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# High gear ratio mechanical transmissions for actuators: Simplified models for efficiency under opposing and aiding loads

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**Abstract.** Planetary gear drives are widely employed in electrical and hydraulic actuation systems, to adapt a high speed, low torque motor to a low speed, high torque user, within strict weight and volume constraints. During the early design phases of these devices, accurate yet simple simulation models are required to evaluate the performance of a given configuration of the device. Similar models are also useful within diagnostic and health monitoring analyses of existing machines, as a discrepancy between the actual behaviour of the physical system and that predicted by its digital twin may be the effect of a damage. This work compares different models available in literature for the efficiency of high gear ratio mechanical transmissions; the models are applied to multiple arrangements common for planetary drives, and the results in form of an efficiency map for the transmissions are compared and discussed. The simulations provide different levels of detail, and require different levels of knowledge about the specific architecture of the system. All of them are able to deal with dry friction; additionally, the different behaviour of the transmission under the effect loads aligned in the same direction of speed or in the opposite one is accounted for.

## 1. Introduction

Planetary gear trains combine high gear ratio and good efficiency in a compact and lightweight package; for this reason, they are widely employed in several applications where mechanical power needs to be transmitted with strict size and weight constraints for a given torque.

In the aerospace field, planetary gears are commonly used for the actuation of flight control surfaces: traditionally, in current generation commercial aircraft, leading edge and trailing edge high lift devices are driven by a centralized, electrical or hydraulic Power Drive Unit (PDU), and the motion is transmitted to the individual surfaces by a shaft and a set of gearboxes, usually featuring planetary gears. Additionally, the emerging field of electromechanical actuation of primary and secondary flight controls makes wide use of planetary gear sets, in order to amplify the torque of electric motors and achieve a power density suitable for this demanding application [1, 2].

One common drawback of planetary gears is the reduced efficiency with respect to ordinary gearboxes. The power losses that arise in meshing gears (such as frictional losses in teeth sliding, bearing losses, lubrication shaking losses, seals and guide bearing friction) are in the order of 1% of the transmitted power for ordinary gearings. In the case of planetary gearboxes, some elements may experience high torques and high speed at the same time: hence, the mechanical power recirculating through these elements can be many times larger than the transmitted power, leads to increased losses and brings down the overall efficiency of the gearbox [3-7].



Then, an important tool for the design and analysis of such mechanical devices is a method for predicting their efficiency in different operating conditions; for example, in [8, 9] the efficiency is shown to vary from aiding and opposing load conditions (i.e. loads directed in the same way as the user shaft speed or in the opposite one, respectively). Methodologies for the estimation of gearbox efficiency are presented and discussed by M. del Castillo [10] and Macmillan [11], for a single Degree of Freedom (DoF) planetary drive. A similar strategy, extended to multi-DoF gearings, is provided by Mantriota and Pennestri [12]. This work compares several models for the efficiency estimation of planetary gear drives. These models feature different levels of complexity and accuracy: some are based on the estimation of teeth sliding friction and bearing losses, as proposed by Benedict and Kelley [13], Setharaman and Kahraman [14], and Anderson and Loewenthal [15, 16]; other are more simplified and treat the transmission with a semi-blackbox approach, as discussed by Berri et al. [8]. It should be noted that to better explain the merits, shortcomings, and advantages of the proposed models, in this work, the authors applied the said approaches to two mechanical gearbox types typically employed in aerospace applications (in particular, a Wolfrom and a harmonic drives). The three approaches proposed have been tested to evaluate the effects produced by friction on the aforementioned gearboxes' overall efficiency. It should be noted that, despite being characterized by significantly different levels of complexity, they have given satisfactorily consistent results.

## 2. Models

Simulation models characterized by different level of complexity and fidelity are usually required during the different phases of the engineering process for a product. The detail design phase of a system commonly requires high fidelity representations to achieve highly accurate predictions of the effect of each design parameter on the system behaviour and performances. This accuracy comes at the expense of a long computational time and a high effort for the development and calibration of the model itself: a very deep knowledge about the system is required, and some parameters may be difficult to estimate or measure. For example, in the specific case of mechanical efficiency, several tribology, fluid-dynamics and contact mechanics problems shall be addressed and accounted for.

