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Transportation Research Part E 145 (2021) 102174

Contents lists available at ScienceDirect

Transportation Research Part E

journal homepage: www.elsevier.com/locate/tre

Comparative analysis of models and performance indicators for optimal service facility location

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ARTICLE INFO

Keywords:
Facility location
Key performance indicators
Back-up coverage
Progressive interventions

ABSTRACT

This study investigates the optimal process for locating generic service facilities by applying and comparing several well-known basic models from the literature. At a strategic level, we emphasize that selecting the right location model to use could result in a problematic and possibly misleading task if not supported by appropriate quantitative analysis. For this reason, we propose a general methodological framework to analyze and compare the solutions provided by several models to obtain a comprehensive evaluation of the location decisions from several different perspectives. Therefore, a battery of key performance indicators (KPIs) has been developed and calculated for the different models' solutions. Additional insights into the decision process have been obtained through a comparative analysis. The indicators involve topological, coverage, equity, robustness, dispersion, and accessibility aspects. Moreover, a specific part of the analysis is devoted to progressive location interventions over time and identifying core location decisions. Results on randomly generated instances, which simulate areas characterized by realistic geographical or demographic features, are reported to analyze the models' behavior in different settings and demonstrate the methodology's general applicability. Our experimental campaign shows that the *p-median* model behaves very well against the proposed KPIs. In contrast, the *maximal covering problem* and some proposed *back-up coverage* models return very robust solutions when the location plan is implemented through several progressive interventions over time.

1. Introduction

Facility location is a fundamental strategic aspect in the design of many processes, including logistical operations (e.g., hub location) and specific transportation/routing applications (e.g., the location of charging stations for electric vehicles). Moreover, it is widely applied in cases where some geographical areas must be covered in terms of commercial reachability (e.g., opening new shops by a firm) or public and private services (e.g., the location of the metro stations or transmitting antennas). Therefore, facility location problems have been studied since a long time (see Miehle, 1958 or Cooper, 1963), and still attract considerable research attention (see, e.g., Labbé et al., 2019; Cherkesly et al., 2019; Brandstätter et al., 2020, or Lin et al., 2020). In particular, as described in Section 2, the

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https://doi.org/10.1016/j.tre.2020.102174
Received 7 March 2020; Received in revised form 14 November 2020; Accepted 19 November 2020
Available online 24 December 2020
1366-5545/

most recent literature focuses on particular and tailored models. Instead, in this work, we focus on fundamental and general location models.

In the planning phase of a facility-location process, two main common decision scenarios may occur. In the first scenario, the decision-maker is still evaluating higher-level plans. Therefore, it is interesting to know, for example, the minimum number of facilities to locate or the minimum budget to allocate to achieve a certain level of service or coverage. In the second scenario, the decision-maker has a pre-allocated budget for the facilities and their operations, implying an a priori knowledge of the exact number of facilities to locate. In this study, we only focus on the latter scenario and leave the former for future studies. The decision to consider first the situation in which the number of facilities to locate is an input for the problem is motivated by several reasons: a) it represents a more practical operational setting, given a preliminary economic analysis of the investment; b) it can support the study of a decision process divided into several planned interventions (see Section 4.4); and c) it can support the analysis of the robustness of the solutions (back-up coverage) against congestion or other external disruptions.

At a strategic level or during the exploratory phases of a decision process, selecting the *right* location model to use could result in a problematic and possibly misleading task if not supported by appropriate quantitative analysis. In fact, in real settings, all the requirements and factors to consider inside the optimization process are often not completely clear (due to externalities, uncertainty, etc.). Several incomparable objectives should be taken into account (conflicting interests of different stakeholders). Finally, especially in industrial applications (see, e.g., Fadda et al., 2018, 2019a; Giusti et al., 2019), it is a good practice for the management to consider a joint evaluation, through several performance indicators of interest, of the behavior of the solutions coming from different models. For this reason, we propose a general methodological framework to analyze and compare the solutions provided by several models from the literature to obtain a comprehensive evaluation of the location process from several different perspectives and thus to provide decision-makers with insights on strategic location management. Different models address different objectives, but they can be reasonably comparable in terms of solution features if the returned solution contains the same type of decisions, i.e., a subset of facilities to locate. Since a single model takes care of a single indicator (its objective function), we believe it is crucial to understand how the returned solutions behave against several other indicators (even if the model does not explicitly consider them). Therefore, a battery of key performance indicators (KPIs) has been developed and calculated for the different models' solutions. The analyzed KPIs include topological aspects, covering capabilities, robustness, and accessibility of the resulting solutions. Besides topological and coverage measures, we mainly focus on back-up models (to address congestion issues implicitly) and equity measures. Moreover, as some types of service infrastructures are commonly supposed to be located through several progressive interventions over a defined time horizon, we also provide ad-hoc KPIs measuring the flexibility of the solutions regarding location changes and objectives changes over time. Furthermore, performing a comparative analysis of several models allows us to provide additional insights for the decision process itself via the study of the percentage of locations that always (or almost always) appear in the optimal solutions. Through this analysis, the decision-maker can choose the favorite location model by looking at several features of the possible solutions provided and not just by assuming that a particular specific indicator is the correct one to optimize.

It is essential to state that our perspective and thus our models and KPIs are related to the location of *service facilities*, i.e., those facilities that the decision-maker prefers to locate, ideally, at the least possible distance from demand centers. Here, the best coverage and accessibility of a service facility concerning the territory is pursued. Instead, our analysis is not directly applicable for *obnoxious facilities*, i.e., those facilities that the users would like to have as far as possible (e.g., nuclear waste processing plants). However, we remark that our general evaluation framework could consider tailored models and KPIs for obnoxious facilities.

The main research questions addressed in this paper are: *(i)* How to support the management in selecting the most appropriate location model for the application at hand? *(ii)* Can we derive general managerial insights for service location processes without considering strongly customized input data? To this aim, we *(i)* propose a general methodological framework to quantitatively analyze and compare the solutions provided by different location models and *(ii)* experimentally validate such a method to provide managerial insights for service facility location processes. These points represent the principal contributions of our study. The administrations can use the analysis to decide how to locate their available facilities to optimize the *quality* of their service from several perspectives. It is worth noting that this approach cannot be found in the location literature, despite it has proven to be valuable and appreciated by the management and a useful tool in real applications. Moreover, several classes of proposed KPIs (e.g., equity, progressive interventions, and core measures) may be seen as contributions themselves.

The general applicability of our analytical approach is proved through three types of datasets simulating realistic features (e.g., densely or poorly populated areas and regions with particular geographical features) to provide strategic insights for location planning. Eventually, our experimental campaign will show that the *p-median* model results in very well concerning both topological and equity aspects. In contrast, the *p-center* and the *maximal covering problem* outperform the other models in terms of coverage capabilities. Finally, the *maximal covering problem* and some proposed *back-up coverage* models return very robust solutions when the location plan is implemented through several progressive interventions over time. However, with an appropriate calibration of the framework components (i.e., models, KPIs, and their importance), similar insights can be easily gathered by the management in any specific real applications.

The remainder of this paper is organized as follows. In Section 2, we review the recent scientific literature related to facility location. Section 3 is devoted to presenting and discussing several well-known basic models from the literature with different features and objectives. In Section 4, we propose and discuss different topological, coverage, equity, progressive intervention, and core solution indicators. In Section 5, the experimental results obtained are analyzed in terms of possible insights into the decision process. Finally, conclusions are presented in Section 6.

2. Literature review

Location theory is one of the most important and oldest branches of logistics, with more than a century of history (Laporte et al., 2015). Therefore, the available solution methods and modern solvers can quickly solve large-sized classical location problems (p-center, p-median, etc.). Furthermore, the importance of location choices, deeply influence lower-level decisions as routing (see Korte and Vygen, 2008 and Perboli et al., 2018) and flow optimization (Giusti et al., 2021). Hence, in recent years, the researchers involved in the field have mainly focused on the definition and solution of complex location problems inspired by real applications.

We can identify four principal branches of research: bilevel, capacitated, stochastic, and work-balance models. The most recent bilevel models are described in Dan and Marcotte (2019), Liu et al. (2019), Farahani et al. (2019), Arulselvan et al. (2019), Guo et al. (2018), Wenxuan et al. (2019), Lin et al. (2019), Chen et al. (2020). All these papers define a bilevel problem in which the first level considers the facilities' location, and the second one considers the optimization of the flow. The solution methods proposed are heuristics due to the problems' combinatorial aspects and the models' complexity. Multilevel models have been considered too (Ortiz-Astorquiza et al., 2019). These models are more complex than the bi-level ones; thus, the solution method presented is usually heuristic. The most recent papers considering capacitated models are Irawan et al. (2019), Raghavan et al. (2019). They address real-sized problems in logistics characterized by a network with capacitated arcs. As for bilevel programs, the authors develop heuristics to solve the problems. In the stochastic branch, the models assume that the network is subject to stochastic variations in the flows once the locations have been selected. Those variations are usually represented by a discretization of the probability distributions using scenarios. This choice requires a large number of new variables, which increases the complexity of the model. Therefore, to solve the problem, the authors implement heuristics (Yu and Zhang, 2018). Finally, workload balance features have been studied within location problems (see, e.g., Davoodi, 2019). Once used only in telecommunications, this aspect is now gaining much interest because of its smart cities' applications. As in the other branches, the complexity of the problems considering workload balance requires the definition of ad-hoc heuristics to solve real-size instances.

All the works just described develop tailored models and techniques suitable for a particular application. For example, the most trending applications of the location theory are related to charging stations for electric vehicles (Cai et al., 2014; Shahraki et al., 2015), electrical batteries in an electric grid (Bose et al., 2012), and sensors in distributed networks (Muradore et al., 2006). It is important to note that it is not easy to generalize solutions tailored to problems in a specific setting to other applications. Furthermore, decision-makers often want to evaluate the optimization process's solution in several real-life applications and projects. They do that often through many features that are exogenous to the model itself (i.e., KPIs) and possibly related to political preferences or human factors (see, e.g., Tadei et al., 2016; Fadda et al., 2016, 2017, 2018, 2019b; Giusti et al., 2019). Therefore, in this study, we develop a framework for comparing, under different perspectives, the solutions obtained by solving different location problems. In particular, we focus on common location problems because they can be solved several times in a few seconds. It is important to note that the framework developed is model-independent and can be used for other models. In a similar spirit, Ertugrul and Karakasoglu (2008) develop a fuzzy multi-criteria decision-making method to overcome the inadequacy of standard mathematical models for facility location selection due to the imprecise or vague nature of linguistic assessment.

To our knowledge, a general methodological framework to quantitatively analyze and compare the solutions coming from different models has never been proposed in the location literature. A few papers have explicitly compared location models on just some particular aspects (like coverage capabilities or equity). However, their vision is quite limited and tailored for a specific application only. For example, in van den Berg et al. (2016), the authors compare four static ambulance location models according to coverage and response time criteria, proving that two models perform exceptionally well overall the considered criteria. Instead, Yu and Solvang (2018) compare the solutions provided by p-median and maximal covering problem for locating post offices. The results provide optimal relocation plans concerning different scenarios. Besides, a comparison between the optimal strategy and the current relocation plan is given. However, in this work, the authors cannot identify a strictly better model than the others. In Karatas et al. (2016), the authors propose a more general comparison between p-median and maximal coverage problems and assess their performance. They concern five decision criteria under redundant coverage requirements, namely, mean distance to facility, mean distance to primary and back-up, mean distance to back-up(s), the ratio of demand served by both primary and back-up coverage within a range threshold, and the ratio of demand served by at least primary coverage within a range threshold. Their methodology involves generating multiple scenarios and solving the two models under each scenario. The experimental results show that, in general, p-median outperforms maximal coverage in four criteria out of five. Tansel et al. (1983) did a similar evaluation, where he compared the p-center and the p-median solutions over instances for several different settings to find common network features of the solutions (this is similar to what we are going to call the *core solution* in Section 4.5). Their comparison is made by just comparing the results coming from 60 references papers from 1978 on. Note that, even if the last two mentioned papers follow our work's same philosophy, they lack a comprehensive, flexible, and structured procedure for comparing many different models under many different perspectives.

We finally remark that the comparison of different available models (or solving techniques) thought suitable frameworks is a very common practice out of the pure optimization literature, as in Data Analytics, Machine Learning (ML), Artificial Intelligence (AI), Computer Science, and Network Theory ones (see, e.g., Cordier et al., 2004; Ghanbari et al., 2010; Kumar and Singh, 2018; Cuzzocrea et al., 2019; Castrogiovanni et al., 2020). Interesting enough, in the AI and ML fields, location problems have been considered from the point of view of prediction (see, e.g., Anagnostopoulos et al., 2011 and Cho, 2015) but, to our knowledge, there is no specific studies in which the goal is the optimal location of facilities.

3. Mathematical models for optimal location

In this section, we propose and discuss several well-known basic location models from the literature. In our analysis, we have decided to include models for which a mixed-integer linear programming (MILP) formulation is available to be able to optimally solve relatively large cases in a reasonable amount of time by merely inputting the explicit models into some powerful MILP solvers available (e.g., Cplex or Gurobi). We focus on two families of problems: basic location models (*p-median*, *p-center*, *p-centdian*, and *maximal covering problem*) and back-up coverage models (*BACOP1* and *BACOP2*). We do not consider ad-hoc models because we aim to measure how effective the standard model solutions result in the general setting. Furthermore, we do not consider covering problems because they do not allow enforcing the number of facilities to locate so to consider the decision-maker's budget limit.

Throughout the study, we will use the following general notation (specific additional notation will be presented below as necessary):

- $G = (N,E)$: complete undirected graph with a set of nodes N representing possible locations for the facilities and a set of edges $E = \{(i,j)|i,j \in N, i \leqslant j\}$;
- d_{ij}: distance between node i and node $j \in N$ (for the sake of simplicity, we will assume that the triangular inequality holds for the distances, that is, $d_{ij} \leqslant d_{iq} + d_{jq}, \forall i < j < q \in N$);
- $h_i := Q_i / \sum_{j \in N} Q_j$: demand rate of node $i \in N$, where Q_i is the service demand in node i;
- p: predefined number of facilities to locate, with $p \leqslant |N|$;
- $\overline{d}$: coverage radius, that is, the threshold distance to define the covering (it could represent, for example, the maximum distance that an electric vehicle can travel or that a user is willing to drive to reach a facility);
- $\mathcal{C}_i = \{j \in N, d_{ij} \leqslant \overline{d}\}$: covering set of $i \in N$, that is, the set of all facilities closer than $\overline{d}$ to node i.

Please note that set E also contains self-loops (i,i) on each node $i \in N$, i.e., we do not assume that the internal distances are strictly equal to 0. In several applications, for example, where nodes represent non-punctual areas such as districts inside a town, the internal distances d_{ii} between a service center and the actual demand might be non-negligible.

3.1. Basic location models

The following models are called *basic* because they do not explicitly pursue any redundancy or robustness of the solution. Note that all the formulations include the requirement to locate exactly p facilities. We define y_j as a binary decision variable taking the value 1 if a facility is located in node $j \in N$, and 0 otherwise.

3.1.1. p-median

The *p-median* problem is to find p nodes of the network where to locate the facilities minimizing the weighted average distance between the located facilities and demand nodes. It can be stated as

$$\min \sum_{i \in N} h_i \sum_{j \in N|(i,j) \in E} d_{ij} x_{ij} \tag{1}$$

subject to

$$\sum_{j \in N|(i,j) \in E} x_{ij} = 1 \quad \forall i \in N \tag{2}$$

$$\sum_{j \in N} y_j = p \tag{3}$$

$$\sum_{i \in N|(i,j) \in E} x_{ij} \leqslant |N| y_j \quad \forall j \in N \tag{4}$$

$$y_j \in \{0,1\} \quad \forall j \in N \tag{5}$$

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in E \tag{6}$$

where x_{ij} is a binary variable for edge $(i,j) \in E$, which takes the value 1 if and only if the demand in node $i \in N$ is served by a facility located in $j \in N$. Minimizing the objective function (1) involves minimizing the average distance traveled by the total demand flow toward the facilities. The constraints (2) ensure that each demand node is served by exactly one facility. The constraint (3) ensures that exactly p facilities are located. The logical constraints (4) ensure that all facilities to which demand nodes are assigned need to be located/built. Finally, (5) and (6) state binary conditions on the variables.

3.1.2. p-center

In the *p-center* problem, we want to find p nodes to locate facilities to minimize the maximum distance between a demand node and its closest facility. In the proposed version of the problem, sometimes called the *vertex restricted p-center* problem, the facilities can be located only at the nodes of the graph. The problem is focused on the worst-case and can be stated as

$$\min M \tag{7}$$

subject to

$$M \geqslant \sum_{j\in N|(i,j)\in E} h_i d_{ij} x_{ij} \quad \forall i \in N \tag{8}$$

and the already presented constraints (2)–(6). Minimizing the objective function (7) involves minimizing the auxiliary variable M, which, according to the constraints (8), takes the maximum value of the expression $\sum_{j\in N} h_i d_{ij} x_{ij}$ over all nodes $i \in N$.

3.1.3. p-centdian

In the *p-centdian* problem, we want to find p nodes where to locate facilities to minimize a linear combination of the objective function of the *p-median* and *p-center* problems presented above. The formulation is then

$$\min \lambda M + (1-\lambda)\sum_{i\in N} h_i \sum_{j\in N|(i,j)\in E} d_{ij} x_{ij} \tag{9}$$

subject to

$$M \geqslant \sum_{j\in N|(i,j)\in E} h_i d_{ij} x_{ij} \quad \forall i \in N \tag{10}$$

and the constraints (2)–(6). Through the parameter λ, with $0 \leqslant \lambda \leqslant 1$, it is possible to define the relative importance of one objective with respect to the other one.

3.1.4. Maximal covering problem

In the *maximal covering problem* (MCP), differently from the well-known set covering problem, there is a predefined number p of facilities to locate to maximize the coverage (Church and ReVelle, 1974). It can be stated as

$$\max \sum_{i\in N} h_i w_i \tag{11}$$

subject to

$$\sum_{j\in N} y_j = p \tag{12}$$

$$w_i \leqslant \sum_{j\in C_i} y_j \quad \forall i \in N \tag{13}$$

$$y_i, w_i \in \{0,1\} \quad \forall i \in N \tag{14}$$

where w_i is a binary variable taking the value 1 if node i is covered, and 0 otherwise. Maximizing the objective function (11) involves maximizing the total demand covered by the located facilities. The constraint (12) ensures that exactly p facilities are located whereas the constraints (13) represent the logical link between the w and y variables.

3.2. Back-up coverage models

Models explicitly pursuing coverage redundancy are useful for creating solutions robust to congestion at the facilities or other unpredictable events like failures or temporary unavailability. Back-up coverage problems have been extensively treated in Hogan and ReVelle (1986) and still attract attention concerning modern applications (see Johnson et al., 2020). In the following, two classical back-up models (again requiring that exactly p facilities must be located) are considered.

3.2.1. BACOP1

The *back-up coverage problem of type 1* (BACOP1) can be stated as

$$\max \sum_{i\in N} h_i u_i \tag{15}$$

subject to

$$\sum_{j\in N} y_j = p \tag{16}$$

$$u_i + 1 \leqslant \sum_{j \in \mathcal{C}_i} y_j \quad \forall i \in N \tag{17}$$

$$y_i, u_i \in \{0, 1\} \quad \forall i \in N \tag{18}$$

where u_i is a binary variable taking the value 1 if demand node i is covered at least twice, and 0 otherwise. Maximizing the objective function (15) involves maximizing the number of demand nodes that are covered twice by a facility. The constraint (16) ensures that exactly p facilities are located whereas the constraints (17) ensure that $u_i = 0$ when location i is not covered by at least two facilities in $\mathcal{C}_i$.

It is essential to notice that, in contrast to the other location models presented, the BACOP1 returns feasible solutions only if there are at least enough facilities to cover all the demand nodes once. It is easy to see that the constraints (17) cannot be satisfied when their right-hand sides are strictly less than 1.

3.2.2. BACOP2

The *back-up coverage problem of type 2* (BACOP2) represents a trade-off between basic and back-up coverage. It can be stated as

$$\max \alpha \sum_{i \in N} h_i u_i + (1 - \alpha) \sum_{i \in N} h_i w_i \tag{19}$$

subject to

$$\sum_{j \in N} y_j = p \tag{20}$$

$$u_i \leqslant w_i \quad \forall i \in N \tag{21}$$

$$u_i + w_i \leqslant \sum_{j \in \mathcal{C}_i} y_j \quad \forall i \in N \tag{22}$$

$$y_i, w_i, u_i \in \{0, 1\} \quad \forall i \in N \tag{23}$$

where w_i is a binary variable taking the value 1 if demand node i is covered at least once, and 0 otherwise. Maximizing the objective function (19) involves maximizing a linear combination of nodes covered once and twice, weighted by a parameter $0 \leqslant \alpha \leqslant 1$. The constraints (21) ensure that if a node is covered at least two times, then it is also covered at least one. The constraints (22) ensure that the sum $u_i + w_i$ cannot exceed the number of times that location i is covered by facilities in $\mathcal{C}_i$. This means that, when $u_i = 1$ then also $w_i = 1$ and therefore the number of covering facilities must be greater or equal than two, while when $w_i = 1$ the number of covering facilities must be greater or equal than one. It is easy to see that, when $\alpha = 0$, the BACOP2 reduces to the MCP.

We want to indicate that, originally, the BACOP2 emerged as a pure multi-objective problem. To maintain uniformity with the other problems, we provide and solve a formulation in which the objective function has been linearized. This practice is common as per literature (see, e.g., Aringhieri et al., 2007; Aringhieri et al., 2016; Kahraman and Topcu, 2017). Moreover, α can be seen as a further possible parameter controlled by the decision-maker, and therefore, we will obtain BACOP2 results for several values of α without considering the extreme degenerate cases. We will call each version of the problem BACOP2-α, with $\alpha = \{0.01, 0.25, 0.50, 0.75, 0.99\}$.

4. KPIs

This section defines an extensive battery of KPIs, thus providing several different noteworthy measures of the goodness of a location solution. The identified KPIs are intended to outline an overview of the advantages and drawbacks of the different possible solutions to help managements decide which model would be the best to implement. We emphasize that our KPIs consider spatial measures (dispersion, average distance, etc.) as well as coverage, equity, and accessibility indicators (which are particularly crucial for services offered to communities) and solution robustness measures in future expansions of the facilities.

The considered KPIs have been selected by looking at the desired features that a decision-maker would like to find in a location plan. In particular, we believe that a solution could be analyzed from a topological perspective and in terms of coverage qualities and equity measures in services for communities. Whenever possible, we will study the worst, best, and average perspectives of a measure. Moreover, the behavior of such solutions over time takes particular importance in long-term implementations. Finally, finding the core solution raises several insights on the riskiness of the decision-making process itself.

