

Superconductor in static gravitational, electric and magnetic fields with vortex lattice

Original

Superconductor in static gravitational, electric and magnetic fields with vortex lattice / Ummarino, G.A., Gallerati, A.. - In: RESULTS IN PHYSICS. - ISSN 2211-3797. - 30:(2021), p. 104838. [10.1016/j.rinp.2021.104838]

Availability:

This version is available at: 11583/2931660 since: 2021-10-15T10:59:42Z

Publisher:

Elsevier

Published

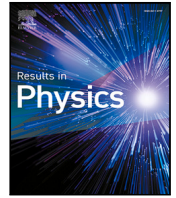
DOI:10.1016/j.rinp.2021.104838

Terms of use:

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

(Article begins on next page)



Superconductor in static gravitational, electric and magnetic fields with vortex lattice

Giovanni Alberto Ummarino^{a,b}, Antonio Gallerati^{a,c,*}

^a Politecnico di Torino, Dipartimento di Scienza Applicata e Tecnologia, Corso Duca degli Abruzzi 24, 10129 Torino, Italy

^b National Research Nuclear University MEPhI, Kashirskoe hwy 31, 115409 Moscow, Russia

^c Istituto Nazionale di Fisica Nucleare, Sezione di Torino, via Pietro Giuria 1, 10125 Torino, Italy

ARTICLE INFO

Keywords:

Gravitomagnetism
GravitoMaxwell
Ginzburg–Landau

ABSTRACT

We estimate the conjectured interaction between the Earth's gravitational field and a superconductor immersed in external, static electric and magnetic field. The latter is close to the sample upper critical field and generates the presence of a vortex lattice. The proposed interaction could lead to multiple, measurable effects. First of all, a local affection of the gravitational field inside the superconductor could take place. Second, a new component of a generalized electric field parallel to the superconductor surface is generated inside the sample.

The analysis is performed by using the time-dependent Ginzburg–Landau theory combined with the gravito-Maxwell formalism. This approach leads us to analytic solutions of the problem, also providing the average values of the generated fields and corrections inside the sample. We will also study which are the physical parameters to optimize and, in turn, the most suitable materials to maximize the effect.

Introduction

The possible interaction between superconductors and gravitational field is an intriguing field of research, providing an interesting connection between condensed matter systems and gravitational interaction, with beneficial effects both in theoretical and applied physics. The seminal paper [1] set the stage for a deeper analysis of the phenomenon, while, in the following years, a certain amount of scientific literature on the subject was produced [2–11]. The underlying idea behind this line of research is that, under certain conditions, a gravity/supercondensate interplay should exist, resulting in a slight affection of the local gravitational field through the interaction with suitable condensate systems. Finally, in 1992 Podkletnov and Nieminen proposed a laboratory experimental configuration to detect the conjectured mutual interplay [12, 13]. Due to the complexity of the experimental setting and the high costs involved, the experiment was then repeated only in simplified versions [14], without achieving conclusive results.

After the above experiments, a series of theoretical explanations have been proposed in the literature [15–18], including the first and most convincing interpretation [19,20] given in terms of a coupling of the gravitational field with an unconventional state of matter (superfluid) in a quantum gravity framework. In all these works, the contribution of the superfluid is included in the energy momentum tensor, exploiting the formalism of general relativity. Unfortunately, this approach does not allow to extrapolate quantitative predictions

to be put in relation with possible laboratory experiments: one is then led to consider alternative, phenomenological evidences to better understand the proposed interplay.

Parallel to DeWitt (and subsequent) studies about the coupling between supercondensates and gravity, other theoretical [21–25] and experimental [26–28] researches were conducted about generalized electric-type fields induced in (super)conductors by the presence of the Earth's weak gravitational field. The main result of those studies was the introduction of a generalized electric-type field, characterized by an electrical component and a gravitational one, leading to detectable corrections to the free fall of charged particles. These results then led to the definition of a set of generalized, fundamental fields featuring both gravitational and electromagnetic components [29–34]. This gravito-Maxwell formalism, valid in the weak gravity regime, is particularly suitable for treating the behaviour of a superfluid immersed in the Earth's gravitational field [35–38].

In this paper we again use the time-dependent Ginzburg Landau theory combined with the gravito-Maxwell formulation. With respect to our previous analyses, this time we also consider the presence of external static electric and magnetic fields. In particular, the magnetic field value is very close to the critical magnetic field of the superconductor, and its presence also determines the presence of a vortex lattice. These new ingredients can lead to an enhancement of the interaction

* Corresponding author at: Politecnico di Torino, Dipartimento di Scienza Applicata e Tecnologia, Corso Duca degli Abruzzi 24, 10129 Torino, Italy.
E-mail addresses: giovanni.ummarino@polito.it (G.A. Ummarino), antonio.gallerati@polito.it (A. Gallerati).

with the gravitational field, once chosen appropriate sample parameters and geometry.

Gravito-Maxwell formalism

Here we consider a weak gravitational background, where the (nearly-flat) spacetime metric $g_{\mu\nu}$ is expressed as

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}, \quad (1)$$

with $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$ and where $h_{\mu\nu}$ is a small perturbation of the flat Minkowski spacetime¹. If we now introduce the symmetric traceless tensor

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h, \quad (2)$$

with $h = h^\sigma_\sigma$, it can be shown that the Einstein equations in the harmonic De Donder gauge $\partial^\mu \bar{h}_{\mu\nu} \simeq 0$ can be rewritten, in linear approximation, as [33,35–38]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \partial^\rho \mathcal{G}_{\mu\nu\rho} = 8\pi G T_{\mu\nu}, \quad (3)$$

having also defined the tensor

$$\mathcal{G}_{\mu\nu\rho} \equiv \partial_{[\nu} \bar{h}_{\rho]\mu} + \partial^\sigma \eta_{\mu[\rho} \bar{h}_{\nu]\sigma} \simeq \partial_{[\nu} \bar{h}_{\rho]\mu}. \quad (4)$$

Gravito-Maxwell formulation

We then introduce the fields [29,33]

$$\mathbf{E}_g = -\frac{1}{2} \mathcal{G}_{00i} = -\frac{1}{2} \partial_{[0} \bar{h}_{i]0}, \quad \mathbf{A}_g = \frac{1}{4} \bar{h}_{0i}, \quad \mathbf{B}_g = \frac{1}{4} \varepsilon_i^{jk} \mathcal{G}_{0jk}, \quad (5)$$

for which we get, restoring physical units, the set of equations [29,31–42]

$$\begin{aligned} \nabla \cdot \mathbf{E}_g &= 4\pi G \rho_g, & \nabla \cdot \mathbf{B}_g &= 0, \\ \nabla \times \mathbf{E}_g &= -\frac{\partial \mathbf{B}_g}{\partial t}, & \nabla \times \mathbf{B}_g &= 4\pi G \frac{1}{c^2} \mathbf{j}_g + \frac{1}{c^2} \frac{\partial \mathbf{E}_g}{\partial t}, \end{aligned} \quad (6)$$

having defined the mass density $\rho_g \equiv -T_{00}$ and the mass current density $\mathbf{j}_g \equiv T_{0i}$. The above equations have the same formal structure of the Maxwell equations, with \mathbf{E}_g and \mathbf{B}_g gravitoelectric and gravitomagnetic field, respectively.