On the other hand, lower fidelity models can provide acceptable results with lower computational time, a simpler implementation, and relying on a basic knowledge of the system. This approach is particularly useful in the preliminary sizing of a system, when most of its parameters have not been determined yet, or in health monitoring tasks, when real-time computations are required; additionally, low fidelity models are useful when dealing with off-the-shelf equipment, whose data may be unavailable or incomplete. This work compares three models available in literature [8-11] for the efficiency of high gear ratio mechanical transmissions; they are characterized by different levels of detail, and can account for dry friction and the effect of aiding and opposing loads.

### 2.1. Model 1

The first model is based on the estimation of dry friction on the tooth profile of meshing gears. The approach was initially presented in [8]. As shown in Figure 1, this method evaluates the torques exchanged between a pair of gears as the sum of a component due to normal loads on the teeth sides and a component due to tangential forces. The corresponding transmission ratio is  $R = z_2/z_1$ , where  $z_1$  and  $z_2$  are the numbers of teeth of the motor and driven gears respectively. Referring to Figure 1 we can introduce the following parameters:  $r_p = (m \cdot z)$  is the pitch radius of each wheel,  $r_t = (r_p + a)$  is the top radius of each wheel, where  $a$  is the addendum, and  $r_b = r_p \cos \theta$  is the base radius of each wheel. Always referring to Figure 1, we can introduce a force  $F$  exchanged between the two wheels along with the segments  $A$  and  $B$ , which lie onto the line of action, tangent to both base circles.  $A$  and  $B$  are bounded by the two wheels' top circles and separated by the pitch circles. The lengths of segments  $A$  and  $B$  are respectively defined as  $A = \sqrt{r_{t1}^2 - r_{b1}^2} - d_{p1}$  and  $B = \sqrt{r_{t2}^2 - r_{b2}^2} - d_{p2}$ , where where  $d_p = r_p \sin(\theta)$ . In opposing load condition, the motor wheel turns clockwise, and the driven wheel turns counterclockwise. Therefore, the surfaces of teeth slide towards each other along  $B$ , and slide away from each other along  $A$ .

The main assumption for this model is that the contact force between the gears is constant along the whole meshing line AB. Therefore, the tangential friction force will be constant in modulus along AB, while its direction will be always opposite to the relative sliding speed of the two surfaces. While this is true when the gears have 1 meshing tooth, the distribution of load between multiple teeth is dependent by the stiffness of each individual tooth.

The torque acting on each gear is the superposition of the contributions from normal and tangential force components, and can be written as [8]:

$$T_{1,OL} = Fr_{b1} - \frac{fFB}{A+B} \left( d_{p1} - \frac{B}{2} \right) + \frac{fFA}{A+B} \left( d_{p1} + \frac{A}{2} \right) + \rho_1 \sqrt{F^2 + \left( fF \frac{A-B}{A+B} \right)^2} \quad (1)$$

$$T_{2,OL} = Fr_{b2} - \frac{fFB}{A+B} \left( d_{p2} + \frac{B}{2} \right) + \frac{fFA}{A+B} \left( d_{p2} - \frac{A}{2} \right) - \rho_2 \sqrt{F^2 + \left( fF \frac{A-B}{A+B} \right)^2} \quad (2)$$

for the opposing load condition, and:

$$T_{1,AL} = Fr_{b1} + \frac{fFB}{A+B} \left( d_{p1} - \frac{B}{2} \right) - \frac{fFA}{A+B} \left( d_{p1} + \frac{A}{2} \right) - \rho_1 \sqrt{F^2 + \left( fF \frac{A-B}{A+B} \right)^2} \quad (3)$$

$$T_{2,AL} = Fr_{b2} + \frac{fFB}{A+B} \left( d_{p2} + \frac{B}{2} \right) - \frac{fFA}{A+B} \left( d_{p2} - \frac{A}{2} \right) + \rho_2 \sqrt{F^2 + \left( fF \frac{A-B}{A+B} \right)^2} \quad (4)$$

for the aiding load condition. It should be noted that, in Eq. (1-4),  $f$  represents the tooth-tooth friction coefficient and  $\rho$  is the friction radius of the bearings holding the gears. The subscript *OL* denotes the opposing load (direct) conditions, whereas subscript *AL* denotes the aiding load (inverse) conditions.

