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Bivariate Macromodeling with Passivity Constraints

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***Abstract*—This paper presents a novel theoretical formulation and an associated algorithm for the computation of passive parameterized macromodels from tabulated scattering data. The main contribution is the construction of passivity constraints for parameterized models as a finite set of linear matrix inequalities, thanks to a suitable expansion into Bernstein polynomials. These constraints are embedded in the model identification process, leading to a convex formulation of passivity enforcement, with guaranteed convergence and without requiring any expensive multivariate passivity check. Two numerical examples demonstrate the effectiveness of the proposed approach.**

I. Introduction

In the recent years, parameterized macromodels emerged as a promising tool to reduce the intensive computational burden required by time domain simulations of complex electronic systems. Such macromodels embed in closed form the dependence of a device behavior on external design or physical parameters, allowing for design optimization, sensitivity and what-if analyses.

The generation of parameterized macromodels using data-driven approaches is typically performed by fitting multivariate rational functions to tabulated scattering responses, suitably swept over the parameter space of interest. Model passivity, which is a fundamental requirement for ensuring stable time-domain simulations, can be either ensured a priori by adopting a particular model structure [1], or by a subsequent passivity enforcement stage [2], which iteratively perturbs the model coefficients until uniform passivity is achieved. The former methods may overconstrain model extraction leading to reduced accuracy, whereas the latter methods are extremely expensive in terms of computing resources, due to the need of iterated multivariate passivity checks. Moreover, convergence in passivity enforcement loop is not always guaranteed.

This work proposes a novel technique for the generation of guaranteed passive macromodels depending on a single external parameter. By inducing the parameterization of the poles and the zeros of the model through polynomials represented in Bernstein basis [3], we are able to formulate the parameterized passivity constraints as a finite number of Linear Matrix Inequalities (LMIs). Combining these constraints with a standard multivariate rational fitting process leads to a guaranteed passive macromodel in only one iteration, thanks to a convex formulation. Hence, no expensive passivity checks are required, and no convergence issues arise. The effectiveness of the proposed approach is demonstrated on two relevant test cases.

II. Background and problem setting

We consider a passive p-port electromagnetic structure, whose behavior depends on one real-valued (normalized) design parameter $\theta \in \Theta = [0,1]$. The structure is characterized over a finite bandwidth by its parameterized scattering matrix $\tilde{\mathsf{H}}(s,\theta)$, in terms of the following tabulated samples

$$\tilde{H}_{k,m} = \tilde{\mathsf{H}}(j\omega_k, \theta_m), \quad k = 1,...K, \quad m = 1,...,M. \tag{1}$$

Our goal is to derive a reduced-order rational model $\mathsf{H}(s,\theta)$, to be extracted by enforcing the fitting condition

$$\mathsf{H}(j\omega_k, \theta_m) \approx \tilde{H}_{k,m}, \quad k = 1,...K, \quad m = 1,...,M \tag{2}$$

subject to appropriate passivity constraints.

We assume the following bivariate model structure [4]

$$\mathsf{H}(s,\theta) = \frac{\mathsf{N}(s,\theta)}{\mathsf{d}(s,\theta)} = \frac{\sum_{i=0}^{\bar{n}} \sum_{\ell=1}^{\bar{\ell}} R_{i,\ell}\, b_{\ell,\bar{\ell}}(\theta)\, \varphi_i(s)}{\sum_{i=0}^{\bar{n}} \sum_{\ell=1}^{\bar{\ell}} r_{i,\ell}\, b_{\ell,\bar{\ell}}(\theta)\, \varphi_i(s)}, \tag{3}$$

where the rational form of (3) is induced by the partial fraction basis $\varphi_0(s) = 1$, $\varphi_i(s) = (s - q_i)^{-1}$ where q_i are fixed stable poles. Further, we denote with $b_{\ell,\bar{\ell}}(\theta)$ the ℓ-th Bernstein polynomial [5] of degree $\bar{\ell}$. These polynomials provide a parameterization of both poles and zeros of $\mathsf{H}(s,\theta)$ and retain the following properties

$$\sum_{\ell=0}^{\bar{\ell}} b_{\ell,\bar{\ell}}(\theta) = 1, \quad \text{and} \quad b_{\ell,\bar{\ell}}(\theta) \geq 0, \forall \ell. \tag{4}$$

We will also exploit the so-called *degree elevation property*, which enables to write any Bernstein polynomial of degree $\bar{\ell}$ in terms of a polynomial of degree $\bar{\ell}+1$ as

$$\sum_{\ell=0}^{\bar{\ell}} a_\ell \cdot b_{\ell,\bar{\ell}}(\theta) = \sum_{m=0}^{\bar{\ell}+1} a'_m b_{m,\bar{\ell}+1}(\theta) \tag{5}$$

with $a'_m = \frac{m}{\bar{\ell}+1} a_{m-1} + \frac{\bar{\ell}+1-m}{\bar{\ell}+1} a_m$.

