Relative Impact of Singular Edge and Corner Basis Functions on the Capacitance of Parallel-plate Capacitors

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***Abstract*—Singular edge and corner basis functions are incorporated into an integral equation numerical formulation for the charge density on parallel-plate capacitors. The underlying representation for charge density is either piecewise constant, linear, or quadratic and provided by conventional representation. The singular edge basis functions are shown to play a significant role in accelerating the convergence of the numerical solutions. When used in conjunction with linear or quadratic underlying representations, the singular corner basis functions have a relatively minor effect on the numerical results.**

Keywords—basis functions, edge singularity, method of moments

I. Introduction

Basis functions incorporating singularities have been developed to improve numerical solutions in situations where fields or surface currents are expected to exhibit that behavior [1]. The authors proposed a family of hierarchical, singular basis functions for representing vector surface currents along the edges of plates [2]. Recently, a family of special vector basis functions was proposed for improving the representation at corners of conducting plates [3]. The basis functions in [2-3] are additive, in the sense that singular basis functions can be added to an expansion of polynomial functions without removing any of the existing functions. Preliminary results suggest that these functions improve the accuracy of numerical results for the surface current density on conducting plates [2-3]. Additional work is required to quantitatively assess the relative importance of the special singular functions compared to polynomial basis functions of varying order. One issue to be addressed is whether the corner basis functions offer a more substantial benefit compared to the use of a higher degree polynomial base.

To help explore this issue, in the present work we consider the combination of polynomial, edge-singular, and corner-singular basis functions for representing the surface charge density on parallel-plate capacitors. These are scalar representations associated with the families of vector functions proposed in [2-3].

II. Formulation

The capacitor plates are assumed to be infinitesimally thin and are discretized into rectangular cells. The standard scalar potential integral equation is employed, with a symmetric voltage excitation to ensure that there is no excess capacitance to infinity. The surface charge density is the primary unknown quantity and can be represented by an expansion in basis functions. Scalar polynomial basis functions are used across each plate of the capacitor as an underlying representation. These functions are either constant, linear, or quadratic. The latter two expansions are provided by the usual "node based" Lagrange basis families. The electric charge density at the edges of an infinitesimally-thin plate is infinite; the infinite behavior cannot be described by polynomial basis functions. To include the edge singularity in the representation for charge, a basis function of the local form

$$B_E(u,v) = \frac{3}{2}u^{-1/2} - 3 \tag{1}$$

may be included in rectangular cells along the plate edges, where the domain is ($0 \le u \le 1$, $0 \le v \le 1$) and the edge is located at $u = 0$. At the plate corners, a different singular behavior is encountered with the form [1]

$$B_C(u,v) = \frac{r^{\nu_e}}{\sqrt{uv}} \tag{2}$$

where $r = \sqrt{u^2 + v^2}$ and where the plate corner is located at the origin in the (u,v) system. The exponent ν_e is approximately 0.29658 for a 90 degree plate corner [1]. In this study, the edge and corner bases are superimposed with the polynomial expansion functions.

III. Results

For illustration, the following results for capacitance normalized to permittivity ε are presented for a hypothetical capacitor with plates of dimension 1m by 1m and plate separation of 0.2m. These dimensions produce large fringing fields and a strong tendency for charge to accumulate along the plate edges. Tables I and II show results for constant basis functions alone, and constant basis functions augmented with singular edge functions (1) in every cell along the plate boundaries. Tables III, IV, and V show results for linear basis functions alone, linear basis functions with singular edge basis functions, and linear basis functions with singular edge and corner basis functions. The edge functions are superimposed with the linear basis functions in every boundary cell, while the corner functions in (2) are superimposed with linear and edge functions in the corner cells.

TABLE I. CONSTANT BASIS, NO EDGE OR CORNER FUNCTIONS

plate mesh	unknowns	Capacitance/ε
5 by 5	50	7.3851
10 by 10	200	7.5281
20 by 20	800	7.6312
30 by 30	1800	7.6717
40 by 40	3200	7.6933

TABLE II. CONSTANT BASIS WITH EDGE FUNCTIONS

plate mesh	unknowns	Capacitance/ε
5 by 5	90	7.7327
10 by 10	280	7.7548
20 by 20	960	7.7611
30 by 30	2040	7.7624
40 by 40	3520	7.7629

TABLE III. LINEAR BASIS, NO EDGE OR CORNER FUNCTIONS

plate mesh	unknowns	Capacitance/ε
5 by 5	72	7.5488
10 by 10	242	7.6460
20 by 20	882	7.7023
30 by 30	1922	7.7222
40 by 40	3362	7.7323

TABLE IV. LINEAR BASIS WITH EDGE FUNCTIONS

plate mesh	unknowns	Capacitance/ε
5 by 5	112	7.7296
10 by 10	322	7.7484
20 by 20	1042	7.7564
30 by 30	2162	7.7589
40 by 40	3682	7.7601

TABLE V. LINEAR BASIS WITH EDGE AND CORNER FUNCTIONS

plate mesh	unknowns	Capacitance/ε
5 by 5	120	7.7361
10 by 10	330	7.7500
20 by 20	1050	7.7568
30 by 30	2170	7.7591
40 by 40	3690	7.7601

Similarly, Tables VI, VII, and VIII show results for quadratic basis functions alone, quadratic basis functions with singular edge basis functions, and quadratic basis functions with singular edge and corner basis functions, respectively.

An investigation of the results of these tables suggests that the convergence of the piecewise constant representation is substantially improved by the addition of the edge-singular bases. Similarly, the performance of the linear representation is improved by the addition of the edge bases. For the linear representation, the corner bases provide a slight change in the capacitance associated with the coarser plate meshes, but the difference decreases as the meshes are refined. For the quadratic underlying representation, the edge functions provide a substantial change, while the corner functions provide a minor change. As best as can be determined, the numerical results from all three approaches are converging to similar values for capacitance, in the range $7.76 < C/\varepsilon < 7.77$.

TABLE VI. QUADRATIC BASIS, NO EDGE OR CORNER FUNCTIONS

plate mesh	unknowns	Capacitance/ε
5 by 5	242	7.6433
10 by 10	882	7.7014
20 by 20	3362	7.7320
30 by 30	7442	7.7425

TABLE VII. QUADRATIC BASIS WITH EDGE FUNCTIONS

plate mesh	unknowns	Capacitance/ε
5 by 5	282	7.8037
10 by 10	962	7.7860
20 by 20	3522	7.7754
30 by 30	7682	7.7717

TABLE VIII. QUADRATIC BASIS WITH EDGE AND CORNER FUNCTIONS

plate mesh	unknowns	Capacitance/ε
5 by 5	290	7.7793
10 by 10	970	7.7787
20 by 20	3530	7.7732
30 by 30	7690	7.7706

IV. CONCLUSIONS

The example used for illustration has a strong fringing field and relatively strong edge singularity. For this example, the addition of edge-singular bases has a substantial effect, and accelerates the convergence of the numerical results for capacitance. The corner-singular bases do not have a strong effect on the results. This observation suggests that corner bases be reserved for use in situations where high accuracy is important in the numerical results. In those situations, an underlying representation of higher order than quadratic may also be indicated.

REFERENCES

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