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***Abstract*—This paper analyzes a recently proposed energy harvester model, based on a harmonic chain of coupled oscillators connected to a piezoelectric transducer. We discuss first how random mechanical vibrations can be modelled as a superposition of sinusoidal signals with random amplitude and phase. Using a mechanical-electrical analogy, we derive equivalent circuits in the form of two-port networks, for both the single mass and the N-mass energy harvester. The transfer functions for both the single mass and the N-mass (valid for any number of masses in the harmonic chain) devices are then obtained cascading the two-port representations of the individual sections, and we give expressions for computing the output power and the power efficiency. Pros and cons of the harmonic chain model are also briefly discussed.**

I. Introduction

The quest for renewable energy sources, driven by the now widely recognized necessity to transition from a pure exploitation economic model to the sustainable development paradigm, suggested the research community to explore alternative energy technologies based on tapping the available ambient energies [1]–[5]. As electrical power sources represent a highly flexible energy form perfectly adapt to implement the Internet of Things scenario made of networks of electronic and electro-mechanical systems that are not only miniaturized, but also wireless connected, we focus here on the harvesting of ambient energy sources yielding electrical energy.

Among the various flavors of ambient energy, e.g. electromagnetic radiation, temperature gradients and many others, mechanical energy is the most ubiquitous [6]–[10], and thus it represents probably the most common energy harvesting input. The energy conversion from mechanical to electrical of course calls for the availability of a transducer: the high electro-mechanical coupling factor and piezoelectric coefficient, as compared to other approaches such as electrostatic, electromagnetic, and triboelectric conversion, make piezoelectric transduction the prominent choice for mechanical energy harvesters [11], [12].

From the modeling standpoint, the mathematical representation of the energy sources should reflect their random nature: a stochastic process is the natural candidate, thus leading to the development of the so-called *stochastic energy harvesters*. The piezoelectric harvester, on the other hand, is represented as an oscillator, whose proper frequencies must be adapted to the spectral region where most of the mechanical energy is available [13], [14].

In this contribution, we derive a novel, cascaded two-port model for the analysis of linear, piezoelectric stochastic energy harvesters. After expressing the stochastic process representing random mechanical vibrations as a superposition of stochastic harmonic functions characterized by a random frequency distribution and random amplitudes, we turn the attention to the modeling of the mechanical and electrical sections of the harvester. Single and multi-modal piezoelectric harvesters are described in the frequency domain by means of the corresponding transmission matrix, a choice that matches the representation of the random inputs as a superposition of harmonics. The input and output power can therefore be easily recovered as a function of the relevant transfer functions derived from the equivalent transmission matrix, as well as the harvester efficiency.

The starting point of the present work has been the piezoelectric harvester model discussed in [15], where an harmonic chain of oscillating masses is compared to the customary single mass device: we have modified the original proposal by connecting the chain of oscillating masses to the transducer with a spring, which seems a more physical representation of the device. After describing the representation of the random vibrations as a superposition of sinusoidal signals with random amplitudes and frequency components, we focus the attention on deriving a cascaded two-port description of both the harmonic chain of masses and of the piezoelectric transducer in terms of the transmission matrices of the electrical equivalent for all of the harvester sections. The main advantage of this procedure is the derivation of the relevant transfer functions allowing for the estimation of the output power and of the power collecting efficiency. According to our description of the harvester, the advantages of the multiple masses device are clearly traced back to the presence of several resonant frequencies as opposed to the case of a single resonance structure.

