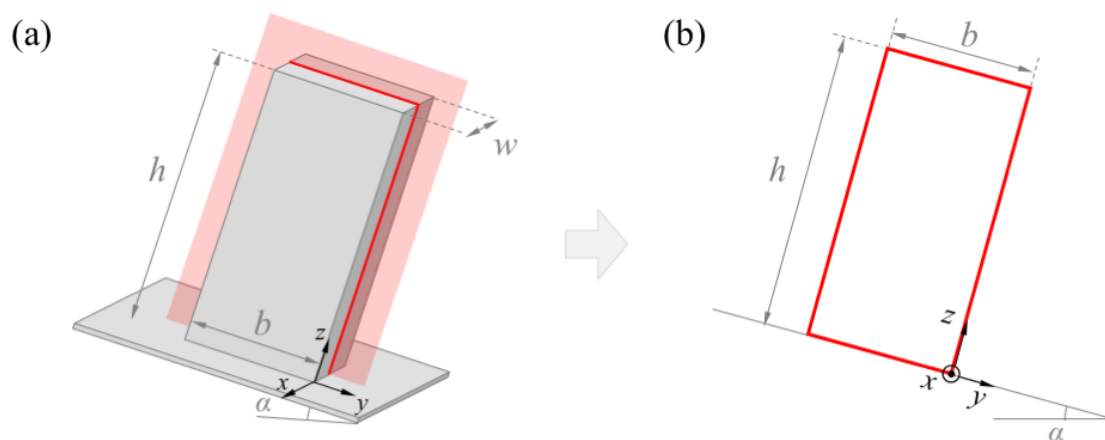
224

$$\frac{b - 2r_c}{h} < \tan \alpha \quad (4)$$

225

226 It must be noted that the approaches presented above should only be applied to 2D
 227 problems or to simplifications of 3D bodies presenting the following features: constant
 228 cross-section, one dimension smaller than the other two (‘thin plate’ assumption), and the
 229 driving force (in this case, that corresponding to the weight of the block only) contained
 230 in the mean plane of the structure, as illustrated in Fig. 4. Additionally, the rotating pivot
 231 is already known, and defined by an identifiable edge.

232



233

234 Fig. 4. (a) 3D body resting on an inclined plane with constant cross-section along the x -direction;
 235 (b) 2D simplification of the problem on the z - y plane.

236

237 Nevertheless, this situation is not commonly found in natural blocks (particularly, in rock
 238 boulders) that usually present irregular shapes and poorly defined contact planes, such
 239 that 2D analysis is unrealistic. In these cases, the third dimension cannot be disregarded
 240 and other approaches are needed for a correct assessment of the toppling mechanism
 241 (Domokos et al., 2012; Yeung and Wong, 2007; Zábranová et al., 2020).

242

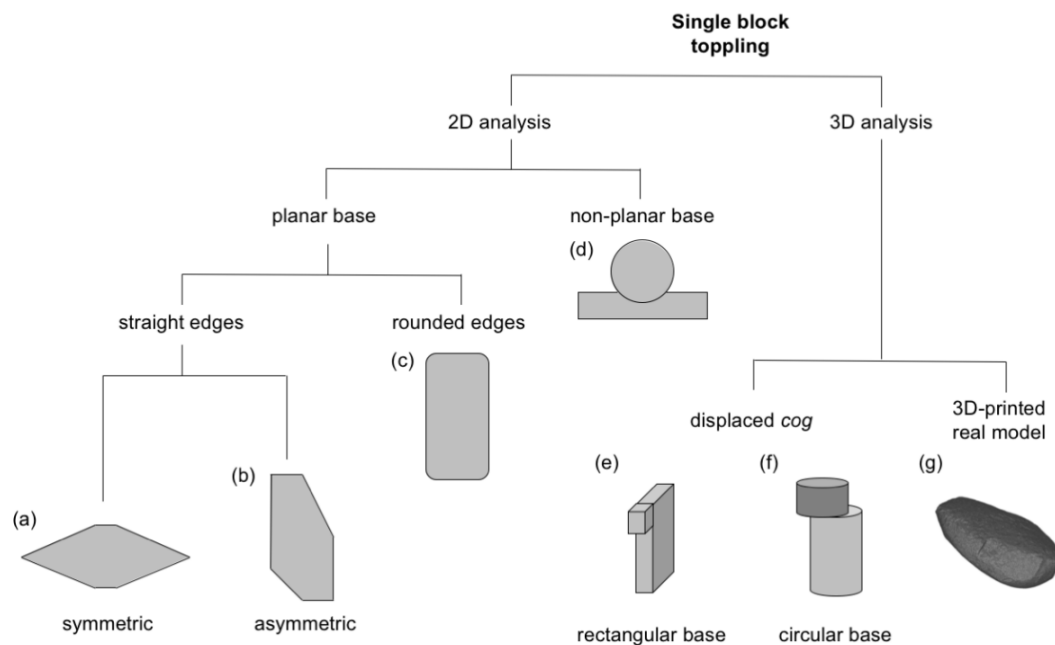
243

244 3. Analytical assessment and laboratory physical modelling

245

246 An experimental program was designed in order to study the critical angle of toppling of
 247 single engineered models subjected to tilt tests under laboratory conditions. The idea was
 248 to test different blocks or assemblies presenting features that determine their kinematic
 249 behavior with respect to toppling: the symmetry of the block section, the edge rounding,
 250 the concavity of the contact base and the position of the center of gravity with respect to
 251 a central cross-section of the block. A sketch showing these features and the models used
 252 is presented in Fig. 5.

253



254

255 Fig. 5. Sketch showing the specimens used for 2D and 3D toppling analyses according to the type
 256 of contact base (planar or non-planar), edge rounding (straight or rounded), symmetry of the
 257 cross-section and position of the cog.

258

259

260

261

262 3.1. Physical models and laboratory testing

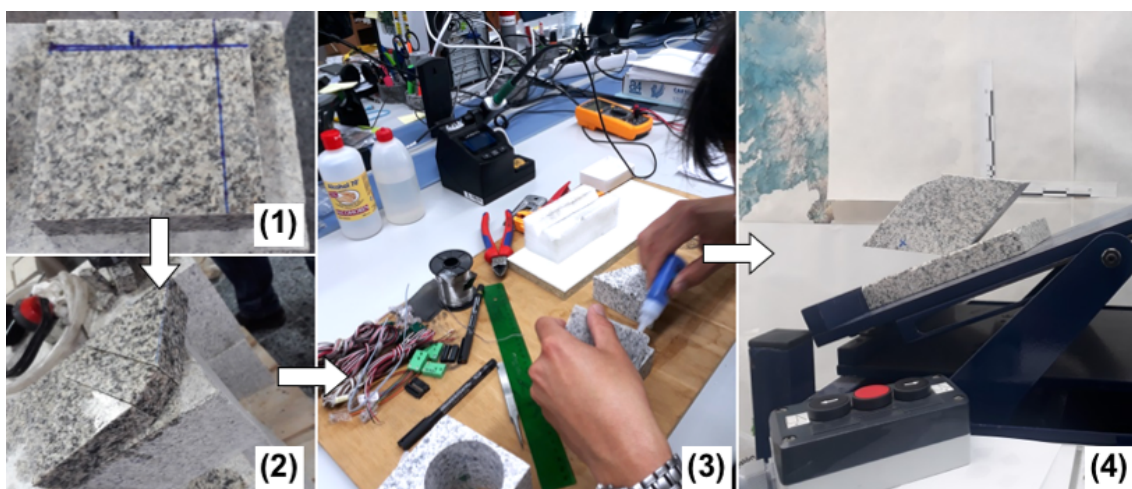
263

264 The models presented in Fig. 5 were selected to perform the laboratory tilt tests. The
265 majority of them (Fig. 5 a–e) consisted of some saw-cut pieces of two igneous rocks
266 (granite and orthogneiss) assembled by gluing them together to form the physical model
267 (Fig. 6). A photo of each model is presented in Fig. 7. In mode case PM-6 (Fig. 5f and
268 Fig. 7e), steel (with a density of 7900 kg/m^3) was used to fabricate the upper cylinder; as
269 can be appreciated, this part of the model was intended to displace the center of gravity
270 of the corresponding assembly. The replica of a real boulder was 3D-printed with PLA
271 (polylactide) plastic based on a 3D point cloud recovered from the original geological
272 structure (Pérez-Rey et al., 2019).

