

The four-tank control problem: Comparison of two disturbance rejection control solutions

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# Final Report on the Force Key Comparison CCM.F-K3

## Measurand Force: 0.5 MN, 1 MN

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This report includes the following sections:

- 1. General information about the CCM.F-K3**
  - 2. Principles of the comparison**
  - 3. Realisation of the comparison**
  - 4. Limitations of the comparison**
  - 5. Uniformity of the measured values**
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  - 7. Results of the measurements: reported deflections and uncertainties, calculated corrections and evaluation of the data**
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### 1. General information about the CCM.F-K3

The CCM force working group decided to carry out CIPM force and torque key comparisons. In force four ranges were agreed – 10 kN, 100 kN, 1 MN and 4 MN. As pilot laboratory for the 1 MN inter-comparisons the force working group of PTB was appointed. This is the report for the 1 MN key comparison denoted as CCM.F-K3. Twelve laboratories - including the pilot - took part in the key comparison (see Table 1).

The intercomparison in the 1 MN range is organised by the pilot laboratory PTB. The intercomparison is carried out in two laboratory groups (Group A and B). In group A the intercomparison is carried out with two compression force transducers of 1 MN nominal force and with two force steps of 500 kN and 1 MN. The name of this Force Key Comparison is CCM.F-K3.a. In group B the intercomparison is carried out with two compression force transducers of 500 kN nominal force and with one force step of 500 kN. The name of this Force Key Comparison is CCM.F-K3.b. It was decided by the CCM Working group that the following countries were invited to participate in these intercomparisons.

Table 1: Participants in the CCM.F-K3 force key comparisons

RMO	Participant	Country	K3.a (500 kN, 1 MN)		K3.b (500 kN)	
				Period of measurements		Period of measurements
SIM	NIST	USA	×	03/2005		
EURAMET	PTB	Germany	×	01/2005 - 08/2011	×	02/2006 - 07/2007
	INRIM	Italy	×	05/2005	×	12/2006
	NPL	United Kingdom	×	01 - 02/2005		
	LNE	France			×	04/2007
	CEM	Spain			×	07/2007
	GUM	Poland			×	07/2006
COOMET	VNIIM	Russia	×	07/2005		
APMP	KRISS	Rep. of Korea			×	09/2006
	NMIA	Australia			×	10 - 11/2006
	NMIJ	Japan			×	02/2007
	NIM	China	×	09/2005 - 06/2011	×	05/2007

## 2. Principles of the comparison

The purpose of key comparisons is to compare the units of the given quantities as realized throughout the world. In the field of force, this is done by using force transducers of high quality, high-precision frequency-carrier amplifiers and very stable bridge standards. The force transducers were subject to similar loading schemes in the force standard machines of the participants following a strict measurement protocol and using similar amplifiers. The loading scheme shown in Figure 1 was agreed.

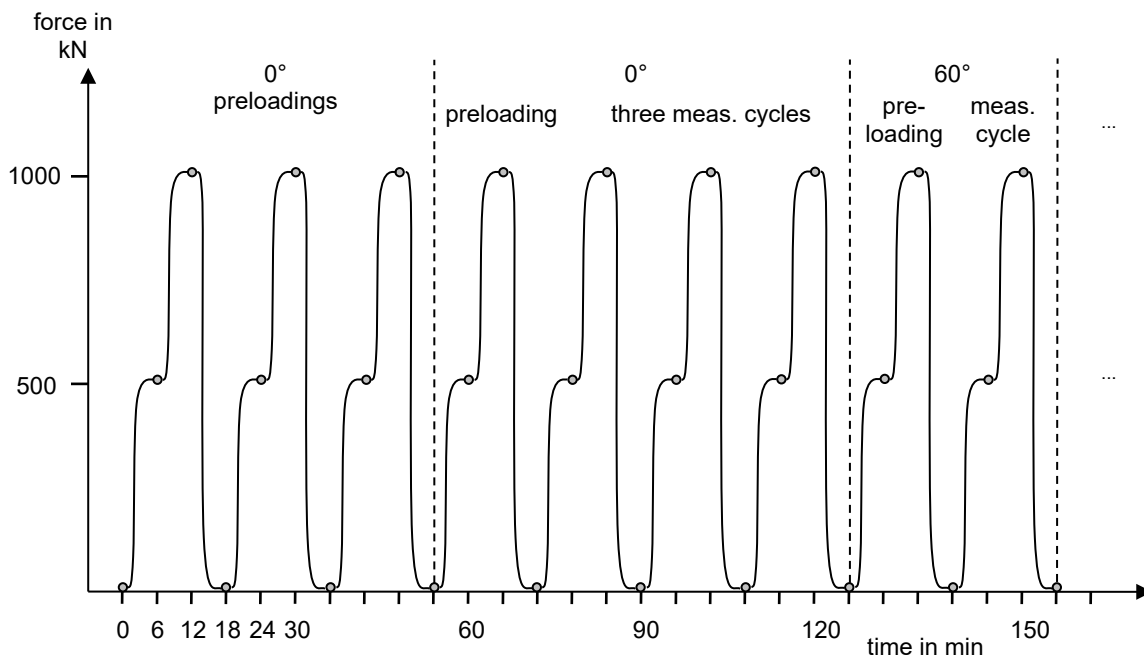


Figure 1: Diagram of the measurement sequence of the CCM.F-K3

The force transducer was rotated from 0° to 720° with 60° steps. Except the first mounting position with seven load cycles – four for stabilization and three for the repeatability measurement - in all other positions one preload and one measurement cycle (as shown for the 60° position in figure 1) were carried out, i.e. at transducer positions of 120°, 180°, 240°, 300°, 360°, 420°, 480°, 540°, 600°, 660° and 720°.

In each group two different force transducers are used.

For the 1 MN comparison PTB has selected two different force transducers which are shown in Figure 2.



Figure 2: Photograph of the two 1 MN force transducers

The comparison measurements had to be done with each of two force transducers having nominal capacities of 1 MN. The first 1 MN transducer (Figure 2, left) was manufactured by GTM, the second one (Figure 2, right) by HBM. The construction principles of the two transducer types are different, to consider different types and different effects of interaction between the force transducers and the force standard machines. The transducers had been selected for their very stable characteristics (T1, Manufacturer GTM, Type KTN-D, S/N 31002), respectively their known history (T2, Manufacturer HBM, Type C12, S/N 1).

For the 500 kN comparison PTB has selected two different force transducers which are shown in Figure 3.

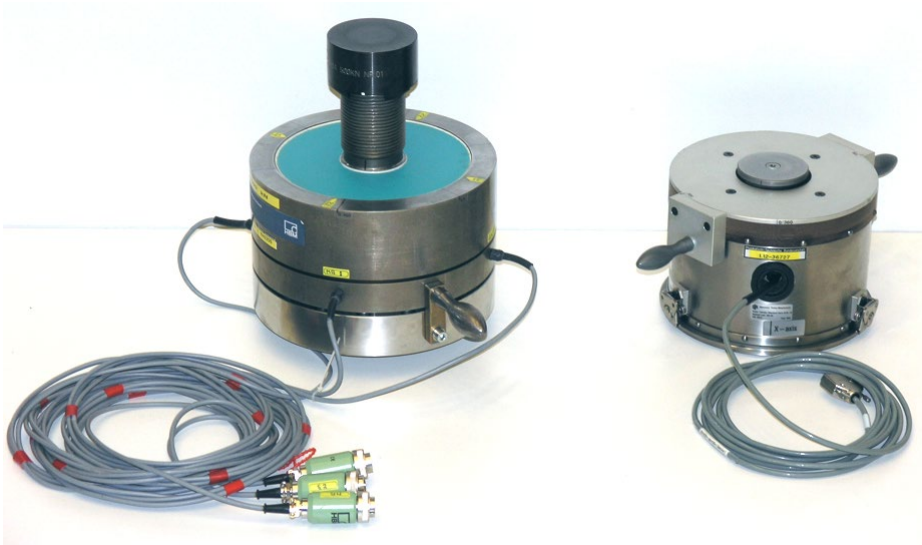


Figure 3: Photograph of the two 500 kN force transducers

The comparison measurements had to be done with each of two force transducers having nominal capacities of 500 kN. The first 500 kN transducer (Figure 3, right) was manufactured by GTM, the second one (Figure 3, left) by HBM. The construction principles of the two transducer types are different, to consider different types. The transducers had been selected for their very stable characteristics (T3, Manufacturer GTM, Type KTN-D, S/N 43010), respectively their known history (T4, Manufacturer HBM, Type Z400, S/N 1).

### 3. Realisation of the comparison

For this key comparison a star type formation had been chosen. That means that the transducers were returned to the pilot laboratory after the measurement at each participant. The pilot repeated all measurements before sending the instruments to the next participant. One complete measurement cycle (pilot – participating laboratory – pilot) is called a loop. The first measurement by the pilot is called the “PTB01” measurement and the second measurement by the pilot after the participating laboratory is called the “PTB02” measurement and sometimes an additional number is used if measurements are repeated to check the stability of the transducer in the pilot.

For all measurements in the pilot PTB's 2 MN deadweight machine was used [1]. The dead-weights are determined with relative expanded ( $k = 2$ ) uncertainties of  $3 \cdot 10^{-6}$ . Because of other influences which must be considered, the uncertainties of  $1 \cdot 10^{-5}$  are theoretical possible for selected force steps. But because of the number of other effects like interaction of the force transducer with the force standard machine which must be taken into consideration in force measurement and the problem of verification of these low uncertainties, the uncertainty over the whole range of the machine is  $\leq 2 \cdot 10^{-5}$ . This value is also used in the evaluation of this comparison to obtain consistency of the data.

Remark: One measurement with the 1 MN transducer T1 at NIM was repeated after all measurements are completed but before draft A was circulated, because probably during the first measurement at NIM there was a contact between the load bottom of this transducer with the temperature isolation.

#### 4. Limitations of the comparison

In 6 it will be shown, that the travelling standards used in this key comparison have a good stability but also drift effects must be considered to obtain consistency in the evaluation of the data.

For one measurement of one laboratory it was decided to repeat one measurement, because the measurement was an outlier. One explanation could be that the transducer which was used in combination with a temperature isolation box was not in the centre of the box so that a contact between the load bottom and the temperature isolation box explains this effect.

In addition to get comparable results some known effects should be taken into consideration. These are possible deviations of the amplifiers (DMP40) of the participating laboratories.

Because there is no real reference value (the transfer transducers do not provide constant values), the following facts should be accepted: there is no absolute numerical reference value and only relative deviations can be compared.

#### 5. Uniformity of the measured values

In practice, it is not possible to calibrate the DMP40 amplifiers of the participating laboratories against an absolute reference standard. The uniformity of the different DMP40s was confirmed with reference to a BN100 bridge standard. Each participating laboratory measured the indication of its own DMP40 against the signal of the pilot's BN100, which was delivered together with the transducers. The pilot monitored the signal of the same BN100 against the same DMP40 amplifier in the pilot laboratory additionally each time when the equipment was back from a participant. The sensitivities of the transducers at nominal force were 2.00 mV/V (T1), 2.74 mV/V (T2), 2.00 mV/V (T3) and 1.94 mV/V (T4). The measurements with the BN100 were carried out with suitably selected voltage ratios near the signals of the transducers for 500 kN and 1000 kN.

#### 6. Characteristics of the transducers

##### Creep effect

To minimize the influence of the creep, a relatively long cycle time of 6 minutes was agreed. This time includes the loading/unloading and the waiting time before the reading. The creep effect should be small enough to eliminate the uncertainty of the time of reading for every loading. The loading diagram in the 2 MN deadweight machine of PTB is seen in Figure 4.

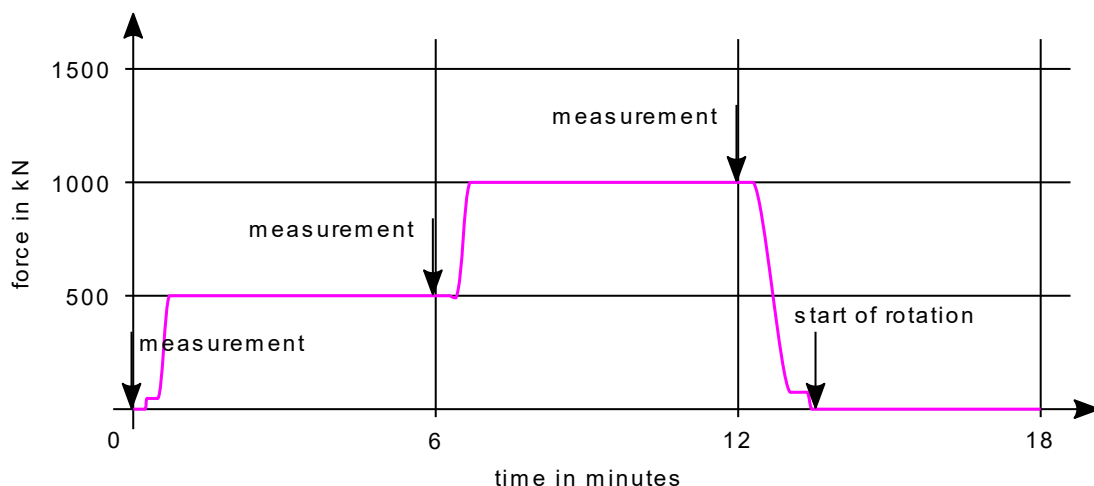


Figure 4: Loading diagram of PTB's 2 MN deadweight machine for the measurement sequence for laboratory group A

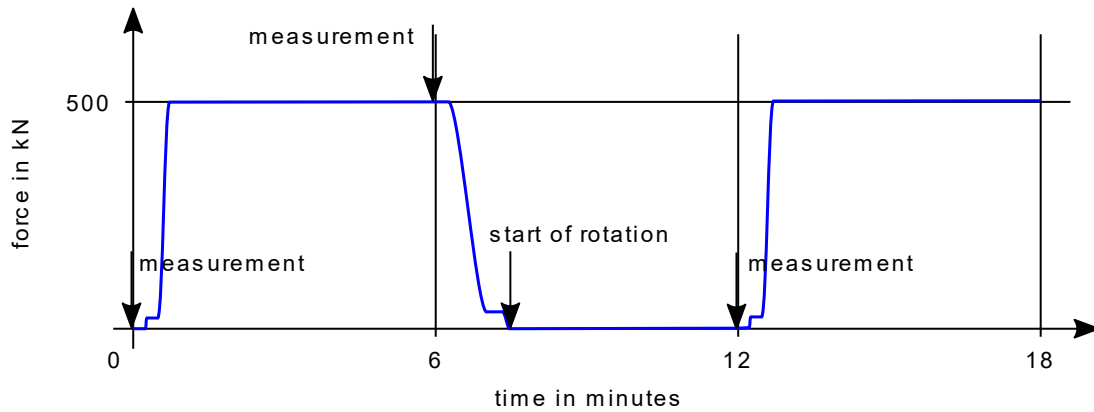


Figure 5: Loading diagram of PTB's 2 MN deadweight machine for the measurement sequence for laboratory group B

### Temperature influences on the sensitivity

The temperature effect on the sensitivity can also be an important factor if the environmental temperature in the participating laboratory is not the same as that at the pilot. But this effect is small and was neglected in the first evaluation. But in one laboratory a temperature correction was performed. Therefore, the temperature coefficients  $K_T$  of all 4 transducers were determined in the pilot laboratory by performing measurements at the following different temperatures 18 °C, 20 °C and 23 °C.