II. Random Vibration modelling

The theory of periodically driven linear systems is well developed, thus following [16], we model random mechanical

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TABLE I
MECHANICAL-ELECTRICAL ANALOGY

Mechanical	**Electrical**
Force, f	Voltage, v
Displacement, x	Charge, q
Momentum $m\dot{x}$	Flux linkage, φ
Mass, m	Inductance L
Compliance, k^{-1}	Capacity, C
Damping, γ	Resistance, R

vibrations with the parametric stochastic process

$$f(t) = \sum_{k=1}^{n} \sigma_k \left(A_k \cos(\omega_k t) + B_k \sin(\omega_k t)\right) \tag{1}$$

where σ_k are constants, while A_k and B_k are Gaussian distributed, uncorrelated random variables, with zero expectation value and unit variance. For the angular frequencies ω_k, different choice criteria have been proposed [17]. To guarantee that the process is not periodic, it is sufficient that ω_m/ω_n is not rational, for at least one pair[1] $m, n \in \mathbb{N}$. For the sake of simplicity, we shall consider random frequencies, uniformly distributed on the interval $]0, \omega_{max}]$. Therefore, the stochastic process (1) is characterized by zero expectation, and by the correlation function

$$r(t_1, t_2) = E[f(t_1) f(t_2)] = \sum_{k=1}^{n} \sigma_k^2 \cos(\omega_k (t_1 - t_2)) \tag{2}$$

where $E[\cdot]$ denotes the expected value operator. Being stationary, we can describe the second order moment through the power spectral density

$$S(\omega) = \sum_{k=1}^{n} \frac{\sigma_k^2}{2} \left(\delta(\omega - \omega_k) + \delta(\omega + \omega_k)\right) \tag{3}$$

Exploiting mechanical-electrical analogies (see table I), forces are replaced by voltages. Thus the stochastic process (1) can be represented by a series of ideal independent voltage sources, as shown in figure 1. We shall make the substitution

$$f(t) \to v_{\text{in}}(t) = \sum_{k=1}^{n} V_{\text{in},k} \cos(\omega_k t + \theta_k) \tag{4}$$

where $V_{\text{in},k} = \sigma_k \sqrt{A_k^2 + B_k^2}$ and $\theta_k = -\arctan(B_k/A_k)$.

III. HARMONIC CHAIN-BASED ENERGY HARVESTER: MODELLING

Any energy harvester for parasitic mechanical vibration scavenging, is composed by three main parts: A mechanical structure designed to capture the kinetic energy of parasitic mechanical vibration; a transducer, responsible for the mechanical-to-electrical energy conversion, and an electrical circuitry for electric power supply.

[1]From the implementation point of view, this condition is never satisfied, as the finite precision of digital computers implies that ratio ω_m/ω_n is always rational. For practical purposes, however, it is sufficient to consider a large enough value for m/n.

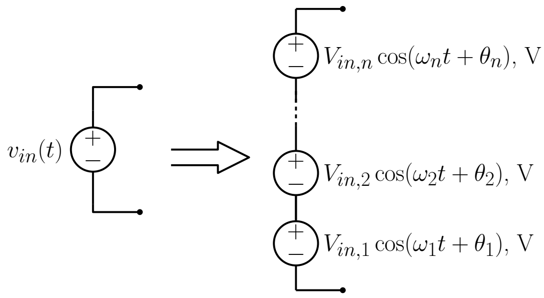

Fig. 1. Equivalent one port for the random vibration force.

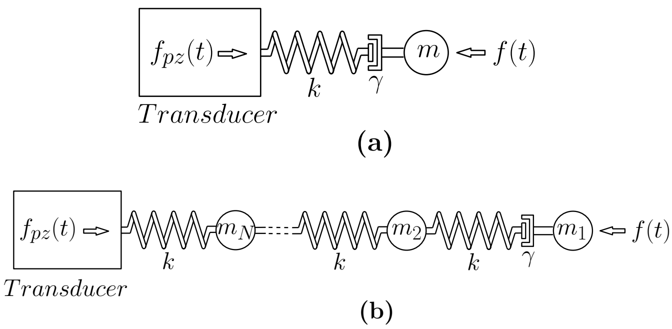

Fig. 2. Schematic representation of a single mass (single mode) **(a)**, and a N-masses harmonic chain based energy harvester **(b)**.