Generalized fields and equations

Now we introduce generalized electric/magnetic fields, scalar and vector potentials, featuring both electromagnetic and gravitational contributions [29,30,33,35]:

$$\mathbf{E} = \mathbf{E}_e + \frac{m}{e} \mathbf{E}_g, \quad \mathbf{B} = \mathbf{B}_e + \frac{m}{e} \mathbf{B}_g, \quad \phi = \phi_e + \frac{m}{e} \phi_g, \quad \mathbf{A} = \mathbf{A}_e + \frac{m}{e} \mathbf{A}_g, \quad (7)$$

where m and e identify the mass and electronic charge, respectively. The generalized Maxwell equations for the new fields read [29,33–38]:

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \left(\frac{1}{\varepsilon_0} + \frac{1}{\varepsilon_g} \right) \rho, & \nabla \cdot \mathbf{B} &= 0, \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, & \nabla \times \mathbf{B} &= (\mu_0 + \mu_g) \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}, \end{aligned} \quad (8)$$

where ε_0 and μ_0 are the vacuum electric permittivity and magnetic permeability. In the above equations, ρ and \mathbf{j} identify the electric charge density and electric current density, respectively, while the mass density and the mass current density vector can be expressed in terms of the latter as

$$\rho_g = \frac{m}{e} \rho, \quad \mathbf{j}_g = \frac{m}{e} \mathbf{j}, \quad (9)$$

while the vacuum gravitational permittivity ε_g and permeability μ_g read

$$\varepsilon_g = \frac{1}{4\pi G} \frac{e^2}{m^2}, \quad \mu_g = \frac{4\pi G}{c^2} \frac{m^2}{e^2}. \quad (10)$$

The model

Time-dependent Ginzburg–Landau formulation

The time-dependent Ginzburg–Landau equations (TDGL) can be written as [43–50]:

$$-\frac{\hbar^2}{2m_\star} \left(\nabla + \frac{2ie}{\hbar} \mathbf{A} \right)^2 \psi + a\psi + b|\psi|^2\psi = -\frac{\hbar^2}{2m_\star D} \left(\frac{\partial}{\partial t} + \frac{2ie}{\hbar} \phi \right) \psi, \quad (11.i)$$

$$\nabla \times \nabla \times \mathbf{A} - \nabla \times \mathbf{B}_0 = \mu_0 (\mathbf{j}_n + \mathbf{j}_s), \quad (11.ii)$$

where \mathbf{B}_0 is the external magnetic field, D is the diffusion coefficient, σ the conductivity in the normal phase and

$$a = a(T) = a_0(T - T_c), \quad b = b(T_c), \quad (12)$$

a_0 , b being positive constants. The contributions related to the normal current and supercurrent densities can be explicitly written as

$$\begin{aligned} \mathbf{j}_n &= -\sigma \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla \phi \right), \\ \mathbf{j}_s &= -i \hbar \frac{e}{m_\star} (\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{4e^2}{m_\star} |\psi|^2 \mathbf{A}. \end{aligned} \quad (13)$$

Let us put ourselves in the London gauge $\nabla \cdot \mathbf{A} = 0$ so that

$$\nabla^2 \mathbf{A} = -\mu_0 (\mathbf{j}_n + \mathbf{j}_s). \quad (14)$$

We now consider a sample of thickness L along the (vertical) \vec{u}_x direction and very large dimensions along \vec{u}_y, \vec{u}_z , see Fig. 1. The sample features a square lattice of vortices, whose axes are directed along the direction of the static magnetic field \mathbf{B}_0 that we choose to be

$$\mathbf{B}_0 = B_0 \vec{u}_z, \quad (15)$$

together with a vector potential \mathbf{A} of the form

$$\mathbf{A} = B_0 x \vec{u}_y. \quad (16)$$

We then consider the presence of a constant external (standard) electric field $\mathbf{E}_0^{(e)}$ along the \vec{u}_x direction, so that a *generalized* static field \mathbf{E}_0 can be expressed as

$$\mathbf{E}_0 = \mathbf{E}_0^{(e)} + \mathbf{E}_0^{(g)} = \left(E_0^{(e)} - E_0^{(g)} \right) \vec{u}_x = \left(E_0^{(e)} - \frac{m}{e} g \right) \vec{u}_x = E_0 \vec{u}_x, \quad (17)$$

with a related scalar potential $\phi_0 = -E_0 x$.

The boundary and initial conditions are

$$\left. \begin{aligned} (\nabla \psi + \frac{2ie}{\hbar} \mathbf{A} \psi) \cdot \mathbf{n} &= 0 \\ \nabla \times \mathbf{A} \cdot \mathbf{n} &= \mathbf{B}_0 \cdot \mathbf{n} \\ \mathbf{A} \cdot \mathbf{n} &= 0 \end{aligned} \right\} \text{on } \partial\Omega \times (0, t), \quad \left. \begin{aligned} \psi(x, 0) &= \psi_0(x) \\ \mathbf{A}(x, 0) &= \mathbf{A}_0(x) \end{aligned} \right\} \text{on } \Omega, \quad (18)$$

where $\partial\Omega$ is the boundary of a smooth and simply connected domain in \mathbb{R}^N . We denote by \mathbf{A}_0 the external vector potential, coinciding with the internal value for $t \leq 0$, i.e. when the sample is in the normal state and the material is very weakly diamagnetic. For $t < 0$ we also have $T < T_c$ and $B > B_{c2}$, while at $t = 0$ we still have $T < T_c$ but $B \simeq B_{c2}$.

If we put ourselves very close to B_{c2} , it is possible to find a solution of the linearized TDGL equations [51,52] for the order parameter of the form

$$\psi(x, y, t) = \sum_{n=-\infty}^{\infty} c_n \exp\left(i q n \left(y + \frac{E_0}{B_0} t\right)\right) \exp\left(-\frac{1}{2\xi(T)} \left(x - \frac{\hbar q n}{2e B_0}\right)^2\right) +$$

¹ Here we work in the ‘mostly plus’ convention and natural units $c = \hbar = 1$

$$+ i \frac{e E_0 \xi^2(T)}{\hbar D} \left(x - \frac{\hbar q n}{2 e B_0} \right). \quad (19)$$

Since we are very close to B_{c_2} , this solution of the linearized TDGL equations describes the behaviour of an ordered vortex lattice, moving under the influence of the external electric field.^{2,3} For a square lattice, q expresses the distance between adjacent vortices [54]

$$q \simeq \frac{2\pi}{\xi(T)}, \quad (20)$$

and we can also replace the general c_n coefficient with the correspondent c_{\square} expression for the square lattice:

$$c_n \rightarrow c_{\square} = \frac{2\sqrt{2}\pi}{\xi^2(T)}, \quad (21)$$

the c_{\square} coefficients being then independent of n .