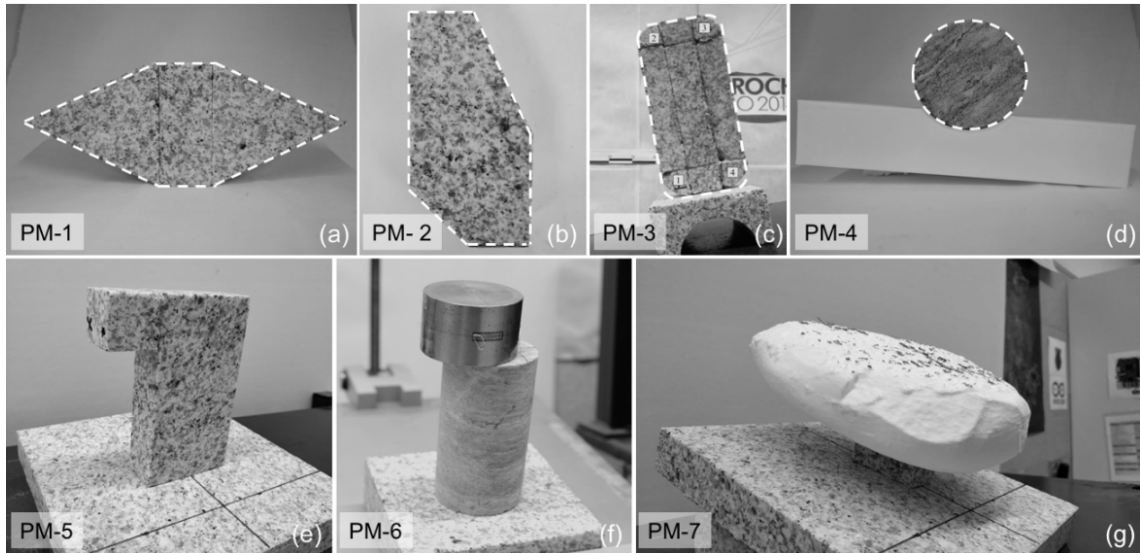
273

274 The density of the plastic is lower than that of the rock. Additionally, the printed version
275 presents an internal plastic pattern with regular hollow zones. Since toppling stability
276 depends in both the stabilizing and overturning moments and they are both proportional
277 to the block weight, density does not influence stability in uniform bodies. The friction in
278 the base is also proportional to the weight in case the contact is considered planar, though
279 it may affect results for rough joints, as the Barton's formula suggest. Since the contacts
280 of the studied elements are typically planar, the weight does not significantly affects the
281 frictional response. When we want to avoid the sliding mechanism, in cases where it can
282 take place at lower tilt angles than toppling, a piece of sand paper can be glued to the base
283 of the element. In conclusion, density, when constant, and friction angle are considered
284 to have a negligible impact on the presented results.

285



286 Fig. 6. Process of fabrication of the rock models (1, 2) dimensioning and parts of a model; (3)
287 gluing of parts to create the complete model and (4) model being tested.



288

289 Fig. 7. Photos of the physical models used in this study (a. symmetric section; b. asymmetric
 290 section; c. rounded edges; d. concave base; e. displaced *cog* (rectangular base); f. displaced *cog*
 291 (*cog* (circular base); g. 3D printed replica of a real boulder). The name assigned to each model is also
 292 provided.
 293

294 Regarding the tilt tests, they were carried out with a custom testing frame designed at the
 295 University of Vigo that consists of a metallic platform able to rotate about a fixed axis
 296 and driven by an asynchronous motor that constant angular lifting velocities (from
 297 0.1°/min to 26°/min) to be maintained. The ‘start’ and ‘stop’ orders can be given to the
 298 machine by a manual control. A constant record of the tilting angle is kept using an
 299 inclinometer (*Leica DISTO D5*) with an accuracy of 0.1° attached to the platform (Fig.
 300 8). The critical angle of toppling for each tested model could be registered by simply
 301 stopping the rotation of the platform when the onset of instability was observed. For that
 302 purpose, the lifting velocity was set at 12°/min for all tests in order to balance precision
 303 in determination of the critical angle with testing time. Each model was tested three times
 304 until toppling occurred, and results were collected for purposes of comparison with
 305 analytical and numerical predictions.
 306



307

308 Fig. 8. Tilting platform used for carrying out tilt tests with the physical models (1. rotating
309 platform; 2. connection box and velocity control; 3. 'start and stop' control; 4. digital
310 inclinometer).

311

312

313 **3.2. Analytical assessment of toppling**

314

315 With the models PM-1 to PM-4 presented in Fig. 5, it is possible to develop simple 2D
316 analytical predictions of the critical angle of toppling. To do that, some cross-sections
317 were divided into simpler shapes (like squares, rectangles and triangles). The driving
318 forces were located at the centroid of each subsection and the stabilizing and overturning
319 moments were calculated by considering the rotation pivot as the lower corner of the
320 model in contact with the tilting table, which was also set as the origin of the coordinate
321 system for calculations. The critical angle of toppling, α_{cr} , can easily be estimated by
322 imposing a factor of safety, $FS = 1$ in Eq. 1.

323

324 Other scenarios like those presented for models PM-5 to PM-7 require a more detailed
325 analysis on the positioning of the *cog* as well as the rotation pivot.

326

327 It is relevant to note that the relative stability of any of the blocks under scrutiny for any
328 position can be both computed in terms of factor of safety and in terms of critical stability
329 angle. Obviously, these two approaches are related to one another, so they can be

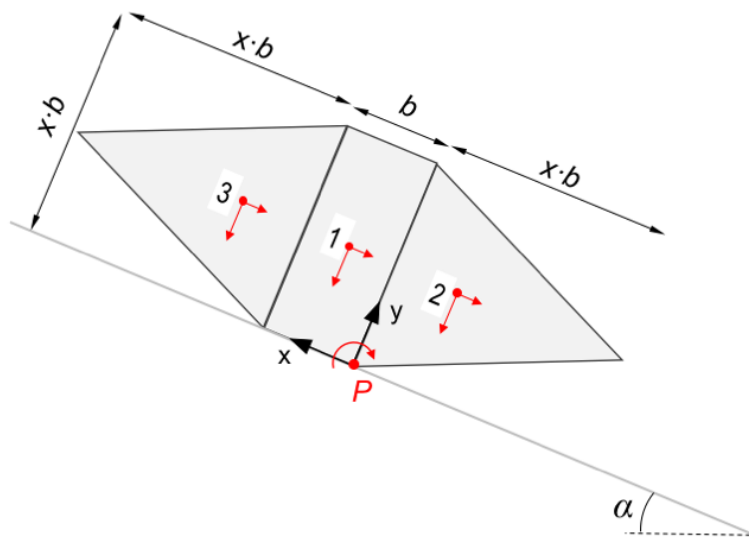
330 computed for different scenarios. For test interpretation purposes where no additional
331 forces exist (water, earthquakes), the authors have selected the critical angle approach,
332 thinking it could be more illustrative for output comparison purposes. However, a factor
333 of safety approach could also be developed.

334

335 3.2.1. Symmetric model with straight edges and planar base

336 The simplest model studied (PM-1), in terms of geometry, consists of a symmetric block
337 with a constant cross-section that can be divided in two triangles and a rectangle. The
338 model is defined in terms of the central block breadth, b . A sketch of this model is
339 presented in Fig. 9, where the relevant force components and the rotation pivot are also
340 indicated for a given inclination of the base (α).

341



342

343 Fig. 9. Sketch of the symmetric model with constant cross-section, where the centroid of each
344 sub-element is shown as well as the rotation pivot and origin of x-y coordinates.

345

346 For demonstrative purposes, we present the derivation of the critical angle formulation,
347 based on the computation of the factor of safety for the rock block illustrated in Fig. 9. If
348 one considers the rotation pivot (P) as the origin, the expression for estimating the factor
349 of safety (FS) dividing the block in its three basic elements is as follows:

350

$$FS = \frac{\sum M_{stabilizing}}{\sum M_{overturning}} = \frac{W_1 \cos \alpha \frac{b}{2} - W_2 \cos \alpha \frac{xb}{3} + W_3 \cos \alpha \frac{(3+x)b}{3}}{W_1 \sin \alpha \frac{xb}{2} + W_2 \sin \alpha \frac{xb}{2} + W_3 \sin \alpha \frac{xb}{2}} \quad (5)$$

351

352 In this case, all forces acting parallel to the x-axis correspond strictly to destabilizing
 353 moments, while those acting parallel to the y-axis contribute to both the stabilization (sub-
 354 sections 1 and 3) and destabilization (sub-section 2) of the block shown in Fig. 9.

355

356 Accounting for the fact that $W_2 = W_3$, and that $W = W_1 + W_2 + W_3$, Eq. 5 can be simplified to:

$$FS = \frac{\sum M_{stabilizing}}{\sum M_{overturning}} = \frac{W \cos \alpha \frac{b}{2}}{W \sin \alpha \frac{xb}{2}} = \frac{1}{x \cdot \tan \alpha} \quad (6)$$

357

358 Equating the FS to 1, the point at which instability initiates, the critical angle for toppling
 359 in Fig. 9 can be derived as Eq. 7:

$$\alpha_{cr} = \text{atan} \left(\frac{1}{x} \right) \quad (7)$$

360

361 This implies that, as mentioned above, the size of the sample does not influence results,
 362 so the physical model represents the behavior of any smaller or larger homothetic block.
 363 For the analyzed case and taking into account that $x = 2.7$, the critical angle in this case
 364 will be $\alpha_{cr} = 20.32^\circ$ as derived from Eq. 7. The influence of the acuteness of the lateral
 365 triangle parts could be easily computed by testing different values of x in Eq. 7. This kind
 366 of analytical solution is therefore quite favorable for evaluation of the influence of some
 367 geometrical parameters of the blocks under scrutiny.

368

369 Alternatively, the FS can be computed for the case the cog is known from the beginning
 370 accounting for the stabilizing and overturning moments of the weight as presented in Eq.
 371 1. For blocks with more complex geometry, it therefore tends to be most convenient to
 372 compute the position of the cog to simplify subsequent computations.