## 2.2. Model 2

The second model is more simplified than that of Section 2.1; it is based on the estimation of waste power in the reference frame of the planet carrier; therefore it is only applicable to planetary gearboxes. The power budget in the reference frame of the planet carrier can be written as:

$$\sum_i (\omega_i - \omega_P) M_i = P_W \quad (5)$$

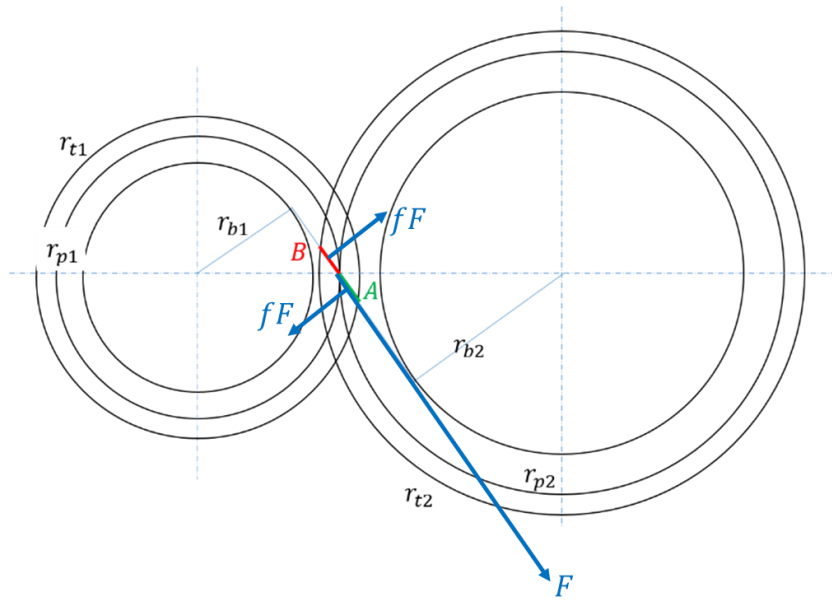
Where  $i$  denotes the  $i$ -th shaft of the transmission,  $\omega_i$  its angular velocity,  $M_i$  its torque, and  $P_W$  is the waste power due to internal frictions. Each component  $(\omega_i - \omega_P) M_i$  can be identified as an input ( $P_{in,P}$ ) or output ( $P_{out,P}$ ) power contribution, and the power budget can be rewritten as:

$$P_{in,P} - P_{out,P} = P_W \quad (6)$$

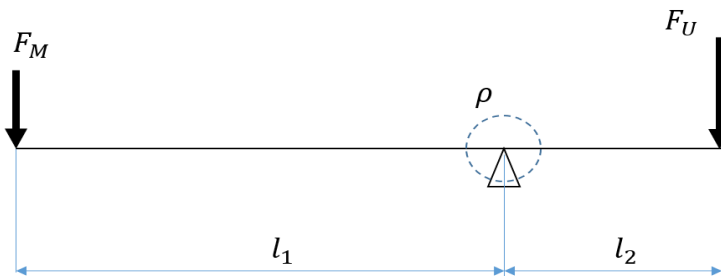
The meshing efficiency in the planet carrier frame  $\eta_P = P_{out,P}/P_{in,P}$  can be estimated based on the number of meshing gears; common values range from 0.992 to 0.995 for each individual meshing [citation needed]. This allows to compute the waste power  $P_W$ . Waste power and torques are frame-invariant, while input and output power contribution depend on angular speeds and by consequence on the reference frame. Then, the efficiency in the fixed reference frame can be computed as:

$$\eta_F = \frac{P_{in,F} - P_W}{P_{in,F}} \quad (7)$$

Depending on the direction of the external load, the input and output power contributions in the different reference frames are switched; then, the efficiency will be different in the two cases. This approach only requires knowledge about the layout of the transmission in terms of number of teeth and arrangement of the gears; the meshing efficiency is estimated from statistical data.



