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# Recognizing the Design Patterns of Complex Vaults: Drawing, Survey and Modeling. Experiments on Palazzo Mazzonis' Atrium in Turin

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**Abstract.** This paper shows the results of research advances on complex vaulted systems produced by the integration of laser scanner survey techniques and three-dimensional modeling for the geometric interpretation of built architecture to recognizing the geometric matrices of the design conception. The integration between TLS techniques and digital modeling methods led to the definition of new workflows, aimed at optimizing the use of data and at refining the quality of the geometrical interpretation. The process incorporates the traditional activities of freehand drawing of eydotypes, aimed at a deep understanding of the peculiar characteristics of the artifact. In particular, from these procedures new opportunities for the research arise to better understand the relationships between survey data, geometric matrices and compositional rules.

The case study presented here, the atrium of Palazzo Mazzonis in Turin was chosen among a small number of atria that present characteristics of originality and uniqueness in a panorama of realizations strongly characterized by compliance with well-established compositional schemes.

**Keywords:** Complex vaults, Drawing, Survey, Modeling, Palazzo Mazzonis.

## 1 Introduction

The research presented in this paper is one of the results of the international collaboration for the project “Nuevas tecnologías para el análisis y conservación del patrimonio arquitectónico”, funded by the Ministry of Science, Innovation and the University of Spain. This allowed Concepción López to join the research group coordinated by Roberta Spallone and Marco Vitali at the Politecnico di Torino. From 2012, the Politecnico’s group has been conducting investigations about bricks-made complex

vaulted systems in several Baroque buildings of Piedmont, aiming to identify the geometric matrices of the vaults shapes and to re-construct them through three-dimensional models, in light of constant comparison with coeval treatises and manuals, and archival sources (when existent).

The present successful collaboration provided the necessary cues to test the value of the digital reconstruction method on data derived from a metric survey campaign carried out using terrestrial laser scanning (TLS). The integration between TLS techniques and digital modeling methods led to the definition of new workflows, aimed at optimizing the use of data and at refining the quality of the geometrical interpretation. In particular, from these procedures, new opportunities for the research arise to better understand the relationships between survey data, geometric matrices, and compositional rules.

The case study presented here, the atrium of Palazzo Mazzonis in Turin was chosen among a small number of atria that present characteristics of originality and uniqueness in a panorama of realizations strongly characterized by compliance with well-established compositional schemes.

## 2 Interpretative Models of Complex Vaults

The intrados surface of the vaulted atria analyzed from 2012 was represented by digital models that give back the idea of the designer, net of asymmetries, irregularities of the plan, and construction accidents. The interpretation work, applied to a relatively large number of vaulted atria, has structured the methodology of geometric decomposition and digital re-construction, successfully applied to the homogeneous classes of vaults [1] [2].

The workflow involves, in the first step, the subdivision of the vaulted surface into elements organized according to a structural and geometrical hierarchy that responds to rules of symmetry (axial longitudinal and transversal) and provides for the identification of the main surface, secondary surfaces (generically oriented along the axes) and angular surfaces. It should be noted that the most delicate phase of this work consists in the recognition of the geometrical matrices of the main surface, of which only portions remain in the built architecture. In particular, the longitudinal and transversal sections of the vaults always intersect only portions of the main surface, making the geometrical interpretation of generatrices and directrices more complex, and requiring additional sections at the imposts. The geometrical recognition of secondary and angular surfaces, rarely referable to geometrical primitives, and more often assimilated to surfaces generated by sections, is simpler.

Then the logic of the geometric composition of the main surface with the secondary and angular surfaces is defined. Digital re-construction can be addressed through two alternative composition rules: a first method, which we can call "by intersections", involves the geometric intersection of the main surface with secondary shapes (cylinders, cones, portions of torus, spheres, ellipsoids or ovaloids); a second method, which we can call "by cuts and fills", consists in cutting portions of the main surface through surfaces with a vertical generatrix (mainly pairs of planes) and filling the space left free

with surfaces generated by sections that rest seamlessly on the edges of the cut of the main surface.

The digital models resulting from this work of geometric interpretation and re-construction have been described through an exploded axonometric that represents the recognized primitive surfaces, the logic of composition, and the sequence of geometric operations necessary for their combination.

The vaulted system of the atrium of Palazzo Mazzonis shows unique features in comparison with well-established patterns recognized in the historical city center. The historical information about the palace is so scarce that it is supposed to be the result of an enlargement of the pre-existing 17th-century building, which belonged to the Marquises Solaro della Chiusa. A survey of the building dated 1845 (fig. 1), was used as the basis for planning the campaign of survey and drafting the eidotypes.

