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Summary To date, many aspects of turbulent mixing require a better and more detailed understanding. Added clarity may be provided by the use of novel approaches such as those based on complex networks, one of which will be presented in this work. The network here proposed aims specifically to describe turbulent mixing in the inhomogeneous direction of a numerically resolved channel flow at $Re_\tau = 950$; this is done using as a starting point Lagrangian data, in the form of the trajectories of a set of tracers released in the channel. Such an approach makes it possible to thoroughly analyse the transient phase before the Taylor regime, the spatial heterogeneities of tracers and the formation of spatial and temporal zoning.

INTRODUCTION

Turbulent mixing is a primary constituent of a huge variety of different phenomena, such as the diffusion of pollutants in the atmosphere or the blending of chemical reactants. Further investigation is needed to better understand its properties, especially its ability to enhance transport [1]. The Lagrangian perspective appears appropriate for this kind of study, since it makes it possible to directly follow the motion of tracers which experience mixing [4]. Complex networks have been already successfully applied to a range of different fields, including biology and social network [2][3]. Their exploitation in the study of turbulence is rather recent but promising, since they allow one to properly represent the temporal and spatial complexity of turbulent flows and are an innovative tool to enrich classical statistical analysis [5][6][7].

DATASET AND NETWORK DEFINITION

The Lagrangian dataset employed in this work consists of the trajectories of $N_p = 10^4$ non inertial tracers released in a turbulent channel flow. The channel flow is simulated by means of direct numerical simulation at a frictional Reynolds number $Re_\tau = \delta u_\tau/\nu = 950$, where δ, u_τ, and ν are the channel half height, the friction velocity and the kinematic viscosity, respectively [8]. The trajectories are integrated for the entire simulation time, which is $T^+ = 15\,200$. At their release, tracers are disposed in $N_l = 100$ equispaced levels along the inhomogeneous direction, which is y; their position is shown in Figure 1(a). The network is specifically designed to describe the mixing of tracers starting from different levels. Indeed, each level constitutes a node in the network, while a connection between nodes is established according to the motion of tracers along y^+. In details, a link between levels i and j exists at any given time if a tracer, released at $t^+ = 0$ inside level i, reaches level j. This leads to the definition of a network composed by $N_l = 100$ nodes, which is *directed* since it retains information about the directional motion of tracers. Moreover, a weight is associated to each link, namely the number of tracers moving between different levels (normalized by the number of tracers present in each level at their release, that is 100). Figure 1(b) contains a simplified representation of the trajectories of particles in and out of levels located at different y coordinates. The mixing process can be fully described by the transport network $\mathbf{W}(t^+)$; the weight matrix of this simplified representation is shown in 1(c), along with the ingoing strength $s_i^{\mathrm{IN}} = \sum_{j=1}^{N_l} W_{ji}$ of the four nodes, which is the sum of weights of incident links and is proportional to the number of tracers moving inside a node at a given time.

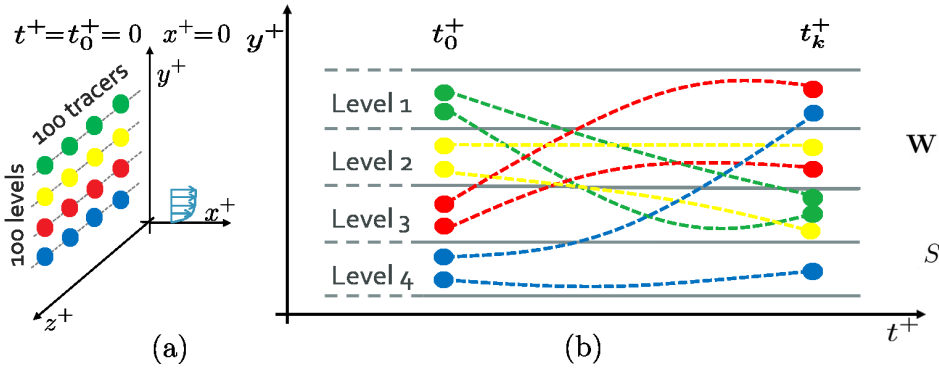

Figure 1: (a) Schematics of the release pattern of tracers, organized in N_l equispaced levels on the $x^+ = 0$ plane; (b) schematics of the motion of tracers between different levels, leading to the definition of the network weight matrix $\mathbf{W}$; (c) network weight matrix $\mathbf{W}$ and ingoing strength s^{IN} for the simplified representation of 1(b). All values in $\mathbf{W}$ are normalized by 2 (*i.e.* the number of particles originally present in each level); the values on the main diagonal are always zero since nodes cannot have self loops.