Dimensionless TDGL. In order to write Eqs. (11) in a dimensionless form, the following expressions are introduced:

$$\begin{aligned} \psi_0^2(T) &= \frac{|a(T)|}{b}, & \xi(T) &= \frac{\hbar}{\sqrt{2} m_{\star} |a(T)|}, \\ \lambda(T) &= \sqrt{\frac{b m_{\star}}{4\mu_0 |a(T)| e^2}}, & \kappa &= \frac{\lambda(T)}{\xi(T)}, \\ \tau(T) &= \frac{\lambda^2(T)}{D}, & \eta &= \mu_0 \sigma D, \\ B_c(T) &= \sqrt{\frac{\mu_0 |a(T)|^2}{b}} = \frac{\hbar}{2\sqrt{2} e \lambda(T) \xi(T)}, \end{aligned} \quad (22)$$

where $\lambda(T)$, $\xi(T)$ and $H_c(T)$ are the penetration depth, coherence length and thermodynamic critical field, respectively. We also define the dimensionless quantities

$$t' = \frac{t}{\tau}, \quad x' = \frac{x}{\lambda}, \quad y' = \frac{y}{\lambda}, \quad x'_0 = \frac{x_0}{\lambda}, \quad \psi' = \frac{\psi}{\psi_0}, \quad (23)$$

and the dimensionless fields can be expressed as:

$$\mathbf{A}' = \frac{\mathbf{A} \kappa}{\sqrt{2} B_c \lambda}, \quad \phi' = \frac{\phi \kappa}{\sqrt{2} B_c D}, \quad \mathbf{E}' = \frac{\mathbf{E} \lambda \kappa}{\sqrt{(2)} B_c D}, \quad \mathbf{B}' = \frac{\mathbf{B} \kappa}{\sqrt{2} B_c}. \quad (24)$$

We now consider the dimensionless version of the London gauge, $\nabla \cdot \mathbf{A}' = 0$. Inserting Eqs. (23) and (24) in Eqs. (11), provides the dimensionless TDGL equations (we drop the primes for the sake of notational simplicity) in a bounded, smooth and simply connected domain in \mathbb{R}^N [43,44]:

$$\frac{\partial \psi}{\partial t} + i \phi \psi + \kappa^2 (|\psi|^2 - 1) \psi + (i \nabla + \mathbf{A})^2 \psi = 0, \quad (25.i)$$

$$\nabla \times \nabla \times \mathbf{A} \nabla \times \mathbf{B}_0 := -\eta \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla \phi \right) - \frac{i}{2\kappa} (\psi^* \nabla \psi - \psi \nabla \psi^*) - |\psi|^2 \mathbf{A}, \quad (25.ii)$$

and the boundary and initial conditions (18) become, in the dimensionless form

$$\left. \begin{aligned} (\nabla \psi + i \mathbf{A} \psi) \cdot \mathbf{n} &= 0 \\ \nabla \times \mathbf{A} \cdot \mathbf{n} &= \mathbf{B}_0 \cdot \mathbf{n} \\ \mathbf{A} \cdot \mathbf{n} &= 0 \end{aligned} \right\} \text{on } \partial \Omega \times (0, t); \quad \left. \begin{aligned} \psi(x, 0) &= \psi_0(x) \\ \mathbf{A}(x, 0) &= \mathbf{A}_0(x) \end{aligned} \right\} \text{on } \Omega.$$

² We should also note that, although this is an exact solution just below B_{c_2} , this formula does not necessarily hold for different values of the magnetic field, such as the lower critical field B_{c_1} ; indeed, close to B_{c_2} the vortices are very close to each other, at distances of the order the coherence length $\xi(T)$ (they are so densely packed that their cores are essentially touching [53]).

³ The motion of the vortices under the influence of the external electric field causes dissipative phenomena even in the superconducting state; one way to avoid the effect is to add defects to anchor the vortices (vortex pinning), in order to reduce or eliminate energy dissipation [51].

Solving dimensionless TDGL

We now discuss how to get an analytic approximate solution to the above dimensionless TDGL.

First of all, we write the following first-order expression for the dimensionless order parameter:

$$\begin{aligned} \psi(x, y, t) &= \sum_{n=-\infty}^{\infty} |c_n| \exp \left(i q n \left(y + \frac{E_0}{B_0} t \right) \right) \\ &\exp \left(-\frac{\kappa^2}{2} (x - n x_0)^2 + i \frac{E_0}{\kappa} (x - n x_0) \right), \end{aligned} \quad (27)$$

with

$$|\psi|^2 = \sum_{n=-\infty}^{\infty} |c_n|^2 \exp \left(-\kappa^2 (x - n x_0)^2 \right). \quad (28)$$

The equations for the vector potential components can be written as

$$\begin{aligned} \frac{\partial^2 A_x(x, t)}{\partial x^2} &= \eta \left(\frac{\partial A_x(x, t)}{\partial t} - E_0 \right) \\ &+ \left(A_x(x, t) - \frac{E_0}{\kappa} \right) \sum_{n=-\infty}^{\infty} |c_n|^2 \exp \left(-\kappa^2 (x - n x_0)^2 \right), \\ \frac{\partial^2 A_y(x, t)}{\partial x^2} &= \eta \frac{\partial A_y(x, t)}{\partial t} \\ &+ \sum_{n=-\infty}^{\infty} (A_y(x, t) - 2\pi \kappa n) |c_n|^2 \exp \left(-\kappa^2 (x - n x_0)^2 \right), \\ \frac{\partial^2 A_z(x, t)}{\partial x^2} &= \eta \frac{\partial A_z(x, t)}{\partial t} + \sum_{n=-\infty}^{\infty} |c_n|^2 \exp \left(-\kappa^2 (x - n x_0)^2 \right). \end{aligned} \quad (29)$$

We now want to obtain explicit solutions for the dimensionless order parameter at first order in E_0 . To this end, we have to estimate the summations

$$\sum_{n=-\infty}^{\infty} |c_n|^2 \exp \left(-\kappa^2 (x - n x_0)^2 \right), \quad \sum_{n=-\infty}^{\infty} n |c_n|^2 \exp \left(-\kappa^2 (x - n x_0)^2 \right). \quad (30)$$