The passing atrium is divided into two main areas by two pairs of columns. Although the surface of the vaulted system is fragmented by the presence of the lintels and arches setting on the columns, this first portion of the atrium produces a unitary perception of space, suggesting a continuity of surface between the three fields in sequence. The caesura between the portion overlooking the street and that on the courtyard is underlined by the presence of the architraved pillars, associated with columns, which define three spans symmetrical with respect to the longitudinal axis.

### **3 Interpretative On-site Drawings and Geometric Reconstruction**

The geometric interpretation and re-construction of the real building need to be based on knowledge phases that start with on-site drawings. Direct contact with the architecture is in fact a very important step in the process, which makes it possible to optimize the subsequent phases of the work. Indeed, this gives the opportunity to investigate modularity and proportioning of spaces, axiality, symmetries, recurrence, geometry of the surfaces that make up the vaulted system, through the drawing intended as a tool for investigation and knowledge. Moreover, the graphic transposition of the spatial characteristics of architecture requires to understand its peculiarities: the graphic activity, in fact, becomes an instrument of verification and control of this understanding and allows, as in the present study, to analyze and solve through the geometric construction problems of intersection and composition of surfaces. The evaluation of these aspects cannot disregard direct observation – it is in fact much more difficult and risky to rely on photographic or video, or even iconographic materials – which makes it possible to constantly change the point of observation, to support evaluations in the execution of some simple measurement operations as well as to make critical selections of information that would not be as easy and safe by other means.

In Palazzo Mazzonis the geometric interpretation method has been supported by the study and the use (on-site) of the archive drawings that were used as a proportioned basis for trying to find proportional criteria or a basis for 2D or 3D geometric graphic interpretative sketches and drawings (fig. 1).



**Fig. 1.** Atrium of Palazzo Mazzonis in Turin; excerpt of the survey drawing dated 1st July 1845 with superimposition of graphical analysis. Source: Archivio Storico del Comune di Torino, Tipi e dis., cart. 63, fasc. 9, dis. 1, tav III. Photo: F. Natta. Graphic processing: F. Ronco.

The analysis of the plan-proportioned drawing suggests the presence of a longitudinal axis that connects the entrance (and the first portion of the atrium, of almost square plan) to the courtyard (and the second portion of the atrium, of rectangular module corresponding to half of the first square), and two transversal axes of symmetry of the two vaulted systems, totally independent, as underlined by the double pillars and the double arches that constitute the main caesura of the space.

The subdivision into sub-modules emphasizes the position in plan of the free supports (pilasters and columns) which are arranged to emphasize an expansion of the unitary space of the first portion of the atrium and a compression and fragmentation of the modules facing the courtyard. The two pseudo-circular decorative elements that characterize the intrados surface of the two main vaults clearly respond to this modular grid. The geometrical analysis of the intrados surface of the two vaulted systems starts from the proportional representation, in a three-dimensional scheme, of the characteristic sections of the vaults: starting from the three sections conducted along the axes of symmetry, the directrices of the surfaces involved in the composition are qualitatively defined and to these are added the longitudinal and transversal sections in correspondence of the pillars (to define the geometrical course of the arches that subdivide the vaulted fields) and the section of the surfaces in correspondence of the perimeter walls.

The construction of this three-dimensional scheme makes it possible to evaluate, surface by surface, which are the additional sections to be studied for each vault (to be produced in the following phase, the digital reconstruction, starting from the point cloud) and to predict the results of the geometrical modelling operations based on the recognized logic of composition. In the specific case of Palazzo Mazzonis the two vaulted systems that characterize the atrium have different features. The portion of the atrium towards the courtyard is covered by the vaults set on the three bays (the central sail vault and the two lateral groin vaults), separated by arches. The portion facing the street presents a decidedly more complex configuration, in which the three portions defined by the two transversal arches are resolved in the central field by intersection of a barrel vault arranged longitudinally and two transversal barrel vaults, in the adjacent fields by intersection of pseudo-conical vaults with veloidic angular surfaces. The joining arch between these latter surfaces would seem to suggest a break defined by vertical planes arranged along the axis of the supports, as suggested by the archive drawing.

