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***Abstract*— Abruptly ended dielectric slabs are important components in several areas of applied electromagnetism. For the study of these geometries, a variety of analytical methods have been proposed in the past. In this paper we formulate the problem in terms of Wiener Hopf equations and we apply the novel and effective semi-analytical solution technique known as Fredholm factorization.**

***Index Terms*— Scattering, Propagation, Wiener-Hopf method, Green's function, Integral equations.**

I. INTRODUCTION

Open dielectric waveguides are important structure to be studied especially in the areas of integrated optics, optoelectronics and millimeter-wave circuits. For the study of these geometries, a variety of analytical methods have been proposed in the past.

Using the Wiener-Hopf (WH) technique, the problem of the diffraction by a semi-infinite dielectric slab has been studied by several authors [1-4].

In this paper we propose a novel and effective technique to study this problem. The proposed methodology is general and it is based on a WH formulation and on a semi-analytical solution procedure called Fredholm factorization [5-6]. This technique has been effectively used to solve diffraction problems in geometries that present angular regions and planar regions filled by arbitrary materials, see for instance [7-10].

The present problem, i.e. the abruptly ended dielectric slab, enlarges the catalog of canonical structures that the method allows to analyze. The method in fact is capable of coupling canonical structures in more complex problems, see for instance [7-10].

With reference to Fig. 1, we define the three regions: 1) x>0, -2d<y<0, 2) x<0, -2d<y<0, 3) y>0 or y<-2d. We assumes that region 2 is filled by dielectric material with relative permittivity εr and regions 1 and 3 are in free space. For simplicity, the source is an Ez-polarized plane wave having the incidence angle φo. Consequently the only not vanishing field components are Ez(x,y), Hx(x,y) and Hy(x,y) independent from z.

To simplify the problem, as it is well known, we can exploit symmetries and reduce the problem to the study of the two diffraction problems where a PEC plane and PMC plane are respectively placed along the symmetry plane at y=-d of the considered geometry (Fig. 2).

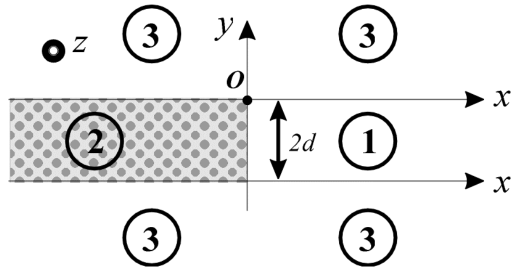

Fig. 1. Scattering by an Abruptly Ended Dielectric Slab Waveguide: we define three regions: 1) free space region x>0, -2d<y<0, 2) dielectric region x<0, -2d<y<0, 3) free space region y>0 or y<-2d

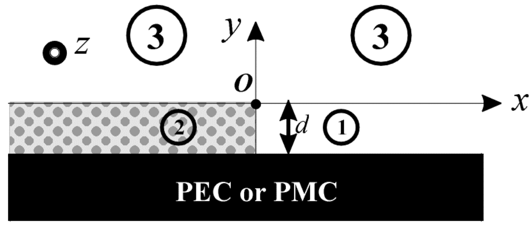

Fig. 2. Scattering by an Abruptly Ended Dielectric Slab Waveguide over a perfect ground. The ground can be PEC or PMC. The combination of the two problems allow to rebuild by symmetry the solution of the original problem reported in Fig. 1 We define the three regions: 1) free space region x>0, -d<y<0, 2) dielectric region x<0, -d<y<0, 3) free space region y>0.

II. FROM WH EQUATIONS TO THE SOLUTION

Both problems reported in Fig. 2 are modelled in terms of WH equations where the unknowns are the Laplace transforms of the electromagnetic field components:

$$V_+(\eta) = \int_0^{\infty} E_z(x,0)e^{j\eta x}dx,\ V_{\pi+}(\eta) = \int_{-\infty}^{0} E_z(x,0)e^{-j\eta x}dx$$
$$I_+(\eta) = \int_0^{\infty} H_x(x,0)e^{j\eta x}dx,\ I_{\pi+}(\eta) = -\int_{-\infty}^{0} H_x(x,0)e^{-j\eta x}dx \quad (1)$$

According to the theory reported in [6] for plane stratified regions we can write a suitable WH equation for region 3 (Fig. 2). In order to get WH equations for semi-layer regions (regions 1 and 2 of Fig. 2) we resort to a generalization of the technique proposed in [11] where the characteristic Green's function procedure is used starting from the wave equation in Laplace domain.

It yields respectively for region 1, 2 and 3 the equations (2), (3) and (4-5).

$$p_o(\tau) + \eta p_1(\tau) - I_+(\eta) = Y_d(\eta)V_+(\eta) \quad (2)$$

$$-p_o(\tau_2) + \eta p_1(\tau_2) + I_{\pi+}(\eta) = Y_{d2}(\eta)V_{\pi+}(\eta) \quad (3)$$

$$-I_{\pi+}(-\eta) + I_+(\eta) = Y_c(\eta)[V_{\pi+}(-\eta) + V_+(\eta)] \quad (4)$$

$$-I_{\pi+}(\eta) + I_+(-\eta) = Y_c(\eta)[V_{\pi+}(\eta) + V_+(-\eta)] \quad (5)$$

where $Y_c(\eta) = \sqrt{k^2-\eta^2}\,/\,k Z_o$, $\tau = \sqrt{k^2-\eta^2}$, $\tau_2 = \sqrt{\varepsilon_r k^2 - \eta^2}$ (k and Z_o are respectively the propagation constant and the impedance of the free space). The spectral admittances Y_d and Y_{d2} are related to equivalent transmission line problems with different characteristics for the cases reported in Fig. 2. For example when the ground is constituted of PEC we have

$$Y_d(\eta) = \frac{-j\tau\cot[\tau d]}{k Z_o},\ Y_{d2}(\eta) = \frac{-j\tau_2\cot[\tau_2 d]}{k Z_o} \quad (6)$$