Some conducted industrial projects support our choices (see, e.g., Fadda et al., 2019a), in which a subset of those KPIs have been presented and appreciated by the management. However, it is essential to remark that other KPIs can be considered for a specific case, or non-interesting KPIs can be eliminated from the analysis. This means that the set of KPIs is a flexible dimension to calibrate to apply our analysis.

The selected KPIs, which we divide in *topological*, *coverage*, *equity*, *progressive intervention*, and *core solution* indicators, are listed and discussed in the following subsections. We will use the following additional notation:

- $\mathcal{L} = \{j \in N \mid y_j = 1\}$: set of nodes where a facility has been located;
- $\mathcal{L}_i = \{j \in \mathcal{C}_i \mid y_j = 1\}$: set of nodes where a facility that covers demand node i has been located;
- $\mathcal{C} = \{i \in N \mid \exists j \in \mathcal{C}_i : \ y_j = 1\}$: set of demand nodes covered by at least one facility.

4.1. Topological KPIs

We first present KPIs based on the solution's topological aspects but not considering the facilities' covering capability (i.e., they do not depend on the coverage radius $\overline{d}$). They mainly concern distance, dispersion, and accessibility aspects.

- WORST-CASE DISTANCE

$$D_{max} := \max_{i \in N} \min_{j \in \mathcal{L}} d_{ij} \tag{24}$$

represents the maximum distance between a demand node and its closest facility. This indicator measures the longest distance that a user of the service has to travel to reach the nearest facility.
- WEIGHT OF THE WORST-CASE DISTANCE

$$D^h_{max} := h_i : \ \arg\max_{i \in N} \min_{j \in \mathcal{L}} d_{ij} \tag{25}$$

represents the demand rate affected by the worst-case scenario in terms of distance.
- BEST-CASE DISTANCE

$$D_{min} := \min_{i \in N} \min_{j \in \mathcal{L}} d_{ij} \tag{26}$$

represents the minimum distance between a demand node and its closest facility. This indicator measures the shortest distance that a user of the service has to travel to reach the nearest facility.
- WEIGHT OF THE BEST-CASE DISTANCE

$$D^h_{min} := h_i : \ \arg\min_{i \in N} \min_{j \in \mathcal{L}} d_{ij} \tag{27}$$

represents the demand rate affected by the best-case scenario in terms of distance.
- AVERAGE DISTANCE

$$D_{avg} := \frac{1}{|N|} \sum_{i \in N} \min_{j \in \mathcal{L}} d_{ij} \tag{28}$$

represents the average distance between a demand node and its closest facility. This indicator measures the average distance that a user has to travel to reach a facility.
- WEIGHTED AVERAGE DISTANCE

$$D^h_{avg} := \frac{1}{|N|} \sum_{i \in N} \min_{j \in \mathcal{L}} h_i d_{ij} \tag{29}$$

represents the average distance, where each node is weighted by its demand rate.
- DISPERSION

$$Disp := \sum_{i \in \mathcal{L}} \sum_{j \in \mathcal{L}} d_{ij} \tag{30}$$

represents the sum of the distances between all the located facilities. It is a measure of the homogeneity of the service from a purely geographical point of view.
- ACCESSIBILITY

$$Acc := \sum_{i \in N} h_i A_i \tag{31}$$

is the total accessibility of the service, where

$$A_i := \sum_{j \in \mathcal{L}} e^{-\beta d_{ij}} \tag{32}$$

is the accessibility of a facility in the sense of Hansen (1959). Accessibility is a measure of each demand node's visibility on the entire set of alternative covering locations. In turn, this accessibility depends on a parameter $\beta > 0$, which must be calibrated, and represents the dispersion of the alternatives in the decision-making process. The calibration is performed according to Tadei et al. (2009) and Fadda et al. (2020). The parameter β can be seen as a distance deterrence factor for locating.

4.2. Coverage KPIs

Here, we present KPIs based on covering aspects of the solution, and therefore, depending on the coverage radius $\overline{d}$. Coverage KPIs are extremely important, since the very final aim of a location problem for service facilities is to make the service available to the demand centers.

- COVERAGE

$$C := 100^*|\mathcal{C}|/|N| \tag{33}$$

represents, as a percentage, the number of covered locations with respect to the total.
- WEIGHTED COVERAGE

$$C^h := 100^*\sum_{i\in\mathcal{C}} h_i \tag{34}$$

represents, as a percentage, the demand rate of the covered locations with respect to the total demand (note that, by definition, $\sum_{i\in N} h_i = 1$).
- WEIGHT OF THE REDUNDANT COVERAGE

$$RC^h := 100^*\sum_{i\in N}\sum_{j\in\mathcal{L}_i} h_i \tag{35}$$

represents, as a percentage, the demand rate of the covered locations multiplied by the times these locations are covered. This indicator measures the weighted redundancy of the coverage.
- WORST-CASE COVERAGE

$$C_{min} := \min_{i\in N}|\mathcal{L}_i| \tag{36}$$

represents the minimum number of facilities covering a demand node. This indicator measures the availability of different choices for the unluckiest user (worst-case scenario).
- WEIGHT OF THE WORST-CASE COVERAGE

$$C^h_{min} := h_i : \arg\min_{i\in N}\left|\mathcal{L}_i\right| \tag{37}$$

represents the demand rate affected by the worst-case scenario in terms of coverage.
- BEST-CASE COVERAGE

$$C_{max} := \max_{i\in N}|\mathcal{L}_i| \tag{38}$$

represents the maximum number of facilities covering a demand node. This indicator measures the availability of different choices for the luckiest user (best-case scenario).
- WEIGHT OF THE BEST-CASE COVERAGE

$$C^h_{max} := h_i : \arg\min_{i\in N}\left|\mathcal{L}_i\right| \tag{39}$$

represents the demand rate affected by the best-case scenario in terms of coverage.
- AVERAGE COVERAGE

$$C_{avg} := \frac{1}{|N|}\sum_{i\in N}|\mathcal{L}_i| \tag{40}$$

represents the average number of facilities covering a demand node.
- WEIGHTED AVERAGE COVERAGE

$$C^h_{avg} := \frac{1}{|N|}\sum_{i\in N} h_i|\mathcal{L}_i| \tag{41}$$

represents the average coverage, where each node is weighted by its demand rate.

4.3. Equity KPIs

Apart from topological and coverage aspects, in several applications, it is essential to consider equity indicators. When the located facilities are public services managed by municipalities, equity is an exciting dimension to study. For a more detailed analysis of equity considerations in location problems, we refer the reader to Barbati and Piccolo (2016). In the following, we present some simple equity KPIs concerning coverage, distance, and accessibility.

- EQUITY OF COVERAGE

$$EC := \frac{\max_{i\in I}(|\mathcal{L}_i|) - \min_{i\in I}(|\mathcal{L}_i|)}{\max_{i\in I}(|\mathcal{L}_i|)} \tag{42}$$

represents the relative difference between the most covered and the least covered demand node.

- EQUITY OF WEIGHTED COVERAGE

$$EC^h := \frac{\max_{i\in I}(|\mathcal{L}_i|/h_i) - \min_{i\in I}(|\mathcal{L}_i|/h_i)}{\max_{i\in I}(|\mathcal{L}_i|/h_i)} \tag{43}$$

is a weighted version of the EC indicator. Here, the cardinality of $\mathcal{L}_i$ is divided by the relative importance h_i of a node to assign a larger weight to nodes with a smaller proportion of the demand.

- MEAN ABSOLUTE DISTANCE DEVIATION

$$D_{mad} := \frac{1}{|N|}\sum_{i\in N}\left|\min_{j\in\mathcal{L}} d_{ij} - D_{avg}\right| \tag{44}$$

represents the mean absolute deviation (MAD), in terms of distance, of the demand nodes with respect to the nearest located facility.

- WEIGHTED MEAN ABSOLUTE DISTANCE DEVIATION

$$D^h_{mad} := \sum_{i\in N}\overline{h}_i\left|\min_{j\in\mathcal{L}} d_{ij} - D_{avg}\right| \tag{45}$$

where $\overline{h}_i := (1-h_i)/(|N|-1)$ is the complementary importance of node i. It is a weighted version of the D_{mad} indicator. Notice that we use these particular weighting factors to maintain their summation equal to 1, as for the h_i. Indeed,

$$\sum_{i\in N}\overline{h}_i = \frac{1}{|N|-1}\sum_{i\in N}(1-h_i) = \frac{1}{|N|-1}\left(|N| - \sum_{i\in N}h_i\right) = \frac{|N|-1}{|N|-1} = 1.$$

- MEAN ABSOLUTE ACCESSIBILITY DEVIATION

$$A_{mad} := \frac{1}{|N|}\sum_{i\in N}|A_i - Acc| \tag{46}$$

represents the MAD in terms of accessibility for a demand node with respect to all the located facilities.

- WEIGHTED MEAN ABSOLUTE ACCESSIBILITY DEVIATION

$$A^h_{mad} := \sum_{i\in N}\overline{h}_i|A_i - Acc| \tag{47}$$

is a weighted version of the A_{mad} indicator.

4.4. KPIs for progressive interventions

The decision-maker often does not have sufficient resources to install all the necessary facilities simultaneously; therefore, installation plans are generally divided into several interventions programmed over time. Let $S = \{1, 2, \ldots, \mathcal{S}\}$ be the set of installation steps over the predefined time horizon and $\overline{p}_s$ the number of facilities to locate at step $s \in S$. Clearly, in the case of progressive interventions, in each step, we can only *upgrade* the solution already implemented in the previous interventions, that is, previously located facilities cannot be uninstalled. Therefore, it is crucial to understand (given a specific location model) how much the upgrading solution of a specific intervention differs, in terms of objective function or structure, from the solution obtained under no conditioning on previous decisions.

Since the simple calculation of the already presented KPIs at different installation steps is not sufficient to capture how the different location models behave against an incremental expansion of the location plan, we propose the following ad-hoc KPIs to evaluate the dynamic performance of a solution.

- DEVIATION OF UPGRADED SOLUTION VALUE (at step $s \geqslant 2$)

$$\frac{DUSV_s := |f(\overline{p}_s) - f(\overline{p}_s|\overline{p}_{s'}, \forall s' \leqslant s-1)|}{f(\overline{p}_s|\overline{p}_{s'}, \forall s' \leqslant s-1)} \tag{48}$$

measures the degree of influence of the previous choices in terms of the optimal objective function. In (48), $f(\overline{p}_s)$ represents the optimal objective function of the problem with parameter $p = \overline{p}_s$ whereas $f(\overline{p}_s|\overline{p}_{s'}, \forall s' \leqslant s-1)$ represents the optimal objective function of the problem with parameter $p = \overline{p}_s$, under the constraint that no facility can be removed from those located in the solutions of all the problems with parameter $p = \overline{p}_{s'}, s' \leqslant s-1$. Because the KPI cannot be calculated for $s = 1$, we simply define $DUSV_1 := 0$.

- DEVIATION OF UPGRADED SOLUTION (at step $s \geqslant 2$)

$$\frac{DUS_s := \langle \mathbf{y}^*(\overline{p}_s), \mathbf{y}^*(\overline{p}_s|\overline{p}_{s'}, \forall s' \leqslant s-1)\rangle}{p_s} \tag{49}$$

measures the degree of influence of the previous choices in terms of the optimal solution, where $\langle \mathbf{v}, \mathbf{v}' \rangle$ is the scalar product between vector $\mathbf{v}$ and $\mathbf{v}'$ in $\mathbb{R}^{|N|}$. Similarly to above, in (49), $\mathbf{y}^*(\overline{p}_s)$ represents the vector of the optimal solution of the problem with parameter $p = \overline{p}_s$ whereas $\mathbf{y}^*(\overline{p}_s|\overline{p}_{s'}, \forall s' \leqslant s-1)$ represents the vector of the optimal solution of the problem with parameter $p = \overline{p}_s$, under the constraint that no facility can be removed from those located in the solution of all the problems with parameter $p = \overline{p}_{s'}, \forall s' \leqslant s-1$. Since the vector components are either 0 or 1, the scalar product is equal to the number of facilities that have been located in both solutions. Again, because this KPI cannot be calculated for $s = 1$, we simply define $DUS_1 := 0$.

It is worth noting that the decision-maker can choose how many steps it makes sense to consider (i.e., $|S|$) and the number of facilities $\overline{p}_s$ to locate in each step $s \in S$.

4.5. KPIs for core solutions

Having chosen several different location models allows us to explore the similarity of the solutions they provide (in terms of located facilities) for any specific instance. We define a core solution $N_c(t)$ as the set of locations $i \in N$ such that $y_i = 1$ in the optimal solution of at least t% of the considered models. Then, the following KPI can assess the importance of a core solution:

- RELATIVE IMPORTANCE OF CORE LOCATIONS

$$KER := |N_c(t)|/|N| \tag{50}$$

represents the proportion of locations included in the core solution for the total number of locations, given a threshold t.

Note that, in addition to the possible insights concerning a location model choice's criticality, identifying a core solution might decrease the computational burden for solving the problem itself. For example, the well-known Kernel Search metaheuristic (see, e.g., Angelelli et al., 2010; Manerba et al., 2018, or Gobbi et al., 2019) is based on the concept of a core solution, that is, the set of variables that have a larger probability of appearing with a non-null value in the optimal solution.

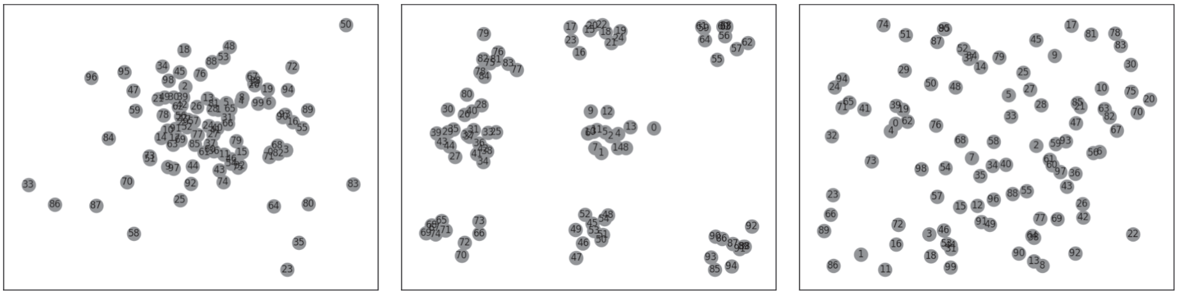
5. Experimental analysis

In this section, we first present the dataset generation procedure and then discuss the experimental results obtained. All the models have been solved optimally by using Gurobi v8.1.0. The computer used for the experiments is an *Intel(R) Core(TM)* i7-5500U CPU@2.40 GHz with 8 GB of RAM and running *Ubuntu* v18.04. However, we are not interested in analyzing the computational performances of the different models in detail; furthermore, the solving times are almost negligible, considering the location decisions' strategic level. To summarize, the time to solve a back-up model never exceeds 1 s of CPU time, and the most time-consuming model seems to be the p-median for a small number of facilities to locate (approximately 15–20 s). In conclusion, given a specific case, the entire set of the models we consider can be solved, and the entire set of KPIs can be calculated in less than 30 s.

5.1. Generation of test datasets

As our goal was to validate our approach, rather than apply it to a specific problem or real-case, we generate a broad set of random instances simulating realistic scenarios as follows.

All the underlying graphs have $|N| = 100$ nodes, and the Euclidean distances between each pair of nodes are calculated. The in-



(a) Mono-polar (b) Multi-polar (c) Uniform

Fig. 1. Examples of the dispersion of nodes in mono-polar, multi-polar, and uniform datasets.

ternal distance d_{ii} for each node $i \in N$ is generated as $d_{ii} = \theta^* \min_{(i,j) \in E | i \neq j} d_{ij}$, where θ is a random variable uniformly distributed in $[0,1]$. Thus, we ensure that the internal demand can be best satisfied by locating a facility in the same node. Each node's coordinates are drawn according to specific probability distributions on a $[-10,10] \times [-10,10]$ square. Clearly, by changing the distributions and their parameters, we can generate datasets with different specific features. In particular, we created three types of datasets with the following characteristics:

- **mono-polar**: datasets that simulate the case of a region where a single main cluster of demand nodes exists, in addition to few sparse nodes around it (e.g., a district where there is one large city and other tiny satellite municipalities). The coordinates of 80% the nodes are drawn from a *Student's t* distribution with 3 degrees of freedom, and those of the remaining 20% are drawn from a *Uniform* distribution;
- **multi-polar**: datasets that simulate the case of a region where there exist several dispersed clusters of demand nodes (e.g., a district with small-medium cities of similar sizes). The nodes' coordinates are drawn from a sum of independent *Multinomial* distributions with random mean value and a unit standard deviation;
- **uniform**: datasets that simulate the case of a region where there is no cluster of demand nodes and the nodes are all dispersed (e.g., an urban area where the demand is spread uniformly across the region). The coordinates of the nodes are drawn according to a *Uniform* distribution.

An example of the dispersion of the nodes in each type of dataset is shown in Fig. 1.

Then, the demand Q_i for each node $i \in N$ is generated randomly. Notice that, because both the models and the KPIs are based only on the demand rate h_i, the exact values of Q_i do not influence the analysis.

Finally, to simulate different possible situations, we consider $p = |N|^*\gamma$ facilities to locate and a coverage radius $\overline{d}$ such that $\mathbb{P}[D < \overline{d}] = \mu$, where D is the random variable based on which the distances between two nodes are generated. In other words, $\overline{d}$ represents the μ-th percentile of the distances. For each one of the three types of datasets above and for every combination of $\gamma = \{0.15, 0.30, 0.45, 0.60\}$ and $\mu = \{0.1, 0.3, 0.5\}$, we generate 10 different datasets. This means we have a total of 360 different datasets.

5.2. Results and discussion

Our discussion of the results is divided into four main parts: a comparison of the proposed models through the selected topological and coverage KPIs (Section 5.2.1), through the equity KPIs (Section 5.2.2), through the ad-hoc KPIs for progressive interventions (Section 5.2.3), and an analysis of the core solutions (Section 5.2.4).

5.2.1. Model comparison through topological and coverage KPIs

In this section, we discuss the performance of the different location models for the selected topological and coverage KPIs, for different values of the coverage radius $\overline{d}$, the number of facilities to locate p, and type of dataset. We calculate their relative absolute deviation (RAD) from the best possible values to compare all KPIs. Since we are considering *service facilities*, such benchmarks are computed for each dataset by simply fixing $y_i = 1, \forall i \in N$ (i.e., all the nodes are located). This means that the RADs for all the KPIs indicate a better performance when they are closer to 0. Note that for some KPIs (namely, all the topological ones except *Disp* and *Acc*), the benchmark solution provides the minimum possible values. In contrast, for the remaining KPIs, opening a facility in all the locations provides the maximum possible values. Finally, it is important to notice that, although each KPI has a possibly different dimension, RADs are dimensionless because they are relative to the corresponding benchmarks.

In Tables A.7–A.15 from Appendix A, for each combination of the value of μ (defining the coverage radius) and type of datasets (mono-polar, multi-polar, and uniform), we present the average RADs of each KPI over the 10 generated datasets. Each row is uniquely identified in these tables by the model used (*Model*) and the percentage (γ) of facilities to locate. Then, there is a specific column for each topological and coverage KPI. For the BACOP1, we calculate the average RADs of the KPIs only for datasets where a feasible solution exists (we report the number of feasible solutions out of 10 in square brackets next to the model name in the first column).

Table 1
Averages and standard deviations of RADs for topological KPIs across all datasets.

RADs	Mono-polar		Multi-polar		Uniform		Tot Avg	
	avg	stdev	avg	stdev	avg	stdev	avg	stdev
p-center	16.61	21.50	12.81	16.43	6.27	7.43	11.90	15.12
p-median	17.23	24.67	10.53	13.00	6.23	6.94	11.33	14.87
p-centdian	15.89	20.90	11.57	13.84	6.21	7.20	11.23	13.98
MCP	19.89	25.68	14.16	16.42	8.31	9.99	14.12	17.36
BACOP1	15.81	18.15	18.31	22.54	8.29	9.73	14.09	16.77
BACOP2-0.01	25.34	37.58	17.95	18.93	15.61	17.45	19.64	24.65
BACOP2-0.25	24.94	37.15	17.11	18.02	15.55	17.36	19.20	24.18
BACOP2-0.50	23.65	34.51	16.59	17.13	15.26	17.08	18.50	22.90
BACOP2-0.75	21.16	29.29	16.51	17.15	14.98	16.71	17.55	21.05
BACOP2-0.99	20.48	27.53	16.25	16.71	13.82	15.84	16.85	20.03
Tot Avg	**20,18**	**27,74**	**16,98**	**20,20**	**7,95**	**9,37**	**15,02**	**19,08**

Table 2
Averages and standard deviations of the RADs for coverage KPIs across all datasets.

RADs	Mono-polar		Multi-polar		Uniform		Tot Avg	
	avg	stdev	avg	stdev	avg	stdev	avg	stdev
p-center	0.81	1.11	0.63	0.83	0.55	0.50	0.66	0.81
p-median	0.87	1.27	0.55	0.56	0.55	0.52	0.66	0.78
p-centdian	0.78	1.01	0.65	0.86	0.54	0.50	0.66	0.79
MCP	0.79	1.11	0.70	0.93	0.53	0.55	0.67	0.87
BACOP1	0.77	1.20	0.74	0.97	0.60	0.71	0.70	0.95
BACOP2-0.01	0.81	1.31	0.81	1.00	0.85	0.98	0.82	1.10
BACOP2-0.25	0.83	1.35	0.81	1.01	0.85	0.98	0.83	1.11
BACOP2-0.50	0.66	0.87	0.83	1.05	0.84	0.95	0.78	0.96
BACOP2-0.75	0.70	0.96	0.82	1.03	0.83	0.94	0.78	0.98
BACOP2-0.99	0.64	0.83	0.73	0.85	0.82	0.94	0.73	0.87
Tot Avg	**0.73**	**1.19**	**0.64**	**0.92**	**0.50**	**0.57**	**0.62**	**0.89**

Table 3
Averages and standard deviations of the WINs for topological KPIs across all datasets.

