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World Heritage and Design for Health
XIX INTERNATIONAL FORUM
NAPLES 17 - CAPRI 18|19 JUNE 2021
Le Vie dei Mercanti
XIX INTERNATIONAL FORUM
Naples 17 - Capri 18 |19 June 2021

Structural design criteria for safety by monitoring of the architectural heritage damage: new proposal

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Abstract
In nature, several types of events undermine structural safety, but precisely the intrinsic characteristics of the struck object play a crucial role in determining its resistance. A historic building has different possibilities of resisting an unexpected event than a more recently constructed building; each category of buildings behaves differently towards an unusual event, and no generalization or transversal homologation is possible. The methodologies to define the structural robustness of such building typologies or the ability to adapt to an unexpected turn of events should be evaluated following different paths. While for steel or reinforced concrete buildings, it is possible to proceed with the concepts of the classical elastic theory, for masonry buildings, such a theory is set aside, being fundamental, as a primary hypothesis, the non-deformability of the material and its non-tensile strength. Heyman's hypotheses can be assumed for this category of masonry constructions: no tensile strength, infinite compressive strength, and no sliding between blocks. Moreover, while the science has been expressed by far in a precise way on the seismic behaviour of steel or reinforced concrete buildings, for historic masonry buildings, it remains to be tested. For this reason, this paper concerns relating the typology of the building to the possible response to the occurrence of a particular type of event, interpolating these categories with that of the possible solutions of structural evaluation.

Keywords: masonry bell-tower; seismic vulnerability; architectural heritage; damages; rigid blocks.

1. Introduction
An inherent vulnerability characterizes historical buildings and monuments to seismic action since most of them are usually designed to withstand gravity loads only and, overtimes, have undergone several transformations with a consequent lack of quality of the masonry or ineffective connections among walls. This is especially true for traditional bell towers, which present a great seismic vulnerability because of their slenderness.
The present paper deals with the structural and seismic evaluation of the fourteenth-century bell tower of St. Chiara in Naples. The bell tower was built in the 14th century together with the adjacent monastic complex and the basilica by Roberto D'Angiò and Regina Sancha d'Aragona [1].
This research work starts from Heyman's study in 1992 [2] to address the issue of leaning towers analytically. According to Heyman on assuming masonry as a unilateral material with no tensile strength, the methodology here adopted proposes to describe the crack curve that delimits the failure mechanism for the bell tower of St. Chiara and evaluate the limit inclination angle associated with the collapse of the structure under gravity loads only. Once the limit inclination angle is identified, the collapse multiplier is derived to assess the vulnerability of the bell tower object of the study.
Future developments of this theme concern, on the one hand, the evaluation of the seismic vulnerability and risk of the bell towers of the historical centre of Naples at large scale, and on the other hand, the possibility of applying the dynamic analysis taking into account the rocking of rigid blocks [3].

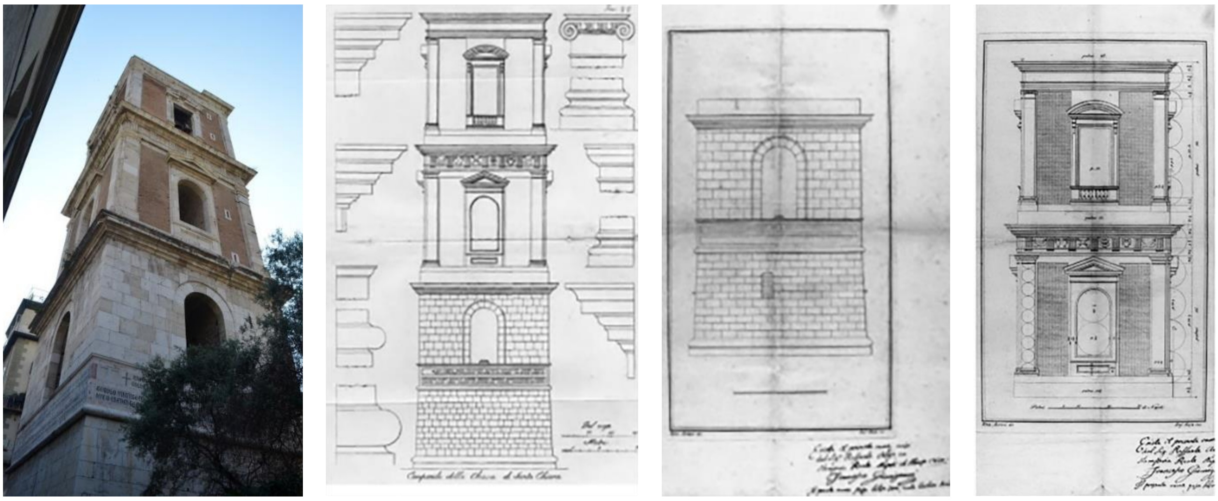
2. The historical bell-tower of St. Chiara in Naples

