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Modified Bresler-Marcuvitz Transverse Equation Theory for Wedge Shaped Regions to derive Generalized Wiener-Hopf Equations

Vito Daniele(1), Guido Lombardi(2)
(1) Politecnico di Torino, Italy, e-mail: vito.daniele@polito.it
(2) Politecnico di Torino, Italy, e-mail: guido.lombardi@polito.it

In this work, we present a novel and general mathematical technique to get Generalized Wiener-Hopf Equations (GWHEs) in angular regions filled by an arbitrary linear medium and with arbitrary boundary conditions [1-2]. We first extend the transverse equation theory developed in [3] by Bresler and Marcuvitz for stratified media. We develop the theory for angular regions in electromagnetics using oblique Cartesian coordinates, starting from Maxwell's equations. It yields a matrix differential problem of first order (1) whose unknowns are the field component ψ_t tangent to the faces of the angular regions (i.e. the field components continuous on the boundaries).

$$-\frac{\partial}{\partial y}\psi_t = \mathcal{M}\left(\frac{\partial}{\partial z},\frac{\partial}{\partial x}\right)\cdot\psi_t \tag{1}$$

The application of oblique Cartesian coordinates, Laplace transform along one face of the angular region, and assuming the problem invariant along the edge, yield a matrix ordinary differential problem of first order.
The characteristic Green's function procedure, as reported in [1, 2, 4], allows to develop a spectral solution before imposing boundary conditions. The projection of the solution on reciprocal eigenvectors of the algebraic matrix operator derived from (1) allows to get a set of functional equations that relate the Laplace transforms of field components along one face of the angular regions to the ones of the other face. The imposition of boundary conditions yields a set of GWHEs for the angular region problem. The boundary conditions can be of any type to model impenetrable or penetrable angular regions [5-6]. The arbitrary of boundary conditions allows to couple angular regions to any other kind of region yielding the possibility to analyze complex scattering problems constituted by canonical sub-regions of different geometry and material, see for instance [7] and references therein. The formulation can be applied to arbitrarily linear media. In isotropic media, an alternative and less flexible method to derive the GWHEs is reported in [5] and it starts form the second order differential wave equation. We notice that the theory developed using the framework of Bresler-Marcuvitz is of first order and it is more general and flexible since it is based on abstract operator theory. It can be extended to complex linear media [2] and other physics such as acoustics or elasticity. Finally, the solution of the system of the GWHEs can obtained using exact or semi-analytical/approximate techniques of factorization, see [5-6] and references therein. We recall that GWHEs differ from the Classical Wiener-Hopf equations (CWHEs) for the definitions of the unknowns in spectral domain. While CWHEs introduce plus and minus functions that are always defined in the same complex plane, the GWHEs present plus and minus functions that are defined in different complex planes but related together. In several important practical cases, suitable mappings allow to redefine the plus and the minus functions of GWHEs in the same complex plane, in particular for isotropic medium. The selection of a general linear medium forbids the reduction to CWHEs but it is possible to work in multiple complex planes with Cauchy decomposition procedures. Further details on the formulation, numerical validations and results will be shown during the presentation. This work is supported by the PRIN Grant 2017NT5W7Z GREEN TAGS.

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