Irrespective of the working principle, energy harvesters typically rely upon oscillators to capture energy from random mechanical vibrations. Efficiency of linear oscillators is restricted by their limited bandwidth, because they act as passband filters. Recently, it has been suggested that energy harvesters based upon a harmonic chain, e.g. a chain of coupled harmonic oscillators as shown in figure 2**(b)**, may offer better performance, exploiting a larger number of operating modes and thus a wider bandwidth [15]. Notice that with respect to [15], we have connected the harmonic chain to the transducer through a spring, so that the single mass device becomes a special case of the harmonic chain ($N = 1$).

In this section, we develop a cascaded two-port network model for energy harvesters with piezoelectric transducers, that exploits a harmonic chain to capture random vibrations kinetic energy. The advantage of using cascaded two-ports is twofold: First, the network model can be easily adapted to any number of oscillators in the chain (and the associated number of admissible oscillation modes). Second, the model can be easily analyzed in the frequency domain to estimate the transfer function and power performance.

A. Piezoelectric transducer: Two-port model

First we derive a two-port network model for piezoelectric transducers. The constitutive equations for a linear piezoelectric material are [18]

$$\begin{bmatrix} \boldsymbol{S} \\ \boldsymbol{D} \end{bmatrix} = \begin{bmatrix} \boldsymbol{s^E} & \boldsymbol{d} \\ \boldsymbol{d}^{\mathrm{T}} & \boldsymbol{\varepsilon^T} \end{bmatrix} \begin{bmatrix} \boldsymbol{T} \\ \boldsymbol{E} \end{bmatrix} \tag{5}$$

where $\boldsymbol{S}$ and $\boldsymbol{T}$ are the mechanical strain and stress tensors, $\boldsymbol{D}$ and $\boldsymbol{E}$ are the dielectric displacement and electric field vectors, $\boldsymbol{s^E}$ and $\boldsymbol{d}$ are the compliance (evaluated at constant electric field) and the piezoelectric charge constants tensors, while $\varepsilon^{\boldsymbol{T}}$ is the absolute permittivity evaluated at constant stress [19]. Finally, $^{\mathrm{T}}$ denotes the transpose.

A lumped parameter model, describing the macroscopic behavior in terms of the force f_{pz} applied to the mechanical part due to the electrical domain, displacement x, charge q and voltage e, can be derived from the microscopic description (5) through spatial integration. In the quasi-static regime, neglecting the stiffness of the piezoelectric material, the governing equations read

$$f_{\mathrm{pz}}(t) = -\alpha\, e(t) \tag{6a}$$

$$q(t) = \alpha\, x(t) - C_p\, e(t) \tag{6b}$$

where α is the electromechanical coupling (in N/V or As/m), and C_p is the electrical capacitance of the mechanical unconstrained system.

System (6) describes an electro-mechanical two port. At the left (input) port the effort (or across) variable is the force $f_{\mathrm{pz}}(t)$, and the flux (or through) variable is the velocity $\dot{x}(t)$. At the right (output) port, the effort is the voltage $e(t)$ and the flux is the current $i(t) = \dot{q}(t)$. Using the mechanical-electrical analogy, an equivalent circuit for the piezoelectric transducer can be derived, as shown in the right part of figure 3. In the complex frequency domain, the state equations for the equivalent two-port network are

$$\begin{bmatrix} V_{\mathrm{pz}}(s) \\ I_{\mathrm{pz}}(s) \end{bmatrix} = \underbrace{\begin{bmatrix} \alpha & 0 \\ \frac{sC_p}{\alpha} & \frac{1}{\alpha} \end{bmatrix}}_{\mathbf{T}_{\mathrm{pz}}} \begin{bmatrix} V'_{\mathrm{pz}}(s) \\ I'_{\mathrm{pz}}(s) \end{bmatrix} \tag{7}$$

where $\mathbf{T}_{\mathrm{pz}}(s)$ denotes the transmission matrix for the two-port model of the piezoelectric transducer.