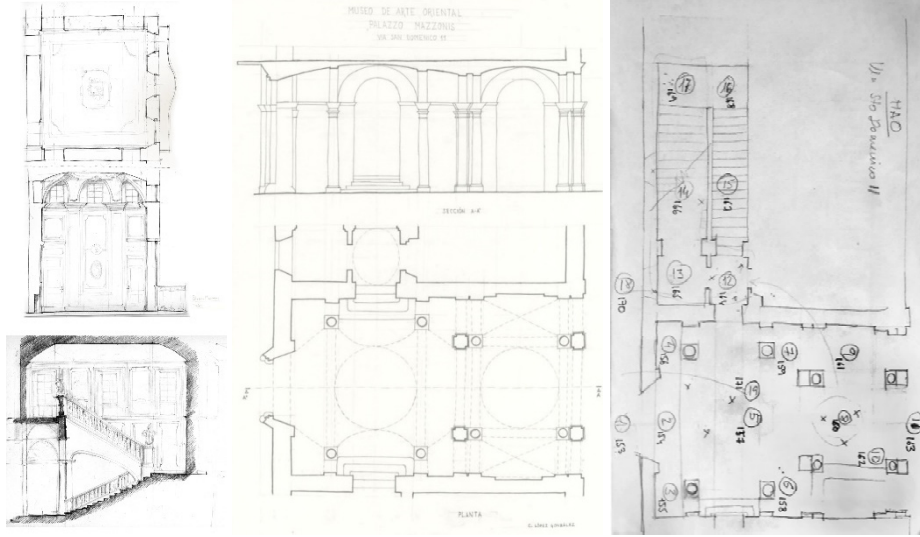
On-site observation also reveals a different arches layout: the projection in plan is curvilinear and the surface that constitutes it has different heights in the two edge curves. The subsequent phases of data extraction from the point cloud therefore constitute a salient step in the definition of the reconstructive geometric model.

#### 4 Eidotypes Aimed to Survey Project

Starting from the geometric reconstruction made on site, we continued to the realization of eidotypes: series of life drawings containing useful information for an in-depth knowledge of the building. They allowed to know and investigate in advance some visible and knowable aspects of the different areas analyzed and defined through the subsequent phases of measurement.

The eidotype, besides being an important guideline during the measurement operations, was also a useful memory support during the following phase of graphic restitution of the survey, in particular during the orientation and point cloud overlapping phase.

The processes and modes of production of the sign are the structural components of the drawing and are therefore closely linked to its purpose and role [3]. For this reason, two types of eidotypes have been realized. The first, made on the A5 format notebook (fig. 2 right), contain a synthetic and schematic representation of the plan of the atrium of Palazzo Mazzonis. This plan was used to design the laser capture sequences, establishing the positioning of the laser scanner, in one or more stations concerning the object to be detected. The choice of the scanning points must guarantee the best and greatest visibility of the area to be detected and reduce shadow phenomena due to the presence of objects. In the case of the atrium, 18 gripping points were required to ensure geometric homogeneity of the metric survey. The second type of eidotypes was instead made on A3 cardboard (fig. 2 left) and the drawings were made on an always approximate but larger scale, to include more information useful for understanding the building such as details, mouldings, architectural orders, projections of the vaulted ceilings and height differences, paying particular attention to the proportional aspects. This operation has attempted to render a work such as architecture in a two-dimensional key to obtain as much information as possible about the shape of the architecture being studied. Particular attention has been paid to geometric features, not trying to reproduce reality “as seen” but introducing geometric laws to better understand it. The geometrical and constructive characteristics and the shape and volume of the rooms have been investigated through the use of the double orthographic projection method, trying to keep on the same sheet the plan, and the section, together with back elevation. As conventionally, we chose to set the horizontal cutting plan just above the windowsill of the ground floor. The order of execution of the eidotypes suggests the order of the scans. It starts from the entrance hall, which can be easily connected through the door to the external facade of the building, and then the other rooms are drawn, following a path that corresponds to that of a person entering the building. For architectural drawings of this type it was necessary to resort to certain graphic conventions, avoiding giving into particular pictorial effects.



**Fig. 2.** Eydotypes of: the staircase and main hall, the atrium, the atrium's plan with scanning stations. Eydotypes: G. Bertola, M. C. López González.

Their correspondence to the truth must manifest itself only through a technical sign in which the shape and structure of the various elements must prevail over the figurative appearance. The drawings, executed by hand without the use of “auxiliary tools” such as rulers and squares, were made using only pencils of three different hardness: one for the elements cut from the section planes and the other for the visible lines and the third for the decorative elements.