WINs	Mono-polar		Multi-polar		Uniform		Tot Avg	
	avg	stdev	avg	stdev	avg	stdev	avg	stdev
p-center	11.46	1.31	11.46	1.31	6.25	0.39	9.72	1.01
p-median	29.17	8.51	34.38	11.82	46.88	21.97	36.81	14.10
p-centdian	10.42	1.09	4.17	0.17	1.04	0.01	5.21	0.42
MCP	3.13	0.10	13.54	1.83	16.67	2.78	11.11	1.57
BACOP1	10.42	1.09	8.33	0.69	2.08	0.04	6.94	0.61
BACOP2-0.01	7.29	0.53	10.42	1.09	5.21	0.27	7.64	0.63
BACOP2-0.25	7.29	0.53	6.25	0.39	9.38	0.88	7.64	0.60
BACOP2-0.50	8.33	0.69	9.38	0.88	12.50	1.56	10.07	1.05
BACOP2-0.75	2.08	0.04	1.04	0.01	0.00	0.00	1.04	0.02
BACOP2-0.99	10.42	1.09	1.04	0.01	0.00	0.00	3.82	0.37

When no feasible solution exists for all the datasets, we report “NAN” for each KPI. However, note that infeasibility affects only two entries in the first table, and therefore, does not bias the general comparisons.

Such detailed results are very hard to read. Therefore, in the following, we propose some ad-hoc aggregate views over all the combinations of KPIs, μ, and γ to find some clear trends in the results:

- average of RADs: since the goal of our analysis is to be as general as possible, we assume that all the KPIs have the same importance, and therefore we consider their simple average. It is worth noting that, since RADs may have very different magnitudes (as in our case), such an aggregate view may present big standard deviations;
- percentage of times that a model provides the best RAD (WIN): this aggregate view looks at estimating the probability that a model achieves the best value of a specific KPI
- percentage of times that a model provides the top three RADs (PODIUM): this aggregate view looks at estimating the probability that a model performs in the top three models of a specific KPI, i.e., it takes into considerations more performance dynamics concerning the WIN aggregator;

Table 4
Averages and standard deviations of the WINs for coverage KPIs across all datasets.

WINs	Mono-polar		Multi-polar		Uniform		Tot Avg	
	avg	stdev	avg	stdev	avg	stdev	avg	stdev
p-center	41.67	17.36	81.25	66.02	81.25	66.02	68.06	49.80
p-median	0.00	0.00	2.08	0.04	0.00	0.00	0.69	0.01
p-centdian	0.00	0.00	0.00	0.00	6.25	0.39	2.08	0.13
MCP	56.25	31.64	16.67	2.78	12.50	1.56	28.47	11.99
BACOP1	2.08	0.04	0.00	0.00	0.00	0.00	0.69	0.01
BACOP2-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BACOP2-0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BACOP2-0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BACOP2-0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BACOP2-0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

- average scoring of a model (SCORE): for each model, it is associated with a score according to its ranking position concerning the goodness of the RAD for a specific KPI (the better the model, the higher the score). Our scoring goes from 9 to 0, with step 1, since we are considering ten models.

It is important to notice that the decision-maker may consider several other aggregate views, particularly those resulting statistically robust in the specific case. Moreover, the view we propose can be calibrated in several ways to cope with real application peculiarities. For instance, the aggregators can be calculated by selecting the most important KPIs for the application at hand. The averages can be computed by weighting the KPIs with their relative importance, or different scoring functions can be adopted. In the following, we analyze in detail the average RADs and the WIN aggregators, while we just report the results obtained for PODIUM (Tables B.19–B.21 in Appendix B) and SCORE (Tables C.23–C.24 in Appendix C) aggregators.

In Tables 1 and 2 we report, for each model, the overall average and standard deviation over the RADs of all the topological and coverage KPIs, respectively, for all the datasets of a given type (mono-polar, multi-polar, and uniform). The total averages are also shown in bold font. Furthermore, with the same layout, we also present in Tables 3 and 4 the WIN aggregator results for topological and coverage KPIs, respectively. As the reader can notice, these last two tables do not show the average by column because it is always equal to 100%.

First, note that the average RADs across all the models (last row of Tables 1 and 2) clearly vary depending on the type of datasets. For both types of KPIs, mono-polar and multi-polar datasets tend to be worse on average in terms of errors for the benchmark and exhibit more evident differences in the models' performance. In particular, the mono-polar datasets seem to be the worst-performing ones for most models; however, some models perform worse in the multi-polar or the uniform case. Uniform datasets, except for a few models, appear to be much easier to deal with in topological and coverage KPIs. This seems to depend on the particular topology of the instances, in which central and peripheral nodes are differently distributed. In particular, peripheral nodes deteriorate most of the KPIs' values because they are distant from the others and thus require ad-hoc facilities to be well served. Therefore, the bad performances of all the methods on mono-polar instances depend on their number of peripheral nodes that are not negligible (as in uniform instances) and far from the high-density region (as in multi-polar instances). Instead, the distribution used in uniform instances ensures an average density of the nodes that guarantees a better homogeneity in the solutions of different models and, in turn, in the KPIs value.

Second, we compare the behavior of different models. Concerning the topological KPIs (Table 1), three different levels of performance can group models. In the best group, containing p-center, p-median, and p-centdian, the latter is the best model, both in terms of average and standard deviation. However, the model is not outperforming p-center and p-median, which show only slightly worse performances. This is not surprising since p-center, p-median, and p-centdian have objective functions considering topological properties. The second group contains MCP and BACOP1, which behave similarly and perform three points worse than the first group. Finally, the third group includes all the BACOP2 models, which perform 2–5 points worse than the second group. A clear trend is that the higher the α, the better the results. The difference in the performance of BACOP2 models may be due to their objective functions, which do not consider topological features of the solution. It is noteworthy that, apart from the total averages, BACOP1 performs better than the other models in the mono-polar datasets, whereas p-median performs better than the other model in the multi-polar datasets. By looking at the WINs results for topological KPIs in Table 3, p-median achieves the best performance, followed by MCP, BACOP2-0.50, and p-center. Instead, on average, BACOP2-α with high values of α and p-centdian show the worst results. These models also show a significant difference in performance concerning the instances type, making them very bad for uniform instances.

Regarding the coverage KPIs, some similar trends can be detected in the RADs results of Table 2, even if the differences between the models are not that evident. p-centian, p-center, and p-median still have the best averages. However, in this set of KPIs, the MCP performance is only slightly worse than that of the best model (outperforming BACOP1). Again, among the BACOP2 models, the best results are obtained for the higher values of α. It is worth noting that BACOP2-0.99 (which performs quite severely in the uniform case) is the best choice for mono-polar datasets. p-median is still the best model for multi-polar datasets. In contrast, MCP is the best model for uniform datasets. From the WINs results shown in Table 4, we can derive that p-center has the best performance, followed by MCP. It is essential to notice that all the other models rarely achieve the best results. If compared with the previous results on topological KPIs, we can enforce the role of p-center while probably MCP should be better considered. The latter achieves the best results several times, which means that the second-class performance obtained on the topological KPIs is probably due to few but awful values in the

Table 5
Averages and standard deviations of the RADs for equity KPIs by dataset type.

RADs	Mono-polar		Multi-polar		Uniform		Tot Avg	
	avg	stdev	avg	stdev	avg	stdev	avg	stdev
p-center	0.14	0.13	0.16	0.13	0.18	0.17	0.16	0.15
p-median	0.16	0.17	0.17	0.18	0.18	0.21	0.17	0.19
p-centdian	0.16	0.16	0.18	0.17	0.19	0.19	0.18	0.18
MCP	0.23	0.19	0.35	0.31	0.24	0.16	0.27	0.22
BACOP1	0.18	0.15	0.33	0.30	0.24	0.18	0.25	0.21
BACOP2-0.01	0.30	0.24	0.45	0.48	0.33	0.29	0.36	0.33
BACOP2-0.25	0.28	0.22	0.41	0.44	0.29	0.24	0.33	0.30
BACOP2-0.50	0.26	0.21	0.44	0.48	0.30	0.25	0.33	0.31
BACOP2-0.75	0.24	0.19	0.43	0.48	0.29	0.25	0.32	0.31
BACOP2-0.99	0.20	0.17	0.39	0.43	0.28	0.23	0.29	0.28
Tot Avg	**0.22**	**0.18**	**0.33**	**0.34**	**0.25**	**0.22**	**0.27**	**0.25**

Table 6
Averages and standard deviations of the WINs for equity KPIs across all datasets.

WINs	Mono-polar		Multi-polar		Uniform		Tot Avg	
	avg	stdev	avg	stdev	avg	stdev	avg	stdev
p-center	5.56	0.31	0.00	0.00	4.17	0.17	3.24	0.16
p-median	18.06	3.26	29.17	8.51	29.17	8.51	25.46	6.76
p-centdian	16.67	2.78	2.78	0.08	9.72	0.95	9.72	1.27
MCP	15.28	2.33	9.72	0.95	9.72	0.95	11.57	1.41
BACOP1	15.28	2.33	9.72	0.95	4.17	0.17	9.72	1.15
BACOP2-0.01	13.89	1.93	5.56	0.31	5.56	0.31	8.33	0.85
BACOP2-0.25	1.39	0.02	4.17	0.17	15.28	2.33	6.94	0.84
BACOP2-0.50	8.33	0.69	13.89	1.93	4.17	0.17	8.80	0.93
BACOP2-0.75	0.00	0.00	0.00	0.00	1.39	0.02	0.46	0.01
BACOP2-0.99	5.56	0.31	25.00	6.25	16.67	2.78	15.74	3.11

RADs for some instances.

Finally, a specific analysis of the back-up models is worthwhile, as they focus on a secondary but still important aspect, namely, redundancy. As already highlighted above, in terms of average RADs, BACOP1 outperforms the other back-up models whereas, for the BACOP2 models, it seems better to choose higher values of α. On the other hand, the WIN results show more robust results when α is 0.50 or less. However, the decision-maker should be aware that BACOP1 may result in infeasible solutions for a small coverage radius or the number of facilities to locate. Therefore, BACOP2, with a suitable value of α (which seems strictly depending on the instance type), is probably the best choice for making a robust decision in terms of redundancy.

5.2.2. Model comparison through equity KPIs

Similarly to the previous section, we discuss the performance of the different location models for the equity KPIs, for different values of coverage radius $\overline{d}$, number of facilities to locate p, and type of datasets. However, in this case, the KPIs do not all have the same benchmark (i.e., the case in which a facility is located in each node). Instead, the best value of each KPI can be calculated by solving the location model where the objective function is the mathematical expression of the KPI in question. Therefore, we decided to calculate RADs from the KPI value of the best model among all the models. In Tables A.16–A.18 in Appendix A, for each dataset type (mono-polar, multi-polar, and uniform), we present the average RADs of each KPI over the 10 generated datasets. Each row in these tables corresponds to a combination of the model used (*Model*) and the percentage (γ) of facilities to locate. There is a column for each equity KPI, clustered by the different values of μ for the coverage radius. In order to have an aggregate view, in Table 5 we report the overall average and standard deviation over the RADs of all equity KPIs by each dataset type (mono-polar, multi-polar, and uniform) while, in Table 6, we show the WINs results.

From Table 5, we can see that, in general, the behavior of the different models in terms of equity KPIs are in line with those observed in the previous section. Again, p-center, p-median, and p-centdian perform very well (exhibiting the lowest RADs). BACOP1 and MCP

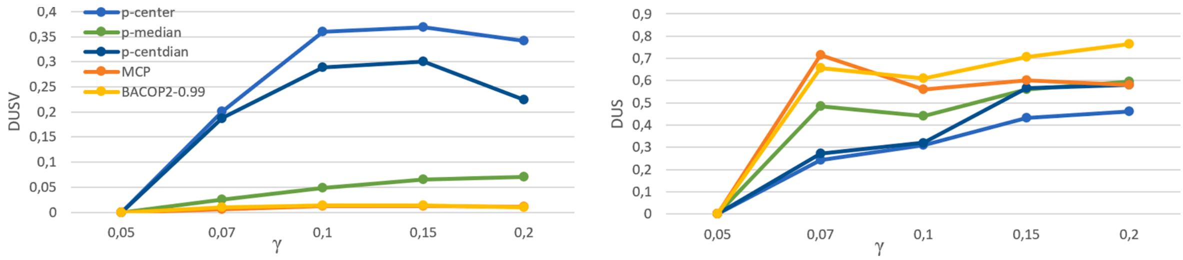


Fig. 2. Average *DUSV* and *DUS* for mono-polar datasets under the exploratory plan.

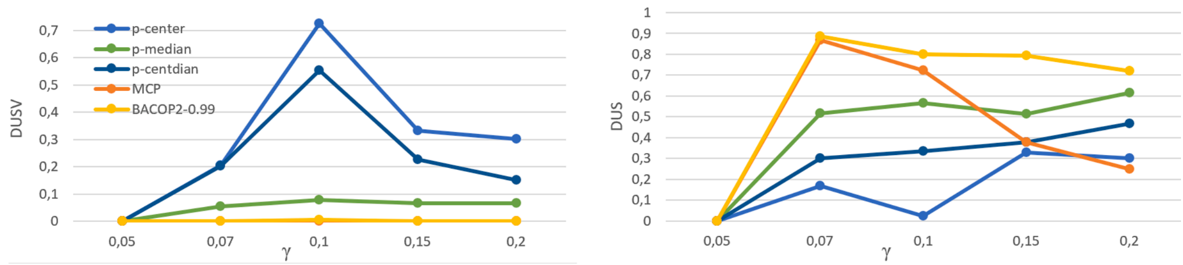


Fig. 3. Average *DUSV* and *DUS* for multi-polar datasets under the exploratory plan.

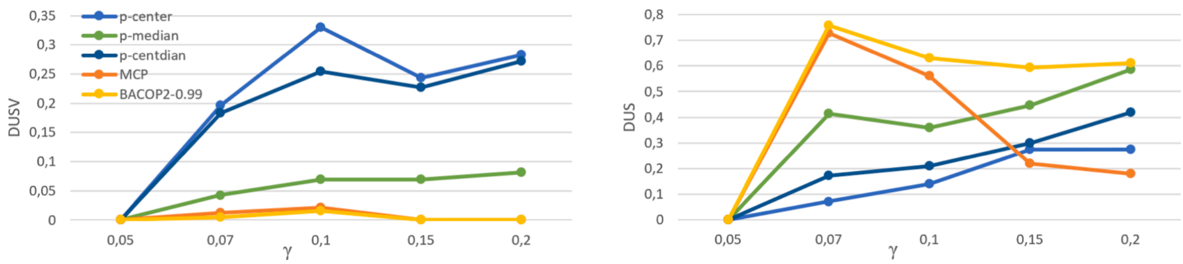


Fig. 4. Average *DUSV* and *DUS* for uniform datasets under the exploratory plan.

have similar performances, whereas BACOP2 performs better with higher values of α. However, some differences can be highlighted. First, p-center is the best equity model (outperforming, although slightly, p-median and p-centdian). This is reasonable because the p-center objective function explicitly addresses the worst case. Second, the difficulty of locating a facility in terms of equity depends on the dataset type differently. In particular, multi-polar datasets are the worst-performing average for all the models, whereas it appears much easier to find a mono-polar datasets solution. Again, this sounds intuitive because it is easier for equity measures to achieve better values when most of the demand is concentrated around a single node. On the contrary, in the presence of several nodes, the service's equity must be addressed more carefully, and a greater effort is necessary to locate the facilities. In Table 6, we can see that the WINs results are different by the RADs one. In particular, p-median confirms the model that achieves the best results most of the time, followed by BACOP2-0.99, MCP, BACOP1, and p-centdian. Interestingly enough, p-centdian has awful performance in WINs only for the multi-polar instances, meaning that it cannot inherit the p-median good characteristics when dealing with many high-density regions.

5.2.3. Analysis of progressive interventions plans

In this section, we analyze the results obtained using the two KPIs proposed in Section 4.4 for a plan composed of progressive interventions over time. Since this analysis involves more operational details than the previous one, we decided to restrict our focus

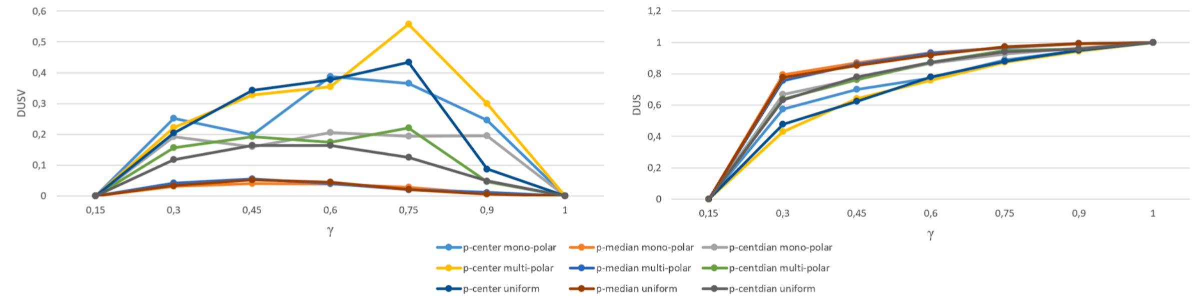


Fig. 5. Average *DUSV* and *DUS* under the slow plan.

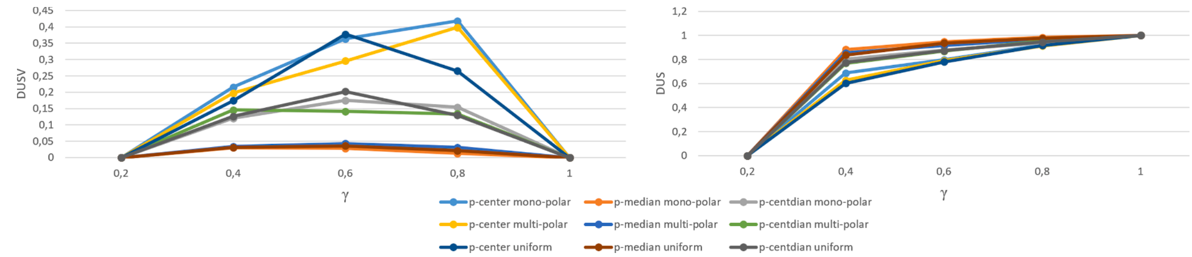


Fig. 6. Average *DUSV* and *DUS* under the accelerated plan.

according to the previous sections' outcomes. More precisely, we decided not to consider the BACOP1 model (because it may result in unfeasible solutions) and all the BACOP2-α models except the one with $\alpha = 0.99$. Finally, we considered only the coverage radius obtained by imposing $\mu = 0.1$, that is, the smallest, and therefore, the more critical in terms of decisions. Large coverage radii tend to provide very similar solutions regardless of the model used. Therefore, it is a very similar performance in KPIs (particularly concerning the coverage ones).

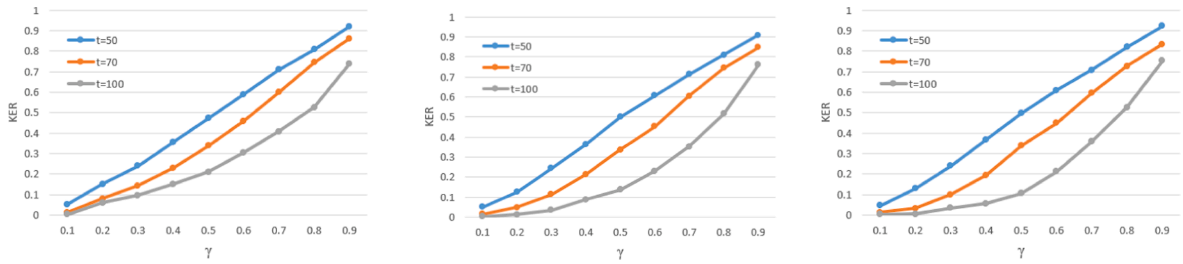
We tested three possible plans, different in terms of the number of installation steps $|S|$ and number of facilities $\overline{p}_s = |N|^*\gamma_s$ to locate in each step $s \in S$:

- **exploratory plan**: $|S| = 5$, with $\gamma_1 = 0.05, \gamma_2 = 0.07, \gamma_3 = 0.1, \gamma_4 = 0.15$, and $\gamma_5 = 0.2$;
- **slow plan**: $|S| = 7$, with $\gamma_1 = 0.15, \gamma_2 = 0.3, \gamma_3 = 0.45, \gamma_4 = 0.6, \gamma_5 = 0.75, \gamma_6 = 0.9$, and $\gamma_7 = 1$;
- **accelerated plan**: $|S| = 5$, with $\gamma_1 = 0.2, \gamma_2 = 0.4, \gamma_3 = 0.6, \gamma_4 = 0.8$, and $\gamma_5 = 1$.

The exploratory plan considers few interventions, up to a small percentage (20%) of facilities to locate out of the total, and it can be seen as a trial set of interventions to understand the feasibility of the whole installation process. Instead, the other two plans look at the entire installation process (up to 100% of facilities located), changing the number and the consistency of the interventions.

Concerning the exploratory plan, the progressive interventions KPIs (*DUSV* and *DUS*) for each considered model and averaged over the 10 generated datasets are shown in Figs. 2–4 for mono-polar, multi-polar, and uniform datasets, respectively. Regarding the *DUSV*, it can be seen that p-center has the worst performance, followed by p-centdian (which seems to inherit such lousy behavior from p-center). Instead, MCP and BACOP2-0.99 have very similar performances and *DUSV* values close to 0. It is worth noting that p-median exhibits more similar results than the latter models to p-center and p-centdian. The results based on *DUS* exhibit similar trends. MCP and BACOP2-0.99 show to be very robust, especially for the very first installation steps, while they tend to have different solutions in the subsequent ones. In particular, MCP steadily decreases in performance after the second step, becoming the worst model at the end of the exploratory plan for multi-polar and uniform datasets. Instead, p-center, p-median, and p-centdian show an almost always increasing trend so that, for example, p-median achieves the same result of BACOP2-0.99 at the end of the plan in the uniform case. It seems that the models which consider topological aspects of the network (p-center, p-median, and p-centdian) reorganize most of their solution in the first interventions. Simultaneously, they are subjected to smaller changes for the subsequent installations, making the marginal contribution of a newly located facility less impacting the intervention plan's progress.

Concerning the slow and the accelerated plan, the *DUSV* and *DUS* for each considered model, dataset type, and averaged over the



(a) Mono-polar (b) Multi-polar (c) Uniform

Fig. 7. *KER* values (y-axis) against different percentages γ of facilities to locate (x-axis), for the different datasets.