B. Single mass energy harvester: Two-port modelling

We shall now discuss the problem of modelling the mechanical part of the energy harvester as a cascaded two-port network. First we consider the case of the single mass (single mode) oscillator shown in figure 2**(a)**. The equation of motion is

$$m\ddot{x}(t) + kx(t) + \gamma\dot{x}(t) = f_{\mathrm{ext}}(t) \tag{8}$$

where m is the inertial mass, k is the stiffness constant of the spring, γ is a dissipation constant and $f_{\mathrm{ext}}(t) = f(t) + f_{\mathrm{pz}}(t)$ is the net force applied to the oscillator. It is convenient to rewrite (8) as a first order system

$$\dot{x} = \frac{y}{m} \tag{9a}$$

$$\dot{y} = -kx - \frac{\gamma}{m}y + f(t) + f_{\mathrm{pz}}(t) \tag{9b}$$

Using the mechanical-electrical analogy in table I, the equivalent two-port network shown in the left part of figure 3 is obtained[2]. The state equations in the complex frequency domain read

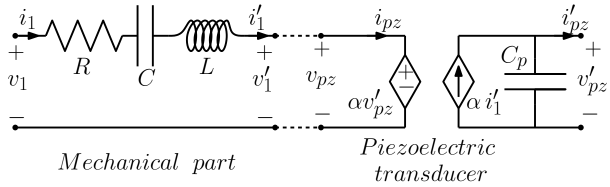

Fig. 3. Equivalent circuit for a single mass energy harvester with piezoelectric transducer.

$$\begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & R + sL + \frac{1}{sC} \\ 0 & 1 \end{bmatrix}}_{\mathbf{T}_1} \begin{bmatrix} V'_1(s) \\ I'_1(s) \end{bmatrix} \tag{10}$$

where $\mathbf{T}_1(s)$ is the transmission matrix for the two-port network model of the single mass oscillator.

When the harmonic oscillator is connected to the piezoelectric transducer, a cascaded two-port network is obtained. The transmission matrix for the cascaded two-port is

$$\begin{aligned} \mathbf{T}(s) &= \mathbf{T}_1(s)\, \mathbf{T}_{\mathrm{pz}}(s) \\ &= \begin{bmatrix} \alpha + \frac{sC_p}{\alpha}(R + sL + \frac{1}{sC}) & \frac{1}{\alpha}(R + sL + \frac{1}{sC}) \\ \frac{sC_p}{\alpha} & \frac{1}{\alpha} \end{bmatrix} \end{aligned} \tag{11}$$

C. Harmonic chain-based energy harvester: Two-port modelling

Now we consider the N-masses harmonic chain shown in figure 2**(b)**. We assume that the external source of mechanical vibrations is applied to the first oscillator, and that the piezoelectric transducer absorbs energy from the N-th oscillator. While we let the masses to be different, we simplify the system considering the connecting springs to have the same stiffness constant k. It is worth mentioning that, differently from the original work [15], the N-th mass is connected to the transducer through a spring, which is absent in [15]. Inclusion of the connecting spring introduces a zero in the transfer function at $\omega = 0$ rad/s. Conversely, if the N-th mass is directly connected to the transducer as in [15], the transfer function has a non-zero gain at $\omega = 0$ rad/s, a condition whose physical consistency seems questionable.

Under these assumptions, the equations of motion take the

[2]It is worth mentioning that a two-port network representation can be derived for the mechanical system directly, in terms of forces and velocity, without reformulating the problem as an electrical equivalent circuit.

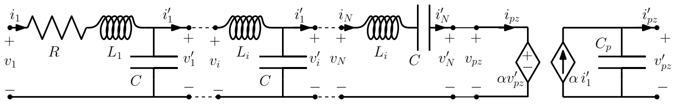

Fig. 4. Equivalent circuit for a N-masses harmonic chain energy harvester with piezoelectric transducer.

form

$$f(t) = m_1\ddot{x}_1 - k(x_2 - x_1) + \gamma\dot{x}_1 \tag{12a}$$

$$\vdots$$

$$0 = m_i\ddot{x}_i - k(x_{i+1} + x_{i-1} - 2x_i) \quad i = 2, \dots, N-1 \tag{12b}$$

$$\vdots$$

$$f_{\text{pz}}(t) = m_N\ddot{x}_N - k(x_{N-1} - 2x_N) \tag{12c}$$

Using the mechanical-electrical analogy, the equivalent circuit for the N-masses harmonic chain based energy harvester as a cascaded two-ports network shown in figure 4 is obtained.

