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# Spectral characteristics of DDPM streams and their application to all-digital amplitude modulation

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**Abstract** A new closed-form expression of the spectral coefficients of the digital streams obtained by dyadic digital pulse modulation is presented and validated in this letter. The new expression provides in-depth insight into the spectral properties of dyadic digital pulse modulation, revealing its applicability as an all-digital bandpass amplitude modulation technique. Simulations and measurements on a proof-of-concept dyadic digital pulse modulation amplitude modulator prototype demonstrate the effectiveness of the approach.

**Introduction:** The dyadic digital pulse modulation (DDPM) has been proposed in [1] to generate digital bitstreams with a pulse density proportional to an input binary code and favourable spectral characteristics for all-digital, low cost, energy and area efficient baseband digital-to-analogue (D/A) conversion [1–4]. More recently, DDPM has also been applied in analogue-to-digital (A/D) conversion [4] and in digitally controlled power converters [5].

Unlike other bistream modulations (e.g. single-bit  $\Sigma\Delta$ ), DDPM is deterministic in nature and the spectra of DDPM streams can be evaluated analytically [1, 2]. Unfortunately, the expression of DDPM spectra reported in [1, 2], although exact, does not provide much insight into the features of the modulation, since it involves nested summations from which the magnitude and phase of the harmonic components and their relations with the digital input are not evident.

This letter proposes a new closed-form expression of the spectra of DDPM streams. The new expression provides in-depth understanding of the spectral characteristics of the modulation and reveals the applicability of DDPM as an all-digital, bandpass amplitude modulation technique, which is confirmed by simulations and experiments on a proof-of-concept DDPM amplitude modulator prototype.

**DDPM streams:** The DDPM modulation [1] associates to an unsigned integer  $n$  on  $N$  bits, expressed in terms of its binary representation  $B_n[N-1:0] = (b_{N-1}, b_{N-2}, \dots, b_1, b_0)$  as  $n = \sum_{i=0}^{N-1} b_i 2^i$ , the periodic digital stream

$$\Sigma_n(t) = \sum_{i=0}^{N-1} b_i S_i(t), \quad (1)$$

obtained by superposition of orthogonal dyadic basis signals (ODBSs)  $S_i(t)$  ( $i = 0, \dots, N-1$ ) defined on the basic period  $(0, 2^N T_{\text{clk}})$  as:

$$S_i(t) = V_{\text{DD}} \sum_{h=0}^{N-1} \Pi\left(\frac{t}{T_{\text{clk}}} - h \cdot 2^{N-i} - 2^{N-i-1} + 1\right), \quad (2)$$

where  $T_{\text{clk}}$  is the clock cycle and  $\Pi(x)$  is the unit pulse ( $\Pi(x) = 1$  for  $0 \leq x \leq 1$  and  $\Pi(x) = 0$  elsewhere).

In other words, ODBSs  $S_i(t)$  are  $N$  non-overlapping, periodically repeated digital streams of  $2^N$  clock cycles, organized so that  $S_{N-1}$  is high (i.e. at  $V_{\text{DD}}$ ) every other clock cycle (i.e. in  $2^{N-1}$  cycles per period),  $S_{N-2}$  is high every other cycle in which  $S_{N-1}$  is low (i.e. in  $2^{N-2}$  cycles),  $S_{N-3}$  is high every other cycle in which both  $S_{N-1}$  and  $S_{N-2}$  are low (i.e. in  $2^{N-3}$  cycles per period) and so on, till  $S_0$ , which is high just in one cycle per period, as shown in Figure 1(a). Being ODBSs  $S_i$  non-overlapping and high in  $2^i$  clock cycles per period, DDPM streams  $\Sigma_n$  defined in (1) are high for exactly  $n$  clock cycles per period and their time average is therefore  $n/2^N V_{\text{DD}}$ , as it can be observed in the same Figure 1(a), where the construction of a DDPM stream by superposition of ODBSs is illustrated for  $n = 10$ .

In practice, DDPM streams can be generated by tiny all-digital hardware (a priority multiplexer in which the data inputs are fed by the sig-

nals  $B_n[N-1:0]$  and the selection inputs are driven by a free-running binary counter, as shown in Figure 1(b) and have been applied thus far in baseband D/A and A/D conversion and in power electronics [1] in view of their low-frequency spectral features, which make it possible to extract their DC component by a low pass filter with non-stringent requirements.