2.2 Graphic description

The construction of the bell tower, located to the left of the basilica of St. Chiara, dates back to 1338. Still, the works were immediately stopped in 1343, after the death of Roberto D'Angiò for lack of funding. At this date, the structure was to a third of its completion. The building work was resumed in the early 15th century, following the earthquake of 1456, when the bell tower almost completely collapsed, with only the marble basement leaving standing. It was later rebuilt in the Baroque style until it was completed around 1604.

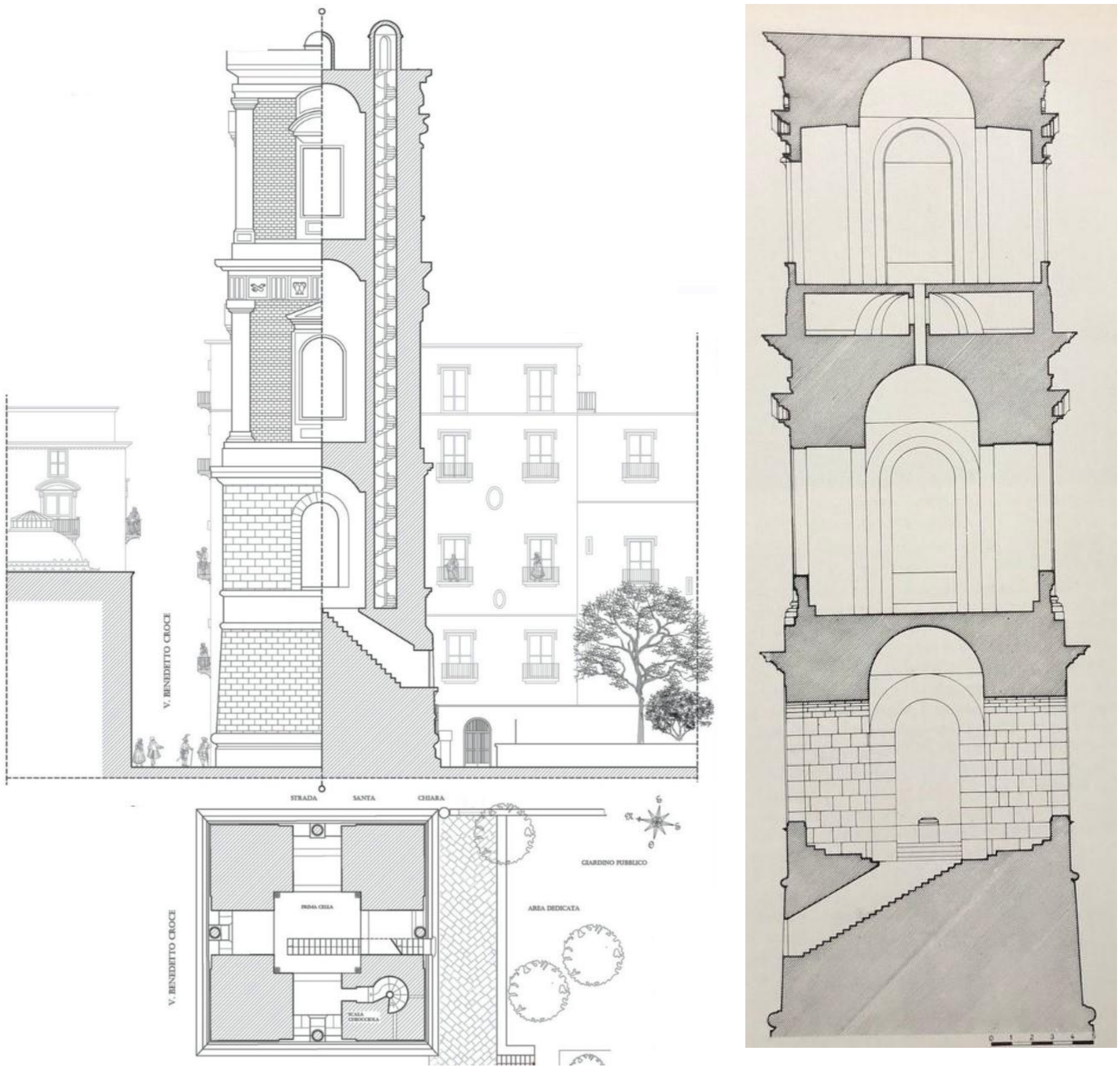
Fig. 1: C. Guerra, Study for the isolation of St. Chiara, June 1954.