373

374

375

376

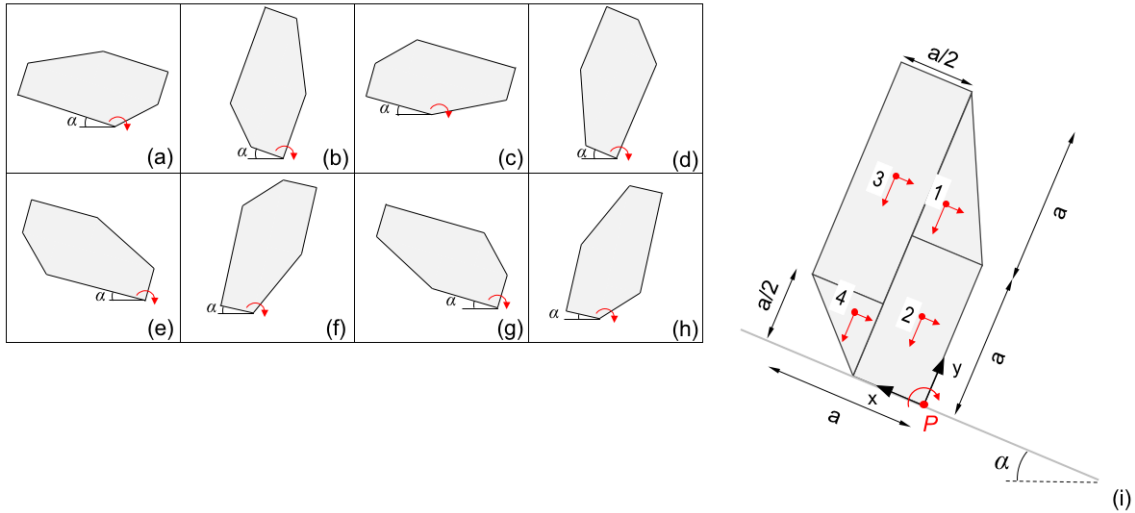
377

378 **3.2.2. Asymmetric model with straight edges and planar base**

379

380 This model (named as PM-2) presents an asymmetric cross-section that can be divided
 381 into simpler shapes (i.e. two rectangles and two triangles) to simplify calculations, as
 382 presented in Fig. 10.

383



384

385 Fig. 10. Sketch of an asymmetric model with constant cross section and straight edges. All the
 386 possible testing positions for toppling analyses are presented (a-h). The sub-sections (1-4)
 387 indicated in the enlarged sketch (i) on the left are kept for all models.

388

389

390 For the model sketched in Fig. 10i (equivalent to that in Fig. 10b), the angle of critical
 391 toppling can be estimated with Eq. 8, in which the cross section was divided in four
 392 simpler subsections.

$$\alpha_{cr} = \arctan \left(\frac{W_1 \frac{a}{3} + W_2 \frac{a}{4} + W_3 \frac{3a}{4} + W_4 \frac{2a}{3}}{W_1 \frac{4a}{3} + W_2 \frac{a}{2} + W_3 \frac{5a}{4} + W_4 \frac{a}{3}} \right) \quad (8)$$

393

394 By modifying the position of the PM-2 block as presented in Fig. 10i, it is possible to get
 395 different scenarios that will change the cross section to be analyzed, as shown in Fig. 10a-
 396 h and, consequently, the critical angle of toppling. This procedure is valuable when
 397 performing the experimental part of the present work, since it will allow testing a single
 398 model in eight different positions.

399

400 To illustrate how the stability against toppling and, ultimately, the critical angle of this
 401 relatively complex geometry block could be computed, the complete set of equations (Eq.

402 9-16) for calculating the analytical critical angle of toppling for each position of the block
 403 is presented in Table 1. Again, in this case, as in any other, the stability against toppling
 404 will be completely independent of the block size.

405

406 Table 1. Equations for estimating the FS against toppling and angles of critical toppling for all
 407 the positions presented in Fig. 10 (a-h).

Block position	Equation
(a)	$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{5a}{6} + W_3 \cdot \frac{3a}{4} - W_4 \cdot \frac{a}{6}}{W_1 \cdot \frac{2a}{3} + W_2 \cdot \frac{3a}{4} + W_3 \cdot \frac{a}{4} + W_4 \cdot \frac{a}{3}} \right) \quad (9)$
(b)	$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{a}{3} + W_2 \cdot \frac{a}{4} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{2a}{3}}{W_1 \cdot \frac{4a}{3} + W_2 \cdot \frac{a}{2} + W_3 \cdot \frac{5a}{4} + W_4 \cdot \frac{a}{3}} \right) \quad (10)$
(c)	$\alpha_{cr} = \arctan \left(\frac{-W_1 \cdot \frac{a}{3} + W_2 \cdot \frac{a}{2} - W_3 \cdot \frac{a}{4} + W_4 \cdot \frac{2a}{3}}{W_1 \cdot \frac{a}{3} + W_2 \cdot \frac{a}{4} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{2a}{3}} \right) \quad (11)$
(d)	$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{2a}{3} + W_2 \cdot \frac{3a}{4} + W_3 \cdot \frac{a}{4} + W_4 \cdot \frac{a}{3}}{W_1 \cdot \frac{2a}{3} + W_2 \cdot \frac{3a}{2} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{5a}{3}} \right) \quad (12)$
(e)	$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{2a}{3} + W_2 \cdot \frac{3a}{2} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{5a}{3}}{W_1 \cdot \frac{2a}{3} + W_2 \cdot \frac{3a}{4} + W_3 \cdot \frac{a}{4} + W_4 \cdot \frac{a}{3}} \right) \quad (13)$
(f)	$\alpha_{cr} = \arctan \left(\frac{-W_1 \cdot \frac{a}{6} - W_2 \cdot \frac{a}{4} + W_3 \cdot \frac{a}{4} + W_4 \cdot \frac{a}{6}}{W_1 \cdot \frac{2a}{3} + W_2 \cdot \frac{3a}{2} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{5a}{3}} \right) \quad (14)$
(g)	$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{4a}{3} + W_2 \cdot \frac{a}{2} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{a}{3}}{W_1 \cdot \frac{a}{3} + W_2 \cdot \frac{a}{4} + W_3 \cdot \frac{3a}{4} + W_4 \cdot \frac{2a}{3}} \right) \quad (15)$
(h)	$\alpha_{cr} = \arctan \left(\frac{W_1 \cdot \frac{a}{6} + W_2 \cdot \frac{a}{4} - W_3 \cdot \frac{a}{4} - W_4 \cdot \frac{a}{6}}{W_1 \cdot \frac{4a}{3} + W_2 \cdot \frac{a}{2} + W_3 \cdot \frac{5a}{4} + W_4 \cdot \frac{a}{3}} \right) \quad (16)$

408

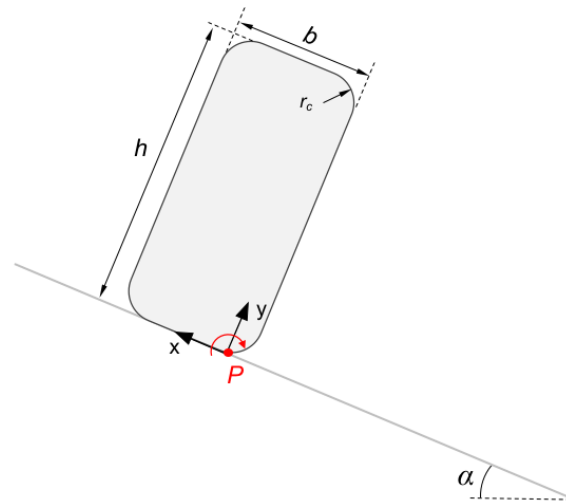
409

410 3.2.3. Symmetric model with rounded edges and planar base

411 Another model presenting a symmetric cross-section is that corresponding to a
 412 rectangular prism with rounded corners (PM-3), as originally analyzed by Alejano et al.
 413 (2015). The cross-section of the model is shown in Fig. 11 and, due to the simplicity of
 414 this section, only the *cog* of the entire model was considered.

415 If the moment equilibrium calculation is performed for the model presented in Fig. 11,
 416 the angle of critical toppling can be estimated (Eq. 17) as rearranged from Eq. 4. Note
 417 that the addition of a non-zero radius of curvature (r) reduces the critical angle of toppling,
 418 since it diminishes the actual contact width by $2r_c$.

$$\alpha_{cr} = \arctan\left(\frac{b - 2r_c}{h}\right) \quad (17)$$



421
 422 Fig. 11. Sketch of the symmetric model with rounded edges (for the studied model: $h/b = 2.125$
 423 and $r_c/h = 0.147$).

424
 425
 426 **3.2.4. Model with non-planar (concave) base**

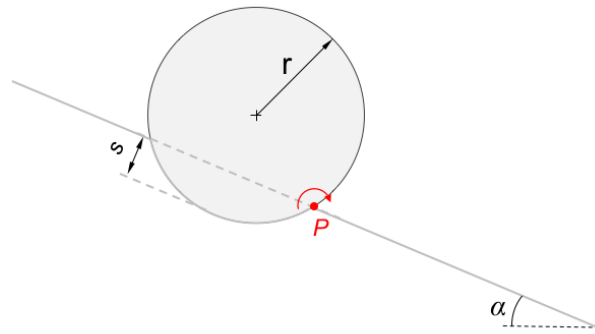
427 The present model is intended to illustrate the effect of a non-planar (concave) base, as a
 428 potentially stabilizing factor. The idea was to study a circular cross-section in which part
 429 of it is embedded in the inclined plane, resting on a concave base that coincides with the
 430 curvature of the cross section (see Fig. 7b and Fig. 12).