A new expression of the spectra of DDPM streams is introduced here to gain more insight into the modulation and into the potential applications of its high-frequency harmonic components.

**Spectra of DDPM streams:** The spectra of a DDPM stream have been evaluated in [1] by the Fourier transform of (1) as:

$$\Sigma_n(f) = \frac{V_{\text{DD}}}{2^N} \sum_{k=-\infty}^{+\infty} c_{k,n} \text{sinc}\left(\frac{k}{2^N}\right) \delta(f - k f_0), \quad (3)$$

where  $f_0 = 1/T_0 = f_{\text{clk}}/2^N$  and

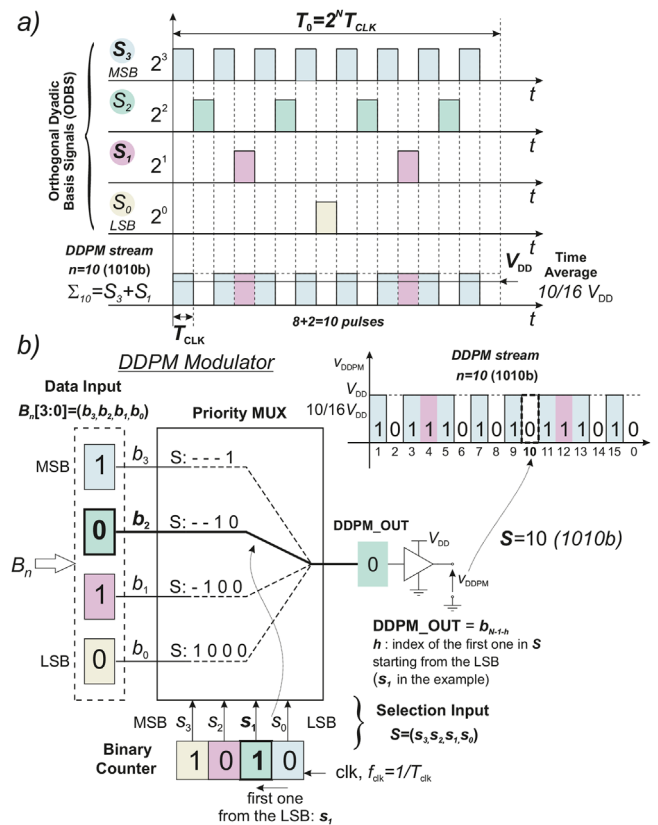
$$c_{k,n} = \sum_{i=0}^{N-1} b_{i,n} 2^i \sum_{m=0}^{2^N-i-1} \delta[k - 2^i m] e^{-j\pi m} \quad (4)$$

in which  $\delta[\cdot]$  is the Kronecker function defined as

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0. \end{cases}$$

Although in closed form, Equation (3) provides a limited insight into the spectral characteristics of DDPM signals since the coefficients  $c_{k,n}$ , which give the magnitude and the phase of the  $k$ -th spectral component of the DDPM stream  $\Sigma_n$ , are expressed in (4) in terms of nested summations, from which their dependence on the input code  $B_n$  is not evident.

**New closed-form expression of the spectral coefficients:** The expression of the DDPM spectral coefficients in (4) can be simplified considering



**Fig. 1** Dyadic digital pulse modulation: a) 4-bit DDPM basis functions and DDPM pattern for  $n = 10$  (1010b); b) hardware architecture of a 4-bit DDPM modulator

that, for a given  $k \neq 0$ , the inner summation in (4) is non-zero only if  $k$  is an integer multiple of  $2^i$ . As a consequence, being

$$v_2(k) = \max \{v \in \mathbb{N} : k|2^v\}, \quad (5)$$

that is, the largest exponent  $v$  such that  $2^v$  divides  $k$ , which is known in number theory [6] as *dyadic order* of  $k \in \mathbb{N}$ , all the terms of the outer summation in (4) for  $i > v_2(k)$  give zero contribution to the spectral coefficient  $c_{k,n}$ .

Moreover, for  $i = v_2(k)$ , the Kronecker delta in the inner summation is non-zero just for one value of  $m$ , which is necessarily *odd*. In fact, if we assume, by contradiction that  $k = m \cdot 2^i$  with  $i = v_2(k)$  and  $m$  even, then  $2^{v_2(k)+1}$  would be also a divider of  $k$  and  $v_2(k)$  would not be the largest exponent such that  $2^{v_2(k)}$  divides  $k$ , in contrast with its definition in (5). Being  $m$  odd, the phase factor  $e^{-j\pi m}$  is equal to  $-1$  and the whole inner summation in (4) evaluates to  $-1$  for  $i = v_2(k)$ .