The temperature sensitivity coefficient  $K_T$  was calculated from the sensitivity determined by measurements performed at temperatures of 18 °C and 23 °C. The measurement values at 20 °C were considered for the estimation of the uncertainty of the temperature coefficients. The results are given in Table 2 and Table 3.

Table 2: Temperature sensitivity coefficients of the transducer T1 and T3

$K_T$	T1, GTM, Type KTN-D, S/N 31002	T3, GTM, Type KTN-D, S/N 43010
Force Step 500 kN	$1.63 \cdot 10^{-5} \text{ K}^{-1} \pm 1.0 \cdot 10^{-5} \text{ K}^{-1}$	$1.93 \cdot 10^{-5} \text{ K}^{-1} \pm 1.0 \cdot 10^{-6} \text{ K}^{-1}$
Force Step 1 MN	$1.27 \cdot 10^{-5} \text{ K}^{-1} \pm 3.5 \cdot 10^{-6} \text{ K}^{-1}$	-

Table 3: Temperature sensitivity coefficients of the transducers T2 and T4

$K_T$	T2, HBM, Type C12, S/N 1	T4, HBM, Type Z400, S/N 1
Force Step 500 kN	$2.10 \cdot 10^{-6} \text{ K}^{-1} \pm 2.3 \cdot 10^{-6} \text{ K}^{-1}$	$1.23 \cdot 10^{-5} \text{ K}^{-1} \pm 1.6 \cdot 10^{-6} \text{ K}^{-1}$
Force Step 1 MN	$1.80 \cdot 10^{-6} \text{ K}^{-1} \pm 2.4 \cdot 10^{-6} \text{ K}^{-1}$	-

### Stability of the transfer transducers

#### a) Stability of the sensitivity over the complete period of the key comparison

Based on the fact that the quality of the comparison substantially depends on the three measurements during the loop, the stability of the transducers is extremely important. The figures below show the stability of the transducers, which is determined as the relative deviations of the resulting deflections for all measurements made by the pilot from their arithmetical mean value.

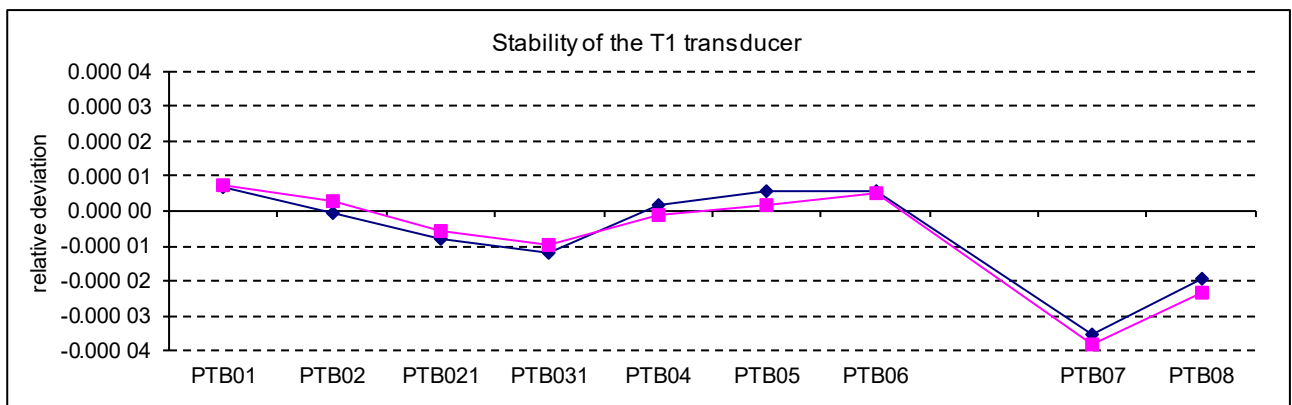


Figure 6: Relative deviations of the deflections for all measurements made by the pilot from the mean value calculated for measurements PTB01 to PTB06: 0.999857 mV/V at 500 kN and 1.999626 mV/V at 1000 kN for transducer T1 ( $\diamond$  – 500 kN,  $\square$  – 1 MN)

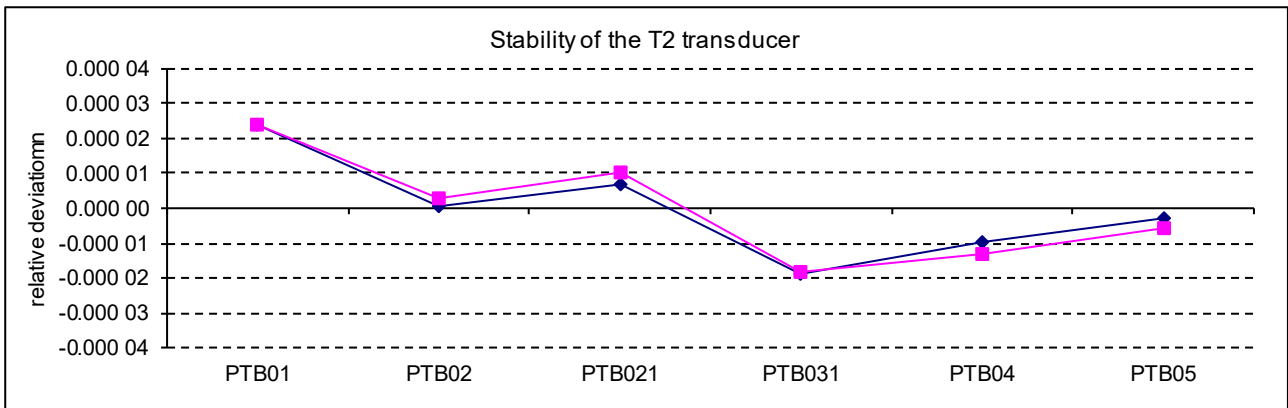


Figure 7: Relative deviations of the deflections for all measurements made by the pilot from their mean value: 1.371644 mV/V at 500 kN and 2.743543 mV/V at 1000 kN for transducer T2 (◇ – 500 kN, □ – 1 MN)

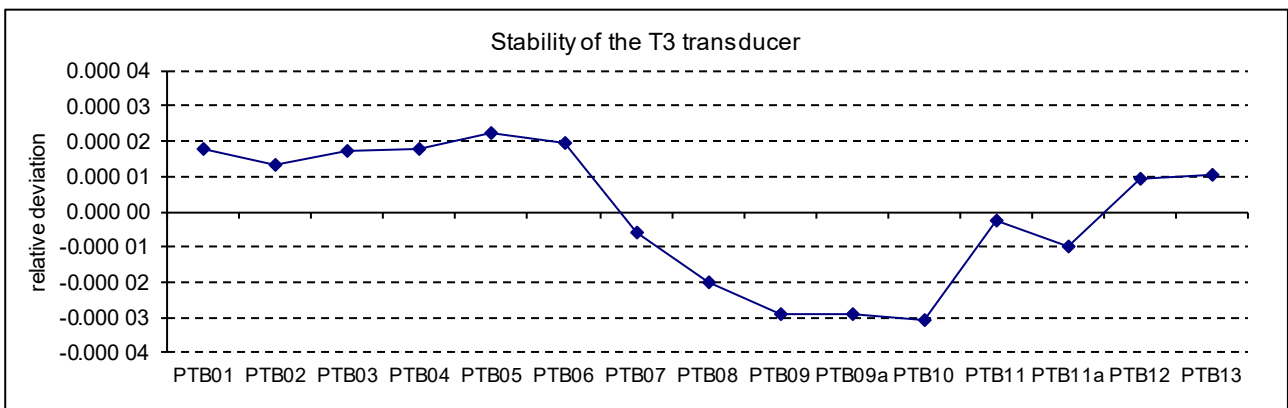


Figure 8: Relative deviations of the deflections for all measurements made by the pilot from their mean value: 1.999089 mV/V at 500 kN for transducer T3

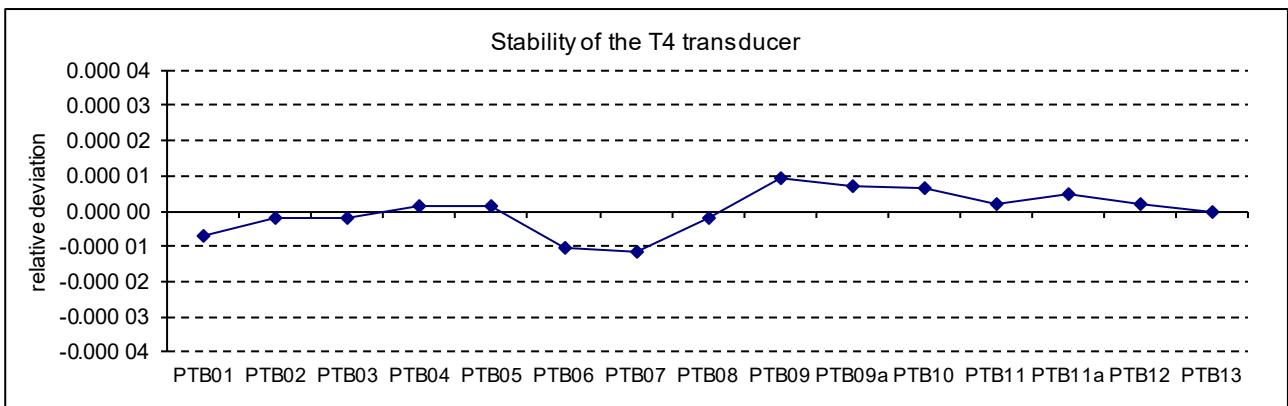


Figure 9: Relative deviations of the deflections for all measurements made by the pilot from their mean value: 1.943314 mV/V at 500 kN for transducer T4

## 7. Results of the measurements: reported deflections and uncertainties, calculated corrections and evaluation of the data

All results are given in the tables in section 7:

- the deflections as reported by the participants and the values with
- corrections for the amplifier.

The pilot reports the arithmetical mean of all measurements made in this laboratory and the arithmetical mean of the corresponding corrected values.

A proposal for the calculation of the weighted mean and a  $\chi^2$  test is given according to procedure A in [2]. In this part the calculation of the key comparison reference values (KCRV), of the relative deviations of the deflections from the corresponding KCRV and of the degrees of equivalence are proposed.

The following influences were considered:

### Sensor drift

Using the relative sensitivity change  $s_i$  of the travelling standard for a given moment of time when the measurements are carried out in the laboratory of the participant, the deflections calculated from the participant's calibration results can be corrected using

$$Y'_i = Y_i \cdot (1 - s_i) \quad (1)$$

with  $Y_i$  being the uncorrected and  $Y'_i$  the corrected deflections. For calculating the relative sensitivity change, a linear trend is assumed. Then  $s_i$  can be found from

$$s_i = \frac{S_{\text{before}} \cdot (T_{\text{after}} - T_i) + S_{\text{after}} \cdot (T_i - T_{\text{before}})}{M \cdot (T_{\text{after}} - T_{\text{before}})} - 1 \quad (2)$$

with  $M$  being the average sensitivity of the travelling standard used for the comparison,  $T_i$  the date of the measurement in the participating laboratory,  $T_{\text{before}}$  the date of the measurement in the pilot laboratory before that participant took place,  $T_{\text{after}}$  the date of the measurement in the pilot laboratory after that participant took place,  $S_{\text{before}}$  and  $S_{\text{after}}$  the corresponding sensitivities found in the pilot measurements before and after the participant.