10 generated datasets, are shown in Figs. 5 and 6, respectively. Note that, in the charts, we have removed the results for MCP and BACOP2-0.99 since their *DUSV* are almost always equal to 0, and in turn, possible *DUS* variations would depend only on the existence of multiple optima (and are thus not of interest in our analysis). This means that MCP and BACOP2-0.99 are the most reliable models when looking at complete installation plans. However, given that the remaining models could be interesting under other perspectives (e.g., the KPIs already presented in the previous sections), we compare their performance in the following. First, note that the results, under both the slow and accelerated plan, do not greatly depend on the dataset type (mono-polar, multi-polar, and uniform). Therefore, the models' behaviors are similar to those under the exploration plan. That is, *DUSV* is very close to 0 for p-median, whereas p-center and p-centdian exhibit worse performances. Interesting enough, while p-median and p-centdian reach their worst performance in the middle of the plan (or even before), p-center tends to have big variations in the solution up to the third or second last intervention. This makes p-center very bad suited for progressive intervention plans. Concerning the *DUS*, it seems that considering the entire horizon of the interventions makes the trends clearer, maintaining the order in terms of performance observed for the *DUSV*.

As expected, *DUS* of all the three models tends to 1 for the last interventions while *DUSV* tends to 0. In fact, if $p_s = |N|$ there is no difference between $\mathbf{y}^*(p_s)$ and $\mathbf{y}^*(p_s|p_{s'},\ \forall s' < s-1)$, thus $\langle \mathbf{y}^*(p_s), \mathbf{y}^*(p_s|p_{s'}, \forall s' < s-1)\rangle = |N|$ and $DUS \rightarrow 1$. If $\mathbf{y}^*(p_s)$ and $\mathbf{y}^*(p_s|p_{s'}, \forall s' < s-1)$ are equal, their value is equal, thus $f(\overline{p}_s) - f(\overline{p}_s|\overline{p}_{s'}, \forall s' \leqslant s-1) = 0$ and $DUSV \rightarrow 0$. Interesting enough, by using basic probability theory, we can compute a lower bound on the speed of the convergence of $DUS \rightarrow 1$. If we consider $\mathbf{y}^*(p_s)$ and $\mathbf{y}^*(p_s|p_{s'}, \forall s' < s-1)$ to be random vectors in which each component is an independently and identically distributed (i.i.d.) Bernoulli random variable with probability π, then

$$\begin{aligned}&\mathbb{E}[\langle \mathbf{y}^*(p_s), \mathbf{y}^*(p_s|p_{s'}, \forall s' < s-1)\rangle] = \\ &= \sum_{i=1}^{|N|} \mathbb{E}[y_i^*(p_s) y_i^*(p_s|p_{s'}, \forall s' < s-1)] = |N|\ \mathbb{E}[y_{\bar{i}}^*(p_s) y_{\bar{i}}^*(p_s|p_{s'}, \forall s' < s-1)] == |N|\ [1 \cdot \pi^2 + 0 \cdot (\pi(1-\pi) + (1-\pi)^2)] = |N|\ \pi^2,\end{aligned} \tag{51}$$

where $y_i^*(p_s)$ and $y_i^*(p_s|p_{s'}, \forall s' < s-1)$ are i-th vector component and $\bar{i}$ is any component. Thus, for each step $s \geqslant 2$, we can define *DUS* of Bernoulli to be

$$DUS_s^{\mathcal{B}} = \frac{\mathbb{E}[\langle \mathbf{y}^*(p_s), \mathbf{y}^*(p_s|p_{s'}, \forall s' < s-1)\rangle]}{p_s} = \frac{|N|\pi^2}{p_s}. \tag{52}$$

In particular, the result comes from the i.i.d. assumption. Note that the probability π can be seen as the γ_s in our experiments, e.g. if $\gamma_s \tilde{} 1$ it means that almost all nodes will be selected, as $\pi \tilde{} 1$ and viceversa. Since the i.i.d. assumption of the vector components is very unlikely to be satisfied, then the theoretical $DUS_s^{\mathcal{B}}$ in Eq. (52) represents a lower bound of any *DUS* coming from a model in which facilities are located though an optimization logic.

5.2.4. Analysis of core solutions

In contrast to the previous analyses, in this section, we focus on finding insights into the location process by examining the similarities in the different models' solution decisions. In particular, we calculate the *KER* indicator presented in Section 4.5 for the same models and coverage radius selected in Section 5.2.3. In Fig. 7(a)–(c), we plot the *KER* values (y-axis) against different percentages γ of facilities to locate (x-axis), for the mono-polar, multi-polar, and uniform datasets, respectively (as always, the values represent averages over the 10 generated datasets).

Each chart shows three colored lines corresponding to three different values for parameter t, required for defining the core solution.

In our specific case, $t = 100$ represents the pure intersection of the solutions obtained by the five models considered. The results show very similar trends for the three dataset types, and (as expected) the *KER* values grow as γ grows. It is noteworthy that, for $t = 50$, the growth is almost linear, whereas, for $t = 100$, it takes the form of a parabolic curve. The uniform datasets seem to be the most difficult to approximate by a core solution for small values of γ, but this is no longer true for γ greater than 0.7. This analysis allows the decision-makers to understand how critical a model's choice is given solutions coming from different models. In fact, the higher the *KER* value, the less critical the choice of a model is. Moreover, studying the core solutions' behavior for several values of the threshold t allows us to provide certain confidence regarding a possible heuristic based on fixing the core locations in the explored solutions. For example, the almost linear behavior of the results for $t = 50$ means that, if we want to locate p facilities, all the locations appear to be selected in the optimal solution by at least three out of the five models.

6. Conclusions

In this study, we have conducted an extensive comparative analysis of models and indicators concerning the optimal location of generic service facilities. Convinced that the right location model is tough to select in many situations, we believe that different models should be jointly considered to address a particular set of requirements and objectives. Therefore, a battery of topological and coverage KPIs have been identified and calculated for the different models' solutions. The analyzed KPIs include the covering capabilities, robustness, equity, dispersion, and accessibility of the resulting solutions. Moreover, since many service infrastructures are commonly supposed to be located through several progressive interventions over a defined time horizon, we also provide ad-hoc KPIs measuring the changes in solutions and their objectives over time. Finally, using a comparative analysis of several models allows us to provide additional insights into the decision process itself (e.g., the decision-making criticality). Our approach has been successfully applied to address specific real cases (see, e.g., Fadda et al., 2019a), and its general applicability is shown based on the results obtained on generated datasets simulating areas characterized by specific geographical and demographic features.

Future research directions can be the following:

- first, a similar analysis can be performed by explicitly including stochasticity into the decision process. For most of the applications, the demand rate h_i of node i is the parameter that makes more sense to represent a stochastic variable. For example, in the location of electric vehicle charging stations, the stochasticity is due to the difficulty of estimating the service demand Q_i for any node i according to static measures (e.g., the number of electric vehicle users living in that node) and to the unpredictable dynamics of traffic flows. When dealing with an explicit stochastic model, it could be interesting to exploit the core solution structure proposed in Section 4.5 to create ad-hoc heuristics-based, for example, on the well-known *Progressive Hedging* framework (see Rockafellar and Wets, 1991; Manerba and Perboli, 2019; Fadda et al., 2019b) for approximating strategic decisions;
- second, as already mentioned in the Introduction, a similar comparative study could be performed on several other location models, where the number of facilities to locate is an output. In that case, tailored KPIs and scenarios of interest should be defined;
- finally, the application of machine learning techniques to location problems is a good topic, which has gained increasing interest recently (see, e.g., Xu et al., 2016 or Glaeser et al., 2019). Machine learning requires an immense amount of data, and in several application domains, gathering such data is not easy. This is where the approach to generate test datasets described in Section 5.1 could help. Changing the nodes' probability distribution in random graphs and the parameters results in a possibly infinite set of cases with specific features within the three realistic types of mono-polar, multi-polar, and uniform districts. The KPIs introduced in Section 4 may help discriminate among different machine learning approaches. In particular, we envisage investigating how to use supervised machine learning to select the best model among the available ones, therefore improving where to locate support facilities or generating solutions that are not model-dependent and compare them with the standard model-dependent ones.

CRediT authorship contribution statement

Edoardo Fadda: Conceptualization, Methodology, Software, Validation, Investigation, Data curation, Writing - original draft. **Daniele Manerba:** Conceptualization, Methodology, Validation, Formal analysis, Investigation, Data curation, Writing - original draft, Writing - review & editing, Visualization. **Gianpiero Cabodi:** Resources, Writing - review & editing. **Paolo Enrico Camurati:** Resources, Writing - review & editing. **Roberto Tadei:** Conceptualization, Methodology, Writing - review & editing, Supervision, Project administration.

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Appendix A. Detailed RADs results

Tables A.7–A.18

Table A.7
Mono-polar, $\mu = 0.1$.

Model	γ	D_{max}	D^h_{max}	D_{min}	D^h_{min}	D_{avg}	D^h_{avg}	*Disp*	*Acc*	*C*	C^h	RC^h	C_{min}	C^h_{min}	C_{max}	C^h_{max}	C_{avg}	C^h_{avg}
p-center	0.15	100.60	1.38	61.25	2.38	50.40	45.96	0.96	0.81	0.31	0.24	0.84	1.00	0.43	0.84	7.65	0.93	0.92
p-median	0.15	144.34	1.40	43.69	2.03	41.80	36.11	0.97	0.76	0.21	0.16	0.84	1.00	0.29	0.81	11.36	0.89	0.89
p-centdian	0.15	96.74	1.09	50.18	2.52	47.15	42.62	0.96	0.79	0.28	0.21	0.84	1.00	0.30	0.85	5.73	0.92	0.91
MCP	0.15	268.48	1.29	75.54	1.49	50.03	47.26	0.97	0.81	0.13	0.09	0.84	1.00	0.27	0.86	4.00	0.90	0.90
BACOP1 **[0]**	0.15	NAN	NAN	NAN	NAN	NAN	NAN	NAN	NAN	NAN	NAN	NAN	NAN	NAN	NAN	NAN	NAN	NAN
BACOP2-0.01	0.15	332.22	1.16	13.89	0.62	80.85	76.17	0.98	0.83	0.27	0.24	0.84	1.00	0.24	0.85	0.59	0.84	0.83
BACOP2-0.25	0.15	317.42	1.16	7.66	0.62	77.87	73.02	0.98	0.82	0.25	0.23	0.84	1.00	0.16	0.84	1.97	0.84	0.83
BACOP2-0.50	0.15	312.06	1.24	6.32	0.87	73.54	68.07	0.98	0.82	0.22	0.19	0.84	1.00	0.16	0.85	1.94	0.84	0.83
BACOP2-0.75	0.15	256.58	1.28	5.07	0.82	63.30	57.17	0.98	0.83	0.16	0.13	0.84	1.00	0.21	0.86	2.12	0.85	0.85
BACOP2-0.99	0.15	244.41	0.97	7.12	0.64	57.82	52.01	0.97	0.83	0.12	0.08	0.84	1.00	0.33	0.90	2.18	0.89	0.89
p-center	0.30	81.70	1.00	13.33	0.55	24.26	18.92	0.87	0.61	0.05	0.02	0.67	1.00	0.62	0.71	5.00	0.82	0.81
p-median	0.30	86.30	1.01	29.24	1.81	22.36	15.76	0.88	0.54	0.06	0.03	0.67	1.00	0.59	0.71	4.49	0.79	0.79
p-centdian	0.30	81.63	1.01	13.96	1.59	23.75	17.52	0.87	0.54	0.05	0.02	0.67	1.00	0.55	0.78	1.95	0.82	0.82
MCP	0.30	39.78	3.75	41.20	1.42	26.44	26.52	0.86	0.68	0.00	0.00	0.67	0.20	0.29	0.78	7.70	0.86	0.86
BACOP1 **[8]**	0.30	27.64	2.92	7.75	1.45	22.73	22.95	0.87	0.64	0.00	0.00	0.67	0.00	0.22	0.76	9.60	0.82	0.81
BACOP2-0.01	0.30	239.54	1.03	6.20	0.68	42.89	41.02	0.90	0.69	0.11	0.09	0.65	1.00	0.12	0.83	0.98	0.80	0.79
BACOP2-0.25	0.30	239.75	1.03	6.11	0.73	42.35	39.70	0.89	0.69	0.10	0.08	0.65	1.00	0.12	0.83	1.96	0.80	0.79
BACOP2-0.50	0.30	227.27	0.62	5.01	0.99	39.70	34.33	0.89	0.66	0.08	0.05	0.65	1.00	0.27	0.83	1.99	0.80	0.80
BACOP2-0.75	0.30	122.98	0.46	5.70	0.87	34.70	30.82	0.88	0.68	0.04	0.01	0.65	1.00	0.43	0.85	2.06	0.81	0.80
BACOP2-0.99	0.30	49.11	1.00	4.98	0.77	30.91	31.13	0.87	0.71	0.00	0.00	0.65	0.30	0.44	0.85	2.05	0.82	0.81
p-center	0.45	42.26	0.73	4.28	1.69	12.87	8.15	0.74	0.42	0.01	0.00	0.49	0.50	0.44	0.68	4.88	0.69	0.68
p-median	0.45	44.10	0.69	2.99	1.69	12.49	7.11	0.74	0.34	0.02	0.00	0.49	0.70	0.54	0.64	8.57	0.67	0.67
p-centdian	0.45	42.14	0.73	4.28	1.69	12.79	7.64	0.74	0.39	0.01	0.00	0.49	0.50	0.44	0.71	4.85	0.69	0.68
MCP	0.45	28.98	3.34	2.14	0.98	16.57	16.50	0.73	0.54	0.00	0.00	0.49	0.00	0.17	0.68	6.73	0.71	0.70
BACOP1	0.45	27.27	3.53	2.97	1.79	16.10	16.20	0.72	0.48	0.00	0.00	0.49	0.00	0.08	0.74	10.52	0.76	0.77
BACOP2-0.01	0.45	35.51	0.78	2.42	0.49	21.58	22.22	0.72	0.56	0.00	0.00	0.48	0.10	0.10	0.83	0.65	0.78	0.77
BACOP2-0.25	0.45	35.51	0.78	2.42	0.49	20.91	21.63	0.72	0.56	0.00	0.00	0.48	0.10	0.10	0.83	0.68	0.78	0.77
BACOP2-0.50	0.45	34.82	0.74	5.35	0.77	20.29	20.87	0.72	0.58	0.00	0.00	0.48	0.10	0.10	0.81	0.28	0.77	0.77
BACOP2-0.75	0.45	34.82	0.74	5.35	0.77	20.29	20.87	0.72	0.58	0.00	0.00	0.48	0.10	0.10	0.81	0.28	0.77	0.77
BACOP2-0.99	0.45	33.23	0.65	5.35	0.77	20.93	21.51	0.72	0.59	0.00	0.00	0.48	0.00	0.00	0.81	0.25	0.77	0.77
p-center	0.60	37.85	0.76	3.77	1.42	7.50	3.41	0.58	0.27	0.01	0.00	0.32	0.40	0.46	0.53	8.36	0.51	0.50
p-median	0.60	37.66	0.78	1.98	0.81	7.29	2.96	0.58	0.19	0.01	0.00	0.32	0.40	0.46	0.53	9.71	0.50	0.50
p-centdian	0.60	37.93	0.77	3.77	1.42	7.49	3.11	0.58	0.22	0.01	0.00	0.32	0.40	0.46	0.53	8.36	0.51	0.50
MCP	0.60	27.80	2.85	1.01	0.85	9.92	9.76	0.57	0.38	0.00	0.00	0.32	0.00	0.15	0.53	6.27	0.52	0.52
BACOP1	0.60	24.93	3.60	1.54	0.92	9.54	9.57	0.55	0.38	0.00	0.00	0.32	0.00	0.00	0.57	4.02	0.59	0.59
BACOP2-0.01	0.60	33.94	1.16	1.00	0.22	14.22	15.09	0.55	0.43	0.00	0.00	0.31	0.00	0.00	0.62	1.27	0.63	0.63
BACOP2-0.25	0.60	33.94	1.16	1.00	0.22	14.22	15.09	0.55	0.43	0.00	0.00	0.31	0.00	0.00	0.62	1.27	0.63	0.63
BACOP2-0.50	0.60	33.36	0.92	1.38	0.24	14.22	15.02	0.54	0.42	0.00	0.00	0.31	0.00	0.00	0.64	2.25	0.64	0.63
BACOP2-0.75	0.60	33.36	0.92	1.38	0.24	14.22	15.02	0.54	0.42	0.00	0.00	0.31	0.00	0.00	0.64	2.25	0.64	0.63
BACOP2-0.99	0.60	33.46	0.99	1.36	0.21	14.31	15.13	0.54	0.42	0.00	0.00	0.31	0.00	0.00	0.64	2.18	0.64	0.63

Table A.8
Mono-polar, $\mu = 0.3$.

Model	γ	D_{max}	D^h_{max}	D_{min}	D^h_{min}	D_{avg}	D^h_{avg}	*Disp*	*Acc*	C	C^h	RC^h	C_{min}	C^h_{min}	C_{max}	C^h_{max}	C_{avg}	C^h_{avg}
p-center	0.15	100.60	1.38	61.25	2.38	50.40	45.96	0.96	0.81	0.02	0.01	0.84	1.00	0.71	0.85	1.86	0.92	0.92
p-median	0.15	144.34	1.40	43.69	2.03	41.80	36.11	0.97	0.76	0.03	0.02	0.84	1.00	0.29	0.82	1.83	0.89	0.89
p-centdian	0.15	96.74	1.09	50.18	2.52	47.15	42.62	0.96	0.79	0.02	0.01	0.84	0.90	1.45	0.85	2.85	0.92	0.92
MCP	0.15	63.28	0.74	25.05	1.38	61.20	62.44	0.96	0.84	0.00	0.00	0.84	0.00	2.59	0.85	1.39	0.94	0.94
BACOP1	0.15	62.78	3.73	37.01	1.16	54.28	55.45	0.96	0.82	0.00	0.00	0.84	0.00	1.08	0.85	1.12	0.93	0.92
BACOP2-0.01	0.15	229.90	0.88	16.24	0.30	76.43	76.09	0.97	0.86	0.03	0.03	0.83	0.95	0.24	0.92	0.92	0.92	0.92
BACOP2-0.25	0.15	227.81	0.94	13.15	0.34	74.92	74.59	0.97	0.85	0.03	0.02	0.83	0.95	0.24	0.92	0.98	0.92	0.92
BACOP2-0.50	0.15	140.84	0.81	20.49	0.53	73.34	71.83	0.97	0.84	0.02	0.02	0.83	0.95	0.51	0.92	1.40	0.92	0.92
BACOP2-0.75	0.15	100.20	0.59	20.49	0.55	70.53	69.43	0.96	0.84	0.01	0.00	0.83	0.75	0.60	0.92	1.43	0.92	0.92
BACOP2-0.99	0.15	80.29	1.08	23.78	0.62	70.20	69.55	0.96	0.84	0.00	0.00	0.83	0.25	0.49	0.93	1.15	0.93	0.92
p-center	0.30	81.70	1.00	13.33	0.55	24.26	18.92	0.87	0.61	0.01	0.00	0.67	0.60	0.44	0.71	1.32	0.80	0.80
p-median	0.30	86.30	1.01	29.24	1.81	22.36	15.76	0.88	0.54	0.01	0.00	0.67	0.80	0.46	0.71	1.09	0.78	0.78
p-centdian	0.30	81.63	1.01	13.96	1.59	23.75	17.52	0.87	0.54	0.01	0.00	0.67	0.60	0.44	0.73	1.53	0.81	0.80
MCP	0.30	58.02	1.01	8.40	1.01	31.95	33.28	0.87	0.69	0.00	0.00	0.67	0.00	0.15	0.66	2.46	0.80	0.80
BACOP1	0.30	53.87	0.42	17.34	1.19	35.36	36.75	0.86	0.69	0.00	0.00	0.67	0.00	0.00	0.76	1.32	0.84	0.84
BACOP2-0.01	0.30	65.44	0.90	7.57	0.62	39.85	40.10	0.87	0.70	0.00	0.00	0.66	0.00	0.45	0.84	0.96	0.84	0.83
BACOP2-0.25	0.30	65.44	0.90	7.57	0.62	39.85	40.10	0.87	0.70	0.00	0.00	0.66	0.00	0.45	0.84	0.96	0.84	0.83
BACOP2-0.50	0.30	65.27	0.96	7.57	0.62	39.94	40.21	0.87	0.71	0.00	0.00	0.66	0.00	0.45	0.84	0.96	0.84	0.84
BACOP2-0.75	0.30	65.27	0.96	7.57	0.62	39.94	40.21	0.87	0.71	0.00	0.00	0.66	0.00	0.45	0.84	0.96	0.84	0.84
BACOP2-0.99	0.30	65.27	1.29	7.69	0.67	40.38	40.66	0.87	0.72	0.00	0.00	0.66	0.00	0.45	0.84	0.99	0.84	0.84
p-center	0.45	42.26	0.73	4.28	1.69	12.87	8.15	0.74	0.42	0.00	0.00	0.49	0.20	0.20	0.62	0.92	0.66	0.66
p-median	0.45	44.10	0.69	2.99	1.69	12.49	7.11	0.74	0.34	0.00	0.00	0.49	0.20	0.20	0.59	0.66	0.65	0.65
p-centdian	0.45	42.14	0.73	4.28	1.69	12.79	7.64	0.74	0.39	0.00	0.00	0.49	0.20	0.20	0.62	0.92	0.66	0.66
MCP	0.45	51.12	3.28	6.93	0.18	18.41	19.21	0.75	0.52	0.00	0.00	0.49	0.00	0.10	0.52	2.30	0.63	0.63
BACOP1	0.45	42.25	2.86	1.50	1.07	17.72	17.64	0.74	0.53	0.00	0.00	0.49	0.00	0.00	0.60	1.25	0.66	0.66
BACOP2-0.01	0.45	59.87	0.92	6.00	0.51	25.81	26.02	0.74	0.57	0.00	0.00	0.49	0.00	0.00	0.66	0.41	0.67	0.66
BACOP2-0.25	0.45	59.87	0.92	6.00	0.51	25.81	26.02	0.74	0.57	0.00	0.00	0.49	0.00	0.00	0.66	0.41	0.67	0.66
BACOP2-0.50	0.45	59.70	0.98	6.00	0.51	25.87	26.03	0.74	0.57	0.00	0.00	0.49	0.00	0.00	0.66	0.41	0.67	0.66
BACOP2-0.75	0.45	59.70	0.98	6.00	0.51	25.87	26.03	0.74	0.57	0.00	0.00	0.49	0.00	0.00	0.66	0.41	0.67	0.66
BACOP2-0.99	0.45	60.97	0.86	6.03	0.56	25.13	25.73	0.74	0.57	0.00	0.00	0.49	0.00	0.45	0.66	0.50	0.67	0.67
p-center	0.60	37.85	0.76	3.77	1.42	7.50	3.41	0.58	0.27	0.00	0.00	0.32	0.10	0.10	0.46	0.84	0.48	0.48
p-median	0.60	37.66	0.78	1.98	0.81	7.29	2.96	0.58	0.19	0.00	0.00	0.32	0.10	0.11	0.48	0.92	0.48	0.48
p-centdian	0.60	37.93	0.77	3.77	1.42	7.49	3.11	0.58	0.22	0.00	0.00	0.32	0.10	0.10	0.46	0.60	0.48	0.48
MCP	0.60	49.24	2.97	0.68	0.08	12.57	13.10	0.59	0.37	0.00	0.00	0.32	0.00	0.04	0.34	1.55	0.45	0.45
BACOP1	0.60	39.92	1.51	1.50	0.19	10.97	11.00	0.58	0.42	0.00	0.00	0.32	0.00	0.00	0.48	0.23	0.48	0.48
BACOP2-0.01	0.60	55.60	1.17	0.51	0.49	15.40	15.51	0.58	0.42	0.00	0.00	0.32	0.00	0.00	0.50	0.45	0.49	0.49
BACOP2-0.25	0.60	55.60	1.17	0.51	0.49	15.40	15.51	0.58	0.42	0.00	0.00	0.32	0.00	0.00	0.50	0.45	0.49	0.49
BACOP2-0.50	0.60	55.43	1.23	0.51	0.49	15.40	15.51	0.58	0.42	0.00	0.00	0.32	0.00	0.00	0.50	0.45	0.49	0.49
BACOP2-0.75	0.60	55.43	1.23	0.51	0.49	15.40	15.51	0.58	0.42	0.00	0.00	0.32	0.00	0.00	0.50	0.45	0.49	0.49
BACOP2-0.99	0.60	54.78	1.15	5.96	0.44	15.42	15.09	0.58	0.41	0.00	0.00	0.32	0.00	0.00	0.50	0.52	0.49	0.49

Table A.9
Mono-polar, $\mu = 0.5$.