431
 432 It can be demonstrated that the angle of critical toppling, α_{cr} , of the model presented in
 433 Fig. 12 solely depends on the depth of the slot (s) relative to the radius of the circular
 434 cross-section (r), n , where $n = s/r$ for and $n \in (0, 1)$. The angle of critical toppling can
 435 therefore be calculated from Eq. 18.

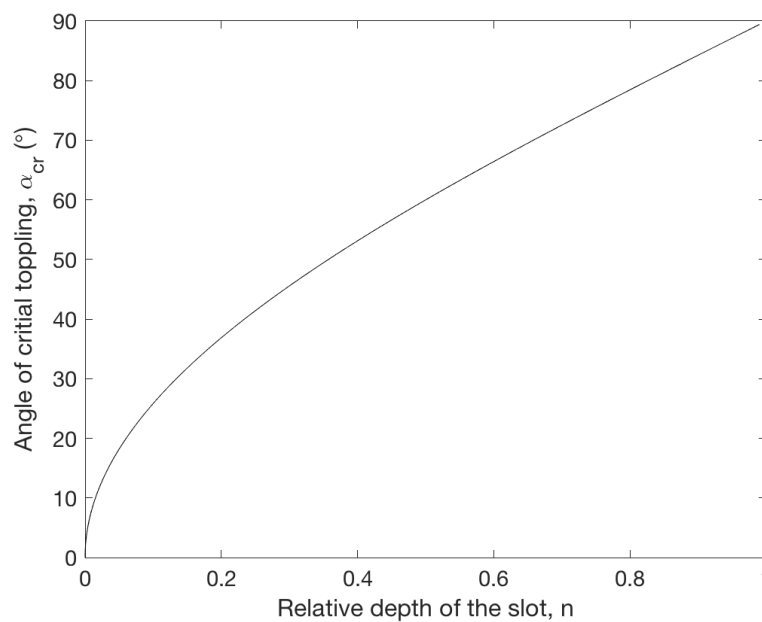
$$\alpha_{cr} = \text{atan}\left\{\frac{\sin[\text{acos}(1 - n)]}{1 - n}\right\} \quad (18)$$

436

437 With the aim of illustrating the effect of a concave contact on the angle of critical toppling
 438 of such a circular section of radius r , the critical angle is plotted against the depth of the
 439 slot expressed as a proportion of the radius (n) in Fig. 13.
 440



441
 442 Fig. 12. Sketch of the model with non-planar (concave) base.
 443



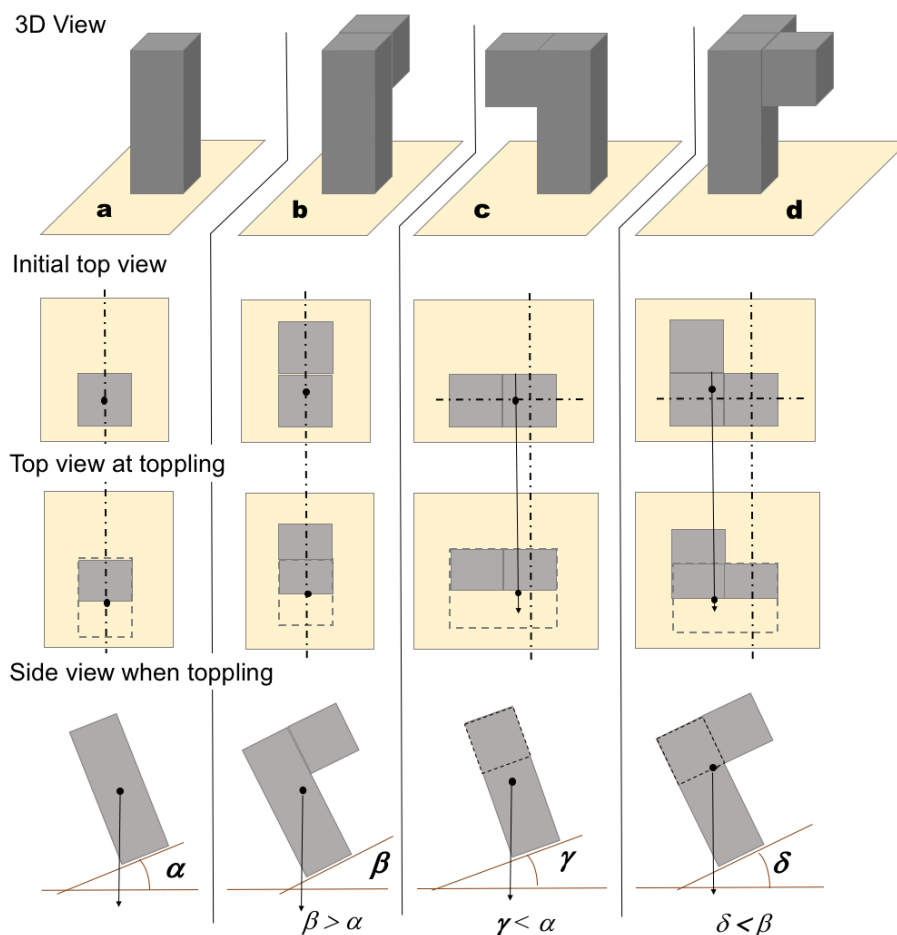
444
 445 Fig. 13. Dependence of the angle of critical toppling (y-axis) with the relative depth of the slot, n
 446 (x-axis), for the model depicted in Fig. 12.
 447

448 As is shown in Fig. 13, the effect of a concave contact clearly influences the angle of
 449 critical toppling of a block with a circular section. This effect is most significant for low
 450 values of n , where the derivative $d\alpha_{cr}/dn$ is maximized; while almost no blocks are
 451 expected to match the geometry shown in Fig. 12 perfectly, in practice, this means that
 452 even relative small degree of concavity in the basal plane for a block of more general
 453 shape has the potential to increase the angle of critical toppling by a few degrees.

454 **3.2.5. Model with the *cog* not contained in a symmetry plane: squared/rectangular**
 455 **base**

456 The position of the center of gravity (*cog*) or centroid clearly influences the toppling
 457 behavior of an element as in the case of the blocks already analyzed in this study. If a
 458 block is positioned onto a flat surface and the vertical projection of its *cog* falls inside the
 459 contact area, the block will remain stable against toppling. Consider the situation
 460 illustrated in Fig. 14a for a squared-based, symmetric and homogeneous block. In this
 461 case, if the *cog* is projected onto the planar base, that projection will fall on the center of
 462 the square section. If the block is progressively tilted, it will topple once the projection of
 463 the *cog* falls out the base.

464



465

466 Fig. 14. Different 3D elements (a, b, c and d) to be subjected to a tilt test to illustrate the role of
 467 geometry on toppling. On the upper row, 3D view of the elements resting on a horizontal base to
 468 be tilted. On the second row, initial top view with the projection including the *cog*. On the third
 469 row, top view of the surface after tilting and in the moment of toppling and, on the last row, side
 470 view of platform and element when toppling.

471

472

473 Fig. 14b shows a similar prism with a cube stuck to its upper back face, a more complex
474 geometry. The *cog* of this model, when placed on a flat surface, will not project on the
475 center of the base but will instead project somewhere behind the center of the base due to
476 the added mass. Because of this, when tilting the plane where this element stands, it will
477 topple at a higher angle than the previous case; in other words, the angle of critical
478 toppling in this case (α) will be higher than the one observed for the element shown in
479 Fig.14a (β).