The uncertainty is calculated according to

$$u^2(Y'_i) = u^2(Y_i) + s_i^2 \cdot u^2(Y_i) + Y_i^2 \cdot u^2(s_i) \quad (3)$$

$$w^2(Y'_i) = \frac{u^2(Y_i) + s_i^2 \cdot u^2(Y_i) + Y_i^2 \cdot u^2(s_i)}{Y_i'^2} \quad (3')$$

### BN100

Using the relative deviations  $d_i(V/V_S)$  for the DMP40 indications of each of the participants at the corresponding voltage ratios  $V/V_S$ , the deflections calculated from the participant's calibration results can be corrected using

$$Y''_i = Y'_i \cdot (1 - d_i(V/V_S)) \quad (4)$$

with  $Y'_i$  being the uncorrected and  $Y''_i$  the corrected deflections. The uncertainty is calculated according to

$$u^2(Y''_i) = u^2(Y'_i) + d_i^2 \cdot u^2(Y'_i) + Y_i'^2 \cdot u^2(d_i) \quad (5)$$

$$w^2(Y''_i) = \frac{u^2(Y'_i) + d_i^2 \cdot u^2(Y'_i) + Y_i'^2 \cdot u^2(d_i)}{Y_i''^2} \quad (5')$$

A - Evaluation of the uncorrected results

Relative deviations from the pilot's mean for transducer T1

Table 4: Uncorrected deflections in mV/V, expanded ( $k = 2$ ) relative and absolute uncertainties as reported by the participants and relative deviations from the pilot's mean for transducer T1 and both force steps

T1	500 kN				1 MN			
	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{mean}$	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{mean}$
NPL	0.999 836	$1.4 \cdot 10^{-5}$	0.000 014	$-2.1 \cdot 10^{-5}$	1.999 604	$1.1 \cdot 10^{-5}$	0.000 023	$-1.1 \cdot 10^{-5}$
NIST	0.999 836	$2.6 \cdot 10^{-5}$	0.000 026	$-2.1 \cdot 10^{-5}$	1.999 572	$2.0 \cdot 10^{-5}$	0.000 041	$-2.7 \cdot 10^{-5}$
INRiM	0.999 862	$2.0 \cdot 10^{-5}$	0.000 020	$5.3 \cdot 10^{-6}$	1.999 626	$2.0 \cdot 10^{-5}$	0.000 041	$2.6 \cdot 10^{-7}$
VNIIM	0.999 866	$2.1 \cdot 10^{-5}$	0.000 021	$9.7 \cdot 10^{-6}$	1.999 623	$2.1 \cdot 10^{-5}$	0.000 041	$-1.2 \cdot 10^{-6}$
NIM	0.999 826	$1.5 \cdot 10^{-5}$	0.000 015	$-3.1 \cdot 10^{-5}$	1.999 567	$1.5 \cdot 10^{-5}$	0.000 030	$-2.9 \cdot 10^{-5}$
PTB ( $P_{mean}$ )	0.999 857	$2.0 \cdot 10^{-5}$	0.000 020	-	1.999 626	$2.0 \cdot 10^{-5}$	0.000 041	-
PTB01	0.999 864	$2.1 \cdot 10^{-5}$	0.000 021	$7.1 \cdot 10^{-6}$	1.999 640	$2.0 \cdot 10^{-5}$	0.000 041	$7.3 \cdot 10^{-6}$
PTB02	0.999 856	$2.1 \cdot 10^{-5}$	0.000 021	$-7.4 \cdot 10^{-7}$	1.999 632	$2.0 \cdot 10^{-5}$	0.000 041	$3.0 \cdot 10^{-6}$
PTB21	0.999 849	$2.0 \cdot 10^{-5}$	0.000 020	$-8.0 \cdot 10^{-6}$	1.999 614	$2.0 \cdot 10^{-5}$	0.000 041	$-5.7 \cdot 10^{-6}$
PTB31	0.999 845	$2.0 \cdot 10^{-5}$	0.000 020	$-1.2 \cdot 10^{-5}$	1.999 606	$2.0 \cdot 10^{-5}$	0.000 041	$-9.9 \cdot 10^{-6}$
PTB04	0.999 858	$2.0 \cdot 10^{-5}$	0.000 020	$1.6 \cdot 10^{-6}$	1.999 623	$2.0 \cdot 10^{-5}$	0.000 041	$-1.3 \cdot 10^{-6}$
PTB05	0.999 863	$2.0 \cdot 10^{-5}$	0.000 020	$5.9 \cdot 10^{-6}$	1.999 629	$2.0 \cdot 10^{-5}$	0.000 041	$1.5 \cdot 10^{-6}$
PTB06	0.999 863	$2.0 \cdot 10^{-5}$	0.000 020	$5.8 \cdot 10^{-6}$	1.999 636	$2.0 \cdot 10^{-5}$	0.000 041	$5.2 \cdot 10^{-6}$

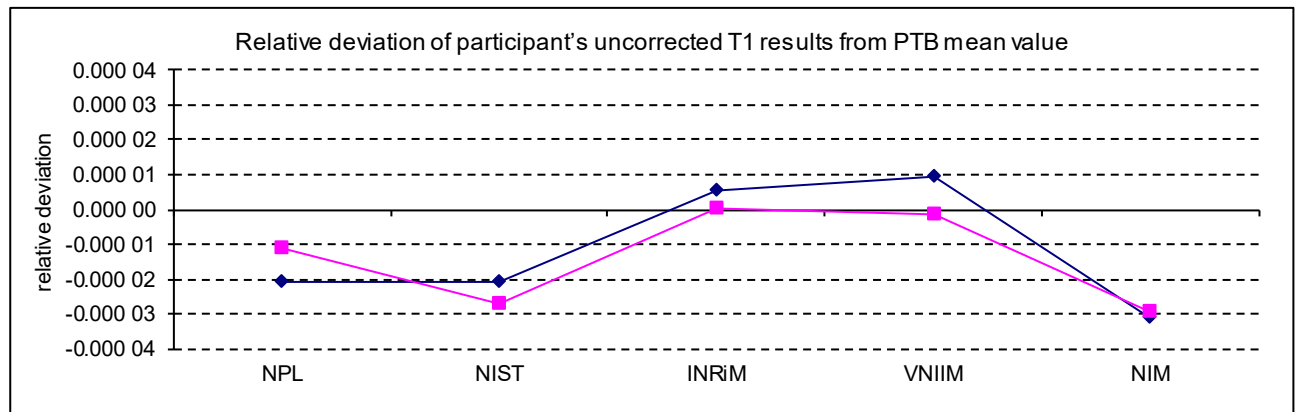


Figure 10: Relative deviation of participant's uncorrected results from PTB mean value (PTB01 to PTB06) for transducer T1 (♦ – 500 kN, ■ – 1 MN)

Relative deviations from the pilot's mean for transducer T2

Table 5: Uncorrected deflections in mV/V, expanded ( $k = 2$ ) relative and absolute uncertainties as reported by the participants and relative deviations from the pilot's mean for transducer T2 and both force steps

T2	500 kN				1 MN			
	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{mean}$	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{mean}$
NPL	1.371 659	$1.0 \cdot 10^{-5}$	0.000 014	$-4.7 \cdot 10^{-6}$	2.743 575 4	$1.0 \cdot 10^{-5}$	0.000 028	$-4.9 \cdot 10^{-6}$
NIST	1.371 652	$2.2 \cdot 10^{-5}$	0.000 031	$2.1 \cdot 10^{-6}$	2.743 569 0	$2.3 \cdot 10^{-5}$	0.000 063	$2.9 \cdot 10^{-6}$
INRiM	1.371 659	$2.3 \cdot 10^{-5}$	0.000 031	$1.6 \cdot 10^{-5}$	2.743 543 9	$2.4 \cdot 10^{-5}$	0.000 065	$3.2 \cdot 10^{-6}$
VNIIM	1.371 641	$2.1 \cdot 10^{-5}$	0.000 028	$1.1 \cdot 10^{-5}$	2.743 495 8	$2.0 \cdot 10^{-5}$	0.000 056	$-2.3 \cdot 10^{-6}$
NIM	1.371 643	$1.4 \cdot 10^{-5}$	0.000 019	$4.8 \cdot 10^{-6}$	2.743 525 4	$1.4 \cdot 10^{-5}$	0.000 038	$2.4 \cdot 10^{-6}$
PTB ( $P_{mean}$ )	1.371 644	$2.1 \cdot 10^{-5}$	0.000 028	-	2.743 542 7	$2.0 \cdot 10^{-5}$	0.000 056	-
PTB01a	1.371 676	$2.0 \cdot 10^{-5}$	0.000 028	$2.4 \cdot 10^{-5}$	2.743 608 5	$2.0 \cdot 10^{-5}$	0.000 056	$2.4 \cdot 10^{-5}$
PTB02	1.371 645	$2.1 \cdot 10^{-5}$	0.000 028	$8.2 \cdot 10^{-7}$	2.743 550 9	$2.0 \cdot 10^{-5}$	0.000 056	$3.0 \cdot 10^{-6}$
PTB03	1.371 653	$2.0 \cdot 10^{-5}$	0.000 028	$6.9 \cdot 10^{-6}$	2.743 570 3	$2.0 \cdot 10^{-5}$	0.000 056	$1.0 \cdot 10^{-5}$
PTB04	1.371 618	$2.1 \cdot 10^{-5}$	0.000 028	$-1.9 \cdot 10^{-5}$	2.743 493 3	$2.0 \cdot 10^{-5}$	0.000 056	$-1.8 \cdot 10^{-5}$
PTB05	1.371 630	$2.1 \cdot 10^{-5}$	0.000 028	$-9.7 \cdot 10^{-6}$	2.743 506 0	$2.0 \cdot 10^{-5}$	0.000 056	$-1.3 \cdot 10^{-5}$
PTB06	1.371 640	$2.1 \cdot 10^{-5}$	0.000 028	$-2.8 \cdot 10^{-6}$	2.743 527 2	$2.0 \cdot 10^{-5}$	0.000 056	$-5.7 \cdot 10^{-6}$

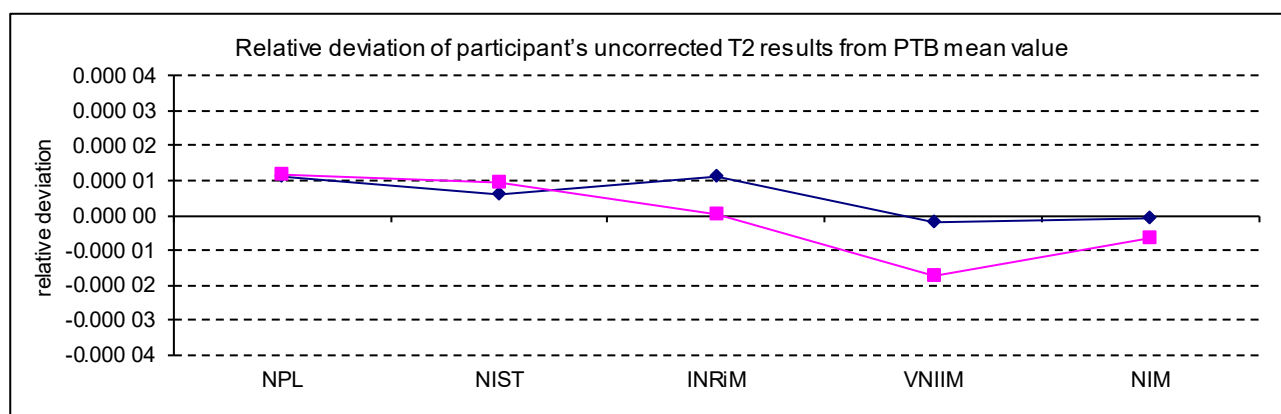


Figure 11: Relative deviation of participant's uncorrected results from PTB mean value (PTB01a to PTB06) for transducer T2 (◆ – 500 kN, ■ – 1 MN)

Relative deviations from the pilot's mean for transducer T3

Table 6: Uncorrected deflections in mV/V, expanded ( $k = 2$ ) relative and absolute uncertainties as reported by the participants and relative deviations from the pilot's mean for transducer T3

T3	500 kN			
	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{mean}$
INRiM	1.999 053	$2.0 \cdot 10^{-5}$	0.000 040	$-1.8 \cdot 10^{-5}$
LNE	1.999 054	$2.1 \cdot 10^{-5}$	0.000 041	$-1.8 \cdot 10^{-5}$
CEM	1.999 113	$2.0 \cdot 10^{-5}$	0.000 041	$1.2 \cdot 10^{-5}$
GUM	1.999 205	$1.2 \cdot 10^{-4}$	0.000 024	$5.8 \cdot 10^{-5}$
KRISS	1.999 250	$2.1 \cdot 10^{-5}$	0.000 043	$8.1 \cdot 10^{-5}$
NMIA	1.999 055	$2.0 \cdot 10^{-5}$	0.000 041	$-1.7 \cdot 10^{-5}$
NMIJ	1.999 022	$1.3 \cdot 10^{-5}$	0.000 025	$-3.4 \cdot 10^{-5}$
NIM	1.999 124	$1.5 \cdot 10^{-5}$	0.000 030	$1.8 \cdot 10^{-5}$
PTB	1.999 089	$2.0 \cdot 10^{-5}$	0.000 040	-
PTB01	1.999 124	$2.0 \cdot 10^{-5}$	0.000 040	$1.8 \cdot 10^{-5}$
PTB02	1.999 116	$2.0 \cdot 10^{-5}$	0.000 040	$1.4 \cdot 10^{-5}$
PTB03	1.999 123	$2.0 \cdot 10^{-5}$	0.000 040	$1.7 \cdot 10^{-5}$
PTB04	1.999 124	$2.0 \cdot 10^{-5}$	0.000 040	$1.8 \cdot 10^{-5}$
PTB05	1.999 134	$2.0 \cdot 10^{-5}$	0.000 040	$2.2 \cdot 10^{-5}$
PTB06	1.999 127	$2.0 \cdot 10^{-5}$	0.000 040	$1.9 \cdot 10^{-5}$
PTB07	1.999 077	$2.0 \cdot 10^{-5}$	0.000 040	$-6.0 \cdot 10^{-6}$
PTB08	1.999 049	$2.0 \cdot 10^{-5}$	0.000 040	$-2.0 \cdot 10^{-5}$
PTB09	1.999 030	$2.0 \cdot 10^{-5}$	0.000 040	$-2.9 \cdot 10^{-5}$
PTB09a	1.999 030	$2.0 \cdot 10^{-5}$	0.000 040	$-2.9 \cdot 10^{-5}$
PTB10	1.999 028	$2.0 \cdot 10^{-5}$	0.000 040	$-3.1 \cdot 10^{-5}$
PTB11	1.999 084	$2.0 \cdot 10^{-5}$	0.000 041	$-2.5 \cdot 10^{-6}$
PTB11a	1.999 069	$2.0 \cdot 10^{-5}$	0.000 040	$-1.0 \cdot 10^{-5}$
PTB12	1.999 107	$2.0 \cdot 10^{-5}$	0.000 040	$9.1 \cdot 10^{-6}$
PTB13	1.999 110	$2.0 \cdot 10^{-5}$	0.000 040	$1.1 \cdot 10^{-5}$

