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# Three-dimensional exotic Newtonian gravity with cosmological constant

Patrick Concha<sup>a,\*</sup>, Lucrezia Ravera<sup>b</sup>, Evelyn Rodríguez<sup>c</sup>

<sup>a</sup> Departamento de Matemática y Física Aplicadas, Universidad Católica de la Santísima Concepción, Alonso de Ribera 2850, Concepción, Chile

<sup>b</sup> INFN, Sezione di Milano, Via Celoria 16, I-20133 Milano, Italy

<sup>c</sup> Departamento de Ciencias, Facultad de Artes Liberales, Universidad Adolfo Ibáñez, Viña del Mar, Chile

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## ABSTRACT

In this work we introduce a cosmological constant in the extended Newtonian gravity theory. To this end, we extend the exotic Newton-Hooke algebra by introducing new generators and central charges. The new algebra obtained here has been denoted as exotic Newtonian algebra and reproduces the extended Newtonian one in the flat limit  $\ell \rightarrow \infty$ . A three-dimensional Chern-Simons action for the exotic Newtonian algebra is presented. We show that the non-relativistic gravity theory proposed here reproduces the most general extended Newtonian gravity theory in the flat limit.

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## 1. Introduction

Newtonian (super)gravity theories have received a growing interest in the recent years due to their utility in condensed matter systems [1–8] and non-relativistic effective field theories [9–12].

The underlying symmetry of Newtonian gravity is given by the so-called Bargmann algebra [13–15], also known as centrally extended Galilean algebra. The construction of such gravity theory requires a geometric framework called Newton-Cartan geometry [13,14,16–25]. This subsequently allowed to formulate a super-symmetric extension of Newton-Cartan gravity [26,27]. Despite the great progress achieved at the bosonic level, an action principle for the Newtonian gravity model has been presented only recently, in [28]. Subsequently, in [29] the authors constructed, using the Chern-Simons (CS) formalism, a three-dimensional (super)gravity theory based on the extended Newtonian algebra. Such algebra requires to extend the so-called extended Bargmann algebra [30–36] by including new generators and central charges. The advantage of adopting a three-dimensional CS formalism relies in the fact that it not only provides with a simpler framework to formulate non-relativistic (super)gravities but also reproduces interesting toy models to approach higher-dimensional theories.

On the other hand, non-relativistic models that include a cosmological constant are described through the Newton-Hooke symmetry, which in the flat limit, leads to the Galilei symmetry [37–45]. However, the incorporation of a cosmological constant in the extended Newtonian gravity remains an open issue.

In this work, we present a novel non-relativistic algebra that allows us to construct a three-dimensional CS exotic Newtonian gravity in the presence of a cosmological constant. To this end, we extend the so-called extended Newton-Hooke algebra [46,47] by introducing new generators. Furthermore, in order to have a well-defined non-degenerate invariant tensor, we require the presence of two central charges. The new algebra is denoted as exotic Newtonian algebra and reproduces the extended Newtonian algebra [29] in the flat limit  $\ell \rightarrow \infty$ . The CS action obtained contains the extended Newton-Hooke gravity Lagrangian as a sub-case. Furthermore, we show that the flat limit reproduces not only the extended Newtonian gravity but also an exotic term. To our knowledge, this is the first report showing an action principle for Newtonian gravity including a cosmological constant. Our results could be useful in the formulation of a non-relativistic supergravity model in the presence of a cosmological constant.

The paper is organized as follows: In Section 2, following [29], we briefly review the extended Newtonian gravity theory and introduce an exotic term in the model. In Section 3, we present a new non-relativistic algebra that we have called exotic Newtonian algebra. Then, the explicit construction of a CS action invariant under this algebra is presented. Section 4 concludes our work with a discussion about possible future developments.

\* Corresponding author.

E-mail addresses: [patrick.concha@ucsc.cl](mailto:patrick.concha@ucsc.cl) (P. Concha), [lucrezia.ravera@mi.infn.it](mailto:lucrezia.ravera@mi.infn.it) (L. Ravera), [evelyn.rodriguez@edu.uai.cl](mailto:evelyn.rodriguez@edu.uai.cl) (E. Rodríguez).

## 2. Extended Newtonian gravity theory

In this section, following [29], we briefly review the so-called extended Newtonian gravity theory. In addition, we present new non-vanishing components of an invariant tensor that allows to write the most general extended Newtonian CS gravity action, the latter involving, in particular, a new exotic term.