**Figure 1.** Force distribution on a pair of meshing gears according to Model 1.



**Figure 2.** Simplified representation of a mechanical transmission used in Model 3.

2.3. Model 3

The third model [8] is the most simplified of the three and treats the transmission with a blackbox approach. No knowledge about the internal layout is required except the transmission ratio. The gearbox is modelled as a lever with friction in the pivot point, as shown in Figure 2. The transmission ratio is  $i = l_1/l_2$ , and the efficiencies in opposing load and aiding load conditions are respectively:

$$\eta_D = \frac{1}{R} \frac{F_{U,D}}{F_{M,D}} = \frac{1}{R} \frac{R-u}{1+u} \tag{8}$$

$$\eta_I = R \frac{F_{M,I}}{F_{U,I}} = R \frac{1-u}{R+u} \tag{9}$$

Where  $u = \rho/l_2$  and  $\rho$  is the friction radius. Eventually, Equations (8) and (9) can be rearranged to write the aiding load efficiency as a function of the opposing load one and the transmission ratio:

$$\eta_I = \frac{2R^2\eta_D - R^2 + R}{R^2\eta_D + R - \eta_D + 1} \tag{10}$$

3. Applications

The three models are compared on two case studies which represent common arrangements for the gear reducers of aerospace electromechanical actuators. These planetary transmissions combine very high gear ratios and good efficiency within a small and compact form factor.

### 3.1. Compound Planetary Gearbox (CPG)

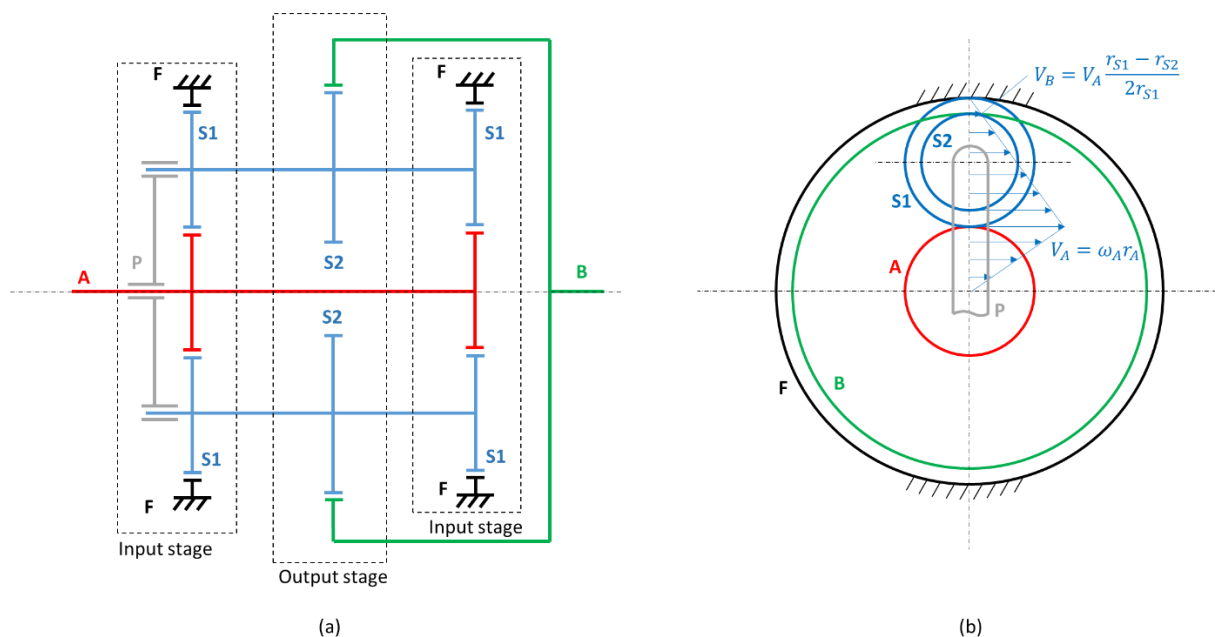
In this work we consider as a case study a specific class of CPG known as Wolfrom drives. The layout of the transmission is shown in Figure 3, with particular reference to the prototype presented in [17]. It comprises of a first stage similar to a common planetary gear set with a fixed ring gear, and a second stage constituted by a rotating ring gear, that carries the output shaft, and a set of planet gears meshing with it and connected to the first stage satellites.