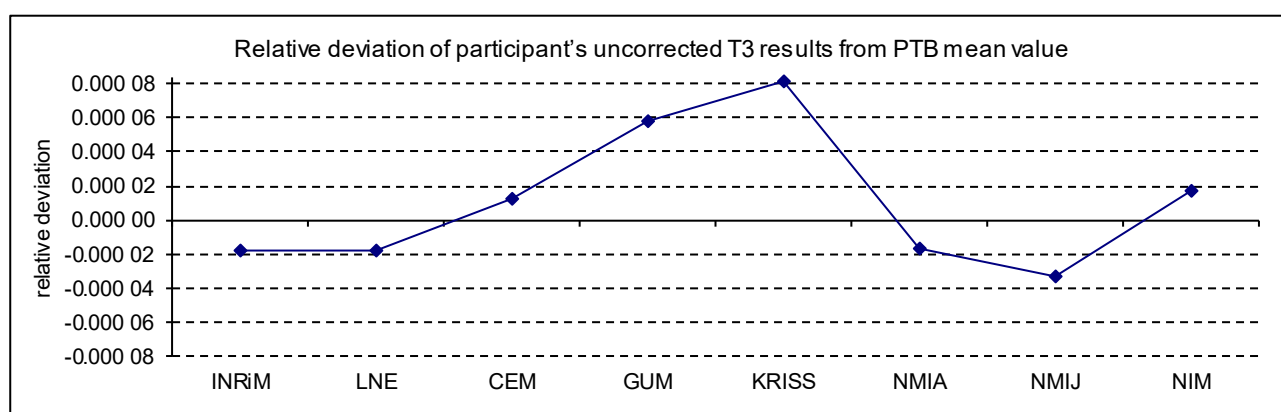


Figure 12: Relative deviation of participant's uncorrected results from PTB mean value (PTB01 to PTB13) for transducer T3

Relative deviations from the pilot's mean for transducer T4

Table 7: Uncorrected deflections in mV/V, expanded ( $k = 2$ ) relative and absolute uncertainties as reported by the participants and relative deviations from the pilot's mean for transducer T4

T4 Participant	500 kN			
	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{mean}$
INRiM	1.943 320	$2.0 \cdot 10^{-5}$	0.000 039	$3.1 \cdot 10^{-6}$
LNE	1.943 303	$2.0 \cdot 10^{-5}$	0.000 039	$-5.5 \cdot 10^{-6}$
CEM	1.943 327	$2.0 \cdot 10^{-5}$	0.000 039	$6.8 \cdot 10^{-6}$
GUM	1.943 364	$1.2 \cdot 10^{-4}$	0.000 233	$2.6 \cdot 10^{-5}$
KRISS	1.943 418	$2.1 \cdot 10^{-5}$	0.000 040	$5.4 \cdot 10^{-5}$
NMIA	1.943 306	$2.0 \cdot 10^{-5}$	0.000 040	$-4.1 \cdot 10^{-6}$
NMIJ	1.943 320	$1.3 \cdot 10^{-5}$	0.000 024	$3.2 \cdot 10^{-6}$
NIM	1.943 300	$1.4 \cdot 10^{-5}$	0.000 027	$-7.0 \cdot 10^{-6}$
PTB	1.943 314	$2.0 \cdot 10^{-5}$	0.000 039	-
PTB01	1.943 300	$2.0 \cdot 10^{-5}$	0.000 039	$-7.1 \cdot 10^{-6}$
PTB02	1.943 310	$2.0 \cdot 10^{-5}$	0.000 039	$-1.9 \cdot 10^{-6}$
PTB03	1.943 310	$2.0 \cdot 10^{-5}$	0.000 039	$-2.0 \cdot 10^{-6}$
PTB04	1.943 316	$2.0 \cdot 10^{-5}$	0.000 039	$1.3 \cdot 10^{-6}$
PTB05	1.943 317	$2.0 \cdot 10^{-5}$	0.000 039	$1.4 \cdot 10^{-6}$
PTB06	1.943 293	$2.0 \cdot 10^{-5}$	0.000 039	$-1.1 \cdot 10^{-6}$
PTB07	1.943 292	$2.0 \cdot 10^{-5}$	0.000 039	$-1.1 \cdot 10^{-6}$
PTB08	1.943 310	$2.0 \cdot 10^{-5}$	0.000 039	$-1.9 \cdot 10^{-6}$
PTB09	1.943 332	$2.0 \cdot 10^{-5}$	0.000 039	$9.5 \cdot 10^{-6}$
PTB09a	1.943 328	$2.0 \cdot 10^{-5}$	0.000 039	$7.4 \cdot 10^{-6}$
PTB10	1.943 327	$2.0 \cdot 10^{-5}$	0.000 039	$6.8 \cdot 10^{-6}$
PTB11	1.943 318	$2.0 \cdot 10^{-5}$	0.000 039	$2.1 \cdot 10^{-6}$
PTB11a	1.943 323	$2.0 \cdot 10^{-5}$	0.000 039	$4.7 \cdot 10^{-6}$
PTB12	1.943 317	$2.0 \cdot 10^{-5}$	0.000 039	$1.8 \cdot 10^{-6}$
PTB13	1.943 300	$2.0 \cdot 10^{-5}$	0.000 039	$-7.1 \cdot 10^{-6}$

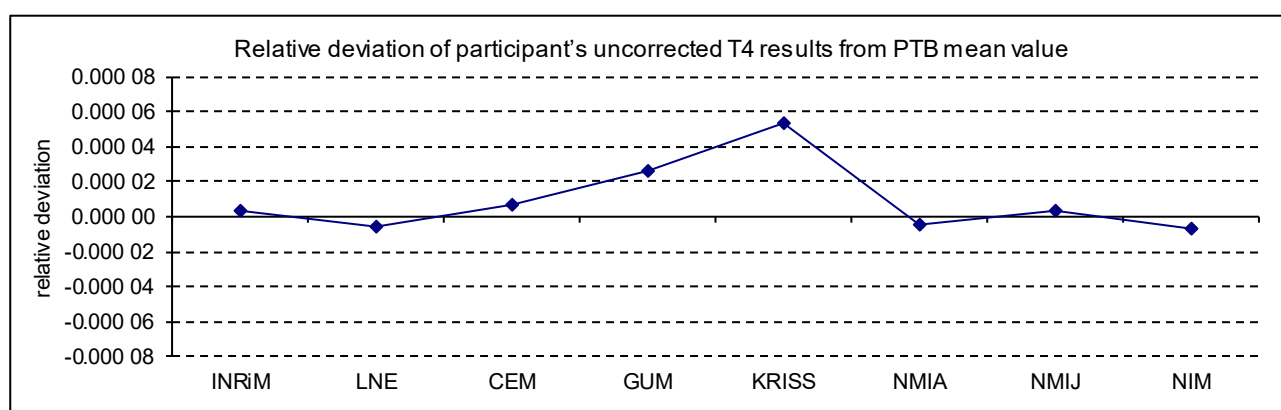


Figure 13: Relative deviation of participant's uncorrected results from PTB mean value (PTB01 to PTB13) for transducer T4

## B - Evaluation of the results corrected for the sensor drift

### Relative deviations from the pilot's mean for transducer T1

Table 8: Drift-corrected deflections in mV/V, expanded ( $k = 2$ ) relative and absolute uncertainties and relative deviations from the pilot's mean for transducer T1 and both force steps

T1	500 kN				1 MN			
	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{\text{mean}}$	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{\text{mean}}$
NPL	0.999 835	$1.4 \cdot 10^{-5}$	0.000 014	$-2.1 \cdot 10^{-5}$	1.999 596	$1.2 \cdot 10^{-5}$	0.000 024	$-1.5 \cdot 10^{-5}$
NIST	0.999 846	$2.7 \cdot 10^{-5}$	0.000 027	$-1.1 \cdot 10^{-5}$	1.999 588	$2.1 \cdot 10^{-5}$	0.000 042	$-1.9 \cdot 10^{-5}$
INRiM	0.999 867	$2.1 \cdot 10^{-5}$	0.000 021	$1.0 \cdot 10^{-5}$	1.999 638	$2.1 \cdot 10^{-5}$	0.000 041	$6.0 \cdot 10^{-6}$
VNIIM	0.999 862	$2.1 \cdot 10^{-5}$	0.000 021	$5.3 \cdot 10^{-6}$	1.999 622	$2.1 \cdot 10^{-5}$	0.000 042	$-1.7 \cdot 10^{-6}$
NIM	0.999 851	$1.6 \cdot 10^{-5}$	0.000 016	$-5.8 \cdot 10^{-6}$	1.999 624	$1.6 \cdot 10^{-5}$	0.000 031	$-6.9 \cdot 10^{-7}$
PTB ( $P_{\text{mean}}$ )	0.999 857	$2.0 \cdot 10^{-5}$	0.000 020	-	1.999 626	$2.0 \cdot 10^{-5}$	0.000 041	-

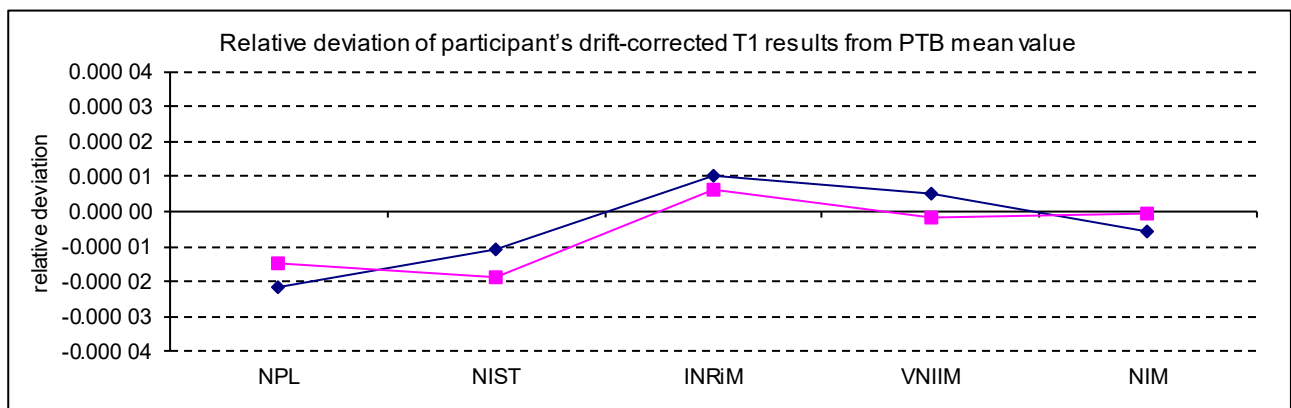


Figure 14: Relative deviation of participant's drift-corrected results from PTB mean value (PTB01 to PTB06) for transducer T1 (♦ – 500 kN, ■ – 1 MN)

### Relative deviations from the pilot's mean for transducer T2

Table 9: Drift-corrected deflections in mV/V, expanded ( $k = 2$ ) relative and absolute uncertainties and relative deviations from the pilot's mean for transducer T2 and both force steps

T2	500 kN				1 MN			
	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{\text{mean}}$	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{\text{mean}}$
NPL	1.371 637	$1.0 \cdot 10^{-5}$	0.000 014	$-4.7 \cdot 10^{-6}$	2.743 529	$1.0 \cdot 10^{-5}$	0.000 028	$-4.9 \cdot 10^{-6}$
NIST	1.371 647	$2.2 \cdot 10^{-5}$	0.000 031	$2.1 \cdot 10^{-6}$	2.743 551	$2.3 \cdot 10^{-5}$	0.000 063	$2.9 \cdot 10^{-6}$
INRiM	1.371 666	$2.3 \cdot 10^{-5}$	0.000 031	$1.6 \cdot 10^{-5}$	2.743 552	$2.4 \cdot 10^{-5}$	0.000 065	$3.2 \cdot 10^{-6}$
VNIIM	1.371 658	$2.1 \cdot 10^{-5}$	0.000 028	$1.1 \cdot 10^{-5}$	2.743 536	$2.0 \cdot 10^{-5}$	0.000 056	$-2.3 \cdot 10^{-6}$
NIM	1.371 650	$1.4 \cdot 10^{-5}$	0.000 019	$4.8 \cdot 10^{-6}$	2.743 549	$1.4 \cdot 10^{-5}$	0.000 038	$2.4 \cdot 10^{-6}$
PTB ( $P_{\text{mean}}$ )	1.371 644	$2.1 \cdot 10^{-5}$	0.000 028	-	2.743 543	$2.0 \cdot 10^{-5}$	0.000 056	-

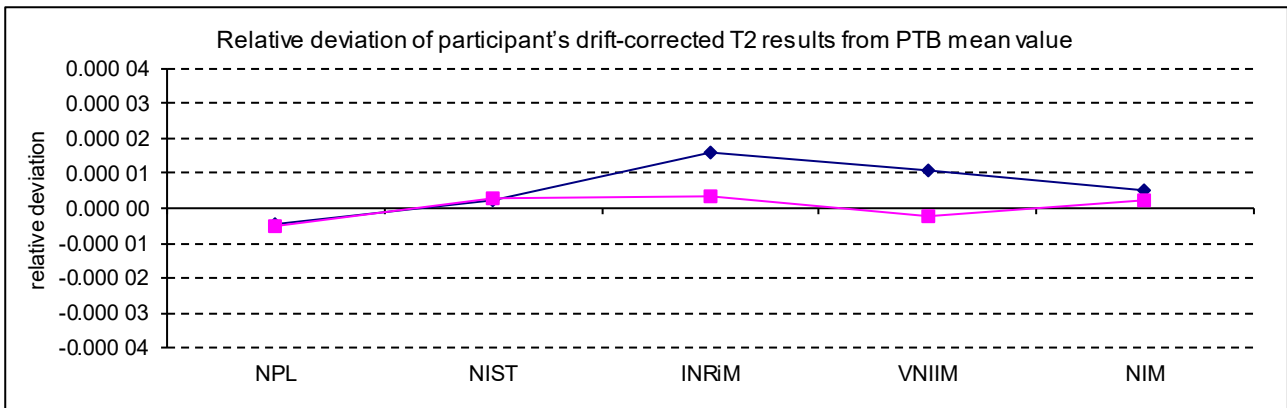


Figure 15: Relative deviation of participant's drift-corrected results from PTB mean value (PTB01a to PTB06) for transducer T2 (◆ – 500 kN, ■ – 1 MN)