The transmission matrices for each of the two-ports representing the harmonic chain are

$$\mathbf{T}_1(s) = \begin{bmatrix} 1 + sC(R + sL_1) & R + sL_1 \\ sC & 1 \end{bmatrix} \tag{13}$$

$$\mathbf{T}_i(s) = \begin{bmatrix} 1 + s^2L_iC & sL_i \\ sC & 1 \end{bmatrix} \quad i = 2, \dots, N-1 \tag{14}$$

$$\mathbf{T}_N(s) = \begin{bmatrix} 1 & sL_N + \frac{1}{sC} \\ 0 & 1 \end{bmatrix} \tag{15}$$

The total transmission matrix between input $(v'_{\text{pz}}, i'_{\text{pz}})$ and output (v_1, i_1) simply becomes

$$\mathbf{T}(s) = \left(\prod_{k=1}^{N} \mathbf{T}_k(s) \right) \mathbf{T}_{\text{pz}}(s) \tag{16}$$

IV. Analysis

We shall now make use of the models derived in the previous section to analyze the response and the power performance of the single mass and of the N-masses harmonic chain-based energy harvester, assuming that a given load is connected to the harvester.

Consider the two-port network closed on a load with impedance $Z_L(s)$, as shown in figure 5, where the transmission matrix $\mathbf{T}$, given by (11) or by (16), respectively, is rewritten in the general form[3]

$$\mathbf{T}(s) = \begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix} \tag{17}$$

[3]The transmission matrix is also known as $ABCD$ matrix.

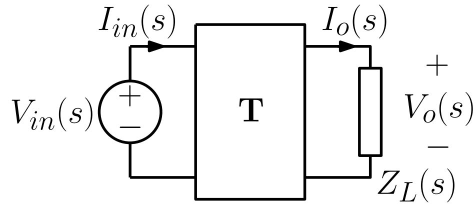

Fig. 5. Equivalent circuit for a N-modes piezoelectric energy harvester.

It is straightforward to derive the relevant transfer functions

$$Y_{\text{in}}(s) = \frac{I_{\text{in}}(s)}{V_{\text{in}}(s)} = \frac{C(s) + D(s)Y_L(s)}{A(s) + B(s)YL(s)} \tag{18}$$

$$H(s) = \frac{V_{\text{o}}(s)}{V_{\text{in}}(s)} = \frac{1}{A(s) + B(s)Y_L(s)} \tag{19}$$

where $Y_L(s) = Z_L(s)^{-1}$ is the load admittance.

It should now be clear the advantage of modelling random vibrations as the sum of harmonics presented in section II. In the frequency ($s = j\omega$, or phasor) domain, for the input voltage described by (4), the output voltage is

$$v_{\text{o}}(t) = \sum_{k=1}^{n} V_{\text{o},k}\cos(\omega_k t + \angle\hat{V}_{\text{o},k}) \tag{20}$$

where $V_{\text{o},k}$ is the amplitude of the k-th harmonic, and the phasors $\hat{V}_{\text{o},k}$ are given by

$$\hat{V}_{\text{o},k} = |H(\omega_k)|V_{\text{in},k}e^{j(\angle H(\omega_k) + \theta_k)} \tag{21}$$

The input current is

$$i_{\text{in}}(t) = \sum_{k=1}^{n} I_{\text{in},k}\cos(\omega_k t + \angle\hat{I}_{\text{in},k}) \tag{22}$$

where the phasors $\hat{I}_{\text{in},k}$ are

$$\hat{I}_{\text{in},k} = |Y_{\text{in}}(\omega_k)|V_{\text{in},k}e^{j(\angle Y_{\text{in}}(\omega_k) + \theta_k)} \tag{23}$$