This transmission can be designed for gear ratios ranging from 10 to several hundreds, while maintaining a compact size and a good efficiency. For this reason, they are employed in several actuators for aircraft secondary flight controls.

The particular gearbox studied in this work has a gear ratio of 124, with the first stage sun gear and planets having 21 teeth each, and the second stage planet gears 20.

### 3.2. Harmonic Drive (HD)

A second case study for this paper is a Harmonic Drive (HD) that shares the same transmission ratio of the CPG. HDs are a group of planetary drives involving the use of a flexible gear, or the flex spline. A fixed ring gear meshes with the flex spline, which carries the output shaft. The flex spline is kept in an elliptical shape by a cam, connected to the input shaft. As the cam rotates, the meshing points between the fixed ring gear and the flex spline move; after a complete revolution of the input shaft, the flex spline has rotated by an angle corresponding to the difference between its number of teeth and that of the fixed ring. The HD can be seen as a planetary drive as the flex spline acts as a pair of satellites, carried by the elliptical cam.

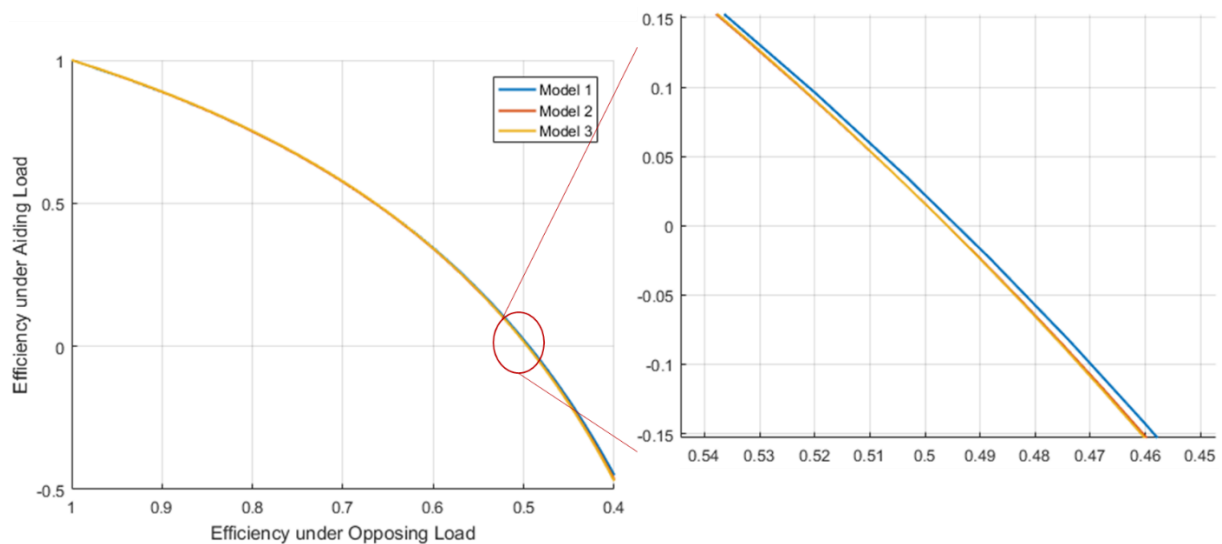


**Figure 3.** (a) Layout of the Compound Planetary Gearbox (CPG) considered in this work; in this specific case, the sun gear A has 21 teeth, the satellites S1 and S2 21 and 20 teeth respectively, the fixed ring gear 63 teeth, and the output ring gear B 62 teeth. (b) Velocity distribution in the gearbox.