Since we are considering a square vortex lattice, we can replace the general coefficients c_n with c_{\square} that, for the case under consideration, can be expressed as [54]

$$c_{\square}^2 = 2 \sqrt{2} \pi \kappa^2, \quad (31)$$

being then a function of κ only. Moreover, we are interested in high- T_c superconductors, so that we also have large values for the κ parameter, $\kappa^2 \gtrsim 10^4$. This gives us the following estimates for the above (30):

$$\begin{aligned} \sum_{n=-\infty}^{\infty} |c_n|^2 \exp \left(-\kappa^2 (x - n x_0)^2 \right) &= c_{\square}^2 e^{-\kappa^2 x^2} \sum_{n=-\infty}^{\infty} e^{-\kappa^2 n^2 x_0^2} e^{2\kappa^2 x n x_0} \\ &\simeq c_{\square}^2 e^{-\kappa^2 x^2}, \\ \sum_{n=-\infty}^{\infty} n |c_n|^2 \exp \left(-\kappa^2 (x - n x_0)^2 \right) &\simeq 0, \end{aligned} \quad (32)$$

where only the $n = 0$ term gives a non negligible contribution in the first summation. Eqs. (29) can be rewritten

$$\begin{aligned} \frac{\partial^2 A_x(x, t)}{\partial x^2} &= \eta \left(\frac{\partial A_x(x, t)}{\partial t} - E_0 \right) + \left(A_x(x, t) - \frac{E_0}{\kappa} \right) c_{\square}^2 e^{-\kappa^2 x^2}, \\ \frac{\partial^2 A_y(x, t)}{\partial x^2} &= \eta \frac{\partial A_y(x, t)}{\partial t} + A_y(x, t) c_{\square}^2 e^{-\kappa^2 x^2}, \\ \frac{\partial^2 A_z(x, t)}{\partial x^2} &= \eta \frac{\partial A_z(x, t)}{\partial t} + c_{\square}^2 e^{-\kappa^2 x^2}. \end{aligned} \quad (33)$$

Since we are dealing with materials featuring a very large value for the κ parameter, $\kappa^2 \geq 10^4$, we can also use the approximation

$$e^{-\kappa^2 x^2} \simeq \frac{\sqrt{\pi}}{\kappa} \delta(x), \quad (34)$$

and our equation for the vector potential components become

$$\frac{\partial A_x(x, t)}{\partial t} \simeq \frac{1}{\eta} \frac{\partial^2 A_x(x, t)}{\partial x^2} - \left(A_x(x, t) - \frac{E_0}{\kappa} \right) c^2 \frac{\sqrt{\pi}}{\eta \kappa} \delta(x) + E_0, \quad (35.i)$$

$$\frac{\partial A_y(x, t)}{\partial t} \simeq \frac{1}{\eta} \frac{\partial^2 A_y(x, t)}{\partial x^2} - A_y(x, t) c^2 \frac{\sqrt{\pi}}{\eta \kappa} \delta(x), \quad (35.ii)$$

$$\frac{\partial A_z(x, t)}{\partial t} \simeq \frac{1}{\eta} \frac{\partial^2 A_z(x, t)}{\partial x^2} - c^2 \frac{\sqrt{\pi}}{\eta \kappa} \delta(x). \quad (35.iii)$$

The initial conditions for the vector potential components are:

$$A_x(x, 0) = 0, \quad A_y(x, 0) = B_0 x, \quad A_z(x, 0) = 0, \quad (36)$$

and the generalized electric field \mathbf{E} inside the superconductor can be obtained from

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi. \quad (37)$$

Averaging over space. Let us now study the average effects of the generalized electric field inside the superconductor. To this end, we integrate the vector potential components over the x -variable [55].

First, we integrate over x Eq. (35.iii), for an interval $x \in [-L/2, L/2]$.⁴ This gives

$$\frac{\partial \bar{A}_z(t)}{\partial t} = -c^2 \frac{\sqrt{\pi}}{\eta \kappa L}, \quad (38)$$

having defined the averaged component $\bar{A}_z(t) = \frac{1}{L} \int_{-L/2}^{L/2} dx A_z(x, t)$ and having used symmetric conditions for the first derivatives with respect to x . The above equation is easily solved as

$$\bar{A}_z(t) = -c^2 \frac{\sqrt{\pi}}{\eta \kappa L} t + \bar{A}_z(0) = -c^2 \frac{\sqrt{\pi}}{\eta \kappa L} t, \quad (39)$$

having used initial conditions (36) to set $\bar{A}_z(0) = 0$. This in turn gives for the averaged, generalized electric field \bar{E}_z component

$$\bar{E}_z = c^2 \frac{\sqrt{\pi}}{\eta \kappa L} = \frac{2\sqrt{2}\pi\kappa}{\eta L}. \quad (40)$$

The averaged differential equation for the $\bar{A}_y(t)$ component is obtained from (35.ii) and reads

$$\frac{\partial \bar{A}_y(t)}{\partial t} = -\bar{A}_y(t) c^2 \frac{\sqrt{\pi}}{\eta \kappa L}, \quad (41)$$

having used the approximation $A_y(0, t) \simeq \bar{A}_y(t)$. The above equation would give for the averaged component

$$\bar{A}_y(t) = \bar{A}_y(0) \exp\left(-c^2 \frac{\sqrt{\pi}}{\eta \kappa L} t\right) = 0, \quad (42)$$

using again initial condition (36), the resulting field $\bar{E}_y(t)$ being then zero.

Finally, for the vertical component we have from (35.i)

$$\frac{\partial \bar{A}_x(t)}{\partial t} = -\left(\frac{\bar{A}_x(t)}{L} - \frac{E_0}{\kappa}\right) c^2 \frac{\sqrt{\pi}}{\eta \kappa} + E_0, \quad (43)$$

⁴ We emphasize that we have dropped the primes for the sake of notational simplicity, so here both L and x correspond to the dimensionless L' and x' of (23); in particular, here one would explicitly have for the dimensionless thickness $L' = L/\lambda$, L being the physical thickness and λ the penetration depth.

that is solved, always using the approximation $A_x(0, t) \simeq \bar{A}_x(t)$, for

$$\begin{aligned} \bar{A}_x(t) &= \bar{A}_x(0) \exp\left(-c^2 \frac{\sqrt{\pi}}{\eta \kappa L} t\right) + E_0 \left(\frac{L}{\kappa} + \frac{\eta \kappa L}{c^2 \sqrt{\pi}}\right) \\ &\times \left(1 - \exp\left(-c^2 \frac{\sqrt{\pi}}{\eta \kappa L} t\right)\right) = \\ &= E_0 \left(\frac{L}{\kappa} + \frac{\eta \kappa L}{c^2 \sqrt{\pi}}\right) \left(1 - \exp\left(-c^2 \frac{\sqrt{\pi}}{\eta \kappa L} t\right)\right) \end{aligned} \quad (44)$$

having again used conditions (36). We finally get for the averaged $E_x(t)$ component of the generalized electric field

$$\begin{aligned} \bar{E}_x(t) &= E_0 - E_0 \left(\frac{L}{\kappa} + \frac{\eta \kappa L}{c^2 \sqrt{\pi}}\right) c^2 \frac{\sqrt{\pi}}{\eta \kappa L} \exp\left(-c^2 \frac{\sqrt{\pi}}{\eta \kappa L} t\right) = \\ &= E_0 - E_0 \left(\frac{L}{\kappa} + \frac{\eta L}{2\sqrt{2}\pi\kappa}\right) \frac{2\sqrt{2}\pi\kappa}{\eta L} \exp\left(-\frac{2\sqrt{2}\pi\kappa}{\eta L} t\right). \end{aligned} \quad (45)$$