By using the *Parameterized Sanathanan-Koerner (PSK)* iteration [4], the model (3) is constructed by iteratively solving the least squares problem

$$\begin{bmatrix} \Psi_x^\mu & \Psi_y^\mu \end{bmatrix} \begin{bmatrix} x^\mu \\ y^\mu \end{bmatrix} \approx h \tag{6}$$

arising from the linearization of (2), where vectors x, y collect the coefficients $R_{i,\ell} \in \mathbb{R}^{p \times p}$ and $r_{i,\ell} \in \mathbb{R}$, respectively, Ψ_x and Ψ_y are regressor matrix blocks, h collects all data (1), and $\mu = 1, 2, \dots$ is the iteration index. See [6] for details.

It is known [7] that model (3) is passive if and only if the following three conditions hold concurrently

1) $\mathsf{H}(s,\theta)$ regular for $\Re\{s\} > 0 \quad \forall\theta \in \Theta$;
2) $\mathsf{H}^*(s,\theta) = \mathsf{H}(s^*,\theta) \quad \forall s \in \mathbb{C}, \forall\theta \in \Theta$;
3) $\mathsf{H}^{\mathsf{H}}(j\omega,\theta)\mathsf{H}(j\omega,\theta) \preceq I_p \quad \forall\omega \in \mathbb{R}, \forall\theta \in \Theta$;

where I_p is the size-p identity matrix, with * and $^{\mathsf{H}}$ denoting complex conjugate and hermitian transpose, respectively. In what follows, we assume condition 1, which is enforced as in [8], with condition 2 implied by the adopted model structure. In this work, we propose a numerically viable strategy to enforce also condition 3 during model generation.

III. Bounded Realness of bivariate PSK models

Based on the model structure (3), the passivity condition 3 can be equivalently rewritten $\forall\omega$ and $\forall\theta \in \Theta$ as

$$\mathsf{N}^{\mathsf{H}}(j\omega,\theta)\mathsf{N}(j\omega,\theta) - \mathsf{D}^{\mathsf{H}}(j\omega,\theta)\mathsf{D}(j\omega,\theta) \preceq 0, \tag{7}$$

where $\mathsf{D}(j\omega,\theta) = \mathsf{d}(j\omega,\theta)\cdot I_p$. Following [4], we express both $\mathsf{N}(s,\theta)$ and $\mathsf{D}(s,\theta)$ through their (parameterized) state-space realizations $\mathsf{Q}(s,\theta) \rightarrow \{A, B, C_{\mathsf{Q}}(\theta), D_{\mathsf{Q}}(\theta)\}$, where Q is a placeholder for $\{\mathsf{N},\mathsf{D}\}$, matrices A, B are constant and common to both $\{\mathsf{N},\mathsf{D}\}$, and where the matrix pair $C_{\mathsf{Q}}(\theta), D_{\mathsf{Q}}(\theta)$ collects numerator and denominator coefficients x, y for $\mathsf{Q}=\mathsf{N}$ and $\mathsf{Q}=\mathsf{D}$, respectively.

Applying the Yakubovich lemma [9, Sec. 3, Theorem 1] leads to the following parameter-dependent non-expansivity condition

$$\forall\theta \in \Theta,\ \exists P(\theta) = P^T(\theta) \quad s.t. \tag{8}$$
$$\begin{bmatrix} A^TP + PA - C_{\mathsf{D}}^TC_{\mathsf{D}} & PB - C_{\mathsf{D}}^TD_{\mathsf{D}} & C_{\mathsf{N}}^T \\ B^TP - D_{\mathsf{D}}^TC_{\mathsf{D}} & -D_{\mathsf{D}}^TD_{\mathsf{D}} & D_{\mathsf{N}}^T \\ C_{\mathsf{N}} & D_{\mathsf{N}} & -I_p \end{bmatrix} \preceq 0.$$

which is equivalent to (7) and provides a purely algebraic (yet parameterized) passivity characterization.