Because frequencies ω_k are all different, the total average power is the sum of the average power at each single frequency. The average input power is therefore

$$P_{\text{in}} = \lim_{T\to+\infty}\frac{1}{T}\int_0^T v_{\text{in}}(t)\, i_{\text{in}}(t)dt \tag{24}$$

$$= \frac{1}{2}\sum_{k=1}^{n} V_{\text{in},k}^2|Y_{\text{in}}(\omega_k)|\cos(\angle Y_{\text{in}}(\omega_k)) \tag{25}$$

while the average output power reads

$$P_{\text{out}} = \lim_{T\to+\infty}\frac{1}{T}\int_0^T v_{\text{o}}(t)\, i_{\text{o}}(t)dt \tag{26}$$

$$= \frac{1}{2}\sum_{k=1}^{n} V_{\text{in},k}^2|H(\omega_k)|^2G_L(\omega_k) \tag{27}$$

where $G_L(\omega) = \text{Re}[Y_L(\omega)]$ is the load conductance. The power efficiency can finally be calculated as

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \tag{28}$$

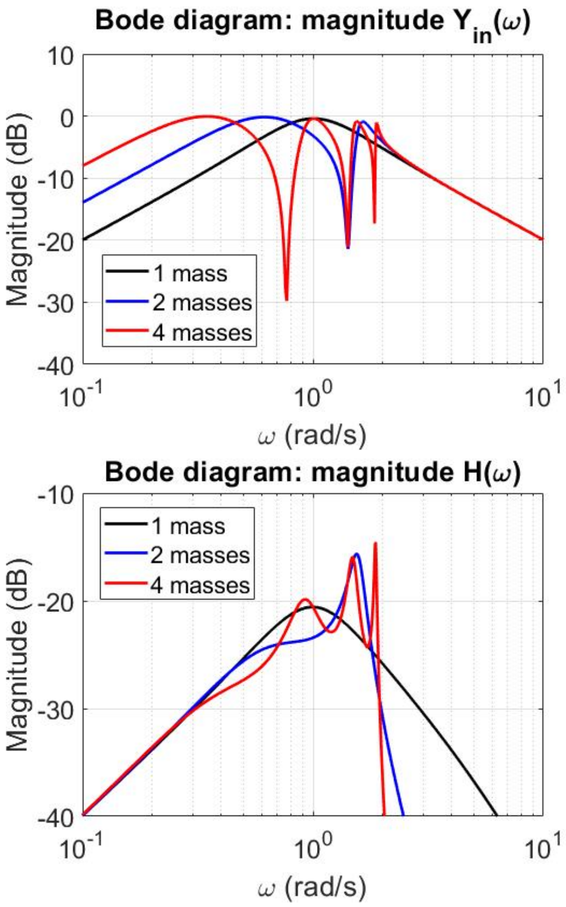

Fig. 6. Bode diagram for the magnitude of the input admittance $Y_{\text{in}}(\omega)$ (above) and the voltage ratio $H(\omega)$ (below), for the circuits shown in figures 3 and 4. For the sake of simplicity all parameters are normalized to 1, except $\alpha = 0.5$.

Figure 6 shows the Bode diagrams for the magnitude of the input admittance $Y_{\text{in}}(\omega)$ and the voltage ratio $H(\omega)$. As expected, the transfer function for the single mass harvester (black line) shows a typical band-pass behavior, with a well defined maximum at the resonant frequency $\omega_1 = 1/\sqrt{L_1 C_1}$. The harvester is very efficient when the energy of mechanical vibrations is concentrated in a narrow band centered at ω_1, but performance degrade rapidly outside the pass-band.