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RESULTS

The geometrization of the particle dynamics provided by the network definition allows us to highlight several features of the mixing phenomenon. The time dependent weight matrices $\mathbf{W}(t^+)$ immediately characterize the extent of mixing between initially distant levels. While for short times (Figure 2(a), $t^+ = 240$) the motion of tracers is confined to levels close to the release location and thus only the elements of $\mathbf{W}$ close to the diagonal are non-zero, for longer times (Figure 2(b), $t^+ = 14\,250$) particles are diffused everywhere across the channel and any information about their starting coordinates is lost. Applying metrics designed for directed networks to the weight matrix $\mathbf{W}$ enables us to thoroughly describe the motion of particles. For instance, the ingoing strength of each node $s^{\mathrm{IN}}(t^+)$ is directly related to the concentration of tracers at any time. As it is shown in Figure 2(c), two main phases in the evolution of the flow are evident: for short times ($t^+ < T_d^+$, $T_d^+ \approx 4000$) the concentration of tracers is highly inhomogeneous, while for larger times ($t^+ > T_d^+$) the vertical mixing of particles is completed and the Taylor dispersion regime is reached. The network formalism is able to capture both the complexity of the irregular, transient phase, and the onset of the asymptotic behaviour. The tools introduced by graph theory may also be exploited to highlight non trivial features which are difficult to notice using statistical methods. The analysis of the physical length of links measures the spatial extent reached by diffusion and also signals the presence of tracers trapped near the channel walls and far away from their starting level. The increase in link length is related to the displacement of tracers towards the channel walls during the initial transient, as can be seen in Figure 2(c). Additionally, common and fast tools developed to find communities in graphs may be used to partition the network into regions of the channel that exchange a reduced amount of tracers between them. Community detection performed in the channel flow is useful to determine the spatial extent of turbulent mixing at different times and highlights the existence of two separate regions near the channel walls.

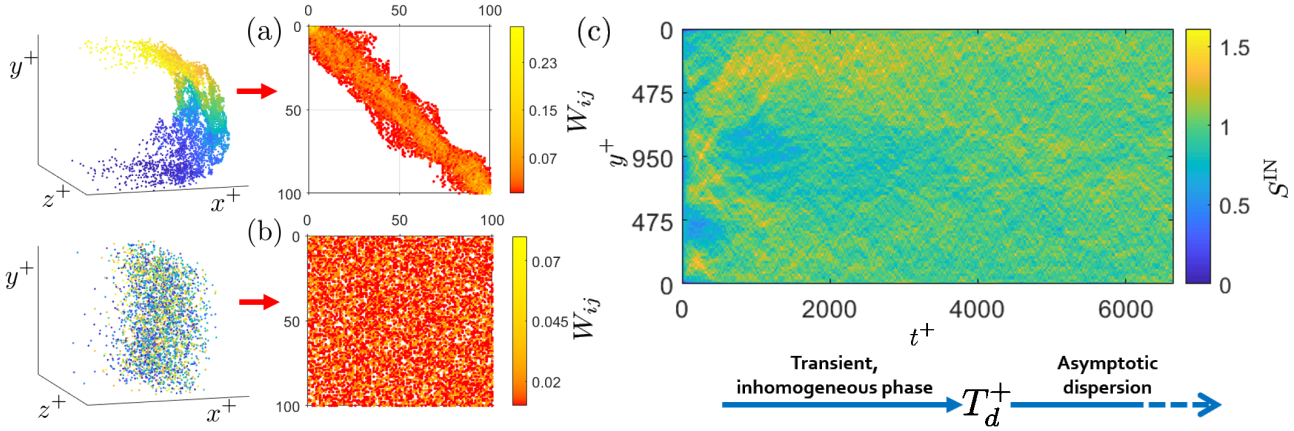

Figure 2: (a) Distribution of particles and network weight matrix at $t^+ = 240$; (b) Distribution of particles and network weight matrix at $t^+ = 14\,250$; (c) the evolution of the ingoing strength s^{IN} of nodes emphasizes both the details of particle motion across the channel and the presence of different regimes in the dispersion process; the onset of the Taylor asymptotic regime ($T_d^+ \approx 4000$) marks the shift between a highly irregular concentration of tracers and a more homogeneous one.

CONCLUSIONS

These findings show both the effectiveness of methods based on network theory and the suitability of the Lagrangian framework for the analysis of turbulent mixing. The complexity of the starting transient and the shift to the asymptotic dispersion regime are well outlined by the network representation of vertical motion inside the channel. Exploitation of the capabilities of the tools presented here offers an in-depth look on the collective and individual behaviour of diffusing tracers in wall bounded turbulence, highlighting their concentration, their trapping in persistent flow structures and their presence in isolated communities [9].

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