## 5 Survey Methodologies: Choice of Tools and Strategies Applied to the Vaulted Systems

This complex framework of vaults above described encloses a beautiful example of compositional design only understandable through the expertise of the geometry and sensitivity by the author. The symmetry in the direction of the access axis produces a sense of depth accentuated by the lateral columns, which approach the center in the second vault. The parallelism of the diaphragmatic arches divides the space through sequential scopes, offering dynamism emphasized by the typological change between the first and second bay. The two solutions used in each of them make these vaults a complex geometric problem. Its study requires a premeditated methodology that contributes to the elaboration of digital models easily analyzed in their geometric, metric, and typological form.

As in any investigation related to the architectural heritage, it is necessary first to carry out an analysis of the written and graphic documentary references to subsequently undertake the virtual reconstruction of the vaulted system. The ideal tools to tackle the data collection of curvilinear surfaces are those that provide clouds of defining points of mentioned surfaces. The most advisable tool is the terrestrial laser scanner (TSL).

This system has an added value because it is a container of qualitative-topological information since, in addition to geometric information, it provides other values such as color or reflectance [4]. However, before using it, it is necessary to consider a series of issues concerning the lighting, the distance up to which the accuracy values of the device can be assumed, and the visibility limitations that require multiple scanning stations. The curvilinear shape of the vaulted surfaces and the bas-relief decoration gives rise to shapes and textures of high reflectivity. Therefore, it is necessary to carry out the scans at a short distance, short duration, and great overlap, since if large sweeps were performed, we would obtain imprecise results. In order to make the complex vaults visible and to explore and manipulate the images obtained from the point cloud, various programs generally linked to the brand of the laser device used can be used.

There are also other generic programs of free access. These programs perfectly manage point clouds and their calculation algorithms, obtaining pinpoint accuracy in the records of the different scans, and facilitating the cleaning of noise. Some allow obtaining orthophotos scaled in JPG or TIF format that can be treated later with Photoshop software improving the results. They are especially advantageous for visualizing the contours of the intrados of the vaults and calculating the radius of curvature in the direction of the main axes (fig. 3).



**Fig. 3.** Orthophotos of the longitudinal and transversal sections. Processing: M. C. López.

Its use is very appropriate in the study of the geometric configuration relating it to the exposed in the treatment, to follow the guidelines of the old architects [5]. In some cases, these programs also allow the creation and optimization of the triangulated mesh known as TIN (Triangulated Irregular Network), which is very convenient in the study of curvilinear surfaces and in the analysis of spatial arrangement through 3D modeling. This mesh of triangles can be imported for visualization, measurement, or analysis purposes to 3D modeling programs and can even be viewed from web browsers.

The resulting cloud can be imported into CAD software proceeding to its vectorization through different formats. In this way a rigorous two-dimensional graphic representation of the real state of the vaults is obtained, that is, of its geometry, not of the geometric ideal. The vault is represented with its deformations, damages, and seats due to the construction process or the passage of time.

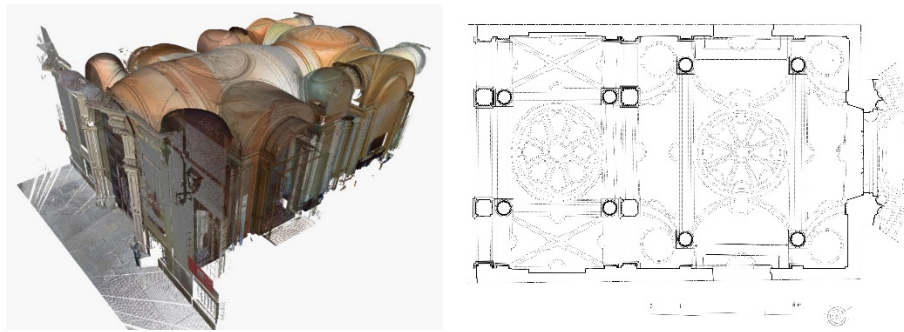
If we were left with only the two-dimensional representation, we would be wasting the potential of this massive data capture system [6]. Therefore, it is necessary to develop strategies that allow obtaining methodologies aimed at improving the expression and understanding of the vaults by extracting a semantic content from the management of the

point cloud. The work methodology used in Palazzo Mazzonis has been based on obtaining graphic representations through the use of the terrestrial laser scanner because it is the ideal tool for taking data from vaulted systems. The Focus 130 x 3D model of the Faro brand has been used for its maneuverability, precision (2mm for 25 m), autonomy (4 hours of battery), and an integrated camera that allows it to assign to each point its natural color.