Finally, for  $i < v_2(k)$ , the inner summation in (4) is non-zero just for one value of  $m$ , which is necessarily *even*. The condition  $i < v_2(k)$  (with strict inequality), in fact, implies that  $2^{i+1}$  also divides  $k$ , which in turn implies that if  $k = m \cdot 2^i$  for an integer  $m$ , then  $m$  must include at least a factor 2 in its prime factor decomposition, that is, it should be even. Being  $m$  even, the phase factor  $e^{-j\pi m}$  is equal to  $+1$ , and the whole inner summation in (4) evaluates to  $+1$  for  $i < v_2(k)$ .

Based on the above considerations, the inner summation in (4) can be explicitly evaluated and the expression of  $c_{k,n}$  for  $k \neq 0$  can be written as

$$c_{k,n} = -b_{v_2(k)}2^{v_2(k)} + \sum_{i=0}^{v_2(k)-1} b_i 2^i, \quad (6)$$

and is very interestingly equal to the value of the binary string<sup>2</sup>

$$B_n[v_2(k) : 0] = (b_{v_2(k)}, \dots, b_0)$$

interpreted as a signed integer in two's complement representation [7].

In other words, Equation (6) reveals that the spectral coefficient  $c_{k,n}$  of the  $k$ -th harmonic of the spectrum of the DDPM stream for the input code  $n$  is always real, is equal for harmonics  $k$  of the same dyadic order and takes the value of the last  $v_2(k) + 1$  LSBs of the binary input string  $B_n$ , interpreted as a signed integer in two's complement, where  $v_2(k)$  is the dyadic order of the harmonic number  $k$ , as illustrated in Figure 2.

Considering the relation between the two's complement notation and the modular arithmetic [7], Equation (6) can be equivalently written as

$$c_{k,n} = [(n + 2^{v_2(k)}) \bmod 2^{v_2(k)+1}] - 2^{v_2(k)}, \quad (7)$$

where  $\bmod$  is the arithmetic modulo operator. The values of the harmonic coefficients  $c_{n,k}$  provided by Equations (6) and (7) are the same obtained directly by taking the fast Fourier transform (FFT) of DDPM streams, which are reported for  $N = 4$  in Figure 3 for direct validation of (6) and (7).

**Application to all-digital amplitude modulation:** From an application perspective, Equation (6) reveals that the amplitude of the harmonics of DDPM streams with dyadic order  $v_2(k)$  can be varied over  $2^{v_2(k)+1}$  uniformly spaced levels by setting the last  $v_2(k) + 1$  LSBs of  $B_n$ .

In particular, by varying the digital input  $B_n$  of a DDPM modulator according to a sequence  $\tilde{n}[h]$  of  $N$ -bit signed integers in two's complement, as illustrated in Figure 4(a), the amplitude of the spectral line at frequency  $2^{N-1}f_0 = f_{\text{clk}}/2$  of the output stream – which can be regarded as a carrier – can be modulated at  $N$ -bit resolution according to the digital modulating sequence  $\tilde{n}[h]$  over a  $1/(2T_0) = f_{\text{clk}}/2^{N+1}$  modulation bandwidth, dictated by the duration of the DDPM pattern  $T_0 = 2^N T_{\text{clk}}$ .

Based on (6), in fact, the spectrum of the modulated DDPM stream in Figure 4(a), which can be generated by inexpensive, fully digital, mismatch insensitive hardware – is equivalent around  $f_{\text{clk}}/2$  to the spectrum