Model	γ	D_{max}	D^h_{max}	D_{min}	D^h_{min}	D_{avg}	D^h_{avg}	*Disp*	*Acc*	*C*	C^h	RC^h	C_{min}	C^h_{min}	C_{max}	C^h_{max}	C_{avg}	C^h_{avg}
p-center	0.15	100.60	1.38	61.25	2.38	50.40	45.96	0.96	0.81	0.00	0.00	0.84	0.30	1.13	0.84	3.54	0.91	0.91
p-median	0.15	144.34	1.40	43.69	2.03	41.80	36.11	0.97	0.76	0.01	0.00	0.84	0.60	1.27	0.82	3.45	0.89	0.89
p-centdian	0.15	96.74	1.09	50.18	2.52	47.15	42.62	0.96	0.79	0.00	0.00	0.84	0.20	1.13	0.84	3.14	0.91	0.91
MCP	0.15	92.09	3.74	51.16	2.11	71.03	70.36	0.96	0.82	0.00	0.00	0.84	0.00	1.30	0.82	2.54	0.94	0.94
BACOP1	0.15	89.39	0.41	40.10	1.80	85.26	87.59	0.96	0.84	0.00	0.00	0.84	0.00	0.74	0.85	3.51	0.94	0.94
BACOP2-0.01	0.15	105.58	1.28	10.14	0.61	98.50	97.86	0.96	0.86	0.00	0.00	0.84	0.07	0.09	0.93	12.91	0.94	0.94
BACOP2-0.25	0.15	105.58	1.28	10.14	0.61	98.50	97.86	0.96	0.86	0.00	0.00	0.84	0.07	0.09	0.93	12.91	0.94	0.94
BACOP2-0.50	0.15	106.82	0.93	10.60	0.62	94.86	94.67	0.96	0.85	0.00	0.00	0.84	0.07	0.05	0.93	9.53	0.94	0.94
BACOP2-0.75	0.15	106.42	0.89	10.60	0.62	96.68	96.24	0.96	0.86	0.00	0.00	0.84	0.07	0.09	0.93	8.83	0.94	0.94
BACOP2-0.99	0.15	107.02	1.31	10.60	0.62	99.59	98.91	0.96	0.86	0.00	0.00	0.84	0.07	0.13	0.93	8.95	0.94	0.94
p-center	0.30	81.70	1.00	13.33	0.55	24.26	18.92	0.87	0.61	0.00	0.00	0.67	0.10	0.01	0.70	4.19	0.78	0.78
p-median	0.30	86.30	1.01	29.24	1.81	22.36	15.76	0.88	0.54	0.00	0.00	0.67	0.10	0.99	0.72	1.50	0.76	0.76
p-centdian	0.30	81.63	1.01	13.96	1.59	23.75	17.52	0.87	0.54	0.00	0.00	0.67	0.10	0.01	0.70	4.64	0.78	0.78
MCP	0.30	76.46	1.91	5.03	0.32	34.88	34.48	0.88	0.64	0.00	0.00	0.67	0.00	1.11	0.69	3.50	0.76	0.76
BACOP1	0.30	69.62	0.92	38.78	0.94	30.38	29.78	0.87	0.66	0.00	0.00	0.67	0.00	1.03	0.68	4.31	0.78	0.78
BACOP2-0.01	0.30	69.58	1.09	10.50	0.27	39.67	39.78	0.87	0.73	0.00	0.00	0.68	0.05	0.10	0.79	11.64	0.79	0.79
BACOP2-0.25	0.30	69.58	1.09	10.50	0.27	39.67	39.78	0.87	0.73	0.00	0.00	0.68	0.05	0.10	0.79	11.64	0.79	0.79
BACOP2-0.50	0.30	70.73	1.09	7.09	0.47	39.18	39.14	0.87	0.70	0.00	0.00	0.68	0.05	0.11	0.79	5.44	0.78	0.78
BACOP2-0.75	0.30	69.57	1.14	7.21	0.31	39.11	39.23	0.87	0.71	0.00	0.00	0.68	0.05	0.10	0.79	6.73	0.78	0.78
BACOP2-0.99	0.30	73.42	0.83	11.49	0.46	41.98	41.57	0.87	0.71	0.00	0.00	0.68	0.07	0.06	0.78	6.38	0.78	0.78
p-center	0.45	42.26	0.73	4.28	1.69	12.87	8.15	0.74	0.42	0.00	0.00	0.50	0.00	0.00	0.57	3.59	0.63	0.63
p-median	0.45	44.10	0.69	2.99	1.69	12.49	7.11	0.74	0.34	0.00	0.00	0.50	0.00	0.00	0.56	3.54	0.63	0.63
p-centdian	0.45	42.14	0.73	4.28	1.69	12.79	7.64	0.74	0.39	0.00	0.00	0.50	0.00	0.00	0.57	3.53	0.63	0.63
MCP	0.45	72.45	1.60	0.56	0.08	23.40	23.40	0.76	0.53	0.00	0.00	0.50	0.00	2.06	0.49	2.76	0.59	0.59
BACOP1	0.45	62.00	0.91	12.61	0.72	19.19	19.32	0.75	0.52	0.00	0.00	0.50	0.00	0.98	0.54	3.85	0.60	0.60
BACOP2-0.01	0.45	62.71	1.59	3.91	0.53	23.39	23.47	0.75	0.56	0.00	0.00	0.51	0.03	0.06	0.61	8.83	0.62	0.62
BACOP2-0.25	0.45	62.71	1.59	3.91	0.53	23.39	23.47	0.75	0.56	0.00	0.00	0.51	0.03	0.06	0.61	8.83	0.62	0.62
BACOP2-0.50	0.45	65.12	1.39	3.82	0.68	23.62	23.79	0.75	0.54	0.00	0.00	0.51	0.03	0.07	0.61	6.73	0.61	0.61
BACOP2-0.75	0.45	64.49	1.38	3.79	0.48	23.54	23.71	0.75	0.54	0.00	0.00	0.51	0.03	0.06	0.61	5.86	0.61	0.61
BACOP2-0.99	0.45	71.48	1.48	1.39	0.20	25.41	25.53	0.75	0.58	0.00	0.00	0.51	0.05	0.06	0.61	5.29	0.61	0.61
p-center	0.60	37.85	0.76	3.77	1.42	7.50	3.41	0.58	0.27	0.00	0.00	0.34	0.00	0.00	0.42	3.31	0.46	0.46
p-median	0.60	37.66	0.78	1.98	0.81	7.29	2.96	0.58	0.19	0.00	0.00	0.34	0.00	0.00	0.42	3.95	0.46	0.46
p-centdian	0.60	37.93	0.77	3.77	1.42	7.49	3.11	0.58	0.22	0.00	0.00	0.34	0.00	0.00	0.42	3.25	0.46	0.46
MCP	0.60	67.44	0.77	0.56	0.08	14.85	14.39	0.61	0.38	0.00	0.00	0.34	0.00	1.02	0.33	2.21	0.43	0.43
BACOP1	0.60	49.89	0.71	1.09	0.95	12.10	11.54	0.60	0.37	0.00	0.00	0.34	0.00	0.00	0.42	3.01	0.44	0.44
BACOP2-0.01	0.60	46.35	1.39	3.03	0.40	13.74	13.12	0.59	0.38	0.00	0.00	0.34	0.03	0.00	0.45	9.23	0.45	0.45
BACOP2-0.25	0.60	46.35	1.39	3.03	0.40	13.74	13.12	0.59	0.38	0.00	0.00	0.34	0.03	0.00	0.45	9.23	0.45	0.45
BACOP2-0.50	0.60	46.90	1.38	3.95	0.61	14.00	13.99	0.59	0.41	0.00	0.00	0.34	0.03	0.02	0.46	1.62	0.46	0.46
BACOP2-0.75	0.60	43.67	1.43	3.92	0.41	13.71	13.61	0.59	0.40	0.00	0.00	0.34	0.03	0.01	0.46	5.09	0.46	0.46
BACOP2-0.99	0.60	57.33	1.87	1.49	0.21	16.37	16.22	0.60	0.41	0.00	0.00	0.34	0.05	0.06	0.44	0.75	0.44	0.44

Table A.10
Multi-polar, $\mu = 0.1$.

Model	γ	D_{max}	D^h_{max}	D_{min}	D^h_{min}	D_{avg}	D^h_{avg}	*Disp*	*Acc*	C	C^h	RC^h	C_{min}	C^h_{min}	C_{max}	C^h_{max}	C_{avg}	C^h_{avg}
p-center	0.15	47.58	0.72	99.26	0.50	36.21	31.82	0.98	0.84	0.04	0.03	0.83	1.00	0.35	0.78	1.78	0.86	0.86
p-median	0.15	49.63	0.66	34.92	1.53	32.76	28.86	0.98	0.77	0.05	0.04	0.83	1.00	3.27	0.78	6.97	0.85	0.85
p-centdian	0.15	48.57	0.72	57.67	1.36	34.21	30.16	0.98	0.80	0.04	0.02	0.83	1.00	1.28	0.78	1.90	0.87	0.86
MCP	0.15	39.97	0.76	57.93	1.08	38.88	38.69	0.98	0.87	0.00	0.00	0.83	0.00	4.46	0.78	11.91	0.87	0.87
BACOP1	0.15	39.87	0.71	150.53	0.71	37.06	36.24	0.98	0.87	0.00	0.00	0.83	0.00	4.54	0.82	10.35	0.84	0.83
BACOP2-0.01	0.15	101.94	19.10	26.02	0.70	41.10	37.21	0.98	0.86	0.15	0.13	0.77	1.00	1.96	0.84	1.05	0.83	0.82
BACOP2-0.25	0.15	81.93	18.99	12.70	0.46	37.51	33.56	0.98	0.84	0.12	0.10	0.77	1.00	1.96	0.86	0.69	0.83	0.82
BACOP2-0.50	0.15	48.16	20.23	12.68	0.70	29.14	28.67	0.98	0.85	0.06	0.06	0.77	1.00	2.33	0.90	1.02	0.83	0.82
BACOP2-0.75	0.15	47.78	24.45	4.04	0.61	27.99	27.43	0.98	0.85	0.04	0.03	0.77	1.00	1.39	0.89	1.12	0.83	0.83
BACOP2-0.99	0.15	31.36	2.59	4.81	0.60	26.10	25.57	0.98	0.85	0.00	0.00	0.77	0.52	2.65	0.89	1.14	0.84	0.84
p-center	0.30	38.65	0.66	7.62	0.97	21.21	16.18	0.91	0.61	0.01	0.00	0.68	0.40	0.43	0.68	7.42	0.74	0.73
p-median	0.30	40.49	1.10	2.37	0.77	19.78	14.11	0.91	0.51	0.01	0.00	0.68	0.50	0.64	0.73	3.61	0.73	0.72
p-centdian	0.30	38.60	0.94	8.72	1.12	20.90	15.04	0.91	0.54	0.01	0.00	0.68	0.40	0.66	0.73	10.17	0.74	0.74
MCP	0.30	39.14	1.89	8.31	0.44	25.83	25.92	0.91	0.68	0.00	0.00	0.68	0.00	0.67	0.58	7.00	0.74	0.74
BACOP1	0.30	36.57	1.86	86.66	0.67	24.69	24.70	0.91	0.73	0.00	0.00	0.68	0.00	1.29	0.74	1.52	0.74	0.74
BACOP2-0.01	0.30	28.12	3.44	16.67	0.67	18.15	17.96	0.90	0.71	0.00	0.00	0.62	0.13	0.74	0.78	0.33	0.76	0.76
BACOP2-0.25	0.30	28.16	3.44	16.67	0.67	18.33	18.13	0.90	0.71	0.00	0.00	0.62	0.13	0.74	0.78	0.33	0.76	0.76
BACOP2-0.50	0.30	28.08	3.41	16.67	0.67	18.31	18.09	0.90	0.70	0.00	0.00	0.62	0.13	0.86	0.78	0.48	0.76	0.76
BACOP2-0.75	0.30	27.57	3.45	16.67	0.67	18.12	18.02	0.90	0.71	0.00	0.00	0.62	0.03	0.86	0.78	0.48	0.76	0.76
BACOP2-0.99	0.30	27.37	3.46	16.70	0.67	18.07	18.04	0.90	0.71	0.00	0.00	0.62	0.03	0.88	0.78	0.39	0.76	0.76
p-center	0.45	34.22	0.65	2.60	0.78	13.51	7.96	0.80	0.41	0.00	0.00	0.45	0.10	0.54	0.59	11.61	0.60	0.58
p-median	0.45	33.46	0.71	2.11	0.74	12.84	7.03	0.80	0.36	0.00	0.00	0.45	0.00	0.68	0.58	0.25	0.59	0.58
p-centdian	0.45	34.54	0.72	2.11	0.74	13.50	7.28	0.80	0.35	0.00	0.00	0.45	0.20	0.71	0.58	11.61	0.60	0.59
MCP	0.45	36.31	2.19	6.12	0.79	17.47	17.83	0.81	0.59	0.00	0.00	0.45	0.00	3.16	0.49	0.94	0.58	0.58
BACOP1	0.45	34.50	1.53	3.89	0.21	17.74	17.86	0.80	0.58	0.00	0.00	0.45	0.00	0.85	0.58	7.03	0.61	0.61
BACOP2-0.01	0.45	25.68	18.81	12.63	0.47	12.79	12.80	0.79	0.59	0.00	0.00	0.42	0.03	1.56	0.56	0.90	0.65	0.65
BACOP2-0.25	0.45	25.68	18.81	12.63	0.47	12.82	12.84	0.79	0.58	0.00	0.00	0.42	0.03	1.56	0.56	0.88	0.65	0.65
BACOP2-0.50	0.45	24.78	18.83	12.58	0.52	12.72	12.63	0.78	0.57	0.00	0.00	0.42	0.03	1.55	0.56	0.96	0.65	0.65
BACOP2-0.75	0.45	24.78	18.83	12.58	0.52	12.72	12.63	0.78	0.57	0.00	0.00	0.42	0.03	1.55	0.56	0.96	0.65	0.65
BACOP2-0.99	0.45	25.16	18.54	12.61	0.52	12.43	12.49	0.78	0.58	0.00	0.00	0.42	0.03	1.68	0.54	1.03	0.64	0.65
p-center	0.60	29.34	0.84	1.50	0.69	8.00	3.20	0.63	0.22	0.00	0.00	0.30	0.00	0.65	0.44	0.73	0.44	0.42
p-median	0.60	29.56	0.76	6.08	0.80	7.83	2.84	0.63	0.19	0.00	0.00	0.30	0.00	0.57	0.39	0.25	0.43	0.42
p-centdian	0.60	29.69	0.82	6.05	0.82	7.95	2.94	0.63	0.21	0.00	0.00	0.30	0.00	0.65	0.44	0.73	0.43	0.42
MCP	0.60	35.04	1.82	4.75	0.76	10.67	10.97	0.64	0.39	0.00	0.00	0.30	0.00	1.17	0.44	3.57	0.42	0.41
BACOP1	0.60	32.67	0.45	0.00	0.56	11.92	11.86	0.64	0.42	0.00	0.00	0.30	0.00	3.75	0.44	6.96	0.45	0.45
BACOP2-0.01	0.60	23.09	18.66	12.21	0.37	9.08	9.19	0.62	0.45	0.00	0.00	0.28	0.03	0.43	0.37	0.70	0.49	0.50
BACOP2-0.25	0.60	23.09	18.66	12.21	0.37	9.06	9.18	0.62	0.45	0.00	0.00	0.28	0.03	0.43	0.37	0.70	0.49	0.50
BACOP2-0.50	0.60	23.09	18.66	12.21	0.37	8.81	8.82	0.62	0.44	0.00	0.00	0.28	0.03	0.64	0.40	0.69	0.49	0.50
BACOP2-0.75	0.60	23.09	18.66	12.21	0.37	8.81	8.82	0.62	0.44	0.00	0.00	0.28	0.03	0.64	0.40	0.69	0.49	0.50
BACOP2-0.99	0.60	24.24	10.67	12.24	0.38	8.52	8.73	0.63	0.43	0.00	0.00	0.28	0.03	0.44	0.38	0.42	0.47	0.48

Table A.11
Multi-polar, $\mu = 0.3$.

Model	γ	D_{max}	D^h_{max}	D_{min}	D^h_{min}	D_{avg}	D^h_{avg}	*Disp*	*Acc*	*C*	C^h	RC^h	C_{min}	C^h_{min}	C_{max}	C^h_{max}	C_{avg}	C^h_{avg}
p-center	0.15	47.58	0.72	99.26	0.50	36.21	31.82	0.98	0.84	0.00	0.00	0.81	0.00	2.26	0.78	0.51	0.86	0.86
p-median	0.15	49.63	0.66	34.92	1.53	32.76	28.86	0.98	0.77	0.00	0.00	0.81	0.00	1.85	0.78	0.57	0.86	0.86
p-centdian	0.15	48.57	0.72	57.67	1.36	34.21	30.16	0.98	0.80	0.00	0.00	0.81	0.00	1.43	0.78	0.56	0.86	0.86
MCP	0.15	77.71	1.45	19.85	0.78	46.37	47.35	0.98	0.87	0.00	0.00	0.81	0.00	1.74	0.76	1.10	0.84	0.84
BACOP1	0.15	94.81	2.13	35.85	0.50	67.56	65.37	0.97	0.83	0.00	0.00	0.81	0.00	1.71	0.78	0.85	0.90	0.90
BACOP2-0.01	0.15	72.88	25.43	17.72	0.62	51.22	52.04	0.97	0.86	0.00	0.00	0.79	0.80	2.97	0.90	1.18	0.90	0.90
BACOP2-0.25	0.15	72.88	25.43	17.72	0.62	51.22	52.04	0.97	0.86	0.00	0.00	0.79	0.80	2.97	0.90	1.18	0.90	0.90
BACOP2-0.50	0.15	72.88	25.43	14.44	0.61	51.14	51.95	0.97	0.86	0.00	0.00	0.79	0.80	2.97	0.89	1.17	0.90	0.90
BACOP2-0.75	0.15	72.88	25.43	14.44	0.61	51.14	51.95	0.97	0.86	0.00	0.00	0.79	0.80	2.97	0.89	1.17	0.90	0.90
BACOP2-0.99	0.15	72.88	25.43	14.44	0.61	50.88	51.79	0.97	0.86	0.00	0.00	0.79	0.80	2.93	0.89	1.17	0.90	0.90
p-center	0.30	38.65	0.66	7.62	0.97	21.21	16.18	0.91	0.61	0.00	0.00	0.65	0.00	0.41	0.67	1.51	0.72	0.72
p-median	0.30	40.49	1.10	2.37	0.77	19.78	14.11	0.91	0.51	0.00	0.00	0.65	0.00	0.54	0.69	1.48	0.72	0.72
p-centdian	0.30	38.60	0.94	8.72	1.12	20.90	15.04	0.91	0.54	0.00	0.00	0.65	0.00	0.55	0.67	0.82	0.72	0.72
MCP	0.30	51.90	1.67	11.24	0.52	25.31	25.37	0.91	0.74	0.00	0.00	0.65	0.00	1.84	0.67	0.52	0.70	0.71
BACOP1	0.30	51.90	1.67	11.24	0.52	25.31	25.37	0.91	0.74	0.00	0.00	0.65	0.00	1.84	0.67	0.52	0.70	0.71
BACOP2-0.01	0.30	53.27	20.73	2.62	0.85	28.59	28.84	0.90	0.73	0.00	0.00	0.63	0.78	3.06	0.77	1.17	0.79	0.80
BACOP2-0.25	0.30	53.27	20.73	2.62	0.85	28.59	28.84	0.90	0.73	0.00	0.00	0.63	0.78	3.06	0.77	1.17	0.79	0.80
BACOP2-0.50	0.30	53.22	20.62	2.62	0.85	28.39	28.57	0.90	0.73	0.00	0.00	0.63	0.78	3.06	0.78	1.17	0.79	0.80
BACOP2-0.75	0.30	53.22	20.62	2.62	0.85	28.39	28.57	0.90	0.73	0.00	0.00	0.63	0.78	3.06	0.78	1.17	0.79	0.80
BACOP2-0.99	0.30	49.93	18.82	16.17	0.46	28.98	29.82	0.89	0.72	0.00	0.00	0.63	0.78	3.54	0.78	1.20	0.80	0.81
p-center	0.45	34.22	0.65	2.60	0.78	13.51	7.96	0.80	0.41	0.00	0.00	0.45	0.00	1.89	0.52	0.31	0.58	0.58
p-median	0.45	33.46	0.71	2.11	0.74	12.84	7.03	0.80	0.36	0.00	0.00	0.45	0.00	1.07	0.55	0.14	0.57	0.57
p-centdian	0.45	34.54	0.72	2.11	0.74	13.50	7.28	0.80	0.35	0.00	0.00	0.45	0.00	1.83	0.52	0.44	0.58	0.58
MCP	0.45	39.47	2.35	9.89	0.42	16.32	16.70	0.81	0.58	0.00	0.00	0.45	0.00	0.51	0.51	0.32	0.56	0.56
BACOP1	0.45	39.47	2.35	9.89	0.42	16.32	16.70	0.81	0.58	0.00	0.00	0.45	0.00	0.51	0.51	0.32	0.56	0.56
BACOP2-0.01	0.45	44.69	25.80	1.21	0.53	16.36	16.59	0.78	0.57	0.00	0.00	0.43	0.72	3.06	0.62	1.15	0.62	0.62
BACOP2-0.25	0.45	44.69	25.80	1.21	0.53	16.36	16.59	0.78	0.57	0.00	0.00	0.43	0.72	3.06	0.62	1.15	0.62	0.62
BACOP2-0.50	0.45	43.10	25.72	1.21	0.53	16.26	16.41	0.78	0.57	0.00	0.00	0.43	0.71	3.06	0.62	1.15	0.62	0.62
BACOP2-0.75	0.45	43.10	25.72	1.21	0.53	16.26	16.41	0.78	0.57	0.00	0.00	0.43	0.71	3.06	0.62	1.15	0.62	0.62
BACOP2-0.99	0.45	35.56	23.76	9.83	0.32	14.33	14.82	0.77	0.57	0.00	0.00	0.43	0.61	1.04	0.65	0.31	0.63	0.64
p-center	0.60	29.34	0.84	1.50	0.69	8.00	3.20	0.63	0.22	0.00	0.00	0.31	0.00	1.23	0.42	1.03	0.41	0.41
p-median	0.60	29.56	0.76	6.08	0.80	7.83	2.84	0.63	0.19	0.00	0.00	0.31	0.00	1.22	0.42	0.94	0.41	0.41
p-centdian	0.60	29.69	0.82	6.05	0.82	7.95	2.94	0.63	0.21	0.00	0.00	0.31	0.00	1.23	0.42	0.94	0.41	0.41
MCP	0.60	36.49	1.75	4.52	0.21	10.46	10.87	0.64	0.41	0.00	0.00	0.31	0.00	1.99	0.38	0.25	0.41	0.41
BACOP1	0.60	36.49	1.75	4.52	0.21	10.46	10.87	0.64	0.41	0.00	0.00	0.31	0.00	1.99	0.38	0.25	0.41	0.41
BACOP2-0.01	0.60	37.10	9.68	0.83	0.52	9.45	9.51	0.62	0.41	0.00	0.00	0.28	0.52	2.74	0.46	1.09	0.45	0.45
BACOP2-0.25	0.60	37.10	9.68	0.83	0.52	9.45	9.51	0.62	0.41	0.00	0.00	0.28	0.52	2.74	0.46	1.09	0.45	0.45
BACOP2-0.50	0.60	35.17	9.59	0.83	0.52	9.33	9.31	0.62	0.41	0.00	0.00	0.28	0.49	2.69	0.46	1.09	0.45	0.45
BACOP2-0.75	0.60	35.17	9.59	0.83	0.52	9.33	9.31	0.62	0.41	0.00	0.00	0.28	0.49	2.69	0.46	1.09	0.45	0.45
BACOP2-0.99	0.60	32.06	23.91	5.81	0.22	8.16	8.75	0.61	0.43	0.00	0.00	0.28	0.42	1.06	0.47	0.19	0.45	0.46

Table A.12
Multi-polar, $\mu = 0.5$.

