The J2 Relativistic Effect and Other Periodic Variations in the Galileo Satellite Clocks

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***Abstract*—The frequency of atomic clocks flying on board the satellites of global navigation satellite systems, as can be observed on the ground, shows periodic variations of different origins. One of them is related to the J2 relativistic effect due to the Earth oblateness, whose amplitude is now measurable thanks to stable clocks like Galileo's passive hydrogen masers. Orbital estimation errors are also translated into periodic frequency variations, whose amplitude changes in time and which are superimposed to the J2 signal. In this work we analyze the data of a Galileo satellite clock by using two different techniques, with a twofold objective: to characterize the most evident periodic variations affecting the apparent clock frequency; to estimate the amplitude of the J2 relativistic effect for a comparison with its theoretical value. This also represents a validation of the J2 correction that should be applied to the space clocks' data, in order to improve the timekeeping/positioning performance of global navigation satellite systems.**

Keywords*—J2 relativistic effect; earth oblateness; general relativity; GNSS; Galileo; space clocks; passive hydrogen maser; periodic variations; spectral analysis

I. Introduction

The frequency observed on the ground from the atomic clocks flying on board the satellites of global navigation satellite systems (GNSS), such as GPS and Galileo, is referred to as apparent frequency. This apparent frequency is affected by periodic variations with different origins. For example, the combined relativistic effect of time dilation and gravitational redshift gives rise to periodic variations which must be corrected at the user level, when computing the navigation solution [1-3]. However, the relativistic corrections currently implemented by most of the users, usually neglect the small contribution due to the Earth's oblateness. The corresponding residual relativistic effect is the so called J2 effect, whose name comes from the J2 coefficient, a parameter that quantifies the Earth's oblateness. As already shown in [1-4], the J2 effect is the sum of a constant frequency offset and a periodic term, whose period is equal to one half of the orbital period of the satellite. The J2 periodic component is also mentioned in [5, 6], and in [4] we already tried to analyze it by using GPS Block IIF clock data and to compare its estimated amplitude with the theoretical value. In this work, we study the J2 signal visible on the passive hydrogen masers flying on the Galileo satellites, applying a deeper data analysis which also takes into account the systematic error due to periodic variations of other origin.

As discussed in [6, 7], orbital estimation errors are also translated into apparent periodic variations of the time and frequency offsets of the satellite clocks, as orbital and clock estimates are usually products of the same algorithm. As a consequence, the first harmonic of this periodic signal has a period corresponding to the satellite's orbital period; the period of the second harmonic then corresponds to the period of the J2 signal, and this makes more difficult to properly estimate the amplitude of the J2 signal only. However, the magnitude of the orbital estimation errors depends on the angle between the sun and the satellite's orbital plane, which changes during the year, therefore it can be found out the period in which such error is smaller. An analysis carried out during the same period should give a better estimate of the amplitude of the J2 signal (i.e., an estimate with a smaller bias).

Besides the relativistic effects and the apparent periodic variations due to the orbital estimation error, we can also have other types of periodic variations of the clocks' frequency. For example, this is the case of the sunlight thermal effect, whose magnitude depends on the orientation of the satellite with respect to the sun and, therefore, on its position along the orbit. However, a detailed discussion of all the possible contributions to the observed periodic variations is out of the scope of this work.

Here, we analyze Galileo space clock data provided by the European Space Agency (ESA). In particular, we consider Galileo's passive hydrogen masers, whose impressive stability makes possible to clearly detect tiny effects as the J2 signal. After a crucial preprocessing stage, we analyze the available data with different techniques, as described in section III. First, with a spectral analysis we identify the main periodic signals affecting the data. Then, we focus on the signal at the orbital period and on the J2 signal, and we fit