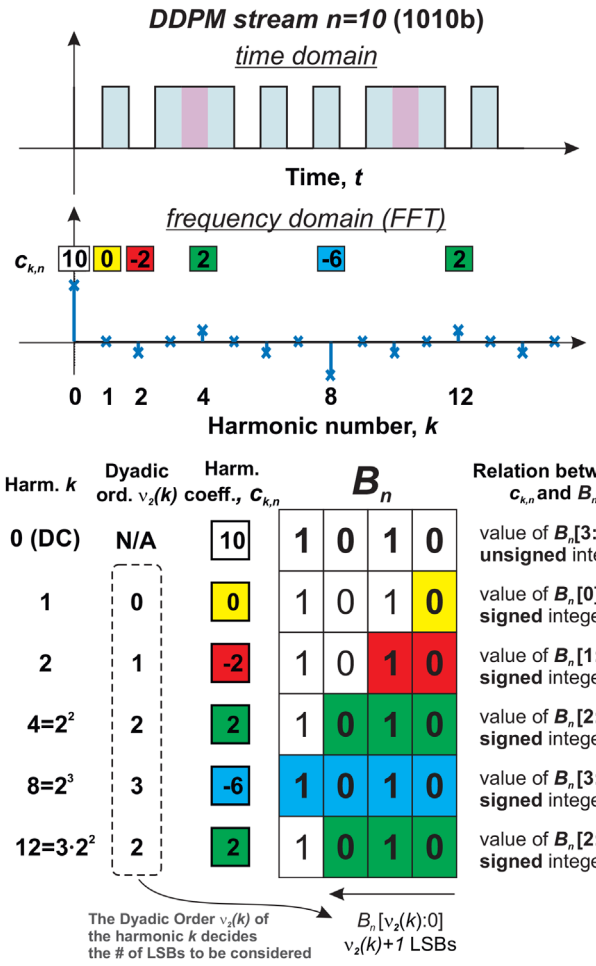


Fig. 2 Relation between the spectral coefficients  $c_{k,n}$  of the DDPM stream associated to  $n = 10$  and its binary representation  $B_n = (b_{N-1}, b_{N-2}, \dots, b_1, b_0) = (1, 0, 1, 0)$  highlighted by (6)

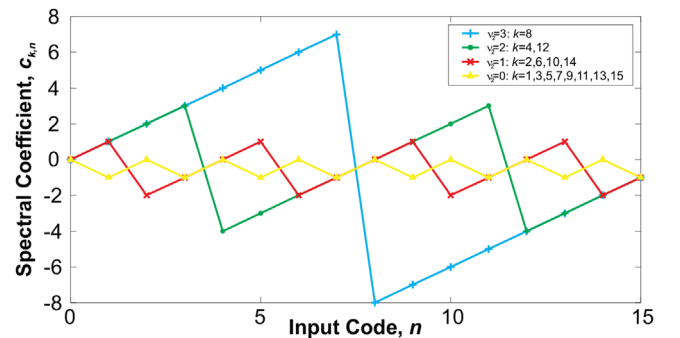


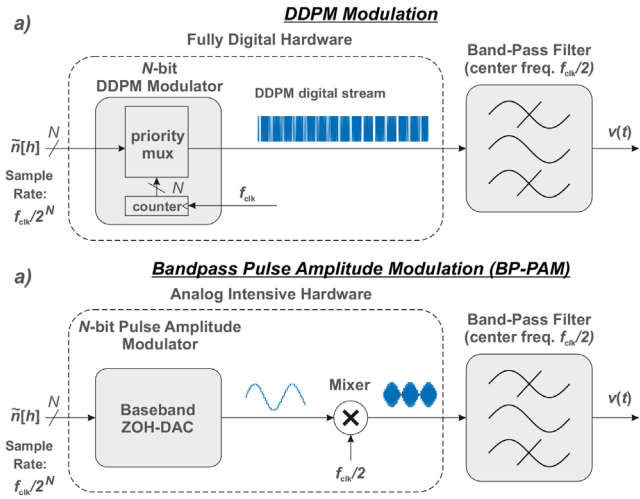
Fig. 3 Harmonic coefficients  $c_{k,n}$  of the DDPM streams for  $N = 4$ , evaluated by FFT and plotted against the input code  $n$  for  $v_2 = 3$  (i.e.  $k = 8$ ),  $v_2 = 2$  (i.e.  $k = 4, 12$ ),  $v_2 = 1$  (i.e.  $k = 2, 6, 10, 14$ ),  $v_2 = 0$  (i.e.  $k = 1, 3, 5, 7, 9, 11, 13, 15$ ). For  $n = 10$ , the coefficients in Figure 2 can be found

of a bandpass, pulse amplitude modulated (BP-PAM) signal obtained by upconversion to  $f_{\text{clk}}/2$  of the output of a zero-order-hold (ZOH)  $N$ -bit analog DAC fed at  $f_{\text{clk}}/2^N$  sample rate, as shown in Figure 4(b), or by using a mixing DAC [8]. The insight gained by (6) therefore reveals the applicability of DDPM as an all-digital, bandpass amplitude modulation technique, for example, in ultra-low cost software-defined radio.