Discussion

The solution of the differential equations for the vector potential gives rise to two very interesting situations. The first one is connected with a non-zero value of the generalized E_z component, determining the emergence of a new, detectable contribution along the direction of the magnetic field. The value of this new electric field is expressed, in dimensional units, as

$$E_z = \frac{4 B_c(T) D}{\eta L}. \quad (46)$$

For an YBCO sample at a temperature $T = 85$ K, this would correspond to a value $E_z = 32$ V/m.

The second remarkable effect is related to the expected variation of the gravitational field along the x direction inside the superconductor. In fact, it is possible to see from Eq. (45) and from Fig. 2 that the field is reduced with respect to its external value. Moreover, in analogy to what we found in [35], for very short time scales ($\sim 10^{-12}$ s), the gravitational field seems to change sign: this could only happen in absence of suitable physical cutoffs preventing arbitrary growth of instabilities, giving rise to negative values [19]. It can also be noted that it is possible to reach observable effects of gravitational field affection in time scales of the order of 10^{-9} s; in principle, when dealing with samples of larger thickness and at temperatures very close to T_c , observation times of the order of microseconds could be reached.

In order to measure the effect it is necessary to determine suitable sample dimensions and chemical composition. In fact, large dimensions of the sample give rise to an increase for the time scales in which the effect occurs, combined with a decreasing intensity of the phenomenon. At the same time, large values of the η parameter (related to the sample characteristics) determines similar effect, that is, a reduced intensity of the correction together with an increase of the observation times. In this regard, the L and η parameters act in the same way: disordered materials (small η values, i.e. bad conductors in the normal state) with negligible thickness dimension (small L values) give rise to larger effects for a shorter time⁵. On the contrary, large dimensions materials with good conductivity in the normal state (large η), would exhibit a weaker effect for a longer time scale. All these observations are in agreement with what we found in [35].

In Table 1 we report typical values for the physical parameters of two common high- T_c cuprates, YBCO and BSCCO [33,56–58]. We also point out that the presence of disorder also determines an increase of the λ penetration depth and, being the square of the latter proportional

⁵ This would be the case, for example, of superconducting films with high disorder

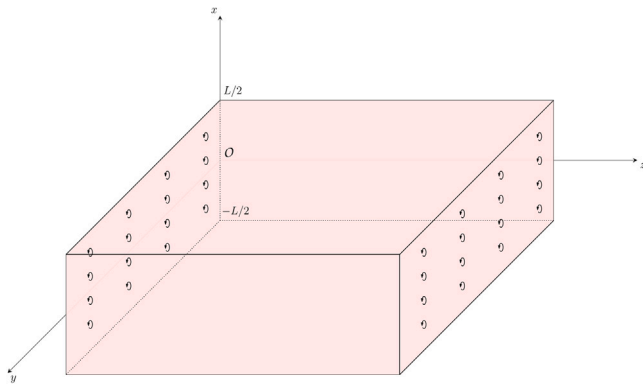


Fig. 1. Physical setup.

to the time scale ($\tau \propto \lambda^2$), the duration of the phenomenon increases with beneficial effects for direct measurements. Finally, if we put ourselves at temperatures very close to T_c , we again find an increase in the λ values together with larger time scales; in the latter case, however, the effects of thermal fluctuations should also be considered [59].

We expect that experimental issues would reside in the very short observation time. Since even detecting a reduced effect would be a remarkable result, it would be better to privilege long time scales setup, rather than configurations leading to larger effects of affection of the gravitational field. The latter would in fact imply strong difficulties in the measurements, since they would manifest themselves only for very short time scales. In light of these considerations, we suggest to consider big samples featuring large η values.

Concluding remarks

A deeper interweaving between condensed matter and gravitational theories has already proven to be a powerful tool for inspecting many aspects of fundamental physics [60–68], while also providing new insights into related unresolved issues.⁶

In this paper we have exploited a multidisciplinary approach to describe a gravity/superfluid interplay, in a specific physical setup involving the presence of external electric and magnetic fields, which in turn determine the presence of a vortex lattice. This situation could induce enhanced effects in the proposed interplay, leading to a non-negligible local affection of the gravitational field within the condensate, together with a predicted electric field generated inside the sample, parallel to the superconductor plane. The experimental verification of the emergence of this new component would result in a great step forward in the study of the interaction of the gravitational field with quantum condensates, opening new and unsuspected horizons both in the theoretical and applicative fields.

CRediT authorship contribution statement

Giovanni Alberto Ummarino: Conceptualization, Methodology, Formal analysis, Writing – original draft. **Antonio Gallerati:** Conceptualization, Methodology, Formal analysis, Writing – original draft.

⁶ See for example ‘analogue gravity’ techniques exploiting a bottom-up formulation for condensed matter systems featuring analogues gravitational effects [69–80], or top-down holographic approaches, where a substrate description comes from the geometric formulation of a suitable gravitational model [81,82].

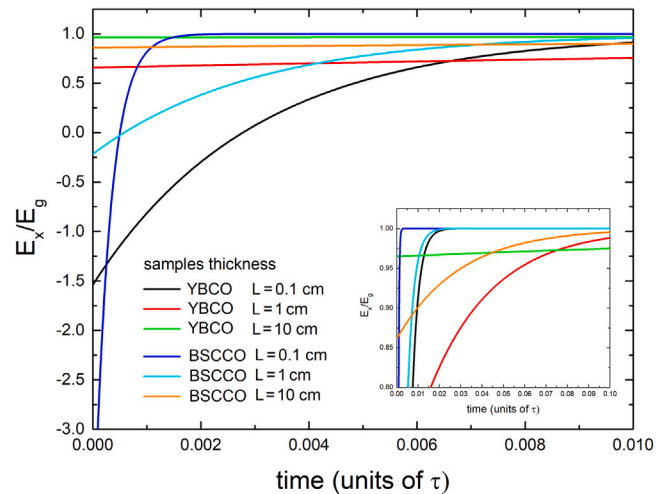


Fig. 2. Variation of gravitational field as a function of time inside superconductive samples of YBCO (black, red and green solid line) and BSCCO (dark blue, blue and orange solid line) for different thickness values ($L = 0.1$ cm, $L = 1$ cm, $L = 10$ cm). In the inset, the same variation is calculated for a larger time range. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)


Table 1
YBCO vs. BSCCO [33,56–58].