Model	γ	D_{max}	D^h_{max}	D_{min}	D^h_{min}	D_{avg}	D^h_{avg}	*Disp*	*Acc*	C	C^h	RC^h	C_{min}	C^h_{min}	C_{max}	C^h_{max}	C_{avg}	C^h_{avg}
p-center	0.15	47.58	0.72	99.26	0.50	36.21	31.82	0.98	0.84	0.00	0.00	0.79	0.00	0.36	0.77	1.61	0.86	0.86
p-median	0.15	49.63	0.66	34.92	1.53	32.76	28.86	0.98	0.77	0.00	0.00	0.79	0.00	0.27	0.77	1.91	0.86	0.86
p-centdian	0.15	48.57	0.72	57.67	1.36	34.21	30.16	0.98	0.80	0.00	0.00	0.79	0.00	0.21	0.76	0.70	0.86	0.86
MCP	0.15	77.71	1.45	19.85	0.78	46.37	47.35	0.98	0.87	0.00	0.00	0.79	0.00	0.44	0.79	1.52	0.85	0.85
BACOP1	0.15	129.80	0.75	19.99	0.74	108.94	109.64	0.98	0.87	0.00	0.00	0.79	0.00	0.44	0.82	1.79	0.90	0.90
BACOP2-0.01	0.15	110.77	13.33	24.03	0.31	91.26	91.03	0.97	0.84	0.00	0.00	0.79	0.84	0.69	0.88	6.03	0.92	0.92
BACOP2-0.25	0.15	112.02	13.29	24.27	0.34	97.53	98.09	0.97	0.85	0.00	0.00	0.79	0.84	0.66	0.88	6.03	0.92	0.92
BACOP2-0.50	0.15	112.51	13.23	24.27	0.34	98.53	99.11	0.97	0.84	0.00	0.00	0.79	0.84	0.67	0.88	6.00	0.91	0.92
BACOP2-0.75	0.15	112.51	13.23	24.27	0.34	98.53	99.11	0.97	0.84	0.00	0.00	0.79	0.84	0.67	0.88	6.00	0.91	0.92
BACOP2-0.99	0.15	111.18	13.14	27.39	0.31	94.72	95.22	0.97	0.85	0.00	0.00	0.79	0.85	0.75	0.88	4.97	0.92	0.92
p-center	0.30	38.65	0.66	7.62	0.97	21.21	16.18	0.91	0.61	0.00	0.00	0.63	0.00	0.26	0.64	0.83	0.72	0.72
p-median	0.30	40.49	1.10	2.37	0.77	19.78	14.11	0.91	0.51	0.00	0.00	0.63	0.00	0.13	0.65	0.75	0.72	0.72
p-centdian	0.30	38.60	0.94	8.72	1.12	20.90	15.04	0.91	0.54	0.00	0.00	0.63	0.00	0.18	0.64	0.83	0.72	0.72
MCP	0.30	51.90	1.67	11.24	0.52	25.31	25.37	0.91	0.74	0.00	0.00	0.63	0.00	0.22	0.65	1.10	0.71	0.71
BACOP1	0.30	51.90	1.67	11.24	0.52	25.31	25.37	0.91	0.74	0.00	0.00	0.63	0.00	0.22	0.64	1.10	0.71	0.71
BACOP2-0.01	0.30	74.66	26.52	11.60	0.28	40.47	40.19	0.89	0.72	0.00	0.00	0.62	0.66	0.61	0.77	8.26	0.79	0.79
BACOP2-0.25	0.30	65.02	26.49	11.64	0.24	32.69	32.97	0.89	0.72	0.00	0.00	0.62	0.64	0.59	0.77	8.36	0.78	0.79
BACOP2-0.50	0.30	63.36	26.62	11.24	0.22	34.37	35.40	0.89	0.72	0.00	0.00	0.62	0.65	0.55	0.77	8.31	0.79	0.79
BACOP2-0.75	0.30	63.36	26.62	11.24	0.22	34.37	35.40	0.89	0.72	0.00	0.00	0.62	0.65	0.55	0.77	8.31	0.79	0.79
BACOP2-0.99	0.30	81.09	14.88	13.71	0.20	38.21	37.61	0.89	0.71	0.00	0.00	0.62	0.63	0.44	0.77	4.26	0.79	0.79
p-center	0.45	34.22	0.65	2.60	0.78	13.51	7.96	0.80	0.41	0.00	0.00	0.45	0.00	0.10	0.51	0.56	0.58	0.58
p-median	0.45	33.46	0.71	2.11	0.74	12.84	7.03	0.80	0.36	0.00	0.00	0.45	0.00	0.12	0.50	0.39	0.57	0.57
p-centdian	0.45	34.54	0.72	2.11	0.74	13.50	7.28	0.80	0.35	0.00	0.00	0.45	0.00	0.12	0.50	0.59	0.58	0.58
MCP	0.45	39.47	2.35	9.89	0.42	16.32	16.70	0.81	0.58	0.00	0.00	0.45	0.00	0.15	0.51	1.04	0.56	0.56
BACOP1	0.45	39.47	2.35	9.89	0.42	16.32	16.70	0.81	0.58	0.00	0.00	0.45	0.00	0.15	0.50	1.04	0.56	0.56
BACOP2-0.01	0.45	51.04	3.40	8.94	0.29	23.03	22.47	0.77	0.57	0.00	0.00	0.43	0.44	0.45	0.60	2.77	0.62	0.62
BACOP2-0.25	0.45	42.64	3.30	5.08	0.33	15.57	15.84	0.78	0.58	0.00	0.00	0.43	0.42	0.44	0.59	2.89	0.61	0.61
BACOP2-0.50	0.45	42.22	3.26	4.72	0.27	16.19	16.64	0.78	0.58	0.00	0.00	0.43	0.43	0.39	0.60	3.09	0.61	0.61
BACOP2-0.75	0.45	42.22	3.26	4.72	0.27	16.19	16.64	0.78	0.58	0.00	0.00	0.43	0.43	0.39	0.60	3.09	0.61	0.61
BACOP2-0.99	0.45	45.12	9.73	4.10	0.31	17.49	17.41	0.77	0.55	0.00	0.00	0.43	0.35	0.42	0.63	5.72	0.63	0.63
p-center	0.60	29.34	0.84	1.50	0.69	8.00	3.20	0.63	0.22	0.00	0.00	0.29	0.00	0.12	0.35	0.59	0.42	0.41
p-median	0.60	29.56	0.76	6.08	0.80	7.83	2.84	0.63	0.19	0.00	0.00	0.29	0.00	0.12	0.36	0.70	0.42	0.41
p-centdian	0.60	29.69	0.82	6.05	0.82	7.95	2.94	0.63	0.21	0.00	0.00	0.29	0.00	0.12	0.35	0.63	0.42	0.41
MCP	0.60	36.49	1.75	4.52	0.21	10.46	10.87	0.64	0.41	0.00	0.00	0.29	0.00	0.29	0.37	0.98	0.41	0.41
BACOP1	0.60	36.49	1.75	4.52	0.21	10.46	10.87	0.64	0.41	0.00	0.00	0.29	0.00	0.29	0.37	0.98	0.41	0.41
BACOP2-0.01	0.60	38.24	18.04	2.14	0.30	9.90	9.96	0.61	0.42	0.00	0.00	0.27	0.26	0.31	0.45	7.31	0.45	0.45
BACOP2-0.25	0.60	35.42	18.05	1.78	0.22	9.38	9.31	0.62	0.43	0.00	0.00	0.27	0.28	0.25	0.43	7.34	0.44	0.44
BACOP2-0.50	0.60	35.85	18.01	2.04	0.23	9.37	9.77	0.62	0.43	0.00	0.00	0.27	0.29	0.35	0.44	8.14	0.44	0.45
BACOP2-0.75	0.60	35.85	18.01	2.04	0.23	9.37	9.77	0.62	0.43	0.00	0.00	0.27	0.29	0.35	0.44	8.14	0.44	0.45
BACOP2-0.99	0.60	34.07	7.26	1.42	0.19	9.24	9.18	0.61	0.42	0.00	0.00	0.27	0.22	0.37	0.46	5.69	0.46	0.46

Table A.13
Uniform, $\mu = 0.1$.

Model	γ	D_{max}	D^h_{max}	D_{min}	D^h_{min}	D_{avg}	D^h_{avg}	*Disp*	*Acc*	*C*	C^h	RC^h	C_{min}	C^h_{min}	C_{max}	C^h_{max}	C_{avg}	C^h_{avg}
p-center	0.15	29.15	0.63	2.79	2.41	22.64	20.27	0.98	0.80	0.05	0.03	0.85	1.00	1.40	0.74	2.13	0.86	0.86
p-median	0.15	28.28	2.00	13.23	3.16	20.30	17.53	0.98	0.78	0.04	0.02	0.85	1.00	1.58	0.74	2.54	0.86	0.85
p-centdian	0.15	29.45	1.17	8.19	2.15	21.64	18.97	0.98	0.80	0.04	0.02	0.85	1.00	2.31	0.74	2.66	0.86	0.86
MCP	0.15	22.59	1.27	5.37	0.39	23.22	23.57	0.98	0.85	0.00	0.00	0.85	0.00	3.58	0.80	1.26	0.87	0.87
BACOP1	0.15	22.60	1.24	4.97	1.90	23.28	23.55	0.98	0.84	0.00	0.00	0.85	0.00	5.12	0.74	0.73	0.83	0.83
BACOP2-0.01	0.15	71.91	1.90	19.26	0.74	41.82	41.01	0.98	0.86	0.13	0.12	0.84	1.00	1.02	0.81	1.66	0.82	0.82
BACOP2-0.25	0.15	68.06	2.88	17.32	0.55	41.18	40.05	0.98	0.87	0.12	0.10	0.84	1.00	0.98	0.81	1.65	0.82	0.82
BACOP2-0.50	0.15	65.75	2.96	16.56	0.49	39.72	38.93	0.98	0.87	0.08	0.07	0.84	1.00	0.60	0.82	2.07	0.82	0.82
BACOP2-0.75	0.15	48.09	1.12	15.22	0.33	36.79	35.63	0.98	0.86	0.03	0.01	0.84	1.00	0.41	0.83	1.70	0.83	0.82
BACOP2-0.99	0.15	36.17	0.73	8.79	0.39	34.96	34.49	0.98	0.85	0.00	0.00	0.84	0.51	0.96	0.83	0.91	0.84	0.83
p-center	0.30	23.63	0.91	3.59	2.53	13.03	9.48	0.91	0.62	0.01	0.00	0.70	0.50	2.41	0.55	2.55	0.72	0.71
p-median	0.30	22.66	0.85	1.81	2.92	11.78	8.44	0.91	0.55	0.01	0.00	0.70	0.50	2.31	0.61	3.34	0.71	0.71
p-centdian	0.30	23.29	0.91	1.75	2.98	12.54	8.94	0.91	0.57	0.00	0.00	0.70	0.40	2.30	0.55	2.02	0.72	0.71
MCP	0.30	22.23	0.67	1.89	0.23	16.43	16.17	0.91	0.70	0.00	0.00	0.70	0.00	3.80	0.37	1.46	0.73	0.73
BACOP1	0.30	20.80	0.78	5.03	1.35	15.27	15.14	0.90	0.72	0.00	0.00	0.70	0.00	4.32	0.62	2.17	0.74	0.74
BACOP2-0.01	0.30	34.36	2.71	9.28	0.85	23.39	22.78	0.90	0.70	0.00	0.00	0.69	0.22	0.78	0.76	1.32	0.75	0.76
BACOP2-0.25	0.30	34.36	2.71	9.28	0.85	23.39	22.78	0.90	0.70	0.00	0.00	0.69	0.22	0.78	0.76	1.32	0.75	0.76
BACOP2-0.50	0.30	34.17	2.54	10.91	1.45	23.12	22.20	0.90	0.70	0.00	0.00	0.69	0.22	0.77	0.76	1.57	0.75	0.75
BACOP2-0.75	0.30	34.17	2.54	10.91	1.45	23.12	22.20	0.90	0.70	0.00	0.00	0.69	0.22	0.77	0.76	1.57	0.75	0.75
BACOP2-0.99	0.30	34.17	2.54	10.91	1.45	23.07	22.15	0.90	0.70	0.00	0.00	0.69	0.22	0.84	0.76	1.57	0.75	0.75
p-center	0.45	19.66	0.94	1.80	2.77	8.02	4.76	0.79	0.40	0.00	0.00	0.55	0.30	1.65	0.49	1.84	0.57	0.56
p-median	0.45	18.79	0.66	1.53	2.76	7.46	4.16	0.79	0.34	0.00	0.00	0.55	0.30	1.52	0.43	2.52	0.57	0.56
p-centdian	0.45	18.79	0.66	1.53	2.76	7.68	4.27	0.79	0.34	0.00	0.00	0.55	0.30	1.65	0.43	1.87	0.57	0.56
MCP	0.45	20.76	2.54	1.91	1.68	10.73	11.02	0.80	0.55	0.00	0.00	0.55	0.00	2.17	0.00	1.57	0.56	0.57
BACOP1	0.45	19.37	0.89	2.05	1.19	10.06	9.88	0.79	0.54	0.00	0.00	0.55	0.00	4.37	0.18	0.89	0.58	0.58
BACOP2-0.01	0.45	35.06	2.73	6.57	0.41	18.42	18.99	0.78	0.55	0.00	0.00	0.55	0.22	0.95	0.57	1.91	0.64	0.64
BACOP2-0.25	0.45	35.06	2.73	6.57	0.41	18.42	18.99	0.78	0.55	0.00	0.00	0.55	0.22	0.95	0.57	1.91	0.64	0.64
BACOP2-0.50	0.45	34.25	2.28	6.27	0.05	18.01	18.50	0.78	0.58	0.00	0.00	0.55	0.22	1.06	0.57	1.97	0.64	0.64
BACOP2-0.75	0.45	34.25	2.28	6.27	0.05	18.01	18.50	0.78	0.58	0.00	0.00	0.55	0.22	1.06	0.57	1.97	0.64	0.64
BACOP2-0.99	0.45	33.79	2.41	6.25	0.04	17.43	18.12	0.78	0.55	0.00	0.00	0.55	0.27	0.86	0.57	1.35	0.63	0.64
p-center	0.60	15.74	0.71	0.94	2.13	4.78	2.06	0.63	0.23	0.00	0.00	0.39	0.00	3.64	0.31	1.62	0.41	0.40
p-median	0.60	15.50	0.74	0.83	2.25	4.56	1.92	0.63	0.18	0.00	0.00	0.39	0.00	3.74	0.25	1.07	0.41	0.40
p-centdian	0.60	15.56	0.74	0.89	1.86	4.73	1.99	0.63	0.21	0.00	0.00	0.39	0.00	3.71	0.31	0.98	0.41	0.40
MCP	0.60	18.97	1.93	1.16	0.64	6.51	6.21	0.64	0.39	0.00	0.00	0.39	0.00	3.85	0.25	0.40	0.40	0.40
BACOP1	0.60	17.02	1.95	0.65	0.63	6.51	6.40	0.63	0.39	0.00	0.00	0.39	0.00	5.42	0.31	0.55	0.42	0.42
BACOP2-0.01	0.60	33.14	2.92	5.69	0.07	13.17	13.49	0.62	0.41	0.00	0.00	0.40	0.22	1.15	0.39	1.40	0.48	0.49
BACOP2-0.25	0.60	33.14	2.92	5.69	0.07	13.17	13.49	0.62	0.41	0.00	0.00	0.40	0.22	1.15	0.39	1.40	0.48	0.49
BACOP2-0.50	0.60	32.48	2.82	2.81	0.34	12.57	13.11	0.62	0.43	0.00	0.00	0.40	0.22	1.22	0.37	1.26	0.48	0.49
BACOP2-0.75	0.60	32.48	2.82	2.81	0.34	12.57	13.11	0.62	0.43	0.00	0.00	0.40	0.22	1.22	0.37	1.26	0.48	0.49
BACOP2-0.99	0.60	29.24	1.98	0.73	0.16	10.74	10.75	0.63	0.38	0.00	0.00	0.40	0.27	0.94	0.37	1.68	0.45	0.46

Table A.14
Uniform, $\mu = 0.3$.