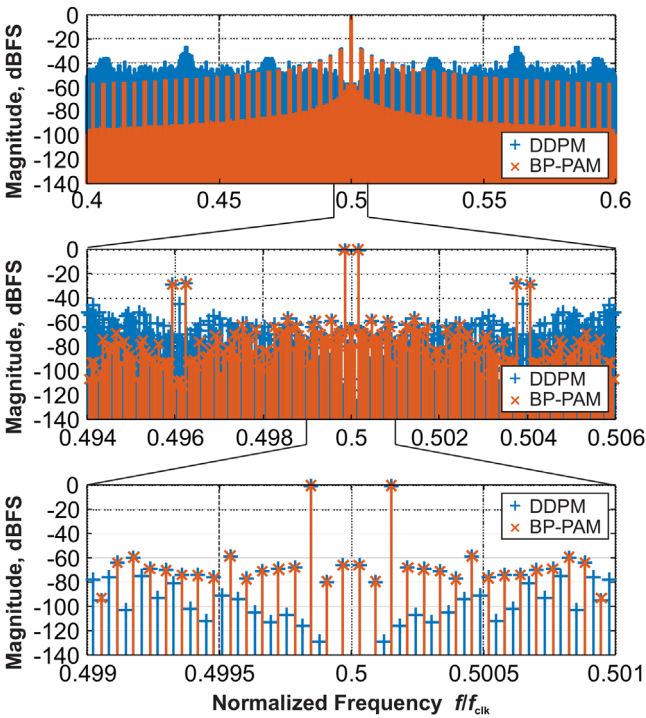
**Simulations:** The equivalence highlighted in Figure 4 is confirmed by the simulation results presented in Figure 5, where the spectra of an 8-bit DDPM stream (Figure 4(a), before the bandpass filter [BPF]) and of a BP-PAM signal (Figure 4(b), before the BPF), both modulated at  $f_{\text{clk}}/2^8$

<sup>1</sup>For  $k = 0$ ,  $c_{k,n} = n$ , as already observed in Figure 1 and discussed in [1].

<sup>2</sup>In this expression, it is assumed  $b_i = 0$  for  $i > N - 1$ .



**Fig. 4** Narrowband equivalence around  $f_{\text{clk}}/2$ , revealed by Equation (6) of: (a) a DDPM stream modulated by the sequence  $\tilde{n}[h]$  and (b) a bandpass pulse-amplitude modulated (BP-PAM) signal obtained upconverting to  $f_{\text{clk}}/2$  the output of a baseband zero-order-hold DAC driven by the same sequence  $\tilde{n}[h]$



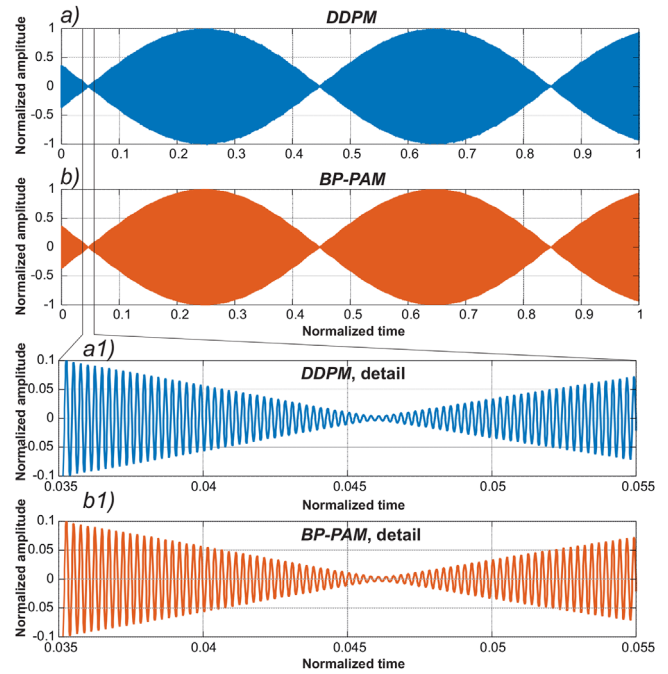
**Fig. 5** Comparison of the spectra of a DDPM stream and of a BP-PAM signal around  $f_{\text{clk}}/2$  for the same modulating sequence  $\tilde{n}[h]$  at the same  $N = 8$  bit resolution and  $f_{\text{clk}}/256$  sample rate

sample rate (i.e.  $f_{\text{clk}}/2^9$  modulation bandwidth), by the same digital sequence of signed integers in two's complement on 8-bit