	YBCO	BSCCO
T_c	89 K	107 K
T_*	85 K	102 K
$\xi(T_*)$	$4.2 \cdot 10^{-9}$ m	$23 \cdot 10^{-10}$ m
$\lambda(T_*)$	$3.947 \cdot 10^{-7}$ m	$5.573 \cdot 10^{-7}$ m
σ^{-1}	$4.0 \cdot 10^{-7} \Omega \text{ m}^{(*)}$	$3.6 \cdot 10^{-6} \Omega \text{ m}^{(**)}$
$B_{c_2}(0)$	61 Tesla	113 Tesla
$B_{c_2}(T_*)$	6 Tesla	11 Tesla
$B_c(T_*)$	0.25 Tesla	0.32 Tesla
$\lambda(0)$	$1.7 \cdot 10^{-7}$ m	$2.4 \cdot 10^{-7}$ m
$\xi(0)$	$1.8 \cdot 10^{-9}$ m	$10 \cdot 10^{-10}$ m
κ	94.4	240.0
$\tau(T_*)$	$5 \cdot 10^{-10}$ s	$3 \cdot 10^{-11}$ s
η	$1.0 \cdot 10^{-3}$	$3.5 \cdot 10^{-4}$
g_*	$1.8 \cdot 10^{-11}$	$1.7 \cdot 10^{-11}$
D	$3.2 \cdot 10^{-4} \text{ m}^2/\text{s}$	$1.0 \cdot 10^{-3} \text{ m}^2/\text{s}$
	^(*) $T = 90$ K	^(**) $T = 108$ K

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

G.A. Ummarino acknowledges partial support from the MEPH. He also thank Fondazione CRT  that partially supported this work for A. Gallerati.

References

- [1] DeWitt Bryce S. Superconductors and gravitational drag. Phys Rev Lett 1966;16:1092–3. <http://dx.doi.org/10.1103/PhysRevLett.16.1092>.
- [2] Papini G. Detection of inertial effects with superconducting interferometers. Phys Lett A 1967;24(1):32–3.
- [3] Papini G. Superconducting and normal metals as detectors of gravitational waves. Lett Nuovo Cimento 1970;4S1:1027–32. <http://dx.doi.org/10.1007/BF02755267>.
- [4] Hirakawa H. Superconductors in gravitational field. Phys Lett A 1975;53(5):395–6.
- [5] Anandan J. Relativistic thermoelectromagnetic gravitational effects in normal conductors and superconductors. Phys Lett A 1984;105(6):280–4.