The extended Newtonian algebra is characterized by the presence of the extended Bargmann generators [30–36], which are given by the set  $\{J, G_a, S, H, P_a, M\}$ , together with a set of additional generators given by  $\{T_a, B_a\}$ . Besides, as shown in [29], the proper construction of a three-dimensional CS action further requires to add two central charges,  $Y$  and  $Z$ . The presence of such central charges assures the non-degeneracy of the invariant tensor, allowing the formulation of a well-defined CS action. The generators of the extended Newtonian algebra satisfy the following non-vanishing commutation relations:

$$\begin{aligned} [J, G_a] &= \epsilon_{ab} G_b, [G_a, G_b] = -\epsilon_{ab} S, [H, G_a] = \epsilon_{ab} P_b, \\ [J, P_a] &= \epsilon_{ab} P_b, [G_a, P_b] = -\epsilon_{ab} M, [H, B_a] = \epsilon_{ab} T_b, \\ [J, B_a] &= \epsilon_{ab} B_b, [G_a, B_b] = -\epsilon_{ab} Z, [J, T_a] = \epsilon_{ab} T_b, \\ [S, G_a] &= \epsilon_{ab} B_b, [G_a, T_b] = \epsilon_{ab} Y, [S, P_a] = \epsilon_{ab} T_b, \\ [M, G_a] &= \epsilon_{ab} T_b, [P_a, B_b] = \epsilon_{ab} Y, \end{aligned} \quad (2.1)$$

where  $a, b = 1, 2$ ,  $\epsilon_{ab} \equiv \epsilon_{0ab}$ ,  $\epsilon^{ab} \equiv \epsilon^{0ab}$ , such that  $\epsilon_{ab}\epsilon^{ac} = -\delta_b^c$ . The extended Newtonian algebra can be seen as the central extension of the algebra introduced in [28] used to define an action principle for Newtonian gravity.

The extended Newtonian algebra admits the following non-degenerate invariant tensor [29]:

$$\begin{aligned} \langle MS \rangle &= \langle HZ \rangle = -\langle JY \rangle = -\beta_1, \\ \langle P_a B_b \rangle &= \langle G_a T_b \rangle = \beta_1 \delta_{ab}, \end{aligned} \quad (2.2)$$

where we have introduced an arbitrary constant  $\beta_1$  in order to distinguish the components in (2.2) from other contributions. Indeed, the extended Newtonian algebra can also be equipped with the extended Bargmann non-vanishing components of the invariant tensor

$$\begin{aligned} \langle JS \rangle &= -\alpha_0, \\ \langle G_a G_b \rangle &= \alpha_0 \delta_{ab}, \\ \langle JM \rangle &= \langle HS \rangle = -\alpha_1, \\ \langle G_a P_b \rangle &= \alpha_1 \delta_{ab}, \end{aligned} \quad (2.3)$$

and with exotic non-vanishing components of the invariant tensor as

$$\begin{aligned} \langle SS \rangle &= \langle JZ \rangle = -\beta_0, \\ \langle G_a B_b \rangle &= \beta_0 \delta_{ab}, \end{aligned} \quad (2.4)$$

being  $\alpha_0$ ,  $\alpha_1$ , and  $\beta_0$  arbitrary independent constants. Observe that the components proportional to  $\alpha_1$  and  $\beta_1$  reproduce, respectively, the usual invariant tensors of the extended Bargmann [46,36] and extended Newtonian algebra [29]. As we shall see below, the respective exotic sectors are related to the constants  $\alpha_0$  and  $\beta_0$  [48].

One can then write the gauge connection one-form  $A = A^A T_A$  for the extended Newtonian algebra as

$$A = \tau H + e^a P_a + \omega J + \omega^a G_a + m M + s S + t^a T_a + b^a B_a + y Y + z Z. \quad (2.5)$$