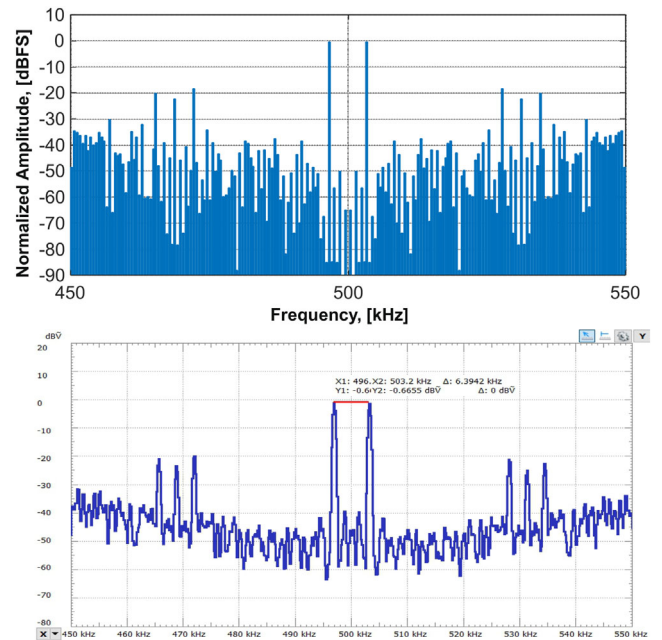
$$\tilde{n}[h] = \lceil 0.95 \cdot 2^7 \cdot \sin(5\pi/64 \cdot h) \rceil,$$

where  $\lceil \cdot \rceil$  is the rounding to the nearest integer operator, are compared around  $f_{\text{clk}}/2$  on three different scales, revealing near identical spectral content in the modulation sidebands.

From Figure 5, it can be observed that the amplitude of the spurious spectral components of the DDPM-modulated stream increase gradually outside the modulation bandwidth and can be therefore suppressed by a BPF with non-stringent requirements, in analogy to what happens in baseband DDPM D/A conversion [1]. This can be more rigorously demonstrated adapting the analysis under dynamic conditions presented in [1] to (6) and is also observed in Figure 6, where the time-domain



**Fig. 6** Time-domain waveforms of the DDPM stream and of the BP-PAM signal considered in Figure 5 after filtering with a second-order BPF with centre frequency  $f_{\text{clk}}/2$  and quality factor  $Q = 40$



**Fig. 7** Simulated (top) and measured (bottom) spectra of a 5-bit DDPM stream at  $f_{\text{clk}} = 1$  MHz

waveforms of the DDPM and BP-PAM signals considered in Figure 5, both filtered by a second-order BPF with  $Q = 40$ , are compared.

**Measurements:** The possibility to apply DDPM as a bandpass amplitude modulation technique has been experimentally verified by measuring the output spectrum of a proof-of-concept  $N = 5$  bit DDPM modulator synthesized on an Altera Cyclone IV FPGA in a DE-2 115 board.

The DDPM modulator operates at  $f_{\text{clk}} = 1$  MHz and is fed by the samples of a digital sine wave at 95% full-swing amplitude and frequency  $f = 7/64 \cdot f_0 = 7/64 \cdot f_{\text{clk}}/2^5 \simeq 3.4$  kHz quantized on  $N = 5$  bits. The measured spectrum around<sup>3</sup>  $f_{\text{clk}}/2 = 500$  kHz of the DDPM stream is

<sup>3</sup>The low frequency of 500 kHz has been chosen in view of the limitation of the available testing equipment and does not reflect a limitation of the proposed approach.

reported in Figure 7 and is in good agreement with the simulated results under the same test conditions (reported in the same figure) and with (6), thus confirming the validity of the analysis and the applicability of DDPM as an all-digital amplitude modulation technique.

*Conclusion:* A new simple, closed-form expression of the spectral coefficients of DDPM streams, which highlights a simple relation between the amplitude of the harmonics of DDPM streams and the binary representation of the DDPM modulator input, has been presented in this letter. The insight gained by the proposed analysis reveals the applicability of DDPM as an all-digital, bandpass amplitude modulation technique, which has been confirmed by simulations and experimental results on a proof-of-concept prototype.

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