- [6] Ross DK. The London equations for superconductors in a gravitational field. *J Phys A: Math Gen* 1983;16(6):1331.
- [7] Dinariev O Yu, Mosolov AB. A relativistic effect in the theory of superconductivity. *Dokl. Akad. Nauk SSSR* 1987;295:98.
- [8] Peng H. A new approach to studying local gravitomagnetic effects on a superconductor. *Gen Relativity Gravitation* 1990;22(6):609–17. <http://dx.doi.org/10.1007/bf00755981>.
- [9] Peng H, Torr DG, Hu EK, Peng B. Electrodynamics of moving superconductors and superconductors under the influence of external forces. *Phys Rev B* 1991;43(4):2700.
- [10] Peng H, Lind G, Chin YS. Interaction between gravity and moving superconductors. *Gen Relativity Gravitation* 1991;23(11):1231–50.
- [11] Li Ning, Torr DG. Effects of a gravitomagnetic field on pure superconductors. *Phys Rev D* 1991;43(2):457.
- [12] Podkletnov E, Nieminen R. A possibility of gravitational force shielding by bulk $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ superconductor. *Physica C: Superconductivity* 1992;203(3–4):441–4.
- [13] Podkletnov E. Weak gravitation shielding properties of composite bulk $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ superconductor below 70 K under EM field, *Cond-Mat/9701074*. 1997.
- [14] Hathaway G, Cleveland B, Bao Y. Gravity modification experiment using a rotating superconducting disk and radio frequency fields. *Physica C: Superconductivity* 2003;385(4):488.
- [15] Li Ning, Noever David, Robertson Tony, Koczor Ron, Brantley Whitt. Static test for a gravitational force coupled to type II YBCO superconductors. *Physica C: Superconductivity* 1997;281(2):260–7.
- [16] Modanese Giovanni. Local contribution of a quantum condensate to the vacuum energy density. *Modern Phys Lett A* 2003;18:683–90. <http://dx.doi.org/10.1142/S0217732303009812>, arXiv:gr-qc/0107073.
- [17] Modanese Giovanni. Possible quantum gravity effects in a charged Bose condensate under variable em field. *Phys. Essays* 2001;14:93–105. <http://dx.doi.org/10.4006/1.3025481>, arXiv:gr-qc/9612022.
- [18] Wu Ning. Gravitational shielding effects in gauge theory of gravity. *Commun Theor Phys* 2004;41:567–72. <http://dx.doi.org/10.1088/0253-6102/41/4/567>, arXiv:hep-th/0307225.
- [19] Modanese Giovanni. Theoretical analysis of a reported weak gravitational shielding effect. *Europhys Lett* 1996;35:413–8. <http://dx.doi.org/10.1209/epl/i1996-00129-8>, arXiv:hep-th/9505094.
- [20] Modanese Giovanni. Role of a ‘local’ cosmological constant in Euclidean quantum gravity. *Phys Rev D* 1996;54:5002–9. <http://dx.doi.org/10.1103/PhysRevD.54.5002>, arXiv:hep-th/9601160.
- [21] Schiff LI, Barnhill MV. Gravitation-induced electric field near a metal. *Phys Rev* 1966;151(4):1067.
- [22] Dessler AJ, Michel FC, Rorschach HE, Trammell GT. Gravitationally Induced Electric Fields in Conductors. *Phys Rev* 1968;168:737–43. <http://dx.doi.org/10.1103/PhysRev.168.737>.
- [23] Torr Douglas G, Li Ning. Gravitoelectric-electric coupling via superconductivity. *Found Phys Lett* 1993;6(4):371–83.
- [24] Mashhoon Bahram. Gravitoelectromagnetism. In: *Reference Frames and Gravitomagnetism*. Singapore: World Scientific Publishing; 2000, p. 121–32. <http://dx.doi.org/10.1142/4710>.
- [25] Mashhoon Bahram. Gravitoelectromagnetism: A Brief review. In: *The Measurement of Gravitomagnetism: A Challenging Enterprise*. New York, USA: Nova Science Publisher Inc; 2007, p. 29–40.
- [26] Witteborn FC, Fairbank WM. Experimental comparison of the gravitational force on freely falling electrons and metallic electrons. *Phys Rev Lett* 1967;19(18):1049.
- [27] Witteborn FC, Fairbank WM. Experiments to determine the force of gravity on single electrons and positrons. *Nature* 1968;220(5166):436–40.
- [28] Herring C. Gravitationally induced electric field near a conductor, and its relation to the surface-stress concept. *Phys Rev* 1968;171(5):1361.
- [29] Agop M, Buzea CGh, Nica P. Local gravitoelectromagnetic effects on a superconductor. *Physica C: Superconductivity* 2000;339(2):120–8.
- [30] Agop M, Ioannou PD, Diaconu F. Some implications of gravitational superconductivity. *Progr Theoret Phys* 2000;104(4):733–42.
- [31] Ruggiero Matteo Luca, Tartaglia Angelo. Gravitomagnetic effects. *Nuovo Cim B* 2002;117:743–68, arXiv:gr-qc/0207065.
- [32] Tartaglia Angelo, Ruggiero Matteo Luca. Gravitoelectromagnetism versus electromagnetism. *Eur J Phys* 2004;25:203–10. <http://dx.doi.org/10.1088/0143-0807/25/2/007>, arXiv:gr-qc/0311024.
- [33] Ummarino Giovanni Alberto, Gallerati Antonio. Superconductor in a weak static gravitational field. *Eur Phys J* 2017;C77(8):549. <http://dx.doi.org/10.1140/epjc/s10052-017-5116-y>, arXiv:1710.01267.
- [34] Behera Harihar. Comments on gravitoelectromagnetism of Ummarino and Gallerati in “Superconductor in a weak static gravitational field” vs other versions. *Eur Phys J* 2017;C77(12):822. <http://dx.doi.org/10.1140/epjc/s10052-017-5386-4>, arXiv:1709.04352.
- [35] Ummarino Giovanni Alberto, Gallerati Antonio. Exploiting weak field gravity-Maxwell symmetry in superconductive fluctuations regime. *Symmetry* 2019;11(11):1341. <http://dx.doi.org/10.3390/sym11111341>, arXiv:1910.13897.
- [36] Ummarino Giovanni Alberto, Gallerati Antonio. Josephson AC effect induced by weak gravitational field. *Classical Quantum Gravity* 2020;37(21):217001. <http://dx.doi.org/10.1088/1361-6382/abb57b>, arXiv:2009.04967.
- [37] Ummarino GA, Gallerati A. Possible alterations of local gravitational field inside a superconductor. *Entropy* 2021;23(2):193. <http://dx.doi.org/10.3390/e23020193>, arXiv:2102.01489.
- [38] Gallerati Antonio. Local affection of weak gravitational field from supercondensates. *Phys Scripta* 2021;96(6):064001. <http://dx.doi.org/10.1088/1402-4896/abeae2>.
- [39] Vieira RS, Brentan HB. Covariant theory of gravitation in the framework of special relativity. *Eur Phys J Plus* 2018;133:165. <http://dx.doi.org/10.1140/epjp/i2018-11988-9>, arXiv:1608.00815.
- [40] Sbitnev Valeriy I. Quaternion algebra on 4D superfluid quantum space-time. *Gravitomagnetonism. Found Phys* 2019;49(2):107–43. <http://dx.doi.org/10.1007/s10701-019-00236-4>, arXiv:1901.09098.
- [41] Giardino Sergio. A novel covariant approach to gravito-electromagnetism. *Braz J Phys* 2020;50(3):372–8. <http://dx.doi.org/10.1007/s13538-020-00746-x>, arXiv:1812.07371.
- [42] Gallerati Antonio. Interaction between superconductors and weak gravitational field. *J Phys Conf Ser* 2020;1690(1):012141. <http://dx.doi.org/10.1088/1742-6596/1690/1/012141>, arXiv:2101.00418.
- [43] Tang Q, Wang S. Time dependent Ginzburg-Landau equations of superconductivity. *Physica D* 1995;88(3–4):139–66.
- [44] Lin Fang-Hua, Du Qiang. Ginzburg-Landau vortices: dynamics, pinning, and hysteresis. *SIAM J Math Anal* 1997;28(6):1265–93.
- [45] Ullah S, Dorsey AT. Effect of fluctuations on the transport properties of type-II superconductors in a magnetic field. *Phys Rev B* 1991;44(1):262.
- [46] Ghinovker M, Shapiro I, Shapiro B Ya. Explosive nucleation of superconductivity in a magnetic field. *Phys Rev B* 1999;59(14):9514.
- [47] Kopnin NB, Thuneberg EV. Time-dependent Ginzburg-Landau analysis of inhomogeneous normal-superfluid transitions. *Phys Rev Lett* 1999;83(1):116.
- [48] Fleckinger-Pellé Jacqueline, Kaper Hans G, Takáč Peter. Dynamics of the Ginzburg-Landau equations of superconductivity. *Nonlinear Anal TMA* 1998;32(5):647–65.
- [49] Du Qiang, Gray Paul. High-kappa limits of the time-dependent Ginzburg-Landau model. *SIAM J Appl Math* 1996;56(4):1060–93.
- [50] Oripov Bakhrom, Anlage Steven M. Time-dependent Ginzburg-Landau treatment of rf magnetic vortices in superconductors: Vortex semiloops in a spatially nonuniform magnetic field. *Phys Rev E* 2020;101(3):033306.
- [51] Kopnin NB, Ivlev BI, Kalatsky VA. The flux-flow Hall effect in type II superconductors. An explanation of the sign reversal. *J Low Temp Phys* 1993;90(1):1–13.
- [52] Kopnin Nikolai. *Theory of Nonequilibrium Superconductivity*. Oxford, UK: Oxford University Press; 2001, <http://dx.doi.org/10.1093/acprof:oso/9780198507888.001.0001>.
- [53] Ketterton JB, Song SN. *Superconductivity*. Cambridge, UK: Cambridge University Press; 1999, <http://dx.doi.org/10.1017/CBO9781139171090>.
- [54] Hoffmann KH, Tang Qi. *Ginzburg-Landau Phase Transition Theory and Superconductivity*. Basel, Switzerland: Springer Basel AG; 2012, <http://dx.doi.org/10.1007/978-3-0348-8274-3>.
- [55] Sanders Jan A, Verhulst Ferdinand, Murdock James. *Averaging Methods in Nonlinear Dynamical Systems*. New York, USA: Springer Verlag; 2007, <http://dx.doi.org/10.1007/978-0-387-48918-6>.
- [56] Weigand M, Eisterer M, Giannini E, Weber HW. Mixed state properties of $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ single crystals before and after neutron irradiation. *Phys Rev B* 2010;81(1):014516.
- [57] Piriou A, Fasano Y, Giannini E, Fischer Ø. Effect of oxygen-doping on $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10+\delta}$ vortex matter: crossover from electromagnetic to Josephson interlayer coupling. *Phys Rev B* 2008;77(18):184508.
- [58] Camargo-Martínez JA, Espitia D, Baquero R. First-principles study of electronic structure of $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$. *Revista Mexicana de Física* 2014;60(1):39–45.
- [59] Larkin Anatoli, Varlamov Andrei. *Theory of Fluctuations in Superconductors*. Oxford, UK: Oxford University Press; 2005, <http://dx.doi.org/10.1093/acprof:oso/9780198528159.001.0001>.
- [60] Anandan J. Detection of gravitational radiation using superconductors. *Phys. Lett. A* 1985;110:446–50. [http://dx.doi.org/10.1016/0375-9601\(85\)90551-1](http://dx.doi.org/10.1016/0375-9601(85)90551-1).
- [61] Zurek WH. Cosmological experiments in condensed matter systems. *Phys Rep* 1996;276:177–221. [http://dx.doi.org/10.1016/S0370-1573\(96\)00009-9](http://dx.doi.org/10.1016/S0370-1573(96)00009-9), arXiv:cond-mat/9607135.
- [62] Jacobson TA, Volovik GE. Event horizons and ergoregions in He-3. *Phys Rev D* 1998;58:064021. <http://dx.doi.org/10.1103/PhysRevD.58.064021>, arXiv:cond-mat/9801308.
- [63] Volovik GE. Superfluid analogies of cosmological phenomena. *Phys Rep* 2001;351:195–348. [http://dx.doi.org/10.1016/S0370-1573\(00\)00139-3](http://dx.doi.org/10.1016/S0370-1573(00)00139-3), arXiv:gr-qc/0005091.
- [64] Kiefer Claus, Weber Carsten. On the interaction of mesoscopic quantum systems with gravity. *Annalen Phys* 2005;14:253–78. <http://dx.doi.org/10.1002/andp.200410119>, arXiv:gr-qc/0408010.
- [65] Minter Stephen J, Wegter-McNelly Kirk, Chiao Raymond Y. Do Mirrors for Gravitational Waves Exist? *Physica E* 2010;42:234. <http://dx.doi.org/10.1016/j.physe.2009.06.056>, arXiv:0903.0661.