Separated from the core of the Basilica and located on the left edge of the monastery walls, the monument looks like a defence tower. The square-plan bell tower has been constructed in three levels, although the original plan probably envisaged the construction of at least five floors [4]. All that remains of the original 14th-century bell tower is the basement made of large blocks of piperno, marble and Tifata travertine, four corner pillars inside the first cell from which the ribs of the lost cross vault started, and four bases, one for each window, which probably included the columns of the old double lancet windows [5].The basement cell underwent significant changes and interventions: a cloister vault over lunettes replaced the cross vault, and the double lancet windows became monofore surmounted by round arches instead of the original ogival arches. Two new cells were added to the first, both characterized by exposed brick walls, alternating with marble cornices and pilasters. The second level, in Doric order, dates back to the 16th century, while the third one, in the Ionic order, to the 17th century (Fig.2). The three levels are reached by a spiral staircase located in one of the corners of the bell tower; it leads up to the attic slab, where there is a small dome with a cylindrical base (Fig. 3). The tower is 50 m high.



(a) (b) (c) (d)

Fig. 2: Bell tower of St. Chiara: (a) picture by the authors; (b) elevation and details from [6]; (c) detail of the first order of the monument; (d) detail of the secondorder of the monument - printingproof for the work by E. Ascione, *De' Migliori monumenti di Napoli*, ASN, Ministero Presidenza del Consiglio.



(a) (b)

Fig. 3: Bell tower of St. Chiara: (a) Half section and half elevation with the plan of the first cell. (b) Survey and section taken from [7].

3. Structural studies

3.1 Empirical rules

The analysis of the equilibrium of masonry towers was approached relatively late. The first scientific report seems to be that of Fresnel (1831) and was followed by few contributions until Rankine (1858) rigorously formulated the theory of the stability of masonry towers and chimneys [8]. Until then, calculations had been based on empirical rules.

Concerning the dimensioning of a tower, once its slenderness has been established (the ratio between the height h and the base side d), the thickness of the wall is the most critical structural parameter to be determined. With reference to the case study, a slenderness $\lambda = 3.08$ and a wall thickness $m = 2.64\mathrm{m}$ have been calculated. This latter value can be compared with empirical rules adopted in the past in the project of towers.

Among the best known empirical rules to determine this thickness are:

- The Gothic rule of the German treatises: $m/h = 1/20$
- The Alberti rule: $m/h = 1/15$
- Rodrigo Gil de Hontañón's rule: $m/h = 1/2$

Indeed, the value we obtained seems to be within the parameters of the Gothic rules.