Relative deviations from the pilot's mean for transducer T3

Table 10: Drift-corrected deflections in mV/V, expanded ( $k = 2$ ) relative and absolute uncertainties and relative deviations from the pilot's mean for transducer T3

T3	500 kN			
	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{\text{mean}}$
INRiM	1.999 101	$2.1 \cdot 10^{-5}$	0.000 041	$6.4 \cdot 10^{-6}$
LNE	1.999 083	$2.1 \cdot 10^{-5}$	0.000 042	$-3.0 \cdot 10^{-6}$
CEM	1.999 093	$2.1 \cdot 10^{-5}$	0.000 042	$2.2 \cdot 10^{-6}$
GUM	1.999 170	$1.2 \cdot 10^{-4}$	0.000 240	$4.1 \cdot 10^{-5}$
KRISS	1.999 209	$2.2 \cdot 10^{-5}$	0.000 043	$6.0 \cdot 10^{-5}$
NMIA	1.999 049	$2.1 \cdot 10^{-5}$	0.000 041	$-2.0 \cdot 10^{-5}$
NMIJ	1.999 081	$1.3 \cdot 10^{-5}$	0.000 026	$-3.7 \cdot 10^{-6}$
NIM	1.999 120	$1.6 \cdot 10^{-5}$	0.000 031	$1.6 \cdot 10^{-5}$
PTB ( $P_{\text{mean}}$ )	1.999 089	$2.0 \cdot 10^{-5}$	0.000 040	-

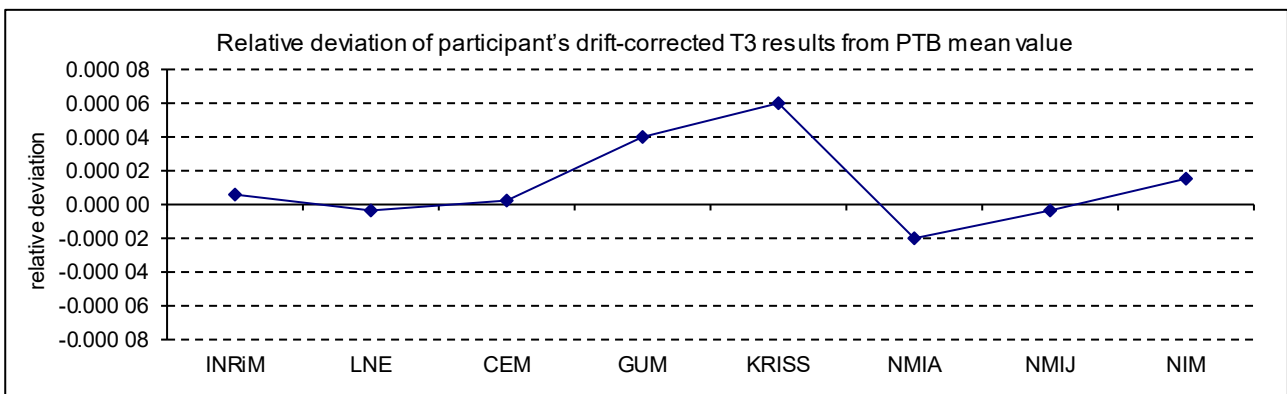


Figure 16: Relative deviation of participant's drift-corrected results from PTB mean value (PTB01 to PTB13) for transducer T3

Relative deviations from the pilot's mean for transducer T4

Table 11: Drift-corrected deflections in mV/V, expanded ( $k = 2$ ) relative and absolute uncertainties and relative deviations from the pilot's mean for transducer T4

T4	500 kN			
Participant	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{\text{mean}}$
INRiM	1.943 315	$2.0 \cdot 10^{-5}$	0.000 040	$8.6 \cdot 10^{-7}$
LNE	1.943 295	$2.1 \cdot 10^{-5}$	0.000 040	$-9.5 \cdot 10^{-6}$
CEM	1.943 321	$2.0 \cdot 10^{-5}$	0.000 040	$3.7 \cdot 10^{-6}$
GUM	1.943 364	$1.2 \cdot 10^{-4}$	0.000 233	$2.6 \cdot 10^{-5}$
KRISS	1.943 429	$2.1 \cdot 10^{-5}$	0.000 041	$5.9 \cdot 10^{-5}$
NMIA	1.943 284	$2.1 \cdot 10^{-5}$	0.000 040	$-1.6 \cdot 10^{-5}$
NMIJ	1.943 306	$1.3 \cdot 10^{-5}$	0.000 026	$-3.9 \cdot 10^{-6}$
NIM	1.943 288	$1.4 \cdot 10^{-5}$	0.000 028	$-1.3 \cdot 10^{-5}$
PTB ( $P_{\text{mean}}$ )	1.943 314	$2.0 \cdot 10^{-5}$	0.000 039	-

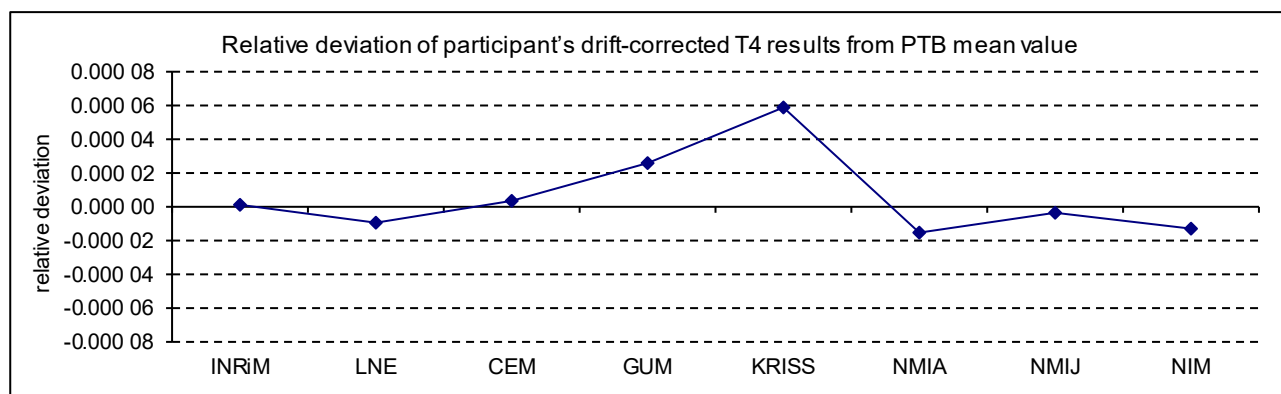


Figure 17: Relative deviation of participant's drift-corrected results from PTB mean value (PTB01 to PTB13) for transducer T4

C - Evaluation of the results corrected for the amplifier deviation

Relative deviations from the pilot's mean for transducer T1

Table 12: Amplifier-corrected deflections in mV/V, expanded ( $k = 2$ ) relative and absolute uncertainties and relative deviations from the pilot's mean for transducer T1 and both force steps

T1 Participant	500 kN				1 MN			
	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{mean}$	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{mean}$
NPL	0.999 830	$1.5 \cdot 10^{-5}$	0.000 015	$-2.5 \cdot 10^{-5}$	1.999 582	$1.5 \cdot 10^{-5}$	0.000 029	$-1.9 \cdot 10^{-5}$
NIST	0.999 846	$2.7 \cdot 10^{-5}$	0.000 027	$-7.9 \cdot 10^{-6}$	1.999 582	$2.2 \cdot 10^{-5}$	0.000 044	$-1.9 \cdot 10^{-5}$
INRiM	0.999 866	$2.1 \cdot 10^{-5}$	0.000 021	$1.1 \cdot 10^{-5}$	1.999 635	$2.2 \cdot 10^{-5}$	0.000 044	$7.3 \cdot 10^{-6}$
VNIIM	0.999 861	$2.2 \cdot 10^{-5}$	0.000 022	$6.7 \cdot 10^{-6}$	1.999 619	$2.2 \cdot 10^{-5}$	0.000 045	$-3.0 \cdot 10^{-7}$
NIM	0.999 855	$1.7 \cdot 10^{-5}$	0.000 017	$1.1 \cdot 10^{-6}$	1.999 633	$1.8 \cdot 10^{-5}$	0.000 035	$-6.4 \cdot 10^{-6}$
PTB ( $P_{mean}$ )	0.999 854	$2.1 \cdot 10^{-5}$	0.000 021	-	1.999 620	$2.2 \cdot 10^{-5}$	0.000 044	-

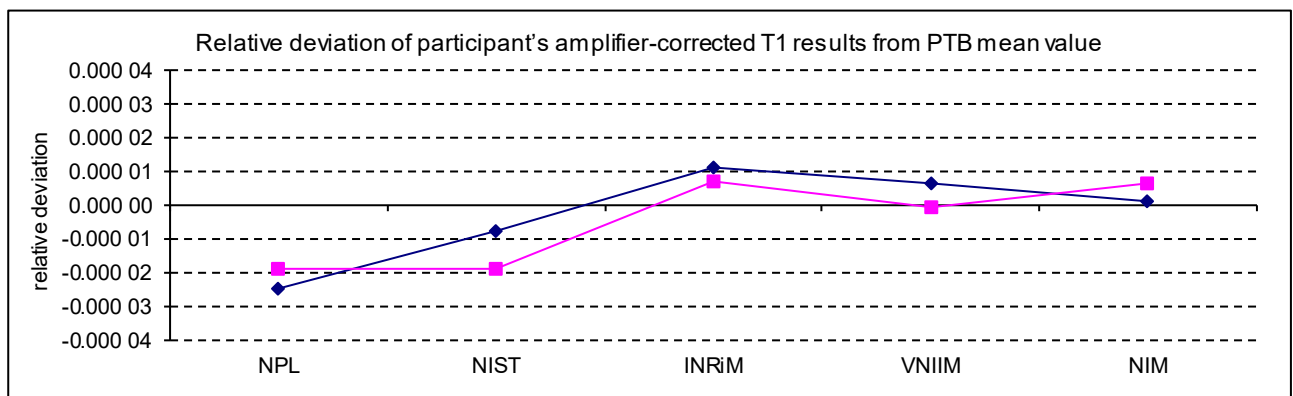


Figure 18: Relative deviation of participant's amplifier-corrected results from PTB mean value (PTB01 to PTB06) for transducer T1 (♦ – 500 kN, ■ – 1 MN)

Relative deviations from the pilot's mean for transducer T2

Table 13: Amplifier-corrected deflections in mV/V, expanded ( $k = 2$ ) relative and absolute uncertainties and relative deviations from the pilot's mean for transducer T2 and both force steps

T2 Participant	500 kN				1 MN			
	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{mean}$	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{mean}$
NPL	1.371 632	$1.1 \cdot 10^{-5}$	0.000 016	$-4.9 \cdot 10^{-6}$	2.743 515	$1.3 \cdot 10^{-5}$	0.000 035	$-6.8 \cdot 10^{-6}$
NIST	1.371 641	$2.3 \cdot 10^{-5}$	0.000 032	$1.6 \cdot 10^{-6}$	2.743 527	$2.4 \cdot 10^{-5}$	0.000 066	$-2.4 \cdot 10^{-6}$
INRiM	1.371 663	$2.3 \cdot 10^{-5}$	0.000 032	$1.8 \cdot 10^{-5}$	2.743 530	$2.5 \cdot 10^{-5}$	0.000 068	$-1.2 \cdot 10^{-6}$
VNIIM	1.371 651	$2.1 \cdot 10^{-5}$	0.000 029	$9.0 \cdot 10^{-6}$	2.743 533	$2.2 \cdot 10^{-5}$	0.000 060	$-3.0 \cdot 10^{-8}$
NIM	1.371 650	$1.5 \cdot 10^{-5}$	0.000 020	$8.4 \cdot 10^{-6}$	2.743 551	$1.6 \cdot 10^{-5}$	0.000 044	$6.5 \cdot 10^{-6}$
PTB ( $P_{mean}$ )	1.371 639	$2.1 \cdot 10^{-5}$	0.000 029	-	2.743 533	$2.2 \cdot 10^{-5}$	0.000 060	-

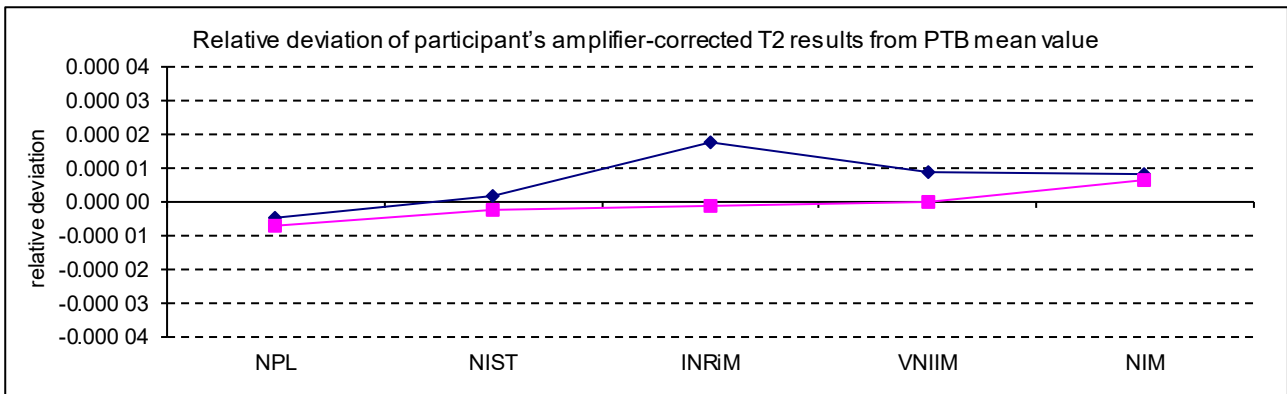


Figure 19: Relative deviation of participant's amplifier-corrected results from PTB mean value (PTB01a to PTB06) for transducer T2 (◆ – 500 kN, ■ – 1 MN)

Relative deviations from the pilot's mean for transducer T3

Table 14: Amplifier-corrected deflections in mV/V, expanded ( $k = 2$ ) relative and absolute uncertainties and relative deviations from the pilot's mean for transducer T3