- [66] Quach James Q. Gravitational Casimir effect. *Phys Rev Lett* 2015;114(8):081104. <http://dx.doi.org/10.1103/PhysRevLett.114.081104>, arXiv:1502.07429; Erratum: *Phys. Rev. Lett.* 2017;118:139901.
- [67] Zaenen Jan, Liu Yan, Sun Ya-Wen, Schalm Koenraad. *Holographic duality in condensed matter physics*. Cambridge, UK: Cambridge University Press; 2015, <http://dx.doi.org/10.1017/CBO9781139942492>.
- [68] Bagchi Bijan, Ghosh Rahul. Dirac Hamiltonian in a supersymmetric framework. *J Math Phys* 2021;62(7):072101. <http://dx.doi.org/10.1063/5.0043854>, arXiv:2101.03922.
- [69] Garay LJ, Anglin JR, Cirac JI, Zoller P. Black holes in Bose-Einstein condensates. *Phys Rev Lett* 2000;85:4643–7. <http://dx.doi.org/10.1103/PhysRevLett.85.4643>, arXiv:gr-qc/0002015.
- [70] Barcelo Carlos, Liberati Stefano, Visser Matt. Analog gravity from Bose-Einstein condensates. *Classical Quantum Gravity* 2001;18:1137. <http://dx.doi.org/10.1088/0264-9381/18/6/312>, arXiv:gr-qc/0011026.
- [71] Novello Mario, Visser Matt, Volovik Grigory E. *Artificial black holes*. Singapore: World Scientific; 2002, <http://dx.doi.org/10.1142/4861>.
- [72] Barcelo Carlos, Liberati Stefano, Visser Matt. Analogue gravity. *Living Rev Rel* 2005;8:12. <http://dx.doi.org/10.12942/lrr-2005-12>.
- [73] Carusotto Iacopo, Fagnocchi Serena, Recati Alessio, Balbinot Roberto, Fabri Alessandro. Numerical observation of Hawking radiation from acoustic black holes in atomic Bose-Einstein condensates. *New J Phys* 2008;10:103001. <http://dx.doi.org/10.1088/1367-2630/10/10/103001>, arXiv:0803.0507.
- [74] Boada O, Celi A, Latorre JI, Lewenstein M. Dirac Equation For Cold Atoms In Artificial Curved Spacetimes. *New J Phys* 2011;13:035002. <http://dx.doi.org/10.1088/1367-2630/13/3/035002>, arXiv:1010.1716.
- [75] Gallerati Antonio. Graphene properties from curved space Dirac equation. *Eur Phys J Plus* 2019;134(5):202. <http://dx.doi.org/10.1140/epjp/i2019-12610-6>, arXiv:1808.01187.
- [76] Capozziello Salvatore, Pincak Richard, Saridakis Emmanuel N. Constructing superconductors by graphene Chern-Simons wormholes. *Ann Physics* 2018;390:303–33. <http://dx.doi.org/10.1016/j.aop.2018.01.010>.
- [77] Franz M, Rozali M. Mimicking black hole event horizons in atomic and solid-state systems. *Nature Rev Mater* 2018;3:491–501. <http://dx.doi.org/10.1038/s41578-018-0058-z>, arXiv:1808.00541.
- [78] Hu Jiazhong, Feng Lei, Zhang Zhendong, Chin Cheng. Quantum simulation of Unruh radiation. *Nat Phys* 2019;15(8):785–9. <http://dx.doi.org/10.1038/s41567-019-0537-1>, arXiv:1807.07504.
- [79] Gallerati Antonio. Negative-curvature spacetime solutions for graphene. *J Phys Condens Matter* 2021;33(13):135501. <http://dx.doi.org/10.1088/1361-648X/abd9a2>, arXiv:2101.03010.
- [80] Kolobov Victor I, Golubkov Katrine, Muñoz de Nova Juan Ramón, Steinhauer Jeff. Observation of stationary spontaneous Hawking radiation and the time evolution of an analogue black hole. *Nat Phys* 2021;17(3):362–7. <http://dx.doi.org/10.1038/s41567-020-01076-0>.
- [81] Andrianopoli L, Cerchiai BL, D'Auria R, Gallerati A, Noris R, Trigiantone M, Zanelli J. \mathcal{N} -extended $D = 4$ supergravity, unconventional SUSY and graphene. *J High Energy Phys* 2020;01:084. [http://dx.doi.org/10.1007/JHEP01\(2020\)084](http://dx.doi.org/10.1007/JHEP01(2020)084), arXiv:1910.03508.
- [82] Gallerati A. Supersymmetric theories and graphene. *PoS* 2021;390:662. <http://dx.doi.org/10.22323/1.390.0662>, arXiv:2104.07420.