Model	γ	D_{max}	D^h_{max}	D_{min}	D^h_{min}	D_{avg}	D^h_{avg}	*Disp*	*Acc*	*C*	C^h	RC^h	C_{min}	C^h_{min}	C_{max}	C^h_{max}	C_{avg}	C^h_{avg}
p-center	0.15	29.15	0.63	2.79	2.41	22.64	20.27	0.98	0.80	0.00	0.00	0.85	0.00	1.07	0.79	1.53	0.86	0.86
p-median	0.15	28.28	2.00	13.23	3.16	20.30	17.53	0.98	0.78	0.00	0.00	0.85	0.00	1.18	0.77	0.51	0.85	0.85
p-centdian	0.15	29.45	1.17	8.19	2.15	21.64	18.97	0.98	0.80	0.00	0.00	0.85	0.00	0.79	0.78	1.36	0.86	0.86
MCP	0.15	40.56	2.34	3.94	1.46	32.55	32.90	0.97	0.82	0.00	0.00	0.85	0.00	1.33	0.73	0.97	0.88	0.88
BACOP1	0.15	39.58	0.88	6.90	1.03	31.50	31.54	0.97	0.82	0.00	0.00	0.85	0.00	1.92	0.81	1.50	0.88	0.88
BACOP2-0.01	0.15	65.47	3.41	37.52	1.73	53.89	52.44	0.97	0.83	0.00	0.00	0.84	0.80	0.82	0.88	3.52	0.89	0.89
BACOP2-0.25	0.15	65.47	3.41	37.52	1.73	53.89	52.44	0.97	0.83	0.00	0.00	0.84	0.80	0.82	0.88	3.52	0.89	0.89
BACOP2-0.50	0.15	61.74	3.20	36.14	1.66	51.46	49.46	0.97	0.83	0.00	0.00	0.84	0.80	1.04	0.88	1.58	0.89	0.89
BACOP2-0.75	0.15	62.54	3.23	36.14	1.66	51.57	49.81	0.97	0.84	0.00	0.00	0.84	0.80	1.04	0.88	1.57	0.89	0.89
BACOP2-0.99	0.15	63.91	3.26	36.70	1.72	52.05	50.09	0.97	0.84	0.00	0.00	0.84	0.80	0.98	0.88	2.45	0.89	0.89
p-center	0.30	23.63	0.91	3.59	2.53	13.03	9.48	0.91	0.62	0.00	0.00	0.70	0.00	0.58	0.66	1.27	0.71	0.71
p-median	0.30	22.66	0.85	1.81	2.92	11.78	8.44	0.91	0.55	0.00	0.00	0.70	0.00	0.36	0.66	1.21	0.71	0.71
p-centdian	0.30	23.29	0.91	1.75	2.98	12.54	8.94	0.91	0.57	0.00	0.00	0.70	0.00	0.39	0.66	1.60	0.71	0.70
MCP	0.30	30.11	1.32	1.97	0.95	16.46	16.39	0.91	0.68	0.00	0.00	0.70	0.00	0.77	0.62	0.61	0.70	0.70
BACOP1	0.30	30.50	1.04	1.43	1.92	18.67	18.97	0.91	0.67	0.00	0.00	0.70	0.00	0.75	0.62	0.75	0.72	0.72
BACOP2-0.01	0.30	49.75	3.01	13.33	1.21	31.48	30.76	0.90	0.68	0.00	0.00	0.70	0.71	1.03	0.73	3.32	0.75	0.75
BACOP2-0.25	0.30	49.75	3.01	13.33	1.21	31.48	30.76	0.90	0.68	0.00	0.00	0.70	0.71	1.03	0.73	3.32	0.75	0.75
BACOP2-0.50	0.30	50.22	2.96	13.16	1.20	31.26	30.59	0.90	0.67	0.00	0.00	0.70	0.72	1.25	0.73	3.32	0.75	0.75
BACOP2-0.75	0.30	49.93	2.96	13.16	1.20	31.05	30.51	0.90	0.67	0.00	0.00	0.70	0.72	1.22	0.72	3.32	0.75	0.75
BACOP2-0.99	0.30	42.43	2.33	6.48	0.91	27.47	26.15	0.90	0.70	0.00	0.00	0.70	0.73	0.66	0.72	1.43	0.74	0.74
p-center	0.45	19.66	0.94	1.80	2.77	8.02	4.76	0.79	0.40	0.00	0.00	0.55	0.00	0.20	0.52	1.04	0.56	0.56
p-median	0.45	18.79	0.66	1.53	2.76	7.46	4.16	0.79	0.34	0.00	0.00	0.55	0.00	0.32	0.51	0.98	0.56	0.56
p-centdian	0.45	18.79	0.66	1.53	2.76	7.68	4.27	0.79	0.34	0.00	0.00	0.55	0.00	0.18	0.51	0.88	0.56	0.56
MCP	0.45	26.01	2.72	1.12	0.60	10.31	10.05	0.80	0.52	0.00	0.00	0.55	0.00	1.04	0.45	0.56	0.54	0.54
BACOP1	0.45	23.74	2.64	1.21	0.90	10.40	10.16	0.80	0.53	0.00	0.00	0.55	0.00	1.04	0.45	0.59	0.55	0.55
BACOP2-0.01	0.45	42.21	2.28	7.32	1.29	20.77	21.28	0.78	0.55	0.00	0.00	0.55	0.59	0.58	0.58	3.59	0.59	0.59
BACOP2-0.25	0.45	42.21	2.28	7.32	1.29	20.77	21.28	0.78	0.55	0.00	0.00	0.55	0.59	0.58	0.58	3.59	0.59	0.59
BACOP2-0.50	0.45	42.13	2.52	7.24	1.21	20.74	21.29	0.78	0.54	0.00	0.00	0.55	0.60	0.88	0.57	3.91	0.60	0.60
BACOP2-0.75	0.45	41.99	2.49	7.24	1.21	20.79	21.32	0.78	0.53	0.00	0.00	0.55	0.60	0.88	0.57	3.91	0.59	0.60
BACOP2-0.99	0.45	34.96	1.93	1.24	0.63	16.35	15.86	0.79	0.54	0.00	0.00	0.55	0.56	0.48	0.58	3.98	0.58	0.58
p-center	0.60	15.74	0.71	0.94	2.13	4.78	2.06	0.63	0.23	0.00	0.00	0.40	0.00	1.07	0.36	0.98	0.41	0.41
p-median	0.60	15.50	0.74	0.83	2.25	4.56	1.92	0.63	0.18	0.00	0.00	0.40	0.00	1.02	0.34	0.82	0.41	0.41
p-centdian	0.60	15.56	0.74	0.89	1.86	4.73	1.99	0.63	0.21	0.00	0.00	0.40	0.00	0.88	0.36	0.98	0.41	0.41
MCP	0.60	19.10	2.04	1.00	0.58	6.57	6.27	0.65	0.38	0.00	0.00	0.40	0.00	1.24	0.34	0.31	0.39	0.39
BACOP1	0.60	19.10	2.04	1.00	0.58	6.57	6.27	0.65	0.38	0.00	0.00	0.40	0.00	1.24	0.34	0.31	0.39	0.39
BACOP2-0.01	0.60	38.73	2.74	4.56	1.17	13.13	13.52	0.63	0.41	0.00	0.00	0.40	0.42	0.11	0.41	1.02	0.43	0.43
BACOP2-0.25	0.60	38.73	2.74	4.56	1.17	13.13	13.52	0.63	0.41	0.00	0.00	0.40	0.42	0.11	0.41	1.02	0.43	0.43
BACOP2-0.50	0.60	37.36	2.87	4.56	1.17	12.97	13.50	0.63	0.41	0.00	0.00	0.40	0.44	0.47	0.40	0.62	0.43	0.43
BACOP2-0.75	0.60	37.22	2.85	4.56	1.17	12.99	13.54	0.63	0.41	0.00	0.00	0.40	0.44	0.50	0.40	0.57	0.43	0.43
BACOP2-0.99	0.60	30.98	1.71	0.24	0.03	10.19	9.94	0.63	0.38	0.00	0.00	0.40	0.39	0.69	0.42	4.59	0.42	0.42

Table A.15
Uniform, $\mu = 0.5$.

Model	γ	D_{max}	D^h_{max}	D_{min}	D^h_{min}	D_{avg}	D^h_{avg}	*Disp*	*Acc*	C	C^h	RC^h	C_{min}	C^h_{min}	C_{max}	C^h_{max}	C_{avg}	C^h_{avg}
p-center	0.15	29.15	0.63	2.79	2.41	22.64	20.27	0.98	0.80	0.00	0.00	0.85	0.00	0.47	0.80	0.46	0.86	0.86
p-median	0.15	28.28	2.00	13.23	3.16	20.30	17.53	0.98	0.78	0.00	0.00	0.85	0.00	0.43	0.77	0.56	0.85	0.85
p-centdian	0.15	29.45	1.17	8.19	2.15	21.64	18.97	0.98	0.80	0.00	0.00	0.85	0.00	0.47	0.79	0.26	0.86	0.86
MCP	0.15	50.08	2.43	1.88	1.06	37.48	38.71	0.98	0.85	0.00	0.00	0.85	0.00	0.52	0.78	0.51	0.86	0.86
BACOP1	0.15	48.60	2.40	5.32	1.42	38.19	38.13	0.98	0.86	0.00	0.00	0.85	0.00	0.59	0.77	0.48	0.87	0.87
BACOP2-0.01	0.15	94.66	3.38	17.01	1.67	82.79	81.14	0.97	0.83	0.00	0.00	0.85	0.90	1.65	0.88	3.56	0.91	0.91
BACOP2-0.25	0.15	94.66	3.38	17.01	1.67	82.79	81.14	0.97	0.83	0.00	0.00	0.85	0.90	1.65	0.88	3.56	0.91	0.91
BACOP2-0.50	0.15	94.21	2.27	17.01	1.67	81.89	79.76	0.97	0.83	0.00	0.00	0.85	0.90	1.53	0.89	3.52	0.91	0.91
BACOP2-0.75	0.15	94.21	2.27	17.01	1.67	81.89	79.76	0.97	0.83	0.00	0.00	0.85	0.90	1.53	0.89	3.52	0.91	0.91
BACOP2-0.99	0.15	92.63	2.18	17.01	1.67	81.60	79.83	0.97	0.83	0.00	0.00	0.85	0.90	1.53	0.89	3.52	0.91	0.91
p-center	0.30	23.63	0.91	3.59	2.53	13.03	9.48	0.91	0.62	0.00	0.00	0.70	0.00	0.29	0.64	0.61	0.71	0.71
p-median	0.30	22.66	0.85	1.81	2.92	11.78	8.44	0.91	0.55	0.00	0.00	0.70	0.00	0.42	0.66	0.45	0.71	0.71
p-centdian	0.30	23.29	0.91	1.75	2.98	12.54	8.94	0.91	0.57	0.00	0.00	0.70	0.00	0.30	0.64	0.63	0.71	0.71
MCP	0.30	34.75	1.19	1.94	0.92	16.99	17.13	0.91	0.70	0.00	0.00	0.70	0.00	0.37	0.63	0.62	0.70	0.70
BACOP1	0.30	34.75	1.19	1.94	0.92	16.99	17.13	0.91	0.70	0.00	0.00	0.70	0.00	0.37	0.63	0.62	0.70	0.70
BACOP2-0.01	0.30	53.32	1.54	7.15	1.64	32.43	31.81	0.89	0.68	0.00	0.00	0.70	0.74	1.52	0.76	6.61	0.76	0.76
BACOP2-0.25	0.30	53.32	1.54	7.15	1.64	32.43	31.81	0.89	0.68	0.00	0.00	0.70	0.74	1.52	0.76	6.61	0.76	0.76
BACOP2-0.50	0.30	53.38	1.54	7.15	1.64	32.49	31.84	0.89	0.68	0.00	0.00	0.70	0.74	1.52	0.76	6.58	0.76	0.76
BACOP2-0.75	0.30	53.38	1.54	7.15	1.64	32.49	31.84	0.89	0.68	0.00	0.00	0.70	0.74	1.52	0.76	6.58	0.76	0.76
BACOP2-0.99	0.30	48.54	2.11	7.81	0.62	29.68	29.07	0.90	0.72	0.00	0.00	0.70	0.73	1.67	0.75	1.93	0.75	0.75
p-center	0.45	19.66	0.94	1.80	2.77	8.02	4.76	0.79	0.40	0.00	0.00	0.55	0.00	0.40	0.51	0.40	0.56	0.56
p-median	0.45	18.79	0.66	1.53	2.76	7.46	4.16	0.79	0.34	0.00	0.00	0.55	0.00	0.19	0.51	0.38	0.56	0.56
p-centdian	0.45	18.79	0.66	1.53	2.76	7.68	4.27	0.79	0.34	0.00	0.00	0.55	0.00	0.40	0.52	0.37	0.56	0.56
MCP	0.45	29.54	2.65	1.16	0.64	10.54	10.37	0.80	0.54	0.00	0.00	0.55	0.00	0.35	0.51	0.57	0.55	0.55
BACOP1	0.45	29.54	2.65	1.16	0.64	10.54	10.37	0.80	0.54	0.00	0.00	0.55	0.00	0.35	0.51	0.57	0.55	0.55
BACOP2-0.01	0.45	36.12	2.31	6.26	0.99	17.79	17.50	0.78	0.54	0.00	0.00	0.55	0.58	1.28	0.61	6.40	0.59	0.60
BACOP2-0.25	0.45	36.12	2.31	6.26	0.99	17.79	17.50	0.78	0.54	0.00	0.00	0.55	0.58	1.28	0.61	6.40	0.59	0.60
BACOP2-0.50	0.45	36.12	2.31	6.26	0.99	17.87	17.62	0.78	0.53	0.00	0.00	0.55	0.58	1.40	0.61	6.40	0.59	0.60
BACOP2-0.75	0.45	36.12	2.31	6.26	0.99	17.87	17.62	0.78	0.53	0.00	0.00	0.55	0.58	1.40	0.61	6.40	0.59	0.60
BACOP2-0.99	0.45	39.06	2.13	6.99	0.27	17.03	16.91	0.78	0.61	0.00	0.00	0.55	0.52	1.72	0.61	6.38	0.59	0.59
p-center	0.60	15.74	0.71	0.94	2.13	4.78	2.06	0.63	0.23	0.00	0.00	0.40	0.00	0.00	0.36	0.33	0.41	0.41
p-median	0.60	15.50	0.74	0.83	2.25	4.56	1.92	0.63	0.18	0.00	0.00	0.40	0.00	0.20	0.36	0.39	0.41	0.41
p-centdian	0.60	15.56	0.74	0.89	1.86	4.73	1.99	0.63	0.21	0.00	0.00	0.40	0.00	0.01	0.36	0.33	0.41	0.41
MCP	0.60	19.10	2.04	1.00	0.58	6.57	6.27	0.65	0.38	0.00	0.00	0.40	0.00	0.30	0.37	0.48	0.39	0.39
BACOP1	0.60	19.10	2.04	1.00	0.58	6.57	6.27	0.65	0.38	0.00	0.00	0.40	0.00	0.30	0.37	0.48	0.39	0.39
BACOP2-0.01	0.60	29.42	3.01	5.10	0.19	10.36	10.49	0.62	0.40	0.00	0.00	0.40	0.36	1.44	0.45	6.31	0.43	0.43
BACOP2-0.25	0.60	29.42	3.01	5.10	0.19	10.36	10.49	0.62	0.40	0.00	0.00	0.40	0.36	1.44	0.45	6.31	0.43	0.43
BACOP2-0.50	0.60	29.42	3.01	5.10	0.19	10.39	10.52	0.62	0.40	0.00	0.00	0.40	0.36	1.44	0.45	6.31	0.43	0.43
BACOP2-0.75	0.60	29.42	3.01	5.10	0.19	10.39	10.52	0.62	0.40	0.00	0.00	0.40	0.36	1.44	0.45	6.31	0.43	0.43
BACOP2-0.99	0.60	30.27	1.72	1.95	0.33	10.26	10.51	0.62	0.47	0.00	0.00	0.40	0.38	1.39	0.44	8.30	0.43	0.43

Table A.16
Mono-polar.

Model	γ	$\mu = 0.1$						$\mu = 0.3$						$\mu = 0.5$					
		EC	EC^h	D_{mad}	D^h_{mad}	A_{mad}	A^h_{mad}	EC	EC^h	D_{mad}	D^h_{mad}	A_{mad}	A^h_{mad}	EC	EC^h	D_{mad}	D^h_{mad}	A_{mad}	A^h_{mad}
p-center	0.15	0.00	0.00	0.43	0.43	0.02	0.16	0.44	0.01	0.52	0.59	0.02	0.16	0.27	0.01	0.49	0.54	0.02	0.16
p-median	0.15	0.00	0.00	0.31	0.57	0.11	0.00	0.44	0.01	0.36	0.70	0.11	0.00	0.33	0.01	0.32	0.62	0.11	0.00
p-centdian	0.15	0.00	0.00	0.45	0.48	0.02	0.13	0.44	0.01	0.52	0.61	0.02	0.13	0.28	0.01	0.52	0.60	0.02	0.13
MCP	0.15	0.00	0.00	0.49	0.55	0.46	0.27	0.00	0.00	0.53	0.43	0.28	0.56	0.10	0.00	0.48	0.41	0.76	1.04
BACOP1	0.15	NAN	NAN	NAN	NAN	NAN	NAN	0.08	0.00	0.34	0.27	0.17	0.35	0.02	0.00	0.37	0.31	0.67	0.91
BACOP2-0.01	0.15	0.00	0.00	0.66	0.62	1.45	0.64	0.47	0.01	0.50	0.41	0.54	0.55	0.03	0.00	0.42	0.37	0.72	1.01
BACOP2-0.25	0.15	0.00	0.00	0.62	0.58	1.43	0.62	0.47	0.01	0.57	0.46	0.51	0.54	0.03	0.00	0.45	0.38	0.66	0.99
BACOP2-0.50	0.15	0.00	0.00	0.48	0.46	1.27	0.53	0.43	0.01	0.62	0.55	0.43	0.48	0.02	0.00	0.45	0.41	0.70	1.04
BACOP2-0.75	0.15	0.00	0.00	0.48	0.45	0.82	0.36	0.43	0.01	0.50	0.41	0.38	0.48	0.02	0.00	0.45	0.41	0.70	1.04
BACOP2-0.99	0.15	0.00	0.00	0.41	0.45	0.47	0.27	0.09	0.00	0.50	0.42	0.12	0.33	0.03	0.00	0.44	0.40	0.55	0.85
p-center	0.30	0.65	0.01	0.20	0.35	0.07	0.08	0.11	0.00	0.18	0.28	0.02	0.07	0.06	0.00	0.19	0.32	0.02	0.07
p-median	0.30	0.65	0.01	0.31	0.60	0.09	0.01	0.13	0.00	0.28	0.49	0.05	0.00	0.07	0.00	0.30	0.56	0.04	0.00
p-centdian	0.30	0.65	0.01	0.27	0.50	0.06	0.06	0.11	0.00	0.25	0.41	0.02	0.05	0.05	0.00	0.27	0.46	0.01	0.05
MCP	0.30	0.01	0.00	0.15	0.16	0.10	0.16	0.08	0.00	0.25	0.18	0.44	0.42	0.03	0.00	0.24	0.25	0.54	0.46
BACOP1	0.30	0.13	0.00	0.20	0.18	0.04	0.08	0.01	0.00	0.17	0.13	0.24	0.33	0.02	0.00	0.20	0.20	0.42	0.40
BACOP2-0.01	0.30	0.65	0.01	0.27	0.27	1.14	0.72	0.02	0.00	0.22	0.19	0.37	0.39	0.01	0.00	0.18	0.17	0.78	0.80
BACOP2-0.25	0.30	0.65	0.01	0.26	0.30	1.09	0.69	0.02	0.00	0.20	0.16	0.31	0.35	0.02	0.00	0.23	0.20	0.56	0.57
BACOP2-0.50	0.30	0.65	0.01	0.25	0.33	0.93	0.60	0.02	0.00	0.19	0.16	0.36	0.39	0.01	0.00	0.18	0.14	0.48	0.51
BACOP2-0.75	0.30	0.65	0.01	0.25	0.32	0.65	0.44	0.02	0.00	0.19	0.16	0.36	0.39	0.01	0.00	0.18	0.14	0.48	0.51
BACOP2-0.99	0.30	0.13	0.00	0.17	0.16	0.02	0.07	0.02	0.00	0.18	0.14	0.33	0.37	0.01	0.00	0.20	0.18	0.38	0.40
p-center	0.45	0.34	0.01	0.10	0.21	0.11	0.10	0.04	0.00	0.12	0.25	0.06	0.06	0.03	0.00	0.18	0.30	0.04	0.05
p-median	0.45	0.36	0.01	0.18	0.37	0.09	0.05	0.04	0.00	0.20	0.41	0.04	0.01	0.03	0.00	0.26	0.47	0.02	0.00
p-centdian	0.45	0.34	0.01	0.14	0.30	0.09	0.07	0.04	0.00	0.16	0.34	0.04	0.03	0.03	0.00	0.22	0.39	0.02	0.02
MCP	0.45	0.16	0.00	0.12	0.08	0.22	0.22	0.02	0.00	0.14	0.12	0.44	0.38	0.03	0.00	0.15	0.13	0.42	0.37
BACOP1	0.45	0.04	0.00	0.10	0.07	0.17	0.19	0.02	0.00	0.13	0.12	0.28	0.27	0.02	0.00	0.22	0.21	0.47	0.44
BACOP2-0.01	0.45	0.15	0.00	0.17	0.16	0.23	0.27	0.01	0.00	0.15	0.14	0.37	0.34	0.01	0.00	0.22	0.20	0.45	0.48
BACOP2-0.25	0.45	0.03	0.00	0.14	0.13	0.20	0.24	0.01	0.00	0.16	0.15	0.34	0.32	0.01	0.00	0.23	0.19	0.36	0.37
BACOP2-0.50	0.45	0.02	0.00	0.10	0.09	0.19	0.22	0.01	0.00	0.13	0.13	0.35	0.31	0.01	0.00	0.19	0.15	0.36	0.36
BACOP2-0.75	0.45	0.02	0.00	0.10	0.09	0.19	0.22	0.01	0.00	0.13	0.13	0.35	0.31	0.01	0.00	0.19	0.15	0.36	0.36
BACOP2-0.99	0.45	0.03	0.00	0.14	0.12	0.18	0.21	0.02	0.00	0.09	0.08	0.43	0.39	0.02	0.00	0.21	0.19	0.40	0.38
p-center	0.60	0.12	0.00	0.10	0.21	0.05	0.04	0.02	0.00	0.13	0.24	0.04	0.04	0.01	0.00	0.12	0.24	0.02	0.02
p-median	0.60	0.12	0.00	0.12	0.26	0.04	0.03	0.02	0.00	0.16	0.30	0.03	0.03	0.01	0.00	0.14	0.29	0.01	0.01
p-centdian	0.60	0.12	0.00	0.11	0.24	0.04	0.03	0.02	0.00	0.15	0.27	0.02	0.02	0.01	0.00	0.13	0.27	0.01	0.01
MCP	0.60	0.05	0.00	0.14	0.13	0.32	0.30	0.01	0.00	0.12	0.10	0.62	0.51	0.01	0.00	0.10	0.11	0.57	0.48
BACOP1	0.60	0.02	0.00	0.07	0.06	0.18	0.18	0.01	0.00	0.16	0.16	0.41	0.36	0.01	0.00	0.12	0.11	0.59	0.53
BACOP2-0.01	0.60	0.01	0.00	0.10	0.10	0.48	0.47	0.00	0.00	0.14	0.12	0.38	0.33	0.00	0.00	0.16	0.16	0.53	0.47
BACOP2-0.25	0.60	0.01	0.00	0.11	0.11	0.50	0.48	0.00	0.00	0.17	0.15	0.39	0.33	0.01	0.00	0.15	0.15	0.52	0.46
BACOP2-0.50	0.60	0.01	0.00	0.09	0.09	0.47	0.46	0.00	0.00	0.14	0.12	0.41	0.35	0.01	0.00	0.09	0.08	0.52	0.47
BACOP2-0.75	0.60	0.01	0.00	0.09	0.09	0.47	0.46	0.00	0.00	0.14	0.12	0.41	0.35	0.01	0.00	0.09	0.08	0.52	0.47
BACOP2-0.99	0.60	0.01	0.00	0.10	0.09	0.44	0.43	0.01	0.00	0.12	0.10	0.49	0.43	0.01	0.00	0.14	0.14	0.57	0.49

Table A.17
Multi-polar.