To ensure the overlap of points and the absence of blind points it was necessary to scan from 18 positions parking lots with an approximate duration of 8 minutes each and an average distance between scans of 3 m. The scanner location and that of the referential spheres were previously programmed.

## 6 From Scanning to 3D Modelling

The registration of point clouds has been carried out with the Lighthouse Scene 19 software. It offers an intuitive interface, has pinpoint accuracy in the records of the different scans, allows obtaining orthophotos scaled in .jpg format, and enables the creation and optimization of the triangulated mesh.

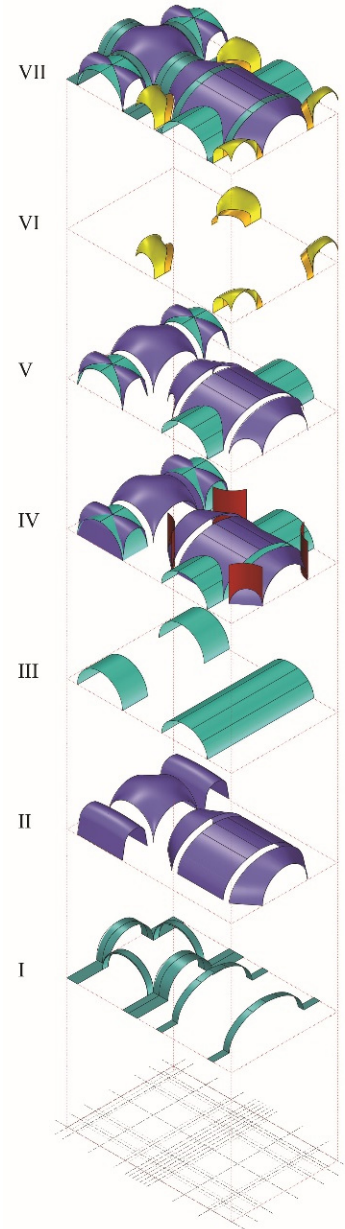


**Fig. 4.** Union of point clouds and surveyed plan drawing in scale 1:50. Scanning: M. C. López. Processing: F. Natta. Drawing: M. Vitali.

The import of the final cloud to AutoCAD 2020 has been done through the .e57 format from where it has been possible to make the two-dimensional drawings that specify the real geometry of the vaulted system (fig. 4). Then, the main sections indicated in the previous paragraph have been carried out. From these sections, the distinctive sections of each vaulted surface have been extracted: if they correspond to the symmetry of the composition, they have been superimposed.

For example, taking into account the vaulted system on the portion of the atrium facing the street, a series of elements can be interpreted starting from pairs of sections, as in the case of arches on columns symmetrical to the central field, while others can be described by the overlapping of four sections symmetrical to the two axes, longitudinal and transversal, as in the case of the four pairs of sections necessary for the correct

geometrization of the perimeter arches of the angular veloidic surfaces. A separate approach has been dedicated to the geometric reconstruction of intersection curves between surfaces, which cannot be derived from flat sections of the point cloud.



**Fig. 5.** De-composition of the atrium's vaulted system. The interpretative geometric model in exploded axonometric view highlights the primitive reference surfaces, the logic, and the hierarchy of composition Modelling: F. Ronco.

In fact, the vaulted system facing the street presents surfaces that seem to follow a combinatorial logic of intersection, as in the case of the barrel vaults that cover the central span. In this case a first possibility of data extraction from the point cloud could be made through the construction of a 3D polyline with object snap on the points of the cloud: this procedure, however, does not allow the geometrization operations necessary for the construction of the interpretative model and has therefore been abandoned. Alternatively, to guarantee, also for these elements, a correct simplification in geometrical terms of reality, the intersection curves have been obtained starting from the plan and elevation projections of the curve, appropriately geometrized. The "geometrical" projections have been extruded perpendicularly to the projection planes: from the intersection of the two surfaces it has been possible to extract the intersection curve in its interpreted and geometrically simplified version.

A similar reasoning was made for the creation of the connection arcs of the pseudo-conical vaults with the veloidic angular vaults. As already mentioned, these arcs do not consist of cylindrical surfaces and have no projection in a linear plan: the procedure previously described was also applied to these elements highlighting, as noted in the on-site geometric interpretation, how the edge curves of the arches do not have the same height. Once all the distinctive sections of each surface have been defined, the interpretative model has been created using modeling tools of Rhinoceros software (fig. 5).