T3	500 kN			
	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{\text{mean}}$
INRiM	1.999 098	$2.1 \cdot 10^{-5}$	0.000 042	$4.6 \cdot 10^{-6}$
LNE	1.999 081	$2.2 \cdot 10^{-5}$	0.000 043	$-4.3 \cdot 10^{-6}$
CEM	1.999 092	$2.1 \cdot 10^{-5}$	0.000 043	$1.6 \cdot 10^{-6}$
GUM	1.999 173	$1.2 \cdot 10^{-4}$	0.000 241	$4.2 \cdot 10^{-5}$
KRISS	1.999 209	$2.2 \cdot 10^{-5}$	0.000 044	$6.0 \cdot 10^{-5}$
NMIA	1.999 050	$2.1 \cdot 10^{-5}$	0.000 043	$-1.9 \cdot 10^{-5}$
NMIJ	1.999 077	$1.4 \cdot 10^{-5}$	0.000 028	$-6.2 \cdot 10^{-6}$
NIM	1.999 143	$1.8 \cdot 10^{-5}$	0.000 036	$2.7 \cdot 10^{-5}$
PTB ( $P_{\text{mean}}$ )	1.999 089	$2.1 \cdot 10^{-5}$	0.000 041	-

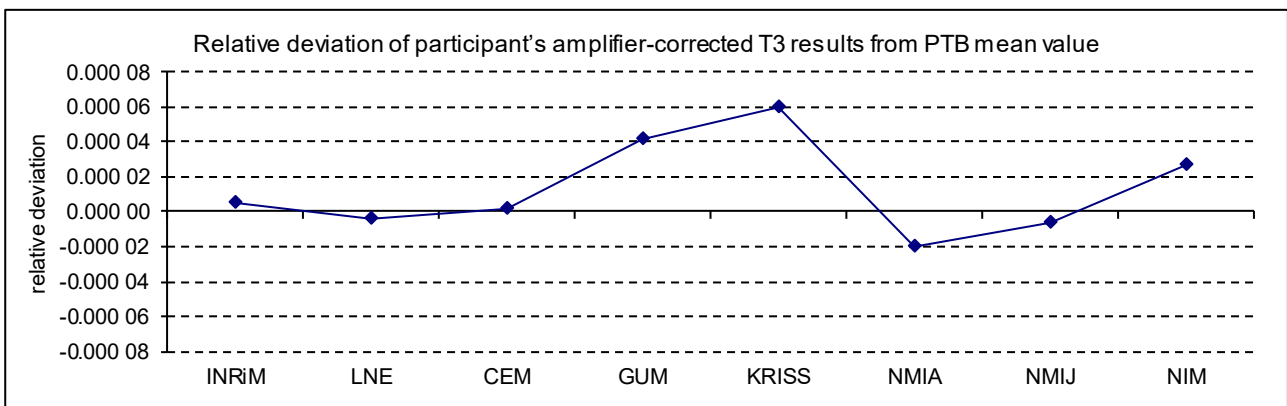


Figure 20: Relative deviation of participant's amplifier-corrected results from PTB mean value (PTB01 to PTB13) for transducer T3

Relative deviations from the pilot's mean for transducer T4

Table 15: Amplifier-corrected deflections in mV/V, expanded ( $k = 2$ ) relative and absolute uncertainties and relative deviations from the pilot's mean for transducer T4

T4	500 kN			
	deflection in mV/V	rel. exp. uncertainty	exp. unc. in mV/V	rel. dev. from $P_{\text{mean}}$
INRiM	1.943 319	$2.1 \cdot 10^{-5}$	0.000 041	$2.9 \cdot 10^{-6}$
LNE	1.943 292	$2.1 \cdot 10^{-5}$	0.000 041	$-1.1 \cdot 10^{-5}$
CEM	1.943 322	$2.1 \cdot 10^{-5}$	0.000 041	$4.7 \cdot 10^{-6}$
GUM	1.943 366	$1.2 \cdot 10^{-4}$	0.000 234	$2.7 \cdot 10^{-5}$
KRISS	1.943 428	$2.2 \cdot 10^{-5}$	0.000 042	$5.9 \cdot 10^{-5}$
NMIA	1.943 283	$2.1 \cdot 10^{-5}$	0.000 041	$-1.6 \cdot 10^{-5}$
NMIJ	1.943 301	$1.4 \cdot 10^{-5}$	0.000 027	$-6.4 \cdot 10^{-6}$
NIM	1.943 302	$1.5 \cdot 10^{-5}$	0.000 030	$-5.6 \cdot 10^{-6}$
PTB ( $P_{\text{mean}}$ )	1.943 313	$2.1 \cdot 10^{-5}$	0.000 040	-

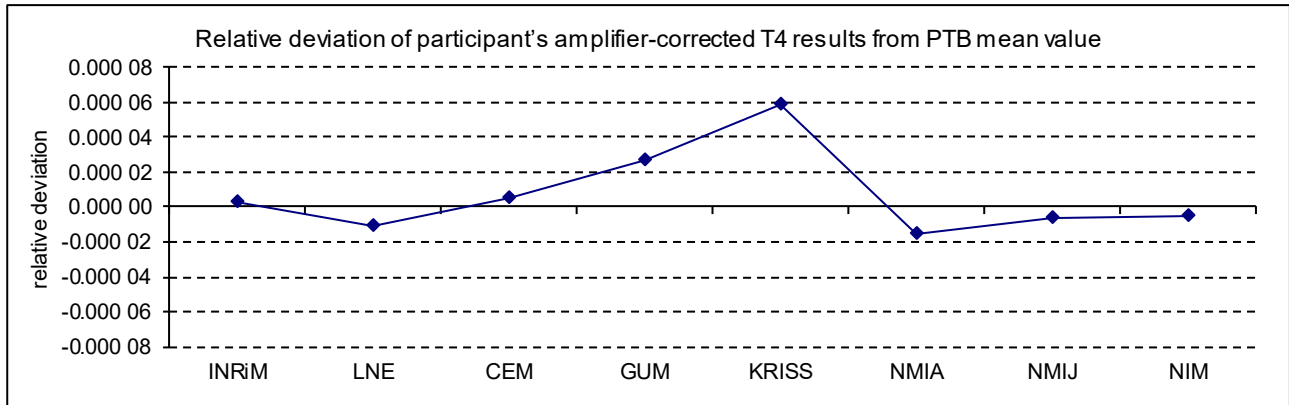


Figure 21: Relative deviation of participant's amplifier-corrected results from PTB mean value (PTB01 to PTB13) for transducer T4

D – Key Comparison Reference Values and evaluation of the corrected results in force units

The corrected (for drift and amplifier deviations) results in mV/V of the participants are the basis for the calculation of the weighted mean. This value in mV/V is considered as an equivalent for the nominal force step in MN, which is taken as the Key Comparison Reference Value (KCRV). This allows the uncertainty in force units to be calculated. This procedure was applied to each of the four transducers and for every force step measured.

This method also allows the results of the different transducers to be compared and combined results for the two force steps to be calculated. Here, it was taken into consideration that the results of a certain participant  $P_i$  are not independent from each other because they were obtained on the same force standard machine. For this purpose, the uncertainty of the results of transducer  $T_j$  at force step  $F_k$  was written as consisting of two parts – a correlated part  $u_{\text{correl}}$  and an uncorrelated part  $u_{\text{uncorrel}}$  – according to (6) with the correlated part being the uncertainty of the force standard machine:

$$u^2(P_i, T_j, F_k) = u_{\text{correl}}^2(P_i, T_j, F_k) + u_{\text{uncorrel}}^2(P_i, T_j, F_k), i = (1, 2, \dots, 12), j = (1, 2, \dots, 4), k = (1, 2) \quad (6)$$

Standard methods were applied to calculate combined results and uncertainties for the case of partly correlated data.

Results for transducer T1

The corrected results of the participants for transducer T1 are given in Table 16. The weighted mean in mV/V and the KCRV in force units are given in Table 17. Table 18 shows the results of the participants in force units. The results are also shown in Figure 22 and Figure 23. For passing the  $\chi^2$ -test, an observed value below 11.07 (probability = 5 %, number of degrees of freedom = 5) was necessary. For the 500 kN force step, this test was almost passed ( $\chi^2_{obs} = 11.09$ ), whereas for the 1 MN force step the test was passed ( $\chi^2_{obs} = 8.46$ ).

Table 16: Corrected deflections and standard uncertainties in mV/V for transducer T1 and both force steps

T1	500 kN		1 MN	
Participant	deflection	std. uncertainty	deflection	std. uncertainty
	in mV/V	in mV/V	in mV/V	in mV/V
NPL	0.999 830	0.000 008	1.999 582	0.000 015
NIST	0.999 846	0.000 014	1.999 582	0.000 022
INRiM	0.999 866	0.000 011	1.999 635	0.000 022
VNIIM	0.999 861	0.000 011	1.999 619	0.000 022
NIM	0.999 855	0.000 008	1.999 633	0.000 018
PTB ( $P_{mean}$ )	0.999 854	0.000 011	1.999 620	0.000 022

Table 17: Weighted means in mV/V with associated standard uncertainty in nV/V and Key Comparison Reference Values in MN with associated standard uncertainty in N for transducer T1 and both force steps

T1	500 kN				1 MN			
	weig. mean	std. uncert.	KCRV	std. uncert.	weig. mean	std. uncert.	KCRV	std. uncert.
	in mV/V	in nV/V	in MN	in N	in mV/V	in nV/V	in MN	in N
	0.999 850	4	0.500 000	2	1.999 608	8	1.000 000	4

Table 18: Calculated forces and expanded ( $k = 2$ ) uncertainties for transducer T1 and both force steps

T1	500 kN		1 MN	
Participant	force	exp. uncertainty	force	exp. uncertainty
	in MN	in N	in MN	in N
NPL	0.499 990	8	0.999 987	15
NIST	0.499 998	14	0.999 987	22
INRiM	0.500 008	11	1.000 013	22
VNIIM	0.500 006	11	1.000 005	22
NIM	0.500 003	8	1.000 012	18
PTB	0.500 002	11	1.000 006	22

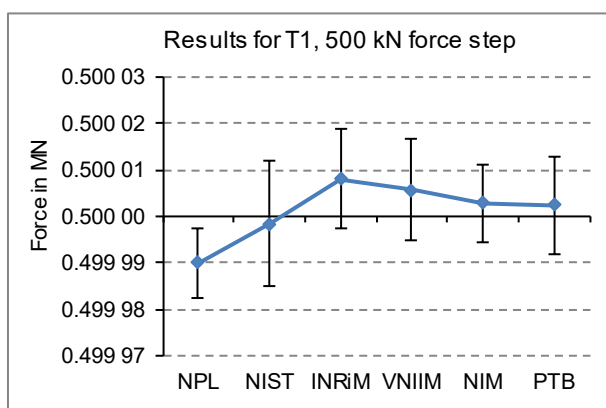


Figure 22: Calculated forces and expanded ( $k = 2$ ) uncertainties for transducer T1 (500 kN force step)

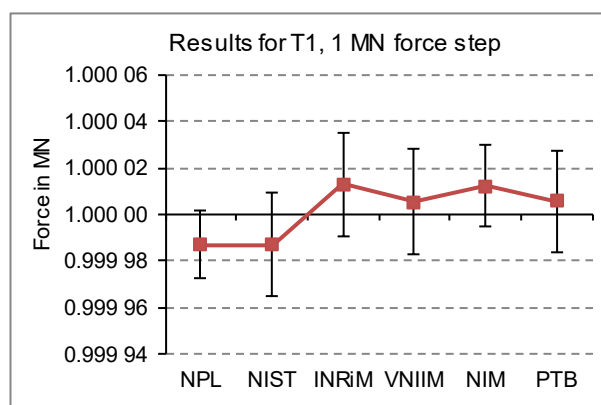


Figure 23: Calculated forces and expanded ( $k = 2$ ) uncertainties for transducer T1 (1 MN force step)

**Results for transducer T2**

The corrected results of the participants for transducer T2 are given in Table 19. The weighted mean in mV/V and the KCRV in force units are given in Table 20. Table 21 shows the results of the participants in force units. The results are also shown in Figure 24 and Figure 25. For passing the  $\chi^2$ - test, an observed value below 11.07 (probability = 5 %, number of degrees of freedom = 5) was necessary. For the 500 kN force step ( $\chi^2_{obs} = 4.50$ ) as well as for the 1 MN force step ( $\chi^2_{obs} = 1.14$ ) the test was passed.