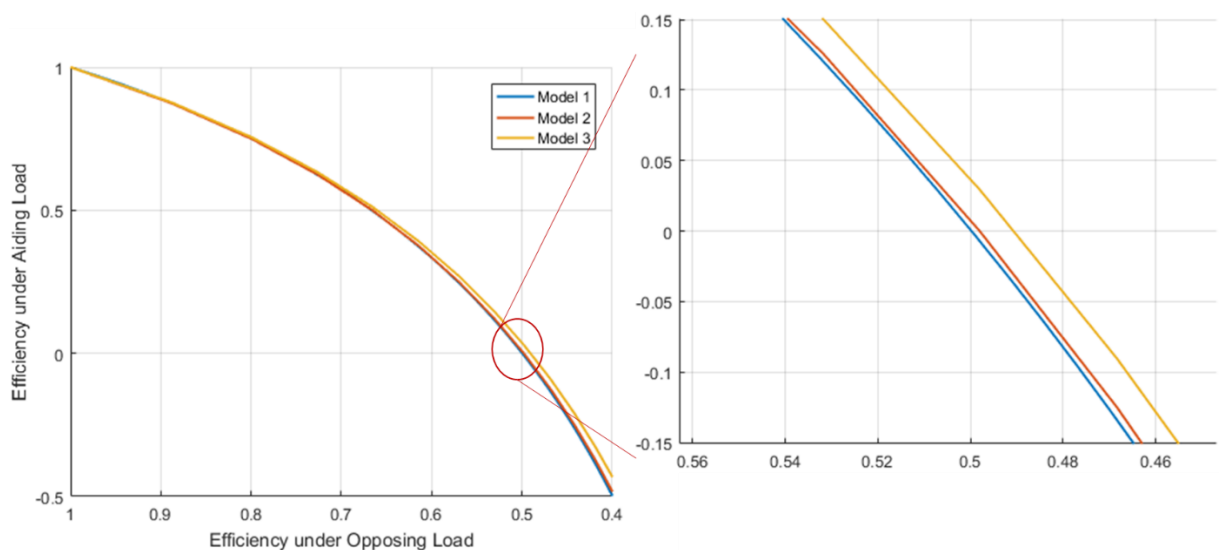
## 4. Results

The first model provides a relationship between gear geometry, friction coefficient, and opposing and aiding load efficiencies; the second one gives a relationship between the meshing efficiency in the carrier reference frame and the aiding and opposing load efficiencies of the transmission; the third model only estimates one efficiency as a function of the other one and the gear ratio.

For this reason, the three models are compared in terms of opposing load efficiency versus aiding load efficiency, as this is information all three can provide. Figure 4 shows the application of the three strategies on the Compound Planetary Gearbox; the second and third model estimates are very close to each other, while the first model, which is the most detailed and arguably the most accurate, predicts a slightly higher aiding load efficiency. Figure 5 reports the results for the Harmonic Drive. The discrepancy between models 1 and 2 is similar to the previous case, while model 3 tends to slightly overestimate aiding load efficiency. However, in both cases the errors between the predictions of the three models are below 3%. The different behaviour between models 1 and 2 is mainly due to the variation of meshing efficiency predicted by model 1 when switching from opposing load to aiding load, whereas model 2 assumes a constant meshing efficiency and ascribes the change in efficiency only to the modification of the input and output power contributions. Model 3 is in general less accurate since it is unable to discriminate the effect of the particular gear layout of the transmission; on the other hand, this same characteristics allows to apply it to a wider variety of case studies, such as worm drives, ballscrews, ordinary gearings, or cable and pushrod transmissions.



**Figure 4.** Comparison of the three models on the CPG.



**Figure 5.** Comparison of the three models on the HD.

## 5. Conclusions

Three models for the efficiency of planetary drives for actuators were discussed and compared. They feature different levels of detail and accuracy, and require different levels of knowledge about the system. The lower fidelity simulations of models 2 and 3 provide quite accurate estimates compared to model 1, despite neglecting some characteristics of the transmission configuration. Then, they can be employed in preliminary design or in real time monitoring in place of the more complex model 1. Future work will include the comparison of these models on a larger number of case studies, as well as an experimental validation of their predictions.

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