The corresponding two-form curvatures read

$$\begin{aligned} F &= R(\tau) H + R^a(e^b) P_a + R(\omega) J + R^a(\omega^b) G_a + R(m) M \\ &\quad + R(s) S + R^a(t^b) T_a + R^a(b^b) B_a + R(y) Y + R(z) Z, \end{aligned} \quad (2.6)$$

with

$$\begin{aligned} R(\tau) &= d\tau, \\ R^a(e^b) &= de^a + \epsilon^{ac} \omega e_c + \epsilon^{ac} \tau \omega_c, \\ R(\omega) &= d\omega, \\ R^a(\omega^b) &= d\omega^a + \epsilon^{ac} \omega \omega_c, \\ R(m) &= dm + \epsilon^{ac} \omega_a e_c, \\ R(s) &= ds + \frac{1}{2} \epsilon^{ac} \omega_a \omega_c, \\ R^a(t^b) &= dt^a + \epsilon^{ac} \omega t_c + \epsilon^{ac} \tau b_c + \epsilon^{ac} s e_c + \epsilon^{ac} m \omega_c, \\ R^a(b^b) &= db^a + \epsilon^{ac} \omega b_c + \epsilon^{ac} s \omega_c, \\ R(y) &= dy - \epsilon^{ac} \omega_a t_c - \epsilon^{ac} e_a b_c, \\ R(z) &= dz + \epsilon^{ac} \omega_a b_c. \end{aligned} \quad (2.7)$$

Then, plugging the connection one-form (2.5) and the non-vanishing components of the invariant tensor given by (2.2), (2.3) and (2.4) into the expression of a three-dimensional CS action, that is

$$I_{CS} = \frac{k}{4\pi} \int \left\langle AdA + \frac{2}{3} A^3 \right\rangle = \frac{k}{4\pi} \int \left\langle AF - \frac{1}{3} A^3 \right\rangle, \quad (2.8)$$

where  $k$  is the CS level of the theory (for gravitational theories  $k$  is related to the gravitational constant  $G$ , that is, specifically,  $k = 1/(4G)$ ), we find the following CS action (written up to boundary terms):

$$\begin{aligned} I_{gEN} &= \frac{k}{4\pi} \int \alpha_0 \left[ \omega_a R^a(\omega^b) - 2sR(\omega) \right] \\ &\quad + 2\alpha_1 \left[ e_a R^a(\omega^b) - mR(\omega) - \tau R(s) \right] \\ &\quad + \beta_0 \left[ b_a R^a(\omega^b) + \omega_a R^a(b^b) - 2zR(\omega) - sds \right] \\ &\quad + 2\beta_1 \left[ e_a R^a(b^b) + t_a R^a(\omega^b) \right] \\ &\quad + yR(\omega) - mR(s) - \tau R(z). \end{aligned} \quad (2.9)$$

The non-relativistic action (2.9) describes the most general action for the extended Newtonian algebra. We have denoted this action by the acronym gEN. One can see that the CS action (2.9) contains four independent sectors proportional to  $\alpha_0$ ,  $\alpha_1$ ,  $\beta_0$ , and  $\beta_1$ , respectively: The first term reproduces the so-called non-relativistic exotic gravity Lagrangian. The name “exotic” in this case is due to the fact that such non-relativistic gravity term can be obtained as a non-relativistic limit of an  $U(1)$ -extension of the so-called exotic Einstein gravity Lagrangian [48] also known as Lorentz or Pontryagin term. The explicit non-relativistic limit reproducing such non-relativistic Lagrangian can be found in [49]. On the other hand, the extended Bargmann gravity term [36] appears considering the term proportional to  $\alpha_1$ . A new “exotic extended Newtonian” gravity term appears along  $\beta_0$ , while the last term coincides with the CS Lagrangian presented in [29]. Here we refer to an exotic extended Newtonian Lagrangian since we conjecture that such term could be alternatively obtained as non-relativistic limit of the exotic Einstein gravity Lagrangian in the Euclidean and Lorentzian signatures. In particular, the existence of such term was

already discussed in [29]. Let us note that the Lagrangian proportional to  $\beta_1$  is different from the Newtonian gravity Lagrangian presented in [28]. Furthermore, as was shown in [29], the coupling to matter of the extended Newtonian gravity of [29] resembles to the matter-coupling of the extended Bargmann gravity [36].

Let us specify that the exotic term introduced here has been obtained by hand. Nevertheless, it would be interesting to recover such term from a non-relativistic limit or through a limit procedure in which the resulting theory exhibits a non-vanishing cosmological constant. In what follows, we shall explore the possibility to include a cosmological constant by introducing an explicit length parameter  $\ell$ . As we will show, the aforementioned exotic contribution can be obtained, at least in our framework, by considering the flat limit  $\ell \rightarrow \infty$  of a sector pertaining to the CS theory we will construct in next section.