When several masses are connected to form the harmonic chain, the performance at the resonant frequency ω_1 gets slightly worse, and new additional resonant frequencies appear. If the energy of the mechanical vibrations is distributed over a wide frequency interval, showing peaks located at two or more frequencies, the harmonic chain may give better performance, provided that the masses are chosen in such a way that the additional resonant frequencies match the spectral components that convey more energy. The number of masses in the harmonic chain equals the number of resonant frequencies (counted with their multiplicity). Thus increasing the number of masses permits, in principle, to match a larger number of spectral components, increasing the total power scavenged by the harvester. Finally, it should be noted that, with respect to the single mass case, the transfer function $H(\omega)$ for the harmonic chain shows a steepest cutoff at high frequency. However, this is not really a limiting factor, as the power density for ambient random vibrations is mostly concentrated at relatively low frequencies.

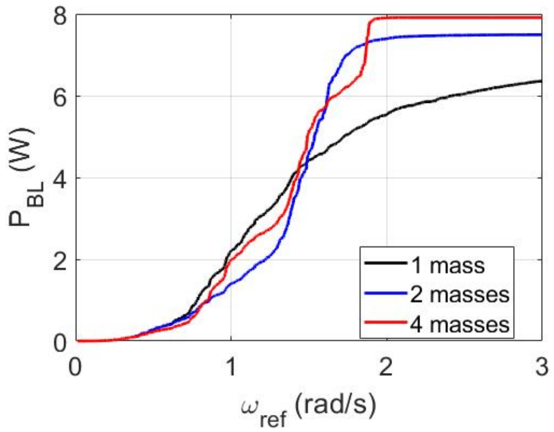

Fig. 7. Band limited power for different number of masses in the harmonic chain. Parameters are the same of figure 6.

Figure 7 shows the cumulative band limited power

$$P_{\text{BL}}(\omega_{\text{ref}}) = \sum_{\omega_k \in]0, \omega_{\text{ref}}]} P(\omega_k) \tag{29}$$

for different number of masses in the harmonic chain. It can be seen that, as expected, increasing the number of masses increases the harvested power, in particular at frequencies different from resonance. It should be noted that in this example, the harmonic chain performs better than the single mass harvester at frequencies higher than resonance. This is however a consequence of the choice made for the parameters. As it is shown in figure 6, the transfer function $H(\omega)$ for the harmonic chain shows peaks at frequencies higher than the resonant frequency. Choosing different values for the masses m_i (inductances L_i), the performance of the harmonic chain can be modified to match the spectrum of the forcing random vibrations. In this example, we assumed that the frequency spectrum of the mechanical vibration is uniformly distributed in the interval $\omega \in]0, 3]$ rad/s. Therefore, P_{BL} is constant for $\omega > 3$ rad/s, irrespective of the number of masses. Finally, our numerical simulations suggest that there are no particular advantages in increasing the number of masses beyond a certain value, at least for the parameters chosen in this example, inasmuch the total harvested power does not vary significantly.

V. Conclusions

In this paper we have proposed a cascade two-port electrical equivalent representation of a recently proposed model [15] for an energy harvester based on an harmonic chain of coupled masses, i.e. a chain of coupled harmonic oscillators, and a piezoelectric transducer for mechanical energy harvesting applications.

We have discussed how random mechanical vibrations can be modelled as a superposition of sinusoidal signals, with random amplitudes and phases. Exploiting the mechanical-electrical analogy, such a model corresponds to ideal voltage sources connected in series. We have also shown that the harmonic chain harvester with piezoelectric transducer can be described as a cascaded two-port network, and we have derived the transmission parameters for both the single mass and the N-masses harmonic chain. In the latter case, the model can be applied to chains with an arbitrary number of masses.

The equivalent voltage source model and the cascaded two-port network model are the ideal tool for the frequency domain analysis of the harvester. The relevant transfer functions have been derived in terms of the transmission parameters, and analytical expressions for the output voltage, output average power and power efficiency have been derived.

The effect of including more masses in the chain, with specific reference to the advantages and disadvantages of the harmonic chain harvesting approach, have been briefly discussed.

References

[1] S. Roundy, P. K. Wright, and J. M. Rabaey, *Energy scavenging for wireless sensor networks*. Springer, 2003.