Model	γ	$\mu = 0.1$						$\mu = 0.3$						$\mu = 0.5$					
		EC	EC^h	D_{mad}	D^h_{mad}	A_{mad}	A^h_{mad}	EC	EC^h	D_{mad}	D^h_{mad}	A_{mad}	A^h_{mad}	EC	EC^h	D_{mad}	D^h_{mad}	A_{mad}	A^h_{mad}
p-center	0.15	1.00	0.01	0.47	0.68	0.07	0.15	0.22	0.00	0.36	0.46	0.05	0.15	0.09	0.00	0.31	0.38	0.05	0.15
p-median	0.15	1.00	0.01	0.50	0.81	0.05	0.00	0.32	0.00	0.36	0.57	0.03	0.00	0.13	0.00	0.33	0.50	0.03	0.00
p-centdian	0.15	1.00	0.01	0.61	0.89	0.04	0.08	0.23	0.00	0.47	0.65	0.03	0.08	0.11	0.00	0.43	0.56	0.03	0.08
MCP	0.15	0.12	0.00	0.29	0.27	0.03	0.16	0.34	0.00	0.24	0.20	1.63	1.49	0.24	0.00	0.34	0.26	2.32	1.37
BACOP1	0.15	0.03	0.00	0.53	0.54	0.05	0.11	0.04	0.00	0.41	0.37	1.02	0.98	0.11	0.00	0.34	0.23	2.59	1.81
BACOP2-0.01	0.15	1.00	0.01	0.45	0.51	1.09	0.66	0.01	0.00	0.29	0.28	0.96	0.91	0.06	0.00	0.27	0.16	3.42	2.78
BACOP2-0.25	0.15	1.00	0.01	0.41	0.46	0.43	0.35	0.01	0.00	0.33	0.32	0.99	0.94	0.05	0.00	0.26	0.15	3.44	2.77
BACOP2-0.50	0.15	1.00	0.01	0.48	0.49	0.15	0.20	0.00	0.00	0.36	0.35	0.98	0.92	0.06	0.00	0.30	0.18	3.63	2.90
BACOP2-0.75	0.15	1.00	0.01	0.40	0.39	0.10	0.14	0.00	0.00	0.36	0.35	0.98	0.92	0.06	0.00	0.30	0.18	3.63	2.90
BACOP2-0.99	0.15	0.13	0.00	0.45	0.42	0.05	0.11	0.00	0.00	0.30	0.29	0.98	0.90	0.09	0.00	0.15	0.08	3.11	2.50
p-center	0.30	0.59	0.01	0.30	0.43	0.07	0.11	0.10	0.00	0.22	0.31	0.07	0.11	0.18	0.00	0.25	0.34	0.07	0.11
p-median	0.30	0.51	0.01	0.45	0.68	0.02	0.00	0.10	0.00	0.34	0.53	0.02	0.00	0.19	0.00	0.39	0.58	0.02	0.00
p-centdian	0.30	0.59	0.01	0.43	0.65	0.06	0.08	0.10	0.00	0.33	0.51	0.06	0.08	0.18	0.00	0.37	0.55	0.06	0.08
MCP	0.30	0.42	0.00	0.28	0.27	0.43	0.49	0.22	0.00	0.16	0.12	0.63	0.49	0.27	0.00	0.20	0.15	0.63	0.49
BACOP1	0.30	0.06	0.00	0.30	0.30	0.23	0.27	0.10	0.00	0.18	0.16	0.78	0.74	0.27	0.00	0.20	0.15	0.63	0.49
BACOP2-0.01	0.30	0.04	0.00	0.28	0.28	0.21	0.25	0.06	0.00	0.18	0.15	1.04	0.97	0.11	0.00	0.20	0.15	2.32	1.96
BACOP2-0.25	0.30	0.03	0.00	0.27	0.25	0.21	0.25	0.06	0.00	0.19	0.15	1.01	0.94	0.10	0.00	0.16	0.13	1.76	1.48
BACOP2-0.50	0.30	0.02	0.00	0.25	0.25	0.22	0.26	0.05	0.00	0.19	0.15	1.06	0.96	0.10	0.00	0.17	0.13	2.41	1.98
BACOP2-0.75	0.30	0.02	0.00	0.25	0.25	0.22	0.26	0.05	0.00	0.19	0.15	1.06	0.96	0.10	0.00	0.17	0.13	2.41	1.98
BACOP2-0.99	0.30	0.07	0.00	0.22	0.22	0.21	0.26	0.04	0.00	0.21	0.18	1.13	1.03	0.09	0.00	0.22	0.17	1.95	1.59
p-center	0.45	0.11	0.00	0.16	0.28	0.02	0.04	0.12	0.00	0.12	0.24	0.02	0.04	0.27	0.00	0.14	0.23	0.02	0.04
p-median	0.45	0.10	0.00	0.22	0.39	0.01	0.01	0.11	0.00	0.18	0.34	0.01	0.01	0.28	0.00	0.20	0.32	0.01	0.01
p-centdian	0.45	0.11	0.00	0.20	0.34	0.02	0.02	0.12	0.00	0.17	0.29	0.02	0.02	0.27	0.00	0.18	0.28	0.02	0.02
MCP	0.45	0.11	0.00	0.22	0.22	0.63	0.63	0.16	0.00	0.17	0.15	0.51	0.41	0.31	0.00	0.19	0.14	0.51	0.41
BACOP1	0.45	0.02	0.00	0.17	0.16	0.48	0.48	0.16	0.00	0.17	0.15	0.51	0.41	0.31	0.00	0.19	0.14	0.51	0.41
BACOP2-0.01	0.45	0.03	0.00	0.16	0.17	0.43	0.42	0.14	0.00	0.12	0.12	0.82	0.77	0.23	0.00	0.12	0.08	1.24	1.08
BACOP2-0.25	0.45	0.03	0.00	0.18	0.15	0.51	0.48	0.14	0.00	0.12	0.11	0.75	0.71	0.24	0.00	0.11	0.07	1.02	0.90
BACOP2-0.50	0.45	0.03	0.00	0.18	0.16	0.51	0.48	0.14	0.00	0.11	0.11	0.75	0.71	0.21	0.00	0.08	0.04	1.19	1.03
BACOP2-0.75	0.45	0.03	0.00	0.18	0.16	0.51	0.48	0.14	0.00	0.11	0.11	0.75	0.71	0.21	0.00	0.08	0.04	1.19	1.03
BACOP2-0.99	0.45	0.04	0.00	0.12	0.11	0.45	0.44	0.03	0.00	0.19	0.15	0.93	0.84	0.08	0.00	0.17	0.13	1.08	0.94
p-center	0.60	0.06	0.00	0.12	0.22	0.04	0.04	0.16	0.00	0.10	0.20	0.04	0.04	0.14	0.00	0.10	0.21	0.04	0.04
p-median	0.60	0.07	0.00	0.15	0.27	0.01	0.01	0.16	0.00	0.13	0.24	0.01	0.01	0.13	0.00	0.13	0.25	0.01	0.01
p-centdian	0.60	0.07	0.00	0.15	0.26	0.04	0.04	0.16	0.00	0.13	0.23	0.04	0.04	0.13	0.00	0.13	0.24	0.04	0.04
MCP	0.60	0.08	0.00	0.16	0.15	0.99	0.87	0.25	0.00	0.13	0.12	0.73	0.60	0.19	0.00	0.14	0.13	0.73	0.60
BACOP1	0.60	0.02	0.00	0.08	0.07	0.74	0.66	0.25	0.00	0.13	0.12	0.73	0.60	0.19	0.00	0.14	0.13	0.73	0.60
BACOP2-0.01	0.60	0.03	0.00	0.14	0.13	0.77	0.70	0.14	0.00	0.06	0.05	0.82	0.72	0.09	0.00	0.07	0.07	1.01	0.88
BACOP2-0.25	0.60	0.02	0.00	0.12	0.11	0.79	0.69	0.19	0.00	0.08	0.07	0.80	0.70	0.07	0.00	0.07	0.06	0.83	0.74
BACOP2-0.50	0.60	0.02	0.00	0.11	0.10	0.78	0.68	0.19	0.00	0.08	0.07	0.80	0.70	0.06	0.00	0.06	0.05	0.96	0.83
BACOP2-0.75	0.60	0.02	0.00	0.11	0.10	0.78	0.68	0.19	0.00	0.08	0.07	0.80	0.70	0.06	0.00	0.06	0.05	0.96	0.83
BACOP2-0.99	0.60	0.04	0.00	0.10	0.10	0.74	0.67	0.05	0.00	0.15	0.12	0.75	0.68	0.05	0.00	0.12	0.11	0.85	0.73

Table A.18
Uniform.

Model	γ	$\mu = 0.1$						$\mu = 0.3$						$\mu = 0.5$					
		EC	EC^h	D_{mad}	D^h_{mad}	A_{mad}	A^h_{mad}	EC	EC^h	D_{mad}	D^h_{mad}	A_{mad}	A^h_{mad}	EC	EC^h	D_{mad}	D^h_{mad}	A_{mad}	A^h_{mad}
p-center	0.15	0.65	0.01	0.46	0.52	0.06	0.12	0.24	0.00	0.70	0.83	0.05	0.12	0.03	0.00	0.65	0.87	0.05	0.12
p-median	0.15	0.65	0.01	0.38	0.64	0.05	0.01	0.27	0.00	0.61	1.02	0.04	0.01	0.08	0.00	0.61	1.09	0.04	0.01
p-centdian	0.15	0.65	0.01	0.47	0.66	0.05	0.07	0.23	0.00	0.72	0.98	0.04	0.07	0.03	0.00	0.69	1.04	0.04	0.07
MCP	0.15	0.00	0.00	0.27	0.30	0.05	0.13	0.37	0.00	0.51	0.48	0.56	0.49	0.17	0.00	0.48	0.47	0.65	0.46
BACOP1	0.15	0.13	0.00	0.46	0.48	0.07	0.12	0.06	0.00	0.29	0.28	0.66	0.59	0.07	0.00	0.61	0.62	0.77	0.67
BACOP2-0.01	0.15	0.65	0.01	0.45	0.51	0.41	0.36	0.05	0.00	0.33	0.37	0.69	0.65	0.07	0.00	0.50	0.57	1.49	1.53
BACOP2-0.25	0.15	0.65	0.01	0.36	0.40	0.35	0.33	0.03	0.00	0.51	0.54	0.67	0.61	0.07	0.00	0.50	0.57	1.49	1.53
BACOP2-0.50	0.15	0.65	0.01	0.40	0.43	0.21	0.24	0.04	0.00	0.51	0.54	0.73	0.64	0.07	0.00	0.48	0.53	1.51	1.56
BACOP2-0.75	0.15	0.60	0.01	0.27	0.31	0.11	0.15	0.04	0.00	0.50	0.52	0.73	0.64	0.07	0.00	0.48	0.53	1.51	1.56
BACOP2-0.99	0.15	0.14	0.00	0.36	0.40	0.08	0.12	0.07	0.00	0.42	0.51	0.59	0.55	0.08	0.00	0.42	0.48	1.33	1.28
p-center	0.30	0.63	0.00	0.21	0.38	0.06	0.08	0.13	0.00	0.26	0.41	0.06	0.08	0.14	0.00	0.27	0.47	0.06	0.08
p-median	0.30	0.53	0.00	0.23	0.52	0.01	0.00	0.11	0.00	0.27	0.55	0.01	0.00	0.13	0.00	0.30	0.65	0.01	0.00
p-centdian	0.30	0.62	0.00	0.20	0.44	0.05	0.05	0.13	0.00	0.24	0.48	0.05	0.05	0.15	0.00	0.26	0.56	0.05	0.05
MCP	0.30	0.54	0.00	0.18	0.20	0.37	0.37	0.26	0.00	0.22	0.19	0.54	0.45	0.22	0.00	0.23	0.26	0.54	0.45
BACOP1	0.30	0.10	0.00	0.14	0.16	0.22	0.23	0.21	0.00	0.21	0.19	0.53	0.46	0.22	0.00	0.23	0.26	0.54	0.45
BACOP2-0.01	0.30	0.07	0.00	0.11	0.13	0.24	0.25	0.17	0.00	0.27	0.26	0.75	0.65	0.12	0.00	0.20	0.23	0.74	0.75
BACOP2-0.25	0.30	0.08	0.00	0.13	0.13	0.22	0.23	0.09	0.00	0.30	0.29	0.56	0.52	0.09	0.00	0.19	0.22	0.70	0.72
BACOP2-0.50	0.30	0.04	0.00	0.19	0.21	0.23	0.27	0.11	0.00	0.34	0.31	0.58	0.53	0.11	0.00	0.22	0.24	0.75	0.76
BACOP2-0.75	0.30	0.04	0.00	0.19	0.21	0.23	0.27	0.11	0.00	0.34	0.31	0.58	0.53	0.11	0.00	0.22	0.24	0.75	0.76
BACOP2-0.99	0.30	0.00	0.00	0.18	0.22	0.22	0.26	0.14	0.00	0.24	0.23	0.59	0.56	0.03	0.00	0.21	0.25	0.61	0.54
p-center	0.45	0.15	0.00	0.21	0.41	0.06	0.06	0.07	0.00	0.23	0.39	0.06	0.06	0.09	0.00	0.24	0.43	0.06	0.06
p-median	0.45	0.13	0.00	0.22	0.46	0.01	0.00	0.05	0.00	0.24	0.45	0.01	0.00	0.13	0.00	0.26	0.49	0.01	0.00
p-centdian	0.45	0.15	0.00	0.21	0.44	0.06	0.06	0.07	0.00	0.23	0.42	0.06	0.06	0.10	0.00	0.24	0.46	0.06	0.06
MCP	0.45	0.19	0.00	0.08	0.09	0.46	0.45	0.13	0.00	0.16	0.15	0.45	0.37	0.15	0.00	0.18	0.19	0.45	0.37
BACOP1	0.45	0.05	0.00	0.16	0.17	0.42	0.41	0.10	0.00	0.17	0.15	0.43	0.36	0.15	0.00	0.18	0.19	0.45	0.37
BACOP2-0.01	0.45	0.01	0.00	0.11	0.11	0.50	0.48	0.18	0.00	0.20	0.18	0.87	0.77	0.11	0.00	0.18	0.18	0.66	0.61
BACOP2-0.25	0.45	0.02	0.00	0.14	0.14	0.49	0.48	0.07	0.00	0.18	0.17	0.48	0.45	0.07	0.00	0.17	0.16	0.55	0.52
BACOP2-0.50	0.45	0.04	0.00	0.14	0.14	0.47	0.46	0.07	0.00	0.20	0.19	0.48	0.45	0.06	0.00	0.20	0.18	0.56	0.54
BACOP2-0.75	0.45	0.04	0.00	0.14	0.14	0.47	0.46	0.07	0.00	0.20	0.19	0.48	0.45	0.06	0.00	0.20	0.18	0.56	0.54
BACOP2-0.99	0.45	0.09	0.00	0.20	0.22	0.49	0.49	0.12	0.00	0.15	0.13	0.66	0.62	0.05	0.00	0.19	0.20	0.52	0.47
p-center	0.60	0.04	0.00	0.09	0.25	0.05	0.04	0.05	0.00	0.11	0.24	0.05	0.04	0.04	0.00	0.08	0.22	0.05	0.04
p-median	0.60	0.05	0.00	0.13	0.29	0.00	0.00	0.04	0.00	0.15	0.28	0.00	0.00	0.04	0.00	0.12	0.25	0.00	0.00
p-centdian	0.60	0.06	0.00	0.11	0.27	0.04	0.04	0.04	0.00	0.13	0.26	0.04	0.04	0.04	0.00	0.10	0.24	0.04	0.04
MCP	0.60	0.11	0.00	0.11	0.11	0.39	0.33	0.13	0.00	0.14	0.11	0.40	0.33	0.11	0.00	0.11	0.10	0.40	0.33
BACOP1	0.60	0.03	0.00	0.12	0.13	0.57	0.50	0.13	0.00	0.14	0.11	0.40	0.33	0.11	0.00	0.11	0.10	0.40	0.33
BACOP2-0.01	0.60	0.02	0.00	0.10	0.10	0.66	0.60	0.15	0.00	0.16	0.11	1.04	0.90	0.06	0.00	0.11	0.09	0.59	0.51
BACOP2-0.25	0.60	0.02	0.00	0.09	0.09	0.63	0.58	0.04	0.00	0.12	0.09	0.46	0.41	0.05	0.00	0.11	0.09	0.51	0.43
BACOP2-0.50	0.60	0.04	0.00	0.10	0.12	0.72	0.65	0.04	0.00	0.13	0.10	0.47	0.41	0.05	0.00	0.12	0.10	0.49	0.41
BACOP2-0.75	0.60	0.04	0.00	0.10	0.12	0.72	0.65	0.04	0.00	0.13	0.10	0.47	0.41	0.05	0.00	0.12	0.10	0.49	0.41
BACOP2-0.99	0.60	0.07	0.00	0.08	0.10	0.60	0.54	0.12	0.00	0.16	0.14	0.59	0.53	0.05	0.00	0.11	0.11	0.54	0.48

Appendix B. Detailed PODIUM aggregator results

Tables B.19–B.21.

Table B.19
Averages and standard deviations of the PODIUMs for topological KPIs across all datasets.

	Mono-polar		Multi-polar		Uniform		Tot Avg	
	avg	stdev	avg	stdev	avg	stdev	avg	stdev
p-center	48.96	23.97	50.00	25.00	50.00	25.00	49.65	24.66
p-median	43.75	19.14	48.96	23.97	55.21	30.48	49.31	24.53
p-centdian	50.00	25.00	44.79	20.06	51.04	26.05	48.61	23.71
MCP	16.67	2.78	18.75	3.52	21.88	4.79	19.10	3.69
BACOP1	20.83	4.34	20.83	4.34	19.79	3.92	20.49	4.20
BACOP2-0.01	17.71	3.14	17.71	3.14	13.54	1.83	16.32	2.70
BACOP2-0.25	25.00	6.25	22.92	5.25	23.96	5.74	23.96	5.75
BACOP2-0.50	26.04	6.78	34.38	11.82	25.00	6.25	28.47	8.28
BACOP2-0.75	26.04	6.78	25.00	6.25	18.75	3.52	23.26	5.52
BACOP2-0.99	25.00	6.25	16.67	2.78	20.83	4.34	20.83	4.46

Table B.20
Averages and standard deviations of the PODIUMs for coverage KPIs across all datasets.

	Mono-polar		Multi-polar		Uniform		Tot Avg	
	avg	stdev	avg	stdev	avg	stdev	avg	stdev
p-center	41.67	17.36	81.25	66.02	81.25	66.02	68.06	49.80
p-median	35.42	12.54	83.33	69.44	81.25	66.02	66.67	49.33
p-centdian	41.67	17.36	81.25	66.02	87.50	76.56	70.14	53.31
MCP	64.58	41.71	18.75	3.52	18.75	3.52	34.03	16.25
BACOP1	50.00	25.00	18.75	3.52	18.75	3.52	29.17	10.68
BACOP2-0.01	22.92	5.25	2.08	0.04	2.08	0.04	9.03	1.78
BACOP2-0.25	8.33	0.69	6.25	0.39	2.08	0.04	5.56	0.38
BACOP2-0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BACOP2-0.75	8.33	0.69	0.00	0.00	0.00	0.00	2.78	0.23
BACOP2-0.99	27.08	7.34	8.33	0.69	8.33	0.69	14.58	2.91

Table B.21
Averages and standard deviations of the PODIUMs for equity KPIs across all datasets.

	Mono-polar		Multi-polar		Uniform		Tot Avg	
	avg	stdev	avg	stdev	avg	stdev	avg	stdev
p-center	44.44	19.75	34.72	12.06	41.67	17.36	40.28	16.39
p-median	38.89	15.12	34.72	12.06	37.50	14.06	37.04	13.75
p-centdian	36.11	13.04	36.11	13.04	41.67	17.36	37.96	14.48
MCP	22.22	4.94	11.11	1.23	26.39	6.96	19.91	4.38
BACOP1	33.33	11.11	13.89	1.93	19.44	3.78	22.22	5.61
BACOP2-0.01	25.00	6.25	19.44	3.78	26.39	6.96	23.61	5.66
BACOP2-0.25	11.11	1.23	29.17	8.51	40.28	16.22	26.85	8.65
BACOP2-0.50	34.72	12.06	44.44	19.75	25.00	6.25	34.72	12.69
BACOP2-0.75	26.39	6.96	37.50	14.06	15.28	2.33	26.39	7.79
BACOP2-0.99	27.78	7.72	38.89	15.12	26.39	6.96	31.02	9.93

Appendix C. Detailed SCORE aggregator results

Tables C.23–C.24.

Table C.22
Averages and standard deviations of the SCOREs for topological KPIs across all datasets.

	Mono-polar		Multi-polar		Uniform		Tot Avg	
	avg	stdev	avg	stdev	avg	stdev	avg	stdev
p-center	5.79	3.23	5.98	3.00	5.57	3.11	5.78	3.11
p-median	6.27	3.21	6.80	3.09	7.05	3.32	6.71	3.21
p-centdian	5.98	3.37	5.80	3.09	6.23	3.06	6.00	3.17
MCP	4.76	2.67	5.09	2.85	5.29	2.91	5.05	2.81
BACOP1	5.47	2.82	5.43	2.73	5.20	2.39	5.36	2.65
BACOP2-0.01	5.15	2.57	5.10	2.93	4.77	2.66	5.01	2.72
BACOP2-0.25	4.99	2.77	5.52	2.47	5.36	2.69	5.29	2.64
BACOP2-0.50	5.70	2.49	6.13	2.53	5.77	2.61	5.86	2.54
BACOP2-0.75	5.30	2.56	5.18	2.50	5.03	2.59	5.17	2.55
BACOP2-0.99	5.59	2.68	3.97	2.68	4.72	2.61	4.76	2.66

Table C.23
Averages and standard deviations of the SCOREs for coverage KPIs across all datasets.

	Mono-polar		Multi-polar		Uniform		Tot Avg	
	avg	stdev	avg	stdev	avg	stdev	avg	stdev
p-center	6.48	3.18	8.90	2.38	8.69	2.79	8.02	2.78
p-median	5.00	3.35	8.04	2.27	7.94	2.47	6.99	2.70
p-centdian	5.27	2.92	7.15	2.02	7.63	1.57	6.68	2.17
MCP	8.79	1.43	7.54	1.15	7.50	1.07	7.94	1.22
BACOP1	7.17	2.36	6.54	1.15	6.50	1.07	6.74	1.52
BACOP2-0.01	4.58	2.51	4.58	1.47	4.83	1.34	4.67	1.77
BACOP2-0.25	4.77	1.97	4.19	1.27	4.10	1.06	4.35	1.43
BACOP2-0.50	4.42	1.49	3.35	1.08	3.27	0.84	3.68	1.14
BACOP2-0.75	4.46	2.16	2.77	1.68	2.69	1.56	3.31	1.80
BACOP2-0.99	4.06	2.96	1.94	2.14	1.85	2.05	2.62	2.38

Table C.24
Averages and standard deviations of the SCOREs for equity KPIs across all datasets.

	Mono-polar		Multi-polar		Uniform		Tot Avg	
	avg	stdev	avg	stdev	avg	stdev	avg	stdev
p-center	5.78	2.77	5.49	2.41	5.67	2.82	5.64	2.67
p-median	4.69	3.91	5.19	3.67	5.21	3.77	5.03	3.78
p-centdian	5.07	3.42	5.00	3.13	5.49	3.24	5.19	3.27
MCP	5.13	2.83	4.29	2.99	5.60	2.95	5.00	2.92
BACOP1	6.26	2.81	4.79	2.72	5.63	2.25	5.56	2.59
BACOP2-0.01	4.99	3.04	5.63	2.57	5.08	3.09	5.23	2.90
BACOP2-0.25	4.76	2.30	6.03	2.18	6.40	2.56	5.73	2.35
BACOP2-0.50	6.31	2.33	6.40	2.74	5.35	2.50	6.02	2.52
BACOP2-0.75	5.72	2.13	5.63	2.72	4.88	2.45	5.41	2.44
BACOP2-0.99	6.29	2.19	6.56	2.72	5.71	2.68	6.19	2.53

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