The functions reported on the left hand side of (2) and (3) in terms of p_o and p_1 are contributions due to the boundary conditions at the interface x=0, -d<y<0. For example their use in (2) is referred to

$$-\frac{\int_{-d}^{0}\sin\left(\tau(y'+d)\right)f_{1\eta}(y')dy}{jkZ_o\sin(\tau d)} = p_o(\tau) + \alpha p_1(\tau) \quad (7)$$

where

$$f_{1\eta}(y) = -j\eta E_z(y,0_+) + j\omega\mu_o H_y(y,0_+) \quad (8)$$

Both the unknown p_o and p_1 functions are even meromorphic functions and thus we can apply Mittag-Leffler expansion with poles

$$\alpha_n = \sqrt{k^2 - (\frac{n\pi}{d})^2},\ \chi_n = \sqrt{\varepsilon_r k^2 - (\frac{n\pi}{d})^2} \quad (9)$$

respectively for equation (2) and (3). The unknown coefficients of the expansions are then related to the estimations of the plus spectral unknowns $V_+(\eta), V_{\pi+}(\eta)$ respectively at $-\alpha_n$ and $-\chi_n$.

The application of the Fredholm factorization [5-6] to the Wiener-Hopf equations (2-5) yields a system of Fredholm integral equations of second kind (FIE) of the form:

$$\begin{pmatrix} V_+(\eta) \\ V_{\pi+}(\eta) \end{pmatrix} + \frac{1}{2\pi j}\int_{-\infty}^{\infty} \underline{\underline{M}}(\eta,\eta')\cdot\begin{pmatrix} V_+(\eta') \\ V_{\pi+}(\eta') \end{pmatrix} d\alpha' = \underline{N}_o(\eta) + \sum_{m=1}^{\infty}\underline{\underline{h}}_m(\eta)\cdot\begin{pmatrix} V_+(-\alpha_m) \\ V_{\pi+}(-\chi_m) \end{pmatrix} \quad (10)$$

where N_o is a unknown term deriving from the source. The matrix kernel M is compact and h_m are known. In particular in this paper the source N_o arises from the illumination of the structure by an Ez- polarized plane wave:

$$E_z^i(\rho,\varphi) = E_o e^{jk\rho\cos(\varphi-\varphi_o)} \quad (11)$$

In order to formulate (10) as a classical vector Fredholm integral equations of second kind we need to explicitly obtain a representation of $V_+(-\alpha_n), V_{\pi+}(-\chi_n)$. For this purpose, we resort to the application of Cauchy representation:

$$F_+(\eta) = \frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{F_+(\eta')}{\eta'-\eta}d\eta' + F_+^{n.s}(\eta), \quad \mathrm{Im}[\eta]>0 \quad (12)$$

where the apex n.s. stands for non-standard portion of the unknowns as defined for example in [11]. We recall that non-standard terms are related to source terms in this case GO that are known a-priori.

Consequently the sum

$$\sum_{m=1}^{\infty}\underline{\underline{h}}_m(\eta)\cdot\begin{pmatrix} V_+(-\alpha_m) \\ V_{\pi+}(-\chi_m) \end{pmatrix} \quad (13)$$

can be rewritten as new portion of integral term plus a source term and it yields a vector FIE of classical form:

$$\begin{pmatrix} V_+(\eta) \\ V_{\pi+}(\eta) \end{pmatrix} + \frac{1}{2\pi j}\int_{-\infty}^{\infty} \underline{\underline{\tilde{M}}}(\eta,\eta')\cdot\begin{pmatrix} V_+(\eta') \\ V_{\pi+}(\eta') \end{pmatrix} d\alpha' = \underline{\tilde{N}}_o(\eta) \quad (14)$$

The major difficulties to solve the vector integral equation (14) arises from the singularities of the Kernel $\underline{\underline{\tilde{M}}}(\eta,\eta')$ located near the integration line, i.e. the real axis. In order to obtain a fast convergence we need to warp the integration line into a suitable path that keeps the singularities far away. We experienced that one of the best integration line is

$$B_\theta : v(u) = u\exp[j\theta], -\infty < u < \infty, 0 < \theta < \pi/2 \quad (15)$$

Warping the real axis η' into B_θ we must change η so that it remains located on the new integration line to avoid kernel singularity lines.

To estimate the warped vector FIE we apply simple sample and hold quadrature (piece-wise constant collocation) in a limited interval where A and h are respectively truncation and step parameters.

The numerical solution yields samples of the plus unknowns $V_+(\eta)$, $V_{\pi+}(\eta)$ along the integration line B_θ that allow to get their reconstruction and analytical continuation.

In planar structures, as the ones considered in this paper, the analytical continuation is valid in the proper sheet of the η-plane, thus the estimation of physical component of field is directly obtained from the reconstructions of $V_+(\eta)$, $V_{\pi+}(\eta)$. In this problem, from the semi-analytical solution, we are able to estimate using asymptotics the reflected and refracted plane waves, the diffracted fields, the surface waves, the leaky waves, and the near field.

III. CONCLUSIONS

The Wiener Hopf technique and the associated Fredholm factorization provide a general efficient theory for the study of the diffraction by an abruptly ended dielectric slab.

In this paper we considered the application of this theory to the cases shown in Fig. 2.

Several numerical simulations obtained by varying geometrical and physical parameters demonstrate the efficiency and the accuracy of the solution obtained with the proposed method. Further details on the formulation, numerical validations and results will be shown during the symposium.

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