480

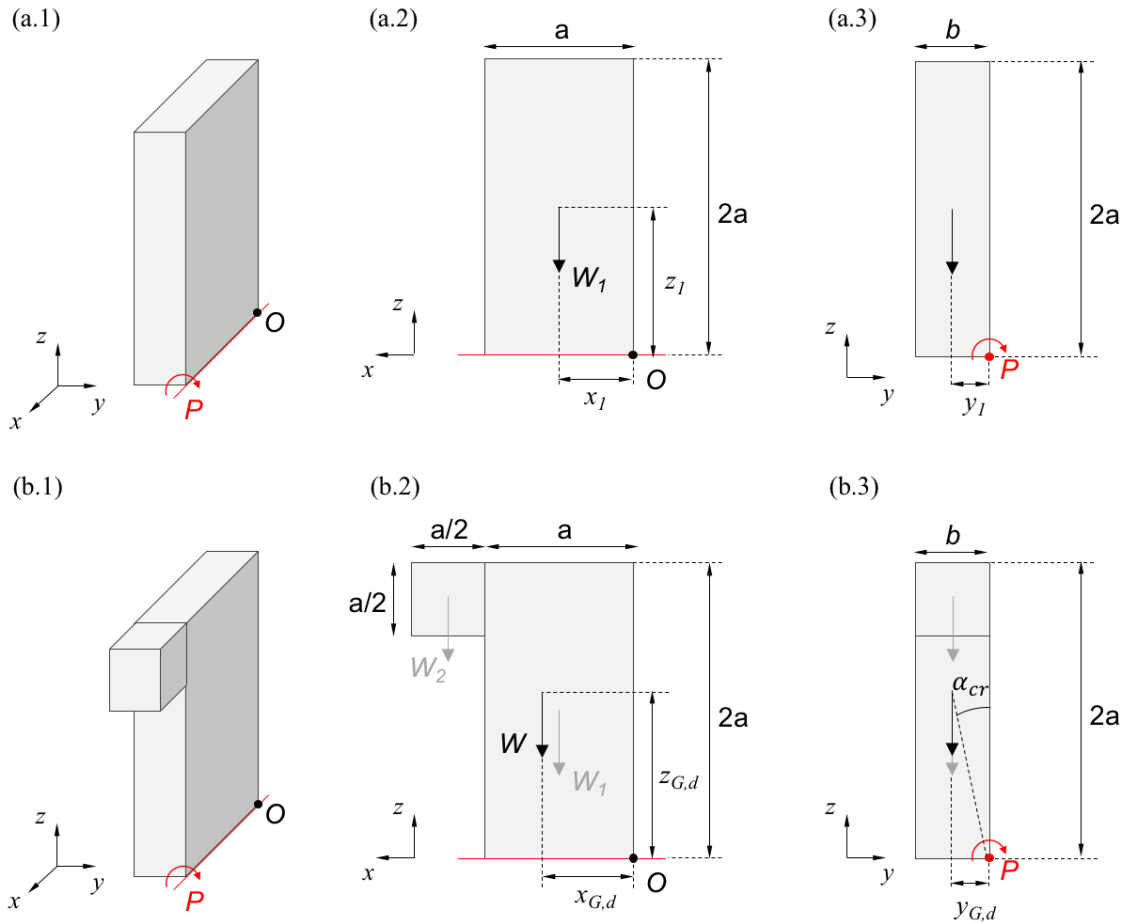
481 The third element (Fig. 14c) is similar to element 'b', but the added cube is now stacked
482 on the upper part of a lateral face. Its *cog* will be at the same height as for element b (since
483 it is the same element in the same position), but its projection onto a horizontal plane will
484 be moved to the left in relation to element 'a' (Fig. 14c, second row). When tilting the
485 platform where element 'c' rests, it will topple at a lower angle than α , because its *cog* is
486 located higher than in case 'a', meaning its projection will fall outside its base at a lower
487 angle γ , which will be also less steep than β .

488

489 Element 'd' is a rectangular prism with a square cross-section where two cubes are
490 attached on the upper part of its lateral backward and leftward faces. In this case, the *cog*
491 of the element will be even more displaced upwards than in the case of elements 'b' and
492 'c' and the *cog* projection on its base will be slightly moved backwards and a little bit to
493 the left in relation to the case of element 'a' in Fig 14. This will clearly be less stable than
494 'b' (since the side-stuck cubes displace the *cog* upwards) but more stable than 'c' (since
495 the back-stuck cube will increase its stability by moving the projection of its *cog*
496 backwards). According to the previous analyses, a hierarchy on the angle of critical
497 toppling can be established for the studied models shown in Fig. 14 as: $\beta > \delta > \alpha > \gamma$.

498

499 With the goal of assessing the effect of a displaced *cog* on the toppling behavior in a more
500 detailed way and following the ideas described in the previous paragraph, the model
501 presented in Fig. 15 has been considered. It consists of a rectangular-based block with a
502 small prismatic slab stuck on the upper part of a lateral face (see Fig. 15b.1).



503
 504 Fig. 15. (a) 3D view of the model with the axis of rotation and origin of coordinates for positioning
 505 the *cog* marked; (b) levelled front view of the assembly and (c) levelled lateral view of the
 506 assembly.
 507

508 As previously mentioned, the effect of adding such a piece to a rectangular-based block
 509 as shown in Fig. 15 implies a displacement of the *cog*, which will move towards the added
 510 mass. This positioning of the *cog* can be easily determined by resorting to Eqs. 19, 20 and
 511 21 for each coordinate, respectively, when considering $m_i = W_i / g$.
 512

$$x_{G,d} = \frac{\sum(m_i \cdot x_i)}{\sum m_i} \quad (19)$$

$$y_{G,d} = \frac{\sum(m_i \cdot y_i)}{\sum m_i} \quad (20)$$

$$z_{G,d} = \frac{\sum(m_i \cdot z_i)}{\sum m_i} \quad (21)$$

513

514

515 Once the 3D coordinates of the displaced *cog* are set ($x_{G,d}$, $y_{G,d}$, $z_{G,d}$), the angle of critical
516 toppling for the model sketched in Fig. 13b.1 can be estimated using Eq. 22:

517

$$\alpha_{cr} = \text{atan}\left(\frac{y_{G,d}}{z_{G,d}}\right) \quad (22)$$

518

519 It has to be highlighted that all the models with a rectangular or squared-base such as
520 those shown in the previous sections and particularly the last shown in Fig. 15 will always
521 have a pre-defined axis of rotation, which coincides with an edge of the block in contact
522 with the rotating base. This is not the case for circular or irregular-based specimens, as
523 considered in Section 3.2.6.

524

525

526 **3.2.6. Model with the *cog* not contained in a symmetry plane: circular base**

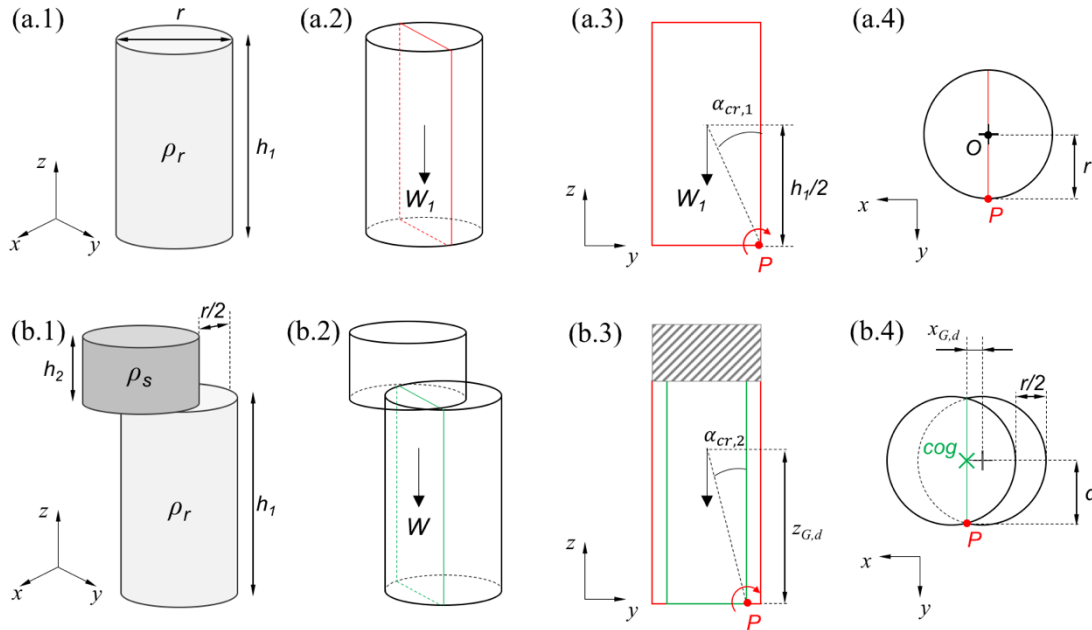
527 Real rock blocks and boulders as found in the field rarely have a *cogs* that project onto
528 the centers of their bases when resting on horizontal surfaces, nor do they have well-
529 defined axes of rotation against toppling mechanism due to their typically irregular
530 shapes. As already noted, for a precise assessment of their stability against toppling, it
531 will be necessary to correctly position the *cog* as well as the pivot or axis around which
532 the toppling mechanism will take place.

533

534 The study of these features has been carried out using a laboratory physical model (as
535 shown in Fig. 7f) consisting of two cylinders composed of different materials (rock and
536 steel with densities of $\rho_r = 2700 \text{ kg/m}^3$ and $\rho_s = 7800 \text{ kg/m}^3$, respectively). Both cylinders
537 have the same radius ($r = 27 \text{ mm}$) but different heights, with the rock cylinder measuring
538 100 mm in height and the steel cylinder measuring 35 mm in height. The assembly is
539 sketched in Fig. 16b.1.

540

541 This test consisted of placing the two specimens as shown in Fig. 16b.1, with the top piece
542 moved outwards a distance of $r/2$. This position of the top steel specimen moves the center
543 of gravity out of the plane of symmetry of the lower specimen, as can be seen in the front
544 view shown in Fig. 7b. Then, the specimen was progressively tilted (being the platform
545 rotation around the x-axis) until toppling of the entire set occurred, when the tilting angle
546 (angle of critical toppling) was achieved.