[2] J. A. Paradiso and T. Starner, "Energy scavenging for mobile and wireless electronics," *IEEE Pervasive computing*, vol. 4, no. 1, pp. 18–27, 2005.

[3] S. P. Beeby, M. J. Tudor, and N. White, "Energy harvesting vibration sources for microsystems applications," *Measurement science and technology*, vol. 17, no. 12, p. R175, 2006.

[4] P. Mitcheson, E. Yeatman, G. Rao, A. Holmes, and T. Green, "Energy harvesting from human and machine motion for wireless electronic devices," *Proceedings of the IEEE*, vol. 96, no. 9, pp. 1457–1486, sep 2008.

[5] X. Lu, P. Wang, D. Niyato, D. I. Kim, and Z. Han, "Wireless networks with RF energy harvesting: A contemporary survey," *IEEE Communications Surveys & Tutorials*, vol. 17, no. 2, pp. 757–789, 2015.

[6] A. Khaligh, P. Zeng, and C. Zheng, "Kinetic energy harvesting using piezoelectric and electromagnetic technologies-state of the art," *IEEE transactions on industrial electronics*, vol. 57, no. 3, pp. 850–860, 2009.

[7] H. Vocca, I. Neri, F. Travasso, and L. Gammaitoni, "Kinetic energy harvesting with bistable oscillators," *Applied Energy*, vol. 97, pp. 771–776, 2012.

[8] X. Wen, W. Yang, Q. Jing, and Z. L. Wang, "Harvesting broadband kinetic impact energy from mechanical triggering/vibration and water waves," *ACS nano*, vol. 8, no. 7, pp. 7405–7412, 2014.

[9] Y. Fu, H. Ouyang, and R. B. Davis, "Nonlinear dynamics and tribo-electric energy harvesting from a three-degree-of-freedom vibro-impact oscillator," *Nonlinear Dynamics*, vol. 92, no. 4, pp. 1985–2004, 2018.

[10] M. F. Daqaq, R. S. Crespo, and S. Ha, "On the efficacy of charging a battery using a chaotic energy harvester," *Nonlinear Dynamics*, vol. 99, no. 2, pp. 1525–1537, 2020.

[11] S. R. Anton and H. A. Sodano, "A review of power harvesting using piezoelectric materials (2003–2006)," *Smart materials and Structures*, vol. 16, no. 3, p. R1, 2007.

[12] N. Sezer and M. Koç, "A comprehensive review on the state-of-the-art of piezoelectric energy harvesting," *Nano Energy*, vol. 80, p. 105567, feb 2021.

[13] M. Bonnin, F. L. Traversa, and F. Bonani, "Analysis of influence of nonlinearities and noise correlation time in a single-DOF energy-harvesting system via power balance description," *Nonlinear Dynamics*, vol. 100, no. 1, pp. 119–133, mar 2020.

[14] ——, "Leveraging circuit theory and nonlinear dynamics for the efficiency improvement of energy harvesting," *Nonlinear Dynamics*, vol. 104, no. 1, pp. 367–382, 2021.

[15] M. Romero-Bastida and J. M. López, "Efficient harmonic oscillator chain energy harvester driven by colored noise," *Scientific Reports*, vol. 10, no. 1, pp. 1–10, 2020.

[16] S. O. Rice, "Mathematical analysis of random noise," *The Bell System Technical Journal*, vol. 23, no. 3, pp. 282–332, 1944.

[17] M. Patzold, U. Killat, and F. Laue, "A deterministic digital simulation model for Suzuki processes with application to a shadowed Rayleigh land mobile radio channel," *IEEE Transactions on Vehicular Technology*, vol. 45, no. 2, pp. 318–331, 1996.

[18] "IEEE standard on piezoelectricity," 1988. [Online]. Available: https://ieeexplore.ieee.org/servlet/opac?punumber=2511

[19] S. Priya and D. J. Inman, *Energy harvesting technologies*. Springer, 2009, vol. 21.