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***Abstract*— The study of the scattering and the radiation problems constituted of a dielectric slit in a thick conducting screen is of great importance in antenna systems, periodic structure, screens and propagation problems. In this paper, we formulate the problem and illustrate the solution's procedure through the Wiener-Hopf method.**

Keywords*— *dielectric-loaded slit, slot, thick metallic shield, screen, Wiener-Hopf method.

I. INTRODUCTION

Antenna systems, periodic structures, screens and propagation obstacles can be constituted by dielectric slits in a thick conducting screens, see Fig. 1. The problem may be classified as canonical; however, it receives a continuous wide attention for practical applications.

Usually, the scattering and the radiation properties of such structures are often studied using the fundamental waveguide mode in the slit.

The literature reports approaches that are based on Green's function formulations [1], integral equations [2], and modal series [3]-[4] together with ray problem outside the slit [5].

As demonstrated in [4] the problem presents special physical properties at narrow band known as extraordinary transmission.

The Wiener-Hopf (WH) technique [6] allows to approach the problem using a comprehensive mathematical-physical model which can be extended to more complex structures involving stratification in the slit region and it is independent form the size of the slit.

The Wiener-Hopf equations obtained by the application of the method are defined in the spectral domain and their solution in terms of spectral transformation of the field components contains all the physical properties of the problem.

In particular in the slit problem we need to apply the Green function's procedure [7]-[9] to make complete the WH equations.

In general, the WH equations cannot be solved exactly. To overcome this limitation we resort to the Fredholm Factorization [10]-[11]. The Fredholm factorization allows to obtain semi-analytical solutions of a given problem with high accuracy and efficiency. The complete solution procedure consists of the following steps: 1) deduction of WH equations, 2) Fredholm factorization, 3) evaluation of field components via inverse spectral transformation and asymptotics.

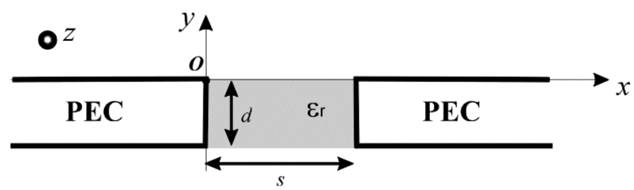

Fig. 1. Dielectric slit in a thick conducting screen.

II. FORMULATION

With reference to Fig. 1, the dielectric slit in a thick perfect electrical conducting (PEC) screen has width s, thickness d and relative dielectric constant εr. In the problem, we consider time-harmonic fields with time dependence $e^{j\omega t}$, which is omitted.

The source considered in this work is an *Ez*-polarized plane wave with incidence angle φ_o:

$$E_z^i(x,y) = E_o e^{jk\rho\cos(\varphi-\varphi_o)} = E_o e^{jk(x\cos\varphi_o + y\sin\varphi_o)} \quad (1)$$

where *k* is the propagation constant of the free space (the region outside the slit).

The W-H equations of the problem can be accomplished by a generalization of the procedure done for the hole problem and proposed in [6], p. 304.

The procedure starts form subdividing the geometry of the problem into sub-regions, which are homogeneous in geometry and material: the top and the bottom sub-regions are homogenous isotropic half-spaces (free space) and the central sub-region is a rectangular region of dimensions d x s made by dielectric.

The WH equations are written in terms of Laplace transforms along x direction of field components at *y=0,d* respectively labelled 1 and 2. The non-null transforms, due to the PEC boundary conditions, are

$$V_{1o}(\eta)=\int_0^s E_z(x,0)e^{j\eta x}dx,$$
$$V_{2o}(\eta)=\int_0^s E_z(x,-d)e^{j\eta x}dx, \tag{2}$$

$$I_{1o}(\eta)=\int_0^s H_x(x,0)e^{j\eta x}dx,$$
$$I_{1+}(\eta)=e^{-j\eta s}\int_s^\infty H_x(x,-d)e^{j\eta x}dx,$$
$$I_{1\pi+}(\eta)=-\int_{-\infty}^0 H_x(x,0)e^{-j\eta x}dx, \tag{3}$$
$$I_{2o}(\eta)=\int_o^s H_x(x,-d)e^{-j\eta x}dx,$$
$$I_{2+}(\eta)=e^{-j\eta s}\int_0^\infty H_x(x,-d)e^{j\eta x}dx,$$
$$I_{2\pi+}(\eta)=-e^{j\eta s}\int_{-\infty}^s H_x(x,-d)e^{-j\eta x}dx.$$

All this quantities are interpreted as WH unknowns.

Using circuital considerations as in multilayered regions, the deduction of the WH equations in the top and bottom sub-regions is systematic and it yields:

$$-I_{1\pi+}(-\eta)+I_{1o}(\eta)+e^{j\eta s}I_{1+}(\eta)=Y_c(\eta)[V_{1o}(\eta)] \tag{4}$$

$$I_{2\pi+}(-\eta)-I_{2o}(\eta)-e^{j\eta s}I_{2+}(\eta)=Y_c(\eta)[V_{2o}(\eta)] \tag{5}$$

with $Y_c(\eta)=\xi(\eta)/k\,Z_o, \xi(\eta)=\sqrt{k^2-\eta^2}$.

By using the alterative unknowns

$$V_{1\pi o}(\eta)=\ e^{j\eta s}V_{1o}(-\eta),\quad V_{2\pi o}(\eta)=e^{j\eta s}V_{2o}(-\eta),$$
$$I_{1\pi o}(\eta)=e^{j\eta s}I_{1o}(-\eta),\quad I_{2\pi o}(\eta)=e^{j\eta s}I_{2o}(-\eta) \tag{6}$$

in (4)-(5) we get two further equations and substituting η with $-\eta$ we obtain a system of 8 equations with the unknowns (2),(3),(6) evaluated in η and $-\eta$.

To complete the WH formulation of the slit problem we need to apply the Green function's function procedure [7]-[9] in the central dielectric rectangular region. Starting from the wave equations and applying the Fourier transform

$$\tilde{E}_{zo}(\alpha,y)=\int_0^s E_z(x,y)e^{j\alpha x}dx \tag{7}$$

we apply the Green's function procedure to get the solution of the second order equation constituted of the particular integral plus the homogenous solution. The imposition of boundary conditions at *y=0,d* and the derivation of the Fourier transform of H_x in *0<x<s* at *y=0,d* yields two incomplete equations that relates $I_{1o}(\eta)$, $I_{2o}(\eta)$ to $V_{1o}(\eta)$, $V_{2o}(\eta)$ plus particular integrals. The incompleteness is due to the presence of the particular integrands that are represented in explicit form using Mittag-Leffler's theorem and Cauchy representation formula [8]-[9]. These two equations of the central regions are doubled substituting η with $-\eta$ and using the unknowns (6). The resulting system of equations allows to compute all the unknowns reported in (2),(3),(6).

The completed WH representation can be semi-analytically solved using the Fredholm Factorization [6], [10]-[11].

III. Solution of the Problems

The adopted procedure reduces the WH equations to a system of Fredholm integral equations of second kind where, after mathematical manipulations, the four unknowns are the spectral "voltage" quantities (2), first line of (6).

Once solved the problem in terms of voltage spectra, via asymptotics we get diffraction coefficients and total fields of the problem for plane wave illumination. The procedure is going to be checked by comparing the numerical results with the current literature reported in the references.

Further details on the formulation, numerical validations and results will be shown during the presentation.

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