Model parameterization is here induced by expansion of all relevant model quantities (3) into Bernstein polynomials. Due to the presence of quadratic terms into (8), we exploit the degree elevation property (5) to cast all expansions into Bernstein polynomials of degree $2\bar{\ell}$. We have

$$Z_{\mathsf{N}}(\theta) = \sum_{\ell=0}^{\bar{\ell}} b_{\ell,\bar{\ell}}(\theta)\cdot Z_{\mathsf{N},\ell} = \sum_{\ell=0}^{2\bar{\ell}} b_{\ell,2\bar{\ell}}(\theta)\cdot Z'_{\mathsf{N},\ell} \tag{9}$$

for numerator state-space matrices $Z = \{C, D\}$,

$$\begin{bmatrix} C_{\mathsf{D}}^T(\theta) \\ D_{\mathsf{D}}^T(\theta) \end{bmatrix} \begin{bmatrix} C_{\mathsf{D}}(\theta) & D_{\mathsf{D}}(\theta) \end{bmatrix} = \sum_{\ell=0}^{2\bar{\ell}} b_{\ell,2\bar{\ell}}(\theta)\cdot X_\ell \tag{10}$$

for quadratic terms, and

$$P(\theta) = \sum_{\ell=0}^{2\bar{\ell}} b_{\ell,2\bar{\ell}}(\theta)\cdot P_\ell, \quad P_\ell = P_\ell^T \quad \forall\ell \tag{11}$$

for the symmetric matrix. Inserting into (8) leads to condition

$$\forall\theta \in \Theta,\ \exists P_\ell = P_\ell^T, \quad \ell = 0,1,...,2\bar{\ell} \quad s.t.$$
$$\Upsilon(\theta) = \sum_{\ell=0}^{2\bar{\ell}} b_{\ell,2\bar{\ell}}(\theta) \underbrace{\begin{bmatrix} \Omega_\ell - X_\ell & K_\ell^T \\ K_\ell & -I_p \end{bmatrix}}_{\Upsilon_\ell} \preceq 0, \tag{12}$$

which implies (8) under the adopted parameterization, with

$$\Omega_\ell = \begin{bmatrix} A^TP_\ell + P_\ell A & P_\ell B \\ B^TP_\ell & 0 \end{bmatrix}, \quad K_\ell = \begin{bmatrix} C'_{\mathsf{N},\ell} & D'_{\mathsf{N},\ell} \end{bmatrix}. \tag{13}$$

We will refer to the matrix coefficients Υ_ℓ as to *control points* for $\Upsilon(\theta)$. Due to property (4), condition (12) is implied by the following *finite* set of $2\bar{\ell}+1$ Linear Matrix Inequalities (LMIs)

$$\exists P_\ell = P_\ell^T \quad s.t. \quad \Upsilon_\ell \preceq 0 \quad \text{for} \quad \ell = 0,1,...,2\bar{\ell}. \tag{14}$$

We are now ready to state our main algorithm for passive model extraction. We note that the control points Υ_ℓ are linear in the unknowns $C_{\mathsf{N},\ell}, D_{\mathsf{N},\ell}$, corresponding to the numerator coefficients $R_{i,\ell}$ and collected in vector x. Therefore,

1) the unconstrained PSK iteration (6) is run until the estimates of both numerator and denominator coefficients stabilize;
2) denoting the last iteration as $\bar{\mu}$, we freeze the denominator coefficients $y^{\bar{\mu}}$ and we solve *only once* the constrained least squares problem

$$\min_{P_\ell,\, x^{\bar{\mu}+1}} ||\Psi_x^{\bar{\mu}+1} x^{\bar{\mu}+1} - \begin{bmatrix} h - \Psi_y^{\bar{\mu}+1} y^{\bar{\mu}} \end{bmatrix}||_2 \tag{15}$$
$$\text{s.t.} \quad \Upsilon_\ell \preceq 0, \quad P_\ell = P_\ell^T, \quad \ell = 0,1,...,2\bar{\ell}$$

to evaluate the numerator coefficients. No iterations are required since the unique solution of (15) provides a model that is uniformly passive throughout the parameter space Θ.

IV. Examples

A. Microstrip Filter

The proposed method was applied to generate a passive parameterized macromodel of a 2-port double-folded microstrip filter (see [10]) parameterized by a stub length $\theta \in [2.08, 2.28]$ mm. The scattering parameters of the structure were sampled by means of an EM solver for 21 linearly spaced values of the design parameter.

A model with $\bar{n} = 9$ poles and polynomial degree $\bar{\ell} = 3$ was generated starting from the available samples. We performed $\bar{\mu} = 10$ initial iterations of the PSK algorithm to generate the model; then, the proposed passivity conditions (14) were enforced in a last iteration, by solving (15). The solution of this constrained problem required approximately 2 s on a common laptop. We show in Fig. 1 (top panel) the accuracy of the model with respect to the raw data for one of the reflection coefficients. The parameterized singular values of the model are shown in the bottom panel of Fig. 1 for 100 different parameter instances, and confirm that passivity holds uniformly throughout the parameter space.