3.2 Stability assessment of the bell-tower

Concerning the stability assessment of a tower, it is a statics problem of statics: at each horizontal section, the moment of stability of the part of the tower above this plane (for a specific geometrical coefficient of safety) must be equal to the moment of wind force W with respect to this plane. The wind force is normally calculated as the product of the dynamic wind pressure and the effective or apparent area (the cross-sectional area normal to the wind direction). Of course, this product may be qualified by a certain factor that considers the particularities of the tower (its round or square shape, the existence of buttresses, etc.) [8].

Considering that the most critical section is at the base, it must therefore be verified:

$$P_t d(q - q') = W h_w \tag{1}$$

where P_t is the total weight of the tower; d is the base diameter in the wind direction; $q = 1/2c$ defines the geometrical safety and q'represents the deviation of the centre of gravity of the tower with respect to the base (e.g. if the tower is leaning); W is the total wind force and h_w is the height at which the wind force acts. Usually, the value of q is taken so that no traction occurs, i.e. q defines the limit of the central core of inertia of the section for the direction considered.

For a tower of uniform cross-section and perfectly vertical ($q' = 0$), the above equation becomes,

$$P_t\,(qd) = W\,(h/2) \tag{2}$$

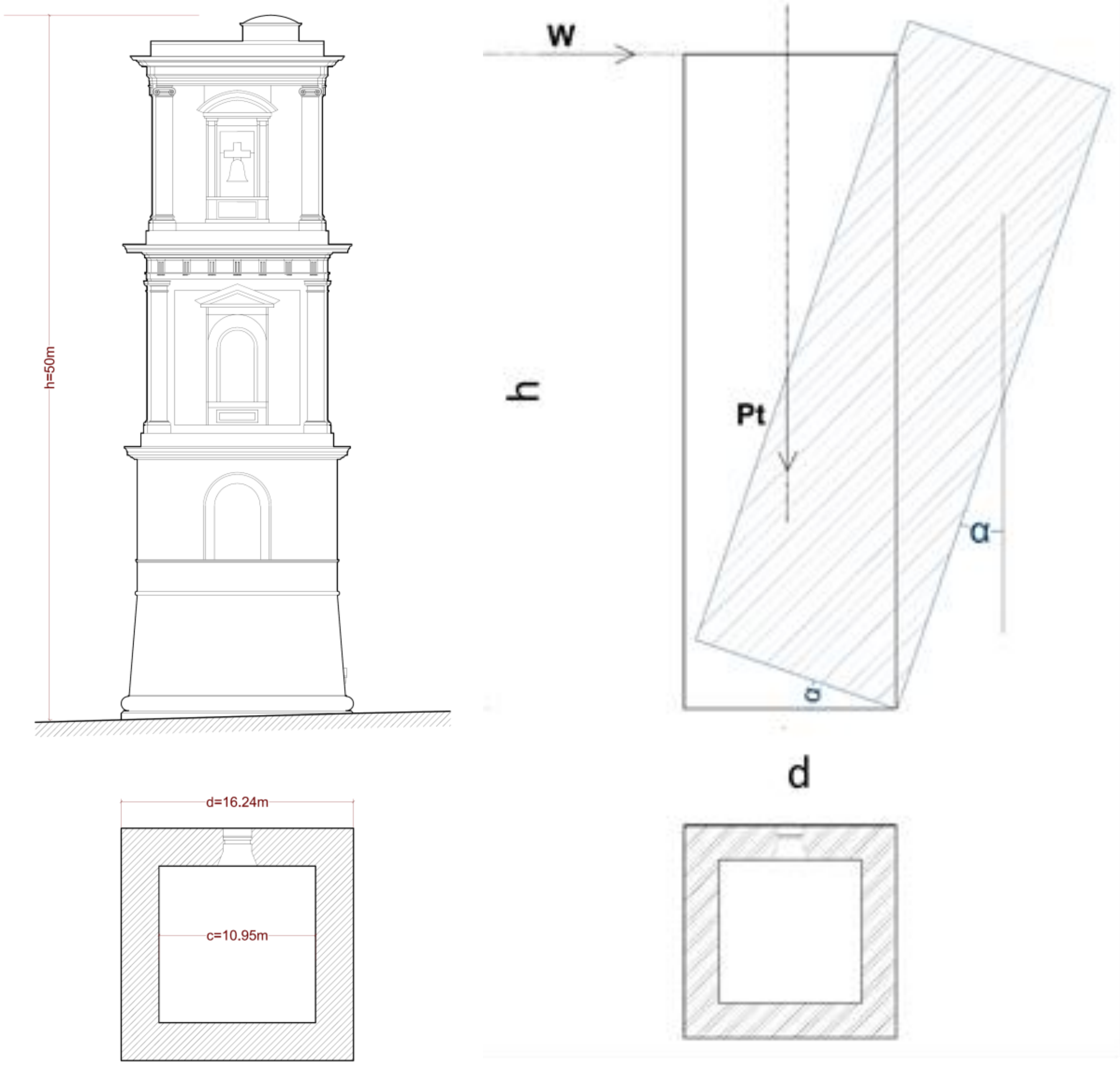
For a hollow square section, like in this case, we have:

$$P_t = \gamma h\,(d^2 - c^2) \tag{3}$$

$$q = (d^2 + c^2)/6d^2 \tag{4}$$

$$W = w\,(hd) \tag{5}$$

where γ is the specific weight of the masonry and w is the wind kinetic pressure.



(a) (b)

Fig. 4: Bell tower of St. Chiara: (a) cross-section of the hollow tower and elevation with main parameters; (b) approximate analysis of the tower, shown at its critical inclination with the identification of the wind force $\boldsymbol{W} = \mathbf{370}\boldsymbol{kN}$ and the angle $\boldsymbol{\alpha} = \mathbf{19}°$ which defines the dangerous limit inclination.

Figure 4a shows the square cross-section of St. Chiara bell-tower with external dimension $d = 16.24m$ and internal dimension $c = 10.95\text{m}$; the area is $(d^2 - c^2)$ and the corresponding section modulus is $(d^2 - c^2)/6d^2$. Thus, this means that cracking will start when the loading becomes eccentric to the centre-line by an amount $(d^2 + c^2)/6d^2$. In the following, equations (3), (4) and (5) have been applied to the case study.

To compute the total weight of the campanile, the specific weight of the masonry has been assumed to be $\gamma = 18\text{kN/m}^3$ for the tuff, obtaining $P_t = 129.45\text{kN}$ approximately equal to 13.20 tons.

The eccentric factor $q = 0.24$, has been obtained from calculations. This is a good value, considering that the geometrical safety value might be 0.29 or 0.30 for a tower of usual wall thickness [2].

To calculate the wind force W, the wind kinetic pressure has been introduced as $w = \frac{1}{2}\rho v_r^2$, where ρ is the air density and v_r is the default reference air speed depending on the site zone. Assuming $\rho = 1.25\text{kg/m}^3$ and $v_r = 27\text{m/s}$, the wind kinetic pressure obtained is $w = 0.456k\text{N/m}^3$. Consequently, by substituting this value into equation (5), the wind force W applied on top of the bell tower has been achieved and its value is $W = 370\text{kN}$.

3.3 A preliminary vulnerability study

The theoretical framework of the structural study proposed in this paper is the Limit Analysis developed by Heyman [9,10]. Following this approach, masonry is considered a unilateral material that can resist compressive stresses but has weak tensile strength. According to Heyman's, the unilateral

model assumes that compressive stresses are very low so that there is no danger of crushing of the material and that tensile stresses cannot be developed.
A masonry tower, subjected to uneven foundation settlements, will crack, and these cracks may lead to overall structural collapse. Often cracks are considered a dangerous sign. Conversely, cracks in masonry construction can be simply attributed to the unilateral behaviour of masonry, which exhibits cracks as soon as tensile stresses appear [11]. Thus, cracking is the way adopted by masonry buildings to stably accommodate small changes in the external environment through a rigid macroblocks partition as clearly described in [9,12;13].
In this section, following the approach introduced by Heyman in [2], a preliminary analysis is developed for the maximum inclination that may be regarded as safe for the masonry tower analysed. Figure 4b shows the exact solution at the point where the bell-tower of St. Chiara is just overturning at an angle α of 19°; it sketches the results corresponding to Equation (6) that defines a hollow masonry tower for dangerous limiting inclination:

$$\tan\alpha = \frac{1}{3}\frac{d}{h} \tag{6}$$

$$\alpha = \tan^{-1}\frac{2}{3}d/h \tag{7}$$

Equation (7) computes the value of the angle at which the hollow tower first develops cracks. In the analysed case this angle is equal to 12.21°: These findings can be compared to the Table I in Fig. 4 elaborated by Heyman in [2] which gives fast results on the value of inclination α of a masonry tower, once known the height to base ratio *h/d* (*H/b* in [2]).

TABLE I
Values of inclination of tower, α

	H/b						
	3	4	5	6	8	10	12
Overturn:							
Solid, Equation (19)	13.4	10.1	8.1	6.8	5.1	4.1	3.4
Hollow, Equation (22)	15.7	11.9	9.6	8.0	6.0	4.8	4.0
First crack:							
Solid	6.3	4.8	3.8	3.2	2.4	1.9	1.6
Hollow	12.5	9.5	7.6	6.3	4.8	3.8	3.2

Fig. 5: Value of inclination of tower, α. This Table provides α-values of for different H/b ratios, according to the solid and thin-walled towers, respectively; the second two lines give values of the angles at which the solid and hollow towers first develop fissures [2].

According to this rule, the Campanile of St. Chiara at Naples, which has a ratio h/d of about 3, should not cause concern. The inclination for overturning is about 15.7° from Table I; the angle at which the hollow tower first should exhibit some cracking is about 12.5°.

4. Conclusions and future works

Damage, failure and collapse mechanisms for the towers, in general, are diverse and depend on slenderness and constructive features such as masonry quality.
Starting from the assumption that masonry structures are made of no tensile-resistant materials, and considering that their low tensile strength does not allow distribution of stresses on the whole structure, in case of seismic actions, local mechanisms of failure/collapse are presented. That is to say that, since a global behaviour does not characterize masonry, it may collapse for loss of stability of limited portions (macro-elements) in certain conditions. Based on the above, a kinematic approach [14] is proposed for evaluating the seismic vulnerability of the surveyed bell tower, as it is introduced and carried out by the research of A. Giuffré [15] and Doglioni [16] on the damage mechanisms studied following the recent earthquakes that occurred in Italy. We mean the kinematic representation model that describes the macro-element's discretization process and its displacements by damage mechanism. With reference to the ‘Guidelines for the assessment and mitigation of seismic risk of the cultural heritage of 2011 [17], the future research aims to identify the primary damage mechanisms

through a 'check-list' of vulnerability identifiers. The bell tower can be divided into two distinct macro-elements consisting of the actual tower and the belfry. In this check-list, vulnerability indicators are classified according to two categories: indicators associated with the geometric configuration of the building and indicators associated with specific construction techniques.
There are several approaches to model the mechanical behaviour of masonry structures (Sarhosis et al. 2016). Housner [3] was the first to investigate the behaviour of rigid single degrees of freedom blocks subjected to horizontal excitations analytically. With formulations derived, it is possible to estimate the minimum horizontal acceleration at the base to cause the overturning of the rigid body.
This paper just represents the starting point of a broader ongoing research project on the vulnerability of masonry towers in Naples. Hence, at this early stage, the study proposed an approximate analysis based on some simplifications: the hypothesis of the square hollow section of uniform thickness and the absence of any irregularity along the height. The authors will further examine these aspects in the future in parallel with the deepening of constructive surveys and aware that, for practical cases, the wall thicknesses could diminish towards the top. Furthermore, the analysis carried out does not account for seismic actions. Anyway, this kind of research allows providing handy hints to insight into the limit inclination angle associated with the collapse of the structure for gravity loads only.

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