### 3. Exotic Newtonian gravity with cosmological constant

In this section we generalize the extended Newtonian gravity algebra introduced in [29] to accommodate a cosmological constant. The new algebra obtained here will be called along the paper as the *exotic Newtonian* algebra. We also provide with the non-vanishing components of the invariant tensor allowing us to construct a CS action invariant under the exotic Newtonian algebra. Remarkably, the extended Newtonian gravity theory presented in [29] appears as a flat limit  $\ell \rightarrow \infty$  of the exotic Newtonian one presented here. The main reason behind our choice of the name “exotic Newtonian” is twofold. On one hand, as we will show, the exotic Newtonian algebra we present contains the “exotic” Newton-Hooke algebra as a subalgebra [50] and reproduces the extended “Newtonian” one in the flat limit. On the other hand, the non-relativistic action based on this novel symmetry contains the non-relativistic exotic gravity and exotic extended Newtonian terms, previously obtained, as particular subcases. As in the case without cosmological constant, the exotic terms appearing here should be related through a proper non-relativistic limit to the exotic AdS Lagrangian [51].

#### 3.1. Exotic Newtonian algebra and flat limit

In order to accommodate a cosmological constant to the extended Newtonian algebra, we require to extend the extended Newton-Hooke algebra [46,47], also known as exotic Newton-Hooke algebra [50], spanned by  $\{J, G_a, H, P_a, M, S\}$  together with an additional set of generators given by  $\{B_a, T_a\}$ . As in the case of the extended Newtonian algebra, we shall also consider the presence of two central charges  $Y$  and  $Z$ . Although the generators are the same as in the extended Newtonian case, the presence of a cosmological constant will imply new non-vanishing commutators involving an explicit scale  $\ell$ . The new algebra, which we dub as the exotic Newtonian algebra, has the following non-vanishing commutation relations:

$$\begin{aligned}
 [J, G_a] &= \epsilon_{ab} G_b, [G_a, G_b] = -\epsilon_{ab} S, [H, G_a] = \epsilon_{ab} P_b, \\
 [J, P_a] &= \epsilon_{ab} P_b, [G_a, P_b] = -\epsilon_{ab} M, [H, B_a] = \epsilon_{ab} T_b, \\
 [J, B_a] &= \epsilon_{ab} B_b, [G_a, B_b] = -\epsilon_{ab} Z, [J, T_a] = \epsilon_{ab} T_b, \\
 [S, G_a] &= \epsilon_{ab} B_b, [G_a, T_b] = \epsilon_{ab} Y, [S, P_a] = \epsilon_{ab} T_b, \\
 [M, G_a] &= \epsilon_{ab} T_b, [P_a, B_b] = \epsilon_{ab} Y, [H, P_a] = \frac{1}{\ell^2} \epsilon_{ab} G_b, \\
 [H, T_a] &= \frac{1}{\ell^2} \epsilon_{ab} B_b, [P_a, P_b] = -\frac{1}{\ell^2} \epsilon_{ab} S, \\
 [M, P_a] &= \frac{1}{\ell^2} \epsilon_{ab} B_b, [P_a, T_b] = -\frac{1}{\ell^2} \epsilon_{ab} Z. \tag{3.1}
 \end{aligned}$$

Interestingly, the limit  $\ell \rightarrow \infty$  reproduces the extended Newtonian algebra of [29]. On the other hand, one can see that setting  $B_a, T_a, Y$ , and  $Z$  to zero one recovers the extended Newton-Hooke algebra [50], which leads to the extended Bargmann algebra in the limit  $\ell \rightarrow \infty$ .

As we shall see, the generators considered here allows to introduce a well-defined non-degenerate invariant tensor which is crucial to formulate a three-dimensional CS action.

#### 3.2. Chern-Simons exotic Newtonian gravity action

Let us now move to the construction of a three-dimensional CS action invariant under the exotic Newtonian algebra introduced previously.