Table 19: Corrected deflections and standard uncertainties in mV/V for transducer T2 and both force steps

T2	500 kN		1 MN	
Participant	deflection	std. uncertainty	deflection	std. uncertainty
	in mV/V	in mV/V	in mV/V	in mV/V
NPL	1.371 632	0.000 008	2.743 515	0.000 021
NIST	1.371 641	0.000 016	2.743 527	0.000 043
INRiM	1.371 663	0.000 016	2.743 530	0.000 044
VNIIM	1.371 651	0.000 014	2.743 533	0.000 040
NIM	1.371 650	0.000 010	2.743 551	0.000 027
PTB ( $P_{mean}$ )	1.371 639	0.000 014	2.743 533	0.000 040

Table 20: Weighted means in mV/V with associated standard uncertainty in nV/V and Key Comparison Reference Values in MN with associated standard uncertainty in N for transducer T2 and both force steps

T2	500 kN				1 MN			
	weig. mean	std. uncert.	KCRV	std. uncert.	weig. mean	std. uncert.	KCRV	std. uncert.
	in mV/V	in nV/V	in MN	in N	in mV/V	in nV/V	in MN	in N
	1.371 642	5	0.500 000	2	2.743 530	13	1.000 000	5

Table 21: Calculated forces and expanded ( $k = 2$ ) uncertainties for transducer T2 and both force steps

T2	500 kN		1 MN	
Participant	force	exp. uncertainty	force	exp. uncertainty
	in MN	in N	in MN	in N
NPL	0.499 996	6	0.999 995	16
NIST	0.499 999	11	0.999 999	32
INRiM	0.500 008	12	1.000 000	32
VNIIM	0.500 003	11	1.000 001	29
NIM	0.500 003	7	1.000 008	20
PTB	0.499 999	11	1.000 001	29

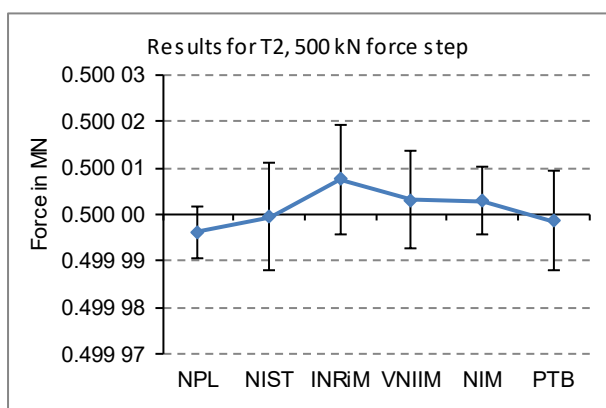


Figure 24: Calculated forces and expanded ( $k = 2$ ) uncertainties for transducer T2 (500 kN force step)

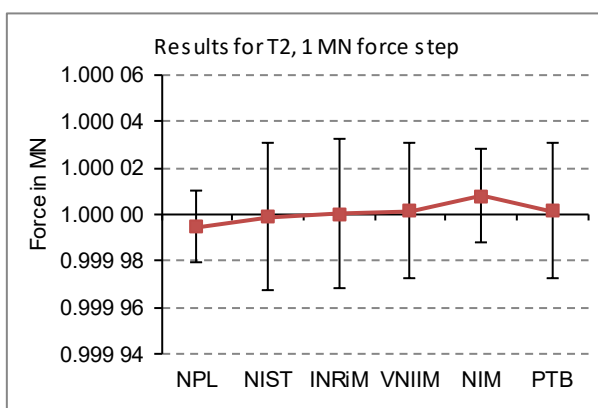


Figure 25: Calculated forces and expanded ( $k = 2$ ) uncertainties for transducer T2 (1 MN force step)

Results for transducer T3

The corrected results of the participants for transducer T3 are given in Table 22. The weighted mean in mV/V and the KCRV in force units are given in Table 23. Table 24 and Figure 26 show the results of the participants in force units. For passing the  $\chi^2$ - test, an observed value below 14.07 (probability = 5 %, number of degrees of freedom = 8) was necessary. For the 500 kN force step ( $\chi_{obs}^2 = 13.68$ ) the test was passed.

Table 22: Corrected deflections and standard uncertainties in mV/V for transducer T3

T3	500 kN	
	deflection	std. uncertainty
	in mV/V	in mV/V
INRiM	1.999 098	0.000 021
LNE	1.999 081	0.000 022
CEM	1.999 092	0.000 021
GUM	1.999 173	0.000 120
PTB (P <sub>mean</sub> )	1.999 089	0.000 021
NMIA	1.999 050	0.000 021
NMIJ	1.999 077	0.000 014
NIM	1.999 143	0.000 018
KRISS	1.999 209	0.000 022

Table 23: Weighted mean in mV/V with associated standard uncertainty in nV/V and Key Comparison Reference Value in MN with associated standard uncertainty in N for transducer T3

T3	500 kN			
	weig. mean	std. uncert.	KCRV	std. uncert.
	in mV/V	in nV/V	in MN	in N
	1.999 091	7	0.500 000	2

Table 24: Calculated forces and expanded ( $k = 2$ ) uncertainties for transducer T3

T3	500 kN	
	force	exp. uncertainty
	in MN	in N
INRiM	0.500 002	11
LNE	0.499 997	11
CEM	0.500 000	11
GUM	0.500 021	60
PTB (P <sub>mean</sub> )	0.500 000	10
NMIA	0.499 990	11
NMIJ	0.499 996	7
NIM	0.500 013	9
KRISS	0.500 029	11

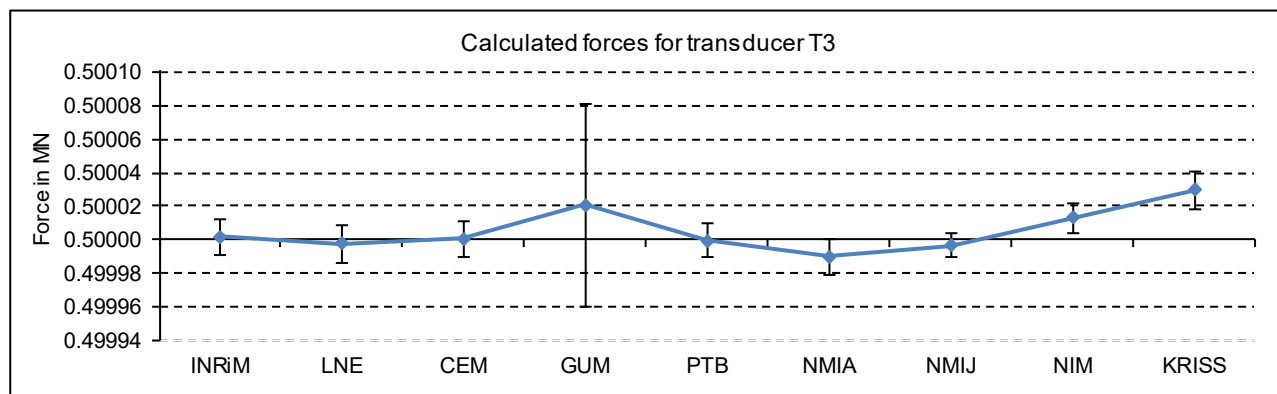


Figure 26: Calculated forces and expanded ( $k = 2$ ) uncertainties for transducer T3

Results for transducer T4

The corrected results of the participants for transducer T4 are given in Table 25. The weighted mean in mV/V and the KCRV in force units are given in Table 26. Table 27 and Figure 27 show the results of the participants in force units. For passing the  $\chi^2$ - test, an observed value below 14.07 (probability = 5 %, number of degrees of freedom = 8) was necessary. For the 500 kN force step ( $\chi_{obs}^2 = 2.20$ ) the test was passed.

Table 25: Corrected deflections and standard uncertainties in mV/V for transducer T4

T4	500 kN	
Participant	deflection	std. uncertainty
	in mV/V	in mV/V
INRiM	1.943 319	0.000 020
LNE	1.943 292	0.000 021
CEM	1.943 322	0.000 020
GUM	1.943 366	0.000 117
PTB (P <sub>mean</sub> )	1.943 313	0.000 020
NMIA	1.943 283	0.000 021
NMIJ	1.943 301	0.000 014
NIM	1.943 302	0.000 015
KRISS	1.943 428	0.000 021

Table 26: Weighted mean in mV/V with associated standard uncertainty in nV/V and Key Comparison Reference Value in MN with associated standard uncertainty in N for transducer T4

T4	500 kN			
	weig. mean	std. uncert.	KCRV	std. uncert.
	in mV/V	in nV/V	in MN	in N
	1.943 304	7	0.500 000	2

Table 27: Calculated forces and expanded ( $k = 2$ ) uncertainties for transducer T4

T4	500 kN	
Participant	force	exp. uncertainty
	in MN	in N
INRiM	0.500 004	11
LNE	0.499 997	11
CEM	0.500 005	11
GUM	0.500 016	60
PTB (P <sub>mean</sub> )	0.500 002	10
NMIA	0.499 995	11
NMIJ	0.499 999	7
NIM	0.500 000	8
KRISS	0.500 032	11

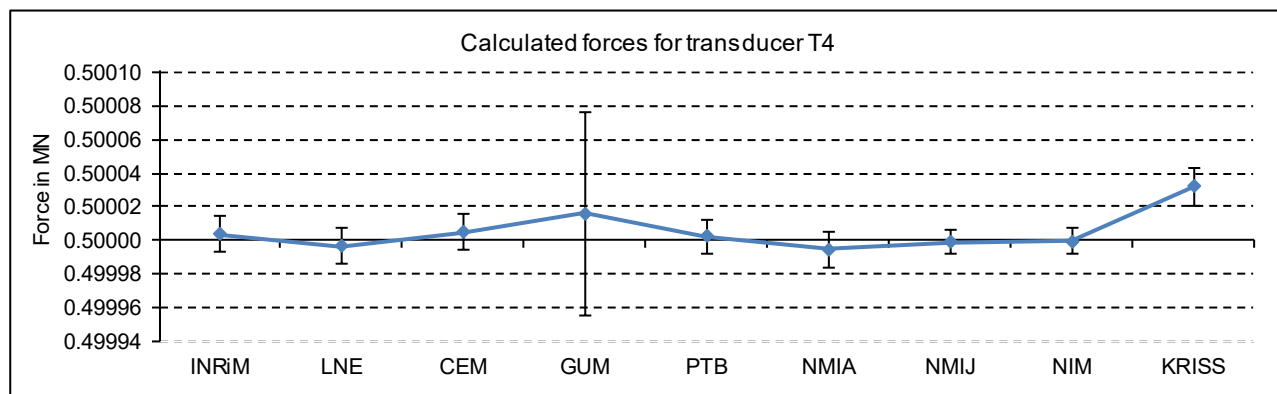


Figure 27: Calculated forces and expanded ( $k = 2$ ) uncertainties for transducer T4

Combined results and degrees of equivalence for the 500 kN force step with transducers T1, T2, T3 and T4

The correlation between the results obtain in one force standard machine is considered and the two contributions to the uncertainty – correlated and uncorelated parts – are shown in Table 28. Based on these vales, the combined result for each of the participants was calculated. It is given in the last two columns of Table 28 and shown in Figure 28.

Table 28: Correlated and uncorrelated uncertainty contributions in N and combined results for the participants for the 500 kN force step

Participant	Transducer	Force	std. uncertainty	$u_{\text{correl}}$	$u_{\text{uncorrel}}$	Force (weig. mean)	std. unc. (weig. mean)
		in MN	in N	in N	in N	in MN	in N
NPL	T1	0.499 990	3.8	2.5	2.8	0.499 994	2.8
	T2	0.499 996	2.8		1.3		
NIST	T1	0.499 998	6.8	2.5	6.3	0.499 999	4.7
	T2	0.499 999	5.7		5.2		
INRiM	T1	0.500 008	5.3	5.0	1.9	0.500 005	5.1
	T2	0.500 008	5.8		3.0		
	T3	0.500 002	5.3		1.7		
	T4	0.500 004	5.3		1.6		
VNIIM	T1	0.500 006	5.4	5.0	2.0	0.500 004	5.2
	T2	0.500 003	5.3		1.7		
NIM	T1	0.500 003	4.2	3.3	2.5	0.500 004	3.5
	T2	0.500 003	3.6		1.4		
	T3	0.500 013	4.5		3.0		
	T4	0.500 000	3.8		1.8		
PTB	T1	0.500 002	5.3	5.0	1.7	0.500 001	5.1
	T2	0.499 999	5.3		1.7		
	T3	0.500 000	5.2		1.4		
	T4	0.500 002	5.2		1.3		
LNE	T3	0.499 997	5.4	5.0	2.0	0.499 997	5.2
	T4	0.499 997	5.3		1.8		
CEM	T3	0.500 000	5.4	5.0	1.9	0.500 003	5.2
	T4	0.500 005	5.3		1.7		
GUM	T3	0.500 021	30.1	30.0	2.4	0.500 018	30.0
	T4	0.500 016	30.0		1.7		
NMIA	T3	0.499 990	5.3	5.0	1.9	0.499 992	5.2
	T4	0.499 995	5.3		1.8		
NMIJ	T3	0.499 996	3.5	2.7	2.3	0.499 998	3.1
	T4	0.499 999	3.5		2.2		
KRISS	T3	0.500 029	5.6	5.0	2.4	0.500 031	5.2
	T4	0.500 032	5.4		2.0		

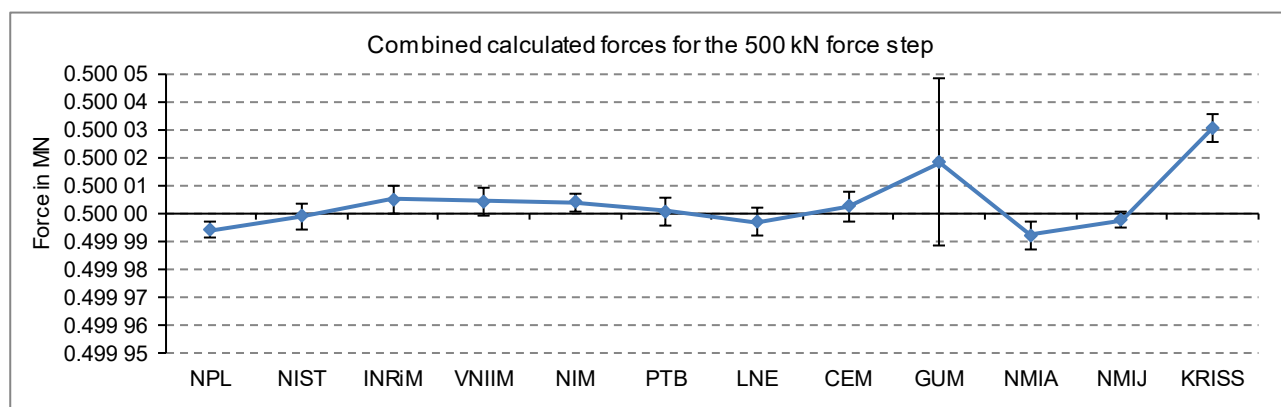


Figure 28: Combined calculated forces and standard uncertainties for the 500 kN force step

For the calculation of the degrees of equivalence of the participant's result, the uncertainty of the combined KCRV was calculated as the mean value of the uncertainties of the KCRVs determined for the four single transducers. The combined KCRV is  $(0.5 \pm 3.6 \cdot 10^{-6})$  MN ( $k = 2$ ).

For each participant, the individual degree of equivalence  $d$  – as relative deviation of the combined result from the KCRV – and the associated expanded ( $k = 2$ ) uncertainty is given in Table 29 and shown in Figure 29.