B. High-speed link

The second example we consider is a high-speed link connecting two multilayer PCBs, which include vertical vias for inner routing, see [11] for full details. The free parameter is the via pad radius, varying in the range $[100, 300]$ μm. The scattering parameters were sampled in correspondence of 9 parameter configurations.

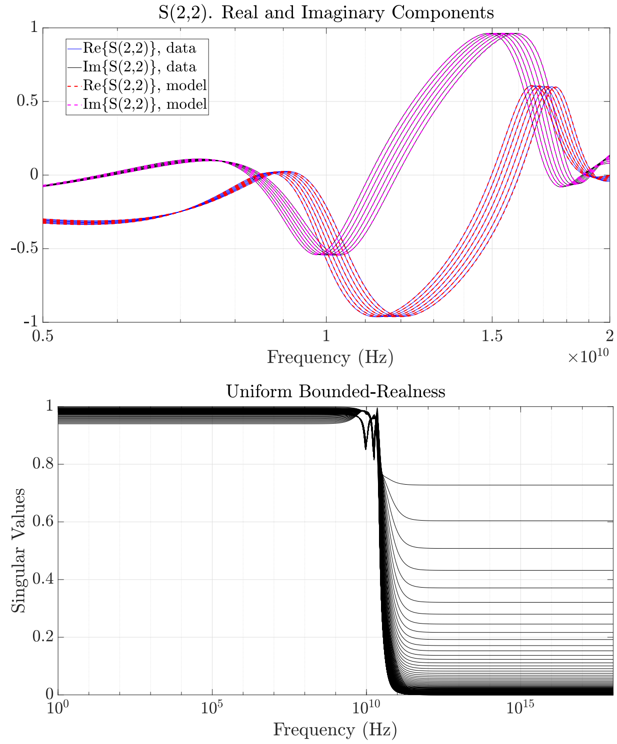

Fig. 1. Microstrip filter. Top panel: comparison of parameterized model responses to raw data for one selected scattering response, evaluated over selected parameter values. Bottom panel: singular values computed over a fine parameter sweep.

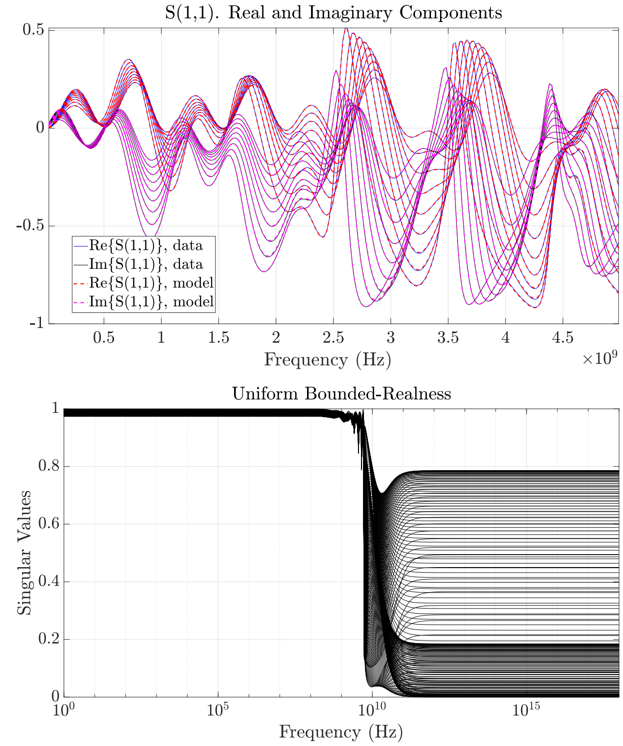

Fig. 2. As in Fig. 1, but for the high-speed link example.

We derived a passive model with $\bar{n} = 25$ poles and polynomial degree $\bar{\ell} = 3$, by applying the PSK algorithm as in the previous example. The passivity-constrained problem (15) required 42 s. The results are shown in Fig. 2 (top panel), which highlights the very large variability induced by the parameter in the responses, as well as the remarkable accuracy of the model when compared to the raw data. Also in this case a fine sweep of the singular values of the model (bottom panel) confirms the uniform passivity throughout the parameter space.

V. Conclusions

This paper introduced a novel strategy for the generation of passive bivariate macromodels from tabulated scattering data. Differently from conventional approaches, based on a two-step process that first computes a macromodel and only in a second stage enforces its passivity by iterative perturbation, the new approach enforces passivity by solving a unique least squares problem constrained by a finite set of linear matrix inequalities. Thanks to this convex formulation, no expensive multivariate passivity checks are required, and convergence is guaranteed in one step. Future investigations will attempt an extension of the proposed strategy in presence of more than one parameter in addition to frequency.

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