## 7 Towards a Scalable Methodology

In recent decades, the rise of metric survey methods using Terrestrial Laser Scanning (and low-cost ones using Structure from Motion photogrammetry) has progressively replaced instrumental survey techniques using Total Stations and stereoscopic photogrammetry, and direct survey using traditional instruments. These last techniques were based on an approach consolidated since the beginning of the practice of metric survey: the critical selection of points, intersections, edges, alignments, which characterize every architecture as a *unicum*. This practice involved a deep architectural culture by the surveyors, based on knowledge of architectural and construction history, which allowed the correct recognizing of architectural elements, building structures, orders, materials, ornamentations, and constructive techniques. Moreover, it was needed a rigorous attention to the formal, structural, and stylistic peculiarities of the building subject of the analysis, to the historical period in which it was realized, also about coeval buildings and to the context in which it rises, to the constructive materials used, and to transformations and calamitous events happened over time. Documentary materials, archival, bibliographic and iconographic sources contribute in a fundamental way to this knowledge. The objective of representing the artefact, as the outcome of the process of knowledge and measurement, through two-dimensional drawings in reduction scales strictly dependent on the aims of the survey (documentation, project, restoration, transformation), addressed the same methodologies, techniques and choices concerning the survey instruments.

All these experiences and practices drawn from the past, can and must be integrated with today's metric survey techniques, and in particular with the TLS techniques, as in

the case study analyzed, which is intended to be used as a paradigm for setting a scalable work process on other similar cases. Indeed, the automatic acquisition of a big amount of points irregularly distributed, that is the point clouds, creates a big database, useful *a posteriori* for multiple interpretations and thematic analyses, for drawing plans, elevations and sections, and, currently, for 3D modelling with different techniques (geometric, parametric, BIM). Nevertheless, it is necessary paying attention to the reduction scales provided in the survey project of laser scanning settings.

3D models, temporally parallel to the rise of laser scanner survey, represent a relatively new product, compared to traditional representations by orthographic projections. These digital 3D models are the result of a synthesis operation performed by the scholar who, pursuing a specific communication objective, selects the information of what is nothing other than a geometric figure representing an architecture [7].

Therefore, consistently with the objectives of the research, the cognitive and heuristic potential of hand-drawing and digital modeling have been used at different times within the workflow that has profitably integrated the database offered by the laser scanner acquisition. The selection of the data for 2D drawings representing the building in today state derived from the horizontal and vertical sections of the point cloud the metric and geometric information which, integrated with the features identified in the preliminary eidotypes, allow to draw up metric reliable and architecturally significant plans and sections. The selection of data for 3D reconstructive models of the design idea presents greater complexity. Indeed, it is a matter of identifying a series of significant sections of the point cloud, suitable to verify the preliminary geometric hypotheses deriving from the consultation of the original design documents (rarely preserved) and from the observations and hypotheses formulated *in situ* about generative shapes, proportions and geometries of the vaulted systems investigated. The workflow set up in this way for the analysis of complex vaulted systems highlights today's relevance of the definition of the architectural survey as an open system of knowledge as stated in the "Chart of Architectural Survey" [8].

## 8 Conclusion

This paper shows the results of research advances on complex vaulted systems produced by the integration of laser scanner survey techniques and three-dimensional modeling for the geometric interpretation of built architecture to recognize the geometric matrices of the design conception. The interpretation process is enriched, concerning the tools used, with new steps for the definition of the geometries supporting the built shape and identifies new procedures, still in a study and refinement phase, for the control of the outcome curves of operations of geometric intersection of surfaces. The verification of the results of the work, carried out in this case study through the control of characteristic sections and intersection curves, requires further investigation in order to find the correspondence between the point cloud and the interpretative model through the overlapping of them, and the evaluation of the precise deviations, currently being elaborated. With regard to the process of geometric interpretation, on the other hand, there are new issues related to the process of conception and realization: some portions

of the complex vaulted systems highlight the relationship between geometry and architectural ideation (in general, surfaces in which it is easier to recognize pure geometric surfaces, or described in treatises or manuals), while others (surfaces that derive from the need to relate even very different curves, as in the case of angular vaults) open reflections on the relationship between architectural design and building techniques, especially the construction and positioning of the ribs and the masonry equipment.

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