The exotic Newtonian algebra (3.1) admits the non-vanishing components of the invariant tensor of the extended Newtonian algebra (2.2)-(2.4) along with

$$\begin{aligned}
 \langle HM \rangle &= -\frac{\alpha_0}{\ell^2}, \\
 \langle P_a P_b \rangle &= \frac{\alpha_0}{\ell^2} \delta_{ab}, \\
 \langle MM \rangle &= -\langle HY \rangle = -\frac{\beta_0}{\ell^2}, \\
 \langle P_a T_b \rangle &= \frac{\beta_0}{\ell^2} \delta_{ab}, \tag{3.2}
 \end{aligned}$$

where  $\alpha_0, \alpha_1, \beta_0$ , and  $\beta_1$  are independent arbitrary constants. It is interesting to note that the components of the invariant tensor proportional to  $\alpha_0$  and  $\alpha_1$  reproduces the invariant non-degenerate bilinear form of the extended Newton-Hooke case [46,47]. On the other hand, those proportional to  $\beta_0$  and  $\beta_1$  are related to the extended Newtonian action besides an exotic term. In particular, both  $\alpha_0$  and  $\beta_0$  are the respective coupling constants of an exotic Lagrangian. Let us further observe that, remarkably, the limit  $\ell \rightarrow \infty$  leads to the invariant tensor of the extended Bargmann algebra [36,47] and the extended Newtonian one [29].

The one-form gauge connection coincides with (2.5) as the field content is the same. However, for the exotic Newtonian algebra, the corresponding two-form curvatures read

$$\begin{aligned}
 \hat{F} &= R(\tau) H + R^a(e^b) P_a + R(\omega) J + \hat{R}^a(\omega^b) G_a + R(m) M \\
 &\quad + \hat{R}(s) S + R^a(t^b) T_a \\
 &\quad + \hat{R}^a(b^b) B_a + R(y) Y + \hat{R}(z) Z, \tag{3.3}
 \end{aligned}$$

with

$$\begin{aligned}
 R(\tau) &= d\tau, \\
 R^a(e^b) &= de^a + \epsilon^{ac} \omega e_c + \epsilon^{ac} \tau \omega_c, \\
 R(\omega) &= d\omega, \\
 \hat{R}^a(\omega^b) &= d\omega^a + \epsilon^{ac} \omega \omega_c + \frac{1}{\ell^2} \epsilon^{ac} \tau e_c, \\
 R(m) &= dm + \epsilon^{ac} \omega_a e_c, \\
 \hat{R}(s) &= ds + \frac{1}{2} \epsilon^{ac} \omega_a \omega_c + \frac{1}{2\ell^2} \epsilon^{ac} e_a e_c, \\
 R^a(t^b) &= dt^a + \epsilon^{ac} \omega t_c + \epsilon^{ac} \tau b_c + \epsilon^{ac} s e_c + \epsilon^{ac} m \omega_c, \\
 \hat{R}^a(b^b) &= db^a + \epsilon^{ac} \omega b_c + \epsilon^{ac} s \omega_c + \frac{1}{\ell^2} \epsilon^{ac} \tau t_c + \frac{1}{\ell^2} \epsilon^{ac} m e_c, \\
 R(y) &= dy - \epsilon^{ac} \omega_a t_c - \epsilon^{ac} e_a b_c, \\
 \hat{R}(z) &= dz + \epsilon^{ac} \omega_a b_c + \frac{1}{\ell^2} \epsilon^{ac} e_a t_c. \tag{3.4}
 \end{aligned}$$

One can then see that the limit  $\ell \rightarrow \infty$  reproduces the diverse curvature two-forms of the extended Newtonian algebra (2.7).

A CS action invariant under the exotic Newtonian algebra introduced here can be obtained by considering the one-form gauge connection for the exotic Newtonian algebra and the non-vanishing components of the invariant tensor (2.2), (2.3), (2.4), and (3.2) in the expression of a three-dimensional CS action (2.8). The exotic Newtonian gravity (exN) action in three spacetime dimensions is given by (up to boundary terms)

$$\begin{aligned}
I_{\text{exN}} = & \frac{k}{4\pi} \int \alpha_0 \left[ \omega_a \hat{R}^a(\omega^b) - 2sR(\omega) \right. \\
& \left. + \frac{1}{\ell^2} e_a R^a(e^b) - \frac{2}{\ell^2} mR(\tau) \right] \\
& + \alpha_1 \left[ e_a \hat{R}^a(\omega^b) + \omega_a R^a(e^b) - 2mR(\omega) - 2sR(\tau) \right] \\
& + \beta_0 \left[ b_a \hat{R}^a(\omega^b) + \omega_a \hat{R}^a(b^b) - 2zR(\omega) - sds \right. \\
& \left. + \frac{2}{\ell^2} yR(\tau) - \frac{1}{\ell^2} mdm \right. \\
& \left. + \frac{1}{\ell^2} t_a R^a(e^b) + \frac{1}{\ell^2} e_a R^a(t^b) \right] + \beta_1 \left[ e_a \hat{R}^a(b^b) + b_a R^a(e^b) \right. \\
& \left. + t_a \hat{R}^a(\omega^b) + \omega_a R^a(t^b) + 2yR(\omega) - 2zR(\tau) - 2m ds \right]. \quad (3.5)
\end{aligned}$$