547

548 Fig. 16. Different views of the model containing the *cog* out of a symmetry axis (model PM-6).

549

550 By considering the origin of the coordinate system at the center of the base of the lower
 551 cylinder (Fig. 16a.4) the angle of critical toppling for the model presented in Fig. 16b.1
 552 when tilted around the x-axis can be calculated using Eq. 23:

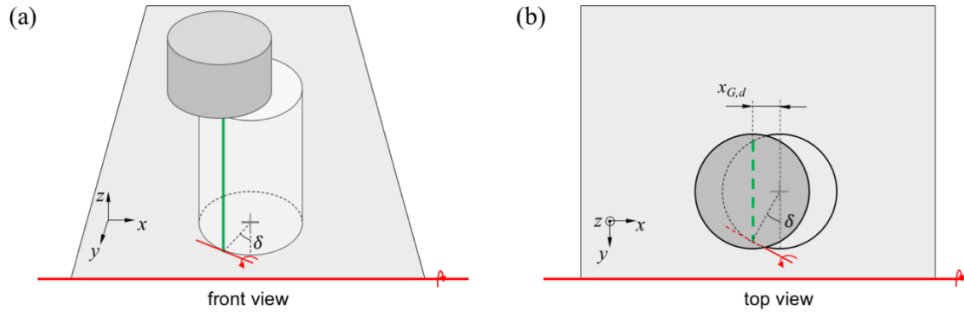
$$\alpha_{cr} = \text{atan} \left(\frac{d}{z_{G,d}} \right) \quad (23)$$

553

554 Unlike the other examples considered in this study, this model will behave differently
 555 once the limit equilibrium for toppling has been reached. Specifically, although the
 556 critical angle of toppling can be calculated in a similar way to that of the other models
 557 (Eq. 12), the reduction of the cross-section (plane) containing the *cog* of the assembly
 558 (shown in green in Fig. 16b.3 and Fig. 16b.4) has to be considered, as it influences the
 559 axis of rotation.

560

561 As shown in Fig. 17, the axis (pivot) of rotation will not coincide with that of the rotating
 562 platform and will move laterally due to the displacement of the *cog* and the circular base
 563 in such a way that the new axis of rotation corresponds to the tangent at the intersection
 564 point between the plane containing the new *cog* (shown in green color in Fig. 17) and the
 565 base of the model. The displacement angle, δ , as shown in Figure 17 can be calculated
 566 using Eq. 24.



567
568
569
570

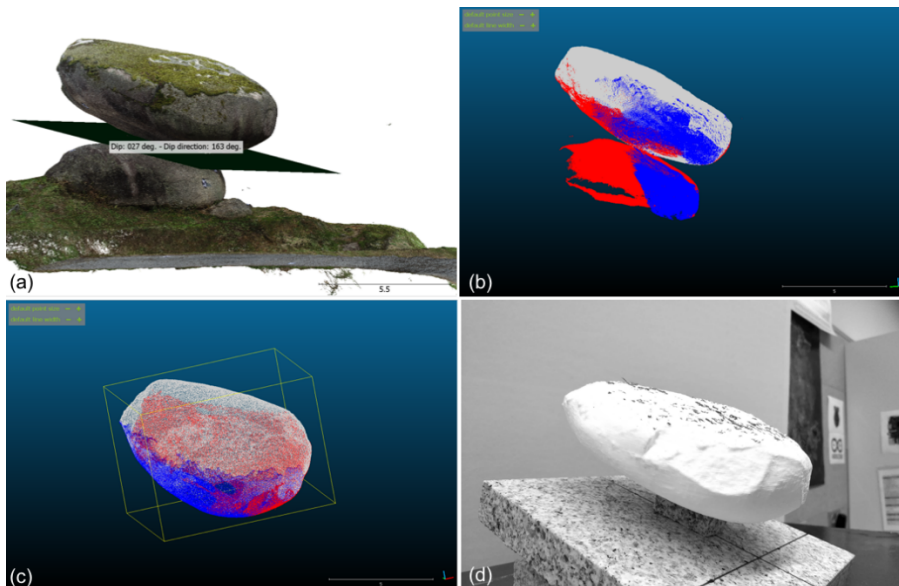
Fig. 17. Different views of the model PM-6 at the equilibrium limit state, where the axis of rotation does not coincide with that of the rotating platform.

$$\delta = \text{asin}\left(\frac{x_{G,d}}{r}\right) \quad (24)$$

571

572 3.2.7. 3D-printed model representative of a real boulder

573 The last model (PM-7) considered in this study corresponds to a plastic (PLA) replica of
574 a real granitic boulder located in the NW of Spain, as studied by Pérez-Rey et al. (2019).
575 The replica, made at a scale of approximately 1:50, was created from an 3D point cloud
576 of the real boulder collected in the field, which was afterwards processed with the
577 software *CloudCompare* (Girardeau-Montaut, 2018) and *Meshlab* (Cignoni et al., 2008)
578 in order to develop the 3D printing stage with a *BCN Sigma 3D* printer (see Fig. 18).

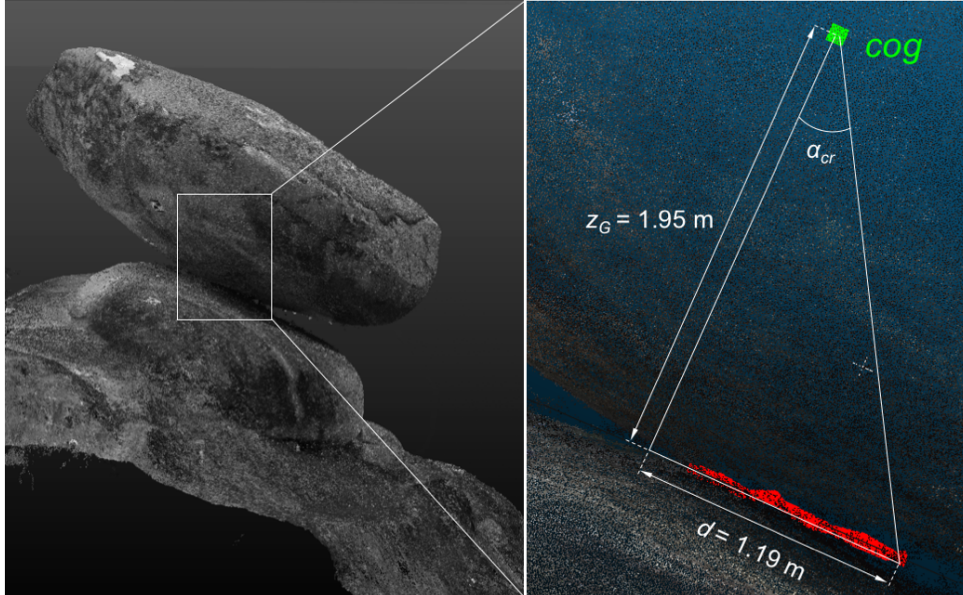


579
580
581
582

Fig. 18 (a, b) Two views of the 3D point cloud of the studied boulder; (c) isolation of the boulder from the rest of the structure and (d) 3D-printed replica on the testing platform.

583 By taking advantage of such a precise 3D point cloud and with the assistance of
584 *CloudCompare* software, it is possible to approximate, in a reasonable manner, the
585 contact area between the boulder and the base and to position the *cog*.

586 Using the relationship presented by Eq. 23 and the geometrical parameters presented in
587 Fig. 19, it is possible to estimate the approximate angle of critical toppling of the boulder
588 to be $\alpha_{cr} = 31.4^\circ$.
589



590 Fig. 19. Detailed view of the position of the *cog* projected onto the contact plane, showing the
591 distance d necessary for estimating the critical angle of toppling.
592
593
594

595 4. Results

596 4.1. Comparison of analytical and experimental results

597 After carrying out all calculations of the angle of critical toppling of each of the models
598 considered for this study, as presented in Section 3, all analytical results are shown in
599 Table 1. Together with these results, the experimental angles of toppling obtained for
600 each series of three tests carried out with the seven models are also provided with an
601 averaged result. As it can be observed in this table, the discrepancy of the analytical and
602 average laboratory results is always less than 1.3° , and the median error is 0.66° .
603

604 It must be noted that some models did not achieve toppling failure in the laboratory tests
605 (in particular, the PM-2 model in some positions). This occurs when the theoretical
606 toppling angle is greater than the basic friction angle of the base contact surface, so the
607 block slides before reaching its toppling angle. These results are indicated in Table 2 with
608 an 's'. It has also been observed that PM-2 model in position (h) was not self-stable in a
609 horizontal position ($\alpha_{cr} < 0$).

610 Table 2. Analytical and experimental results for the angle of critical toppling as obtained
 611 for the seven studied models.