Table 29: Degrees of equivalence  $d$  and expanded ( $k = 2$ ) uncertainty  $U$  of the participant's result for the 500 kN force step

Participant	$d$	$U(d)$
	in ppm	in ppm
NPL	-12.0	11.3
NIST	-1.9	18.9
INRiM	10.4	20.4
VNIIM	8.8	20.7
NIM	7.8	14.0
PTB	1.5	20.2
LNE	-5.8	20.7
CEM	5.1	20.6
GUM	36.6	120.1
NMIA	-15.6	20.7
NMIJ	-4.4	12.5
KRISS	61.3	21.0

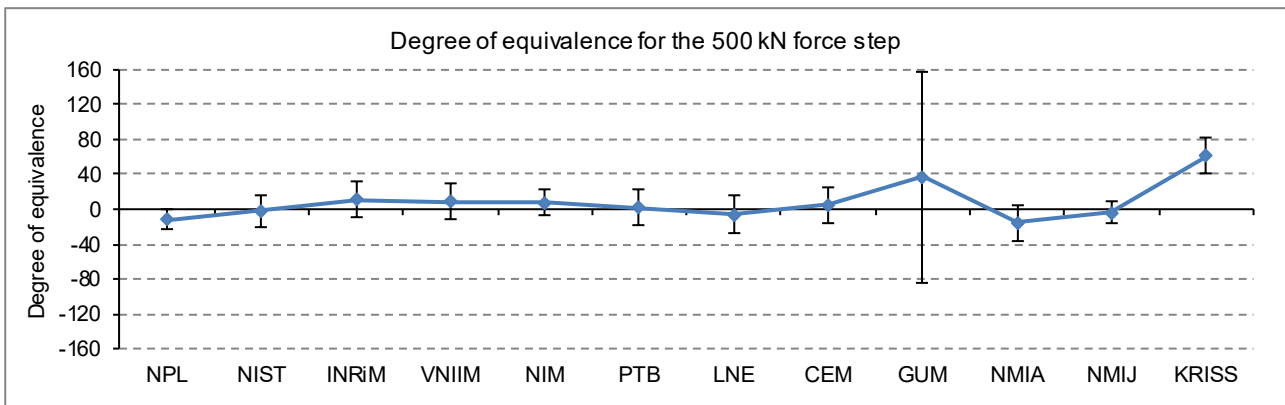


Figure 29: Degrees of equivalence and expanded ( $k = 2$ ) uncertainties for the 500 kN force step

**Remark:**

The measurements of the 500 kN force standard machine from KRISS are not considered in the reference value. This machine is not more available and was replaced in 2016 by a new 1 MN deadweight machine. The new 1 MN deadweight machine will be compared with PTB in the comparison CCM.F-K3.1.

Combined results and degrees of equivalence for the 1 MN force step with transducers T1 and T2

The correlation between the results obtain in one force standard machine is considered and the two contributions to the uncertainty – correlated and uncorelated parts – are shown in Table 30. Based on these vales, the combined result for each of the participants was calculated. The data is given in the last two columns of Table 30 and shown in Figure 30.

Table 30: Correlated and uncorrelated uncertainty contributions in N and combined results for the participants for the 1 MN force step

Participant	Transducer	Force	std. uncertainty	$u_{\text{correl}}$	$u_{\text{uncorrel}}$	Force (weig. mean)	std. unc. (weig. mean)
		in MN	in N	in N	in N	in MN	in N
NPL	T1	0.999 987	7.3	5.0	5.3	0.999 991	6.4
	T2	0.999 995	7.8		5.9		
NIST	T1	0.999 987	11.1	5.0	9.9	0.999 991	9.7
	T2	0.999 999	15.8		14.9		
INRiM	T1	1.000 013	11.1	10.0	4.8	1.000 009	11.3
	T2	1.000 000	16.0		12.5		
VNIIM	T1	1.000 005	11.2	10.0	5.1	1.000 004	11.2
	T2	1.000 001	14.5		10.5		
NIM	T1	1.000 012	8.8	6.7	5.7	1.000 010	8.1
	T2	1.000 008	10.0		7.4		
PTB	T1	1.000 006	11.0	10.0	4.5	1.000 004	11.1
	T2	1.000 001	14.5		10.5		

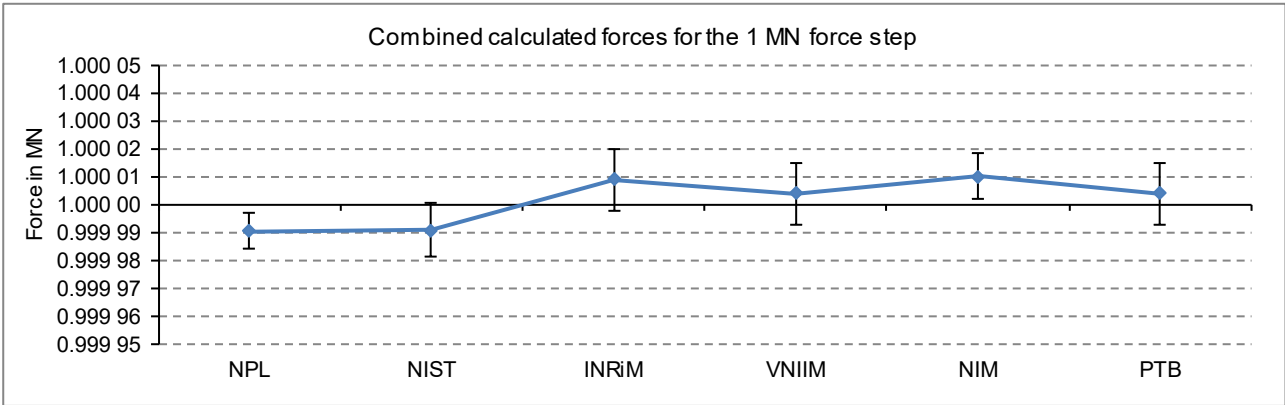


Figure 30: Combined calculated forces and standard uncertainties for the 1 MN force step

For the calculation of the degrees of equivalence of the participant's result, the uncertainty of the combined KCRV was calculated as the mean value of the uncertainties of the KCRVs determined for the two single transducers. The combined KCRV is  $(1 \pm 8.7 \cdot 10^{-6})$  MN ( $k = 2$ ). For each participant, the individual degree of equivalence  $d$  – as relative deviation of the combined result from the KCRV – and the associated expanded ( $k = 2$ ) uncertainty is given in Table 31 and shown in Figure 31.

Table 31: Degrees of equivalence  $d$  and expanded ( $k = 2$ ) uncertainty  $U$  of the participant's result for the 1 MN force step

Participant	$d$	$U(d)$
	in ppm	in ppm
NPL	-9.5	12.7
NIST	-9.2	19.4
INRiM	8.9	22.5
VNIIM	3.9	22.4
NIM	10.3	16.1
PTB	4.2	22.2

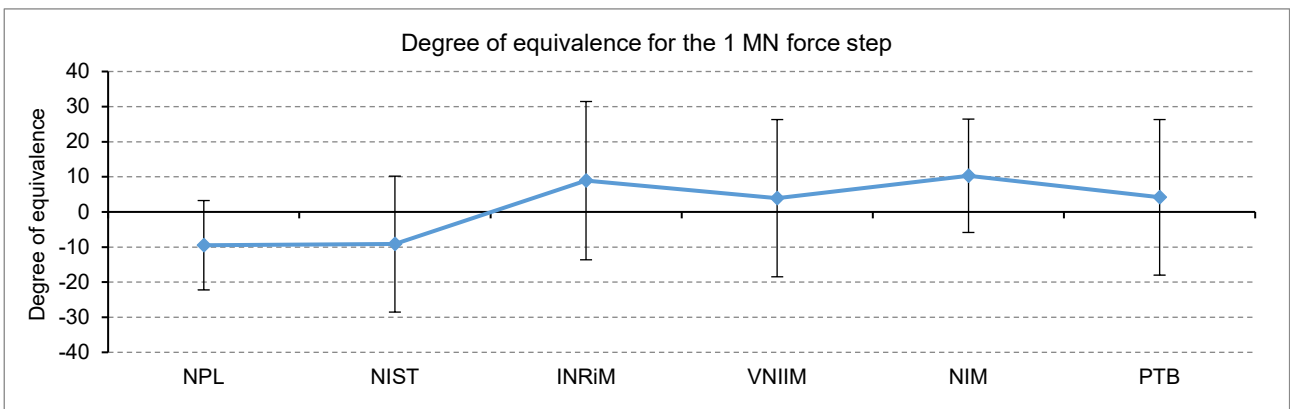


Figure 31: Degrees of equivalence and expanded ( $k = 2$ ) uncertainties for the 1 MN force step

Degrees of equivalence between the participants

Table 32: Degrees of equivalence  $d$  and expanded ( $k = 2$ ) uncertainty  $U$  between the participant's results for the 500 kN force step

	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$
	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm
	NPL		NIST		INRiM		VNIIM		NIM		PTB		LNE		CEM		GUM		NMIA		NMIJ		KRISS	
NPL			-10.1	-22.0	-22.4	-23.3	-20.8	-23.6	-19.9	-18.0	-13.5	-23.2	-6.3	-23.6	-17.2	-23.5	-48.6	-120.7	3.6	-23.6	-7.7	-16.9	-73.4	-23.8
NIST	10.1	22.0			-12.3	-27.8	-10.7	-28.0	-9.8	-23.5	-3.4	-27.7	3.8	-28.0	-7.1	-28.0	-38.5	-121.6	13.7	-28.0	2.4	-22.7	-63.3	-28.2
INRiM	22.4	23.3	12.3	27.8			1.6	-29.1	2.5	-24.8	8.9	-28.7	16.1	-29.1	5.2	-29.0	-26.2	-121.9	26.0	-29.1	14.7	-24.0	-51.0	-29.3
VNIIM	20.8	23.6	10.7	28.0	-1.6	29.1			1.0	-25.0	7.3	-28.9	14.6	-29.3	3.6	-29.2	-27.8	-121.9	24.4	-29.2	13.1	-24.2	-52.6	-29.4
NIM	19.9	18.0	9.8	23.5	-2.5	24.8	-1.0	25.0			6.3	-24.6	13.6	-25.0	2.7	-24.9	-28.8	-121.0	23.4	-25.0	12.2	-18.8	-53.5	-25.2
PTB	13.5	23.2	3.4	27.7	-8.9	28.7	-7.3	28.9	-6.3	24.6			7.3	-29.0	-3.7	-28.9	-35.1	-121.8	17.1	-28.9	5.8	-23.8	-59.9	-29.1
LNE	6.3	23.6	-3.8	28.0	-16.1	29.1	-14.6	29.3	-13.6	25.0	-7.3	29.0			-10.9	-29.2	-42.4	-121.9	9.8	-29.3	-1.4	-24.2	-67.1	-29.5
CEM	17.2	23.5	7.1	28.0	-5.2	29.0	-3.6	29.2	-2.7	24.9	3.7	28.9	10.9	29.2			-31.4	-121.9	20.8	-29.2	9.5	-24.2	-56.2	-29.4
GUM	48.6	120.7	38.5	121.6	26.2	121.9	27.8	121.9	28.8	121.0	35.1	121.8	42.4	121.9	31.4	121.9			52.2	-121.9	40.9	-120.8	-24.8	-122.0
NMIA	-3.6	23.6	-13.7	28.0	-26.0	29.1	-24.4	29.2	-23.4	25.0	-17.1	28.9	-9.8	29.3	-20.8	29.2	-52.2	121.9			-11.3	-24.2	-77.0	-29.4
NMIJ	7.7	16.9	-2.4	22.7	-14.7	24.0	-13.1	24.2	-12.2	18.8	-5.8	23.8	1.4	24.2	-9.5	24.2	-40.9	120.8	11.3	24.2			-65.7	-24.4
KRISS	73.4	23.8	63.3	28.2	51.0	29.3	52.6	29.4	53.5	25.2	59.9	29.1	67.1	29.5	56.2	29.4	24.8	122.0	77.0	29.4	65.7	24.4		

Table 33: Degrees of equivalence  $d$  and expanded ( $k = 2$ ) uncertainty  $U$  between the participant's results for the 1 MN force step

	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$	$d$	$U(d)$
	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm	in ppm
	NPL		NIST		INRiM		VNIIM		NIM		PTB	
NPL			-0.3	-23.2	-18.4	-25.9	-13.4	-25.8	-19.8	-20.6	-13.6	-25.6
NIST	0.3	23.2			-18.1	-29.7	-13.1	-29.6	-19.5	-25.2	-13.3	-29.4
INRiM	18.4	25.9	18.1	29.7			5.0	-31.8	-1.4	-27.7	4.8	-31.6
VNIIM	13.4	25.8	13.1	29.6	-5.0	31.8			-6.4	-27.6	-0.2	-31.5
NIM	19.8	20.6	19.5	25.2	1.4	27.7	6.4	27.6			6.1	-27.4
PTB	13.6	25.6	13.3	29.4	-4.8	31.6	0.2	31.5	-6.1	27.4		

## 8. Summary

The results of the measurements (deflections and uncertainties) reported by the participants of the CIPM key comparison CCM.F-K3 to the pilot laboratory were evaluated. Some known effects were included into the evaluation by correction terms. In detail, corrections for the deviations of the amplifiers of the participating laboratories.

This report contains a calculation of the key comparison reference values in analogy to the torque key comparisons, the corresponding uncertainties, the relative deviations of the values from the reference value and the degrees of equivalence.

## References

- [1] *Peschel, D., Kumme, R., Mauersberger, D., Peters, M.*: PTB's "new" 2 MN dead weight force standard machine, 19<sup>th</sup> International IMEKO TC3 Conference on Force, Mass and Torque, February 19-23, 2005, Cairo/Egypt, <http://www.imeko.org/publications/tc3-2005/IMEKO-TC3-2005-081u.pdf>
- [2] *M. G. Cox*, The Evaluation of key comparison data, 2002 *Metrologia* **39** 589-595  
**DOI**: <https://doi.org/10.1088/0026-1394/39/6/9>