The CS gravity action (3.5) is invariant under the exotic Newtonian algebra introduced previously. One can notice that there are four independent terms proportional to  $\alpha_0$ ,  $\alpha_1$ ,  $\beta_0$ , and  $\beta_1$ , respectively. In particular, the terms proportional to  $\alpha_0$  and to  $\alpha_1$  correspond to the exotic Lagrangian and to the extended Newton-Hooke gravity Lagrangian [46,47], respectively. The latter is known to describe a non-relativistic model with cosmological constant which can appear from the (A)dS algebra after having considered the limit  $c \rightarrow \infty$  and  $\Lambda \rightarrow 0$ , but keeping  $c^2 \Lambda$  finite. On the other hand, the exotic term could be recovered by a suitable non-relativistic limit of the exotic Lagrangian for the AdS algebra also known as Pontryagin and Nieh-Yan Chern-Simons terms [51]. The gauge fields related to the generators  $\{B_a, T_a, Y, Z\}$  appear exclusively along  $\beta_1$  and  $\beta_0$ . In particular, the Lagrangian proportional to  $\beta_1$  generalizes the extended Newtonian gravity Lagrangian presented in [29] by introducing new terms with an explicit scale  $\ell$ .

Let us note that in the limit  $\ell \rightarrow \infty$  of (3.5) we have that the sector involving the contributions proportional to  $\alpha_0$  and  $\alpha_1$  can be rewritten, up to boundary terms, as the Lagrangian invariant under the extended Bargmann (EB) algebra, namely the terms proportional to  $\alpha_0$  and  $\alpha_1$  in (2.9). One can see that the exotic Newtonian curvature  $R(\omega)$  is not modified by the flat limit and coincides with the extended Bargmann one. On the other hand, the limit  $\ell \rightarrow \infty$  applied in the  $\beta_1$  and  $\beta_0$  sectors reproduces, up to boundary terms, the extended Newtonian gravity and the corresponding exotic Lagrangian, which are the terms proportional to  $\beta_1$  and  $\beta_0$  in (2.9), respectively.

One can see that each independent term of the exotic Newtonian gravity action (3.5) is invariant under the gauge transformation laws  $\delta A = d\lambda + [A, \lambda]$ , being

$$\begin{aligned}
\lambda = & \Lambda H + \Lambda^a P_a + \Omega J + \Omega^a G_a + \chi M + \kappa S + \Upsilon^a T_a \\
& + \Sigma^a B_a + \gamma Y + \zeta Z \quad (3.6)
\end{aligned}$$

the gauge parameter. Specifically, the gauge transformations of the theory are given by

$$\begin{aligned}
\delta \tau = & d\Lambda, \quad \delta \omega = d\Omega, \\
\delta e^a = & d\Lambda^a + \epsilon^{ac} \omega \Lambda_c - \epsilon^{ac} \Omega e_c + \epsilon^{ac} \tau \Omega_c - \epsilon^{ac} \Lambda \omega_c,
\end{aligned}$$