Model	Position	Analytical	Experimental (s = slide)			
		α_{cr} (°)	α_1	α_2	α_3	α_{mean}
PM-1	—	20.32	20.2	20.3	20.5	20.3
PM-2	(a)	44.22	30.4 (s)	27.2 (s)	30.1 (s)	29.2 (s)
	(b)	28.66	29.1	29.0	29.2	29.1
	(c)	4.18	3.2	3.2	3.1	3.2
	(d)	24.55	25.4	25.3	25.6	25.4
	(e)	65.45	33.5 (s)	30.9 (s)	27.0 (s)	30.5 (s)
	(f)	1.41	2.6	2.7	2.7	2.7
	(g)	61.34	29.0 (s)	25.6 (s)	26.5 (s)	27.0 (s)
	(h)	< 0	—	—	—	—
PM-3	—	10.01	11.0	11.0	9.67	10.6
PM-4	(a) (n =1/6)	33.56	33.3	32.5	32.8	32.9
	(b) (n=1/3)	48.19	47.3	48.3	48.4	48.0
PM-5	—	11.88	11.6	12.3	12.4	12.1
PM-6	—	17.29	16.5	15.9	16.2	16.2
PM-7	—	31.39	30.8	30.4	30.7	30.6

612

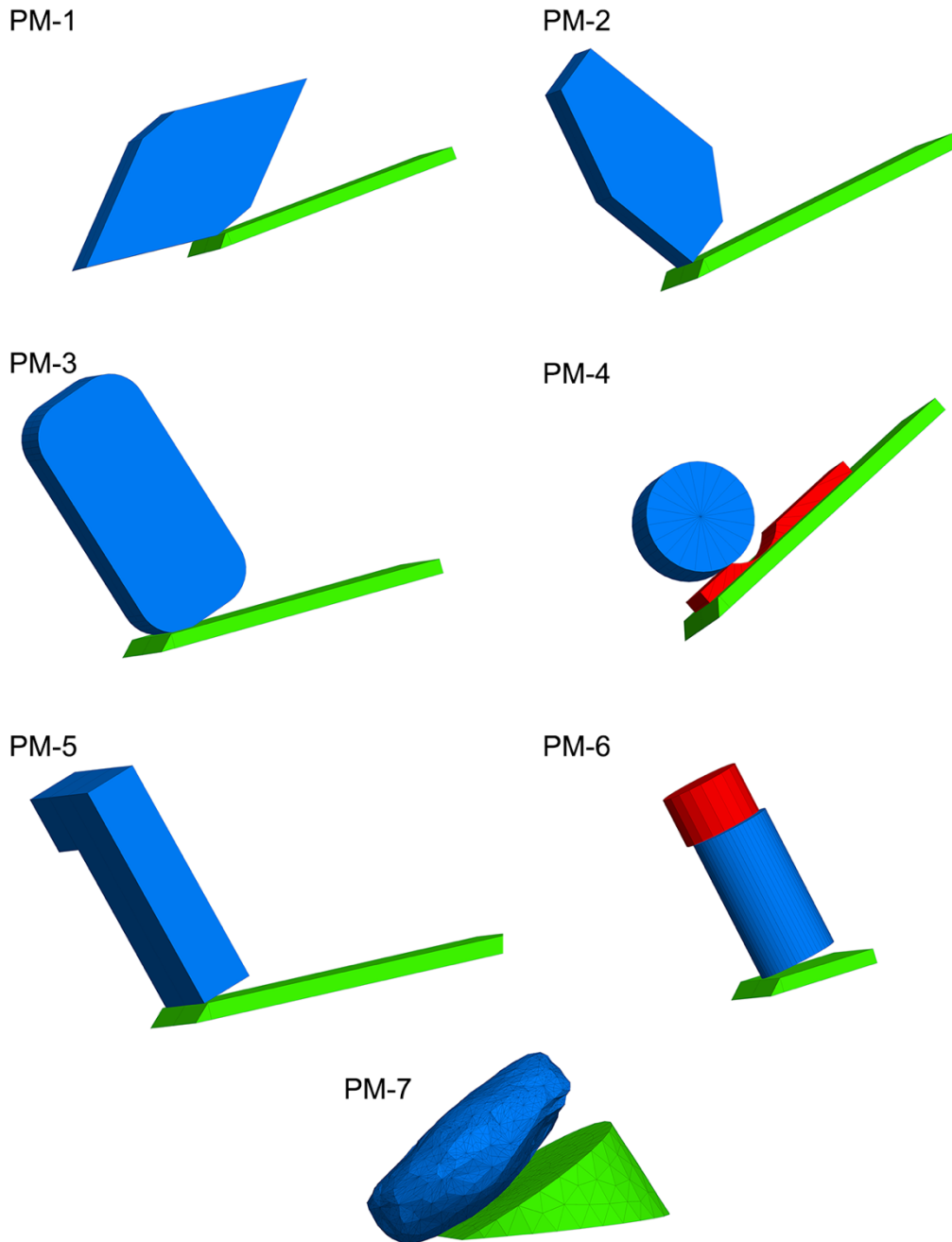
613

614 4.2. 3D discrete numerical modelling

615 Another way to validate the results of the physical models is by comparing these results
 616 with those obtained by numerical analysis. For this part of the study, we have utilized the
 617 Distinct Element Method (DEM), which applies an explicit finite difference method for
 618 modelling large displacements and rotations of block systems (Cundall, 1971). This
 619 method has been used in numerous studies of toppling (Brideau and Stead, 2010; Lanaro
 620 et al., 1997; Pritchard and Savigny, 1990). In this case, we use the DEM as implemented
 621 in the software 3DEC v5.20 (Itasca Consulting Group, 2019).

622 Discrete Element Methods can deal with geological structures of any size and shape, and
 623 with a great variety of constitutive models for both the intact rock and the discontinuities.
 624 They also allow for simulation of complex hydrogeological environments or time-
 625 dependant phenomena like rock-dynamics or creep. In this study, this approach was used
 626 because it does not require the prior definition of a displacement direction (as required in
 627 the analytical calculation), meaning the results of the other methods can be confirmed in

628 cases where there is any doubt about the displacement direction, such as for models with
629 a displaced *cog* (PM-5 and PM-6), or the model with an irregular and complex shape
630 (PM-7). The same tilt tests performed with the physical models were simulated in 3DEC
631 (Fig. 20).



632

633

634

635

Fig. 20. Results for the numerical simulation of the tilt tests performed.

636 In these models, both the tilt table and the specimens were modelled as rigid blocks. The
 637 contact stiffnesses were set to k_n 650 GPa/s and k_s 150 GPa/s. The tilting rate was set
 638 slow enough to ensure that no inertial effect was produced so the test could be considered
 639 static. The results of these models are presented in Table 3, where the maximum
 640 difference observed between results obtained using various methods is indicated.

641

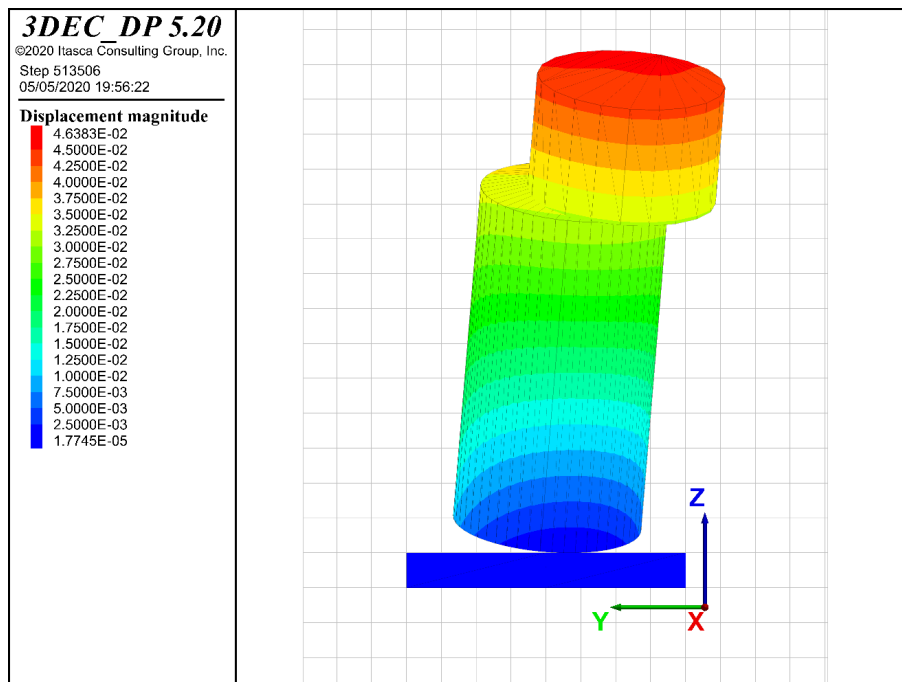
642 Table 3. Angles of critical toppling calculated by different methods and absolute maximum
 643 difference between results. Involved methods indicated in brackets.

Model	Angle of critical toppling (°)			Absolute max. difference (°)
	Experimental (E)	Analytical (A)	DEM (D)	
PM-1	20.3	20.32	20.1	0.22 (E-A)
PM-2	29.1	28.66	28.6	0.5 (E-N)
PM-3	10.6	10.01	10.0	0.6 (E-N)
PM-4	48.0	48.19	48.1	0.19 (E-A)
PM-5	12.1	11.88	11.9	0.22 (E-A)
PM-6	16.2	17.29	17.3	1.1 (E-N)
PM-7	30.6	31.39	31.3	0.79 (E-A)

644

645 The results obtained using the DEM models agree with those obtained by both the
 646 physical models and the analytical method, even in the cases where the centre of gravity
 647 is not located in the plane of symmetry, and the toppling involves complex movement not
 648 parallel to the tilting direction, as in the case of PM-6 (Fig. 21).

649

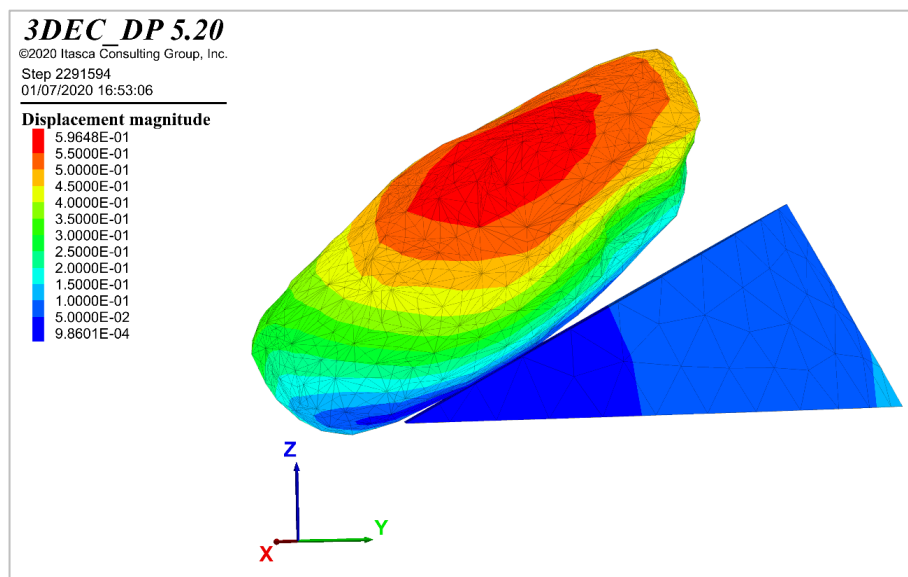


650

651 Fig. 21. 3DEC results of the tilt simulation on a rock + steel set with and upper cylinder uncentered
 652 $r/2$ to the left (PM-6). After the block starts to topple, its movement does not follow the tilting
 653 direction because the *cog* is not located in the symmetry plane as assumed in section 3.2.6

654 It is relevant to note that the DEM and the analytical approach match so closely here (and
655 in fact in general). This suggests that the errors observed in the experimental results are a
656 largely a function of limitations in the “manufacturing” processes used to make the
657 various specimens.

658 The critical angle in models with complex geometries (PM-7) measured by the three
659 methods were also similar, confirming the validity of both the experimental and the
660 analytical approaches. This model (Fig. 22) presents both an asymmetrical geometry and
661 an irregular base shape, resulting in complex movement after destabilization.
662



663

664 Fig. 22. Displacement magnitude after tilt-test of PM-7 calculated by 3DEC.

665

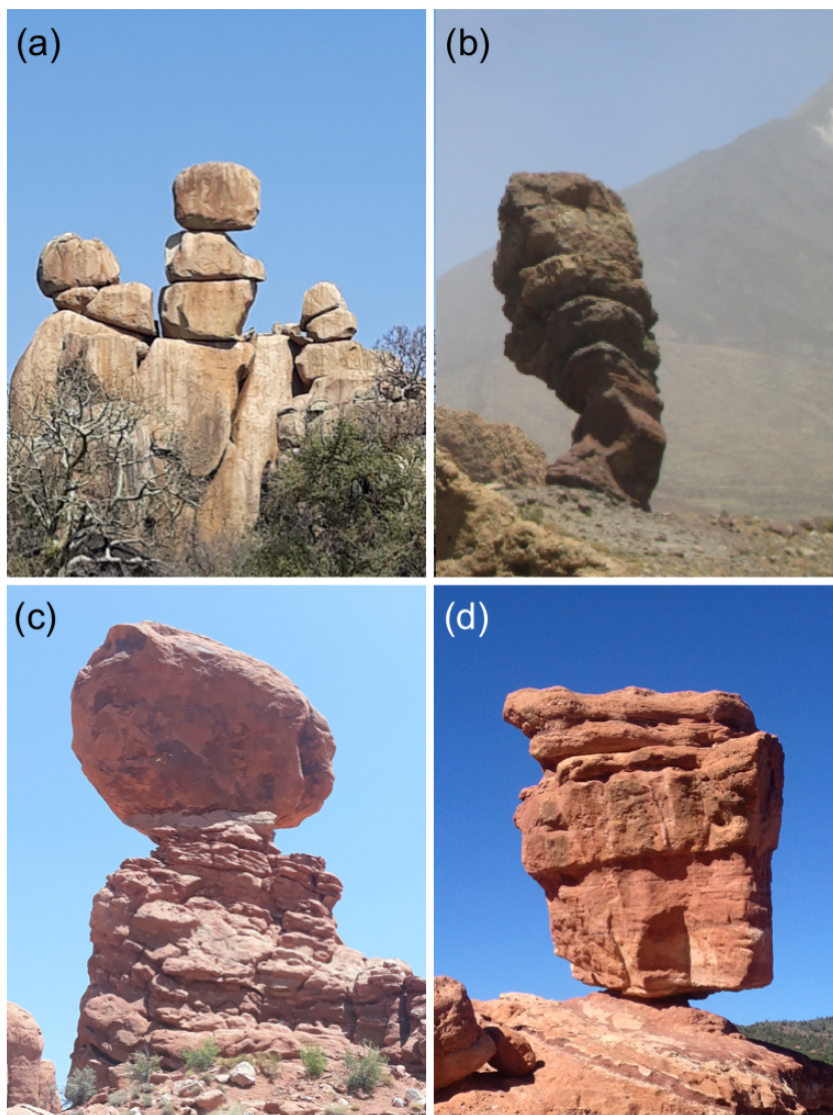
666 5. Discussion

667

668 It is not difficult in nature to find rock blocks or groups of blocks that could potentially
669 become unstable due to toppling. In some case, these blocks are irregular enough so as to
670 be considered heritage or part of natural parks, so they are protected (Fig. 23). On the
671 other hand, the instability of some other less aesthetically appealing rock blocks may
672 jeopardize infrastructure or even people, lives and properties. In any of these cases, it is
673 important to be able to analyze the stability of these blocks under different conditions
674 such that appropriate protective measures can be defined.

675

676 Although some approaches were developed in the past to compute the stability of blocks
677 against toppling, in many cases, and specifically those corresponding to complex
678 geometry blocks, it was indeed difficult to accurately compute stability against toppling.
679 Recent advances in theoretical stability analysis based on idealized geometries (rounded
680 corners, concave or convex surfaces) have contributed to a better understanding of
681 toppling phenomena. The methodology or group of approaches presented here based on
682 modern block geometry reconstruction methods, 3D printing of a block replica and testing
683 of this replica using a tilt table, help to reproduce the potential instability phenomena of
684 these blocks and to assess their degree of stability or instability.
685



686
687 Figure 23. Balanced stones in different Natural Parks. (a) The three sisters balancing rocks,
688 Matopos National Park, Matabeleland, Zimbabwe (b) Roque de García, basaltic horn at Teide
689 National Park, Tenerife, Canary Islands, Spain (c) 3,500 t balanced rock, Arches National Park,
690 Utah, USA and (d) 700 t balanced rock, the garden of gods, Colorado Springs, USA. Photos by
691 the authors.

692 One notable limitation of the approach demonstrated in this study is the lack of knowledge
693 of the geometry (concavity or concaveness and roughness) of the contact between the
694 block and the surface where it rests and its actual frictional behavior. However, the
695 proposed approach, in combination with detailed in-situ characterization and the
696 application of analytical and numerical calculation techniques as illustrated in this
697 document, has the potential to contribute to improved assessments of the stability of
698 irregular rock blocks or boulders.

699

700 **6. Conclusions**

701

702 All over the world, and particularly in mountainous terrain in hot and temperate regions,
703 rock blocks or boulders occur, and may exist in a state of marginal stability. In most of
704 these cases, the potential instability of these blocks does not represent a hazard to human
705 life or property. In some cases, however, it may be important to quantify block stability
706 either due to an associated hazard, or due to its significance to the community or its natural
707 landscape value.

708

709 Analyzing the stability of these blocks is not an easy task, primarily due to their complex
710 geometry and because it is also difficult to characterize in sufficient detail all the features
711 actually affecting their stability, including block geometry, geometry of the contact with
712 the base surface, strength and deformability characteristics of this contact —of particular
713 relevance when considering rough joints and infill material with non-negligible tensile
714 strength—, and potential triggers such as water pressure and earthquake loading.

715

716 Recently developed remote-sensing tools, such as photogrammetry or LiDAR can be used
717 in order to recover a rather accurate geometry of a block of interest as well as an
718 approximate representation of the contact area (typically hidden). Based on the recovered
719 3D point cloud, a scaled version of the rock block or boulder can be 3D-printed, and its
720 toppling behavior physically observed using a tilting platform, since toppling is
721 exclusively dependent on the geometry of the potentially overturning object and the
722 concavity of the base. This approach can be applied in combination with analytical or
723 numerical techniques to study the mechanisms involved and to check physical testing
724 results.

725

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Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

On behalf of the co-authors:

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Laboratory physical modelling of block toppling instability by means of tilt tests

CRedit author statement

Ignacio Pérez-Rey: Conceptualization, Methodology, Writing original draft; **Mauro Muñiz-Menéndez:** Software, Validation; **Javier González:** Visualization, Investigation; **Federico Vagnon:** Investigation; **Gabriel Walton:** Formal analysis, Writing - Review & Editing; **Leandro R. Alejano:** Supervision, Funding acquisition.