$$\begin{aligned}
\delta \omega^a = & d\Omega^a + \epsilon^{ac} \omega \Omega_c - \epsilon^{ac} \Omega \omega_c + \frac{1}{\ell^2} \epsilon^{ac} \tau \Lambda_c - \frac{1}{\ell^2} \epsilon^{ac} \Lambda e_c, \\
\delta m = & d\chi + \epsilon^{ac} \omega_a \Lambda_c - \epsilon^{ac} e_a \Omega_c, \\
\delta s = & d\kappa + \epsilon^{ac} \omega_a \Omega_c + \epsilon^{ac} e_a \Lambda_c, \\
\delta t = & d\Upsilon + \epsilon^{ac} \omega \Upsilon_c - \epsilon^{ac} \Omega t_c + \epsilon^{ac} \tau \Sigma_c - \epsilon^{ac} \Lambda b_c + \epsilon^{ac} s \Lambda_c \\
& - \epsilon^{ac} \kappa e_c + \epsilon^{ac} m \Omega_c - \epsilon^{ac} \chi \omega_c, \\
\delta b^a = & d\Sigma^a + \epsilon^{ac} \omega \Sigma_c - \epsilon^{ac} \Omega b_c + \epsilon^{ac} s \Omega_c - \epsilon^{ac} \kappa \omega_c \\
& + \frac{1}{\ell^2} \epsilon^{ac} \tau \Upsilon_c - \frac{1}{\ell^2} \epsilon^{ac} \Lambda t_c + \frac{1}{\ell^2} \epsilon^{ac} m \Lambda_c - \frac{1}{\ell^2} \epsilon^{ac} \chi e_c, \\
\delta y = & d\gamma - \epsilon^{ac} \omega_a \Upsilon_c + \epsilon^{ac} t_a \Omega_c - \epsilon^{ac} e_a \Sigma_c + \epsilon^{ac} b_a \Lambda_c, \\
\delta z = & d\zeta + \epsilon^{ac} \omega_a \Sigma_c - \epsilon^{ac} b_a \Omega_c + \frac{1}{\ell^2} \epsilon^{ac} e_a \Upsilon_c - \frac{1}{\ell^2} \epsilon^{ac} t_a \Lambda_c. \quad (3.7)
\end{aligned}$$

Naturally, the gauge transformations of the most general extended Newtonian gravity theory are recovered in the limit  $\ell \rightarrow \infty$ . The non-degeneracy of the invariant tensor allows to achieve a well-defined CS gravity action whose equations of motion are given by the vanishing of the respective curvature two-forms (3.4).

#### 4. Discussion

In this work we have introduced a cosmological constant in the extended Newtonian gravity action presented in [29]. To this end, we have presented a new non-relativistic algebra that we have called exotic Newtonian algebra. The new algebra introduced here generalizes the extended Newton-Hooke one by including additional generators. Furthermore, analogously to the extended Newtonian case, we consider extra central charges in order to have a non-degenerate invariant tensor allowing to construct a well-defined three-dimensional CS action. Interestingly, the non-relativistic gravity theory presented here contains the extended Newton-Hooke gravity as a sub-case. In the flat limit  $\ell \rightarrow \infty$ , the exotic Newtonian gravity reproduces the most general extended Newtonian gravity theory. Indeed, we have shown that the limit  $\ell \rightarrow \infty$  lead us not only to the extended Bargmann gravity [46,47] but also to the extended Newtonian gravity [29], each one with their respective exotic sectors.

It is important to clarify that the non-relativistic algebra introduced here has been obtained by hand. Nevertheless, it would be interesting to recover it by a possible non-relativistic limit or contraction procedure of a relativistic theory.<sup>1</sup>

On the other hand, the algebra expansion procedure introduced in [53] and subsequently defined using Maurer-Cartan forms [54,55] and semigroups [56] have not long ago turned out to be useful to get diverse non-relativistic (super)algebras [57–61]. It would be worth it to explore the possibility to obtain the extended Newtonian algebra and the exotic Newtonian one presented here using the expansion procedure [work in progress].

Another aspect that deserves further investigation is the extension of the exotic Newtonian algebra to the supersymmetric case. As it is known, the construction of non-relativistic supergravity models have been very recently approached [36,26,27] (see also [62] for the development of an ultra-relativistic model). Moreover, the inclusion of a cosmological constant in a non-relativistic supergravity theory remains a difficult task [63]. One could expect that, as in the extended Newtonian superalgebra, the presence of additional fermionic generators is required in order to have a well-defined exotic Newtonian superalgebra [work in progress].

<sup>1</sup> Subsequently to our work, in [52] the authors have presented a non-relativistic limit allowing to recover the present exotic Newtonian gravity theory.

Our result can be seen as an extension of the extended Newtonian algebra [29]. However, other generalizations or extensions could be considered. Indeed, it would be interesting to study the construction of a Maxwellian version of the extended Newtonian algebra [work in progress]. The Maxwell algebra, introduced in [64,65], have been of recent interest in the (super)gravity context [66–82]. Its non-relativistic version has been explored in [49] and subsequently recovered from an enlarged extended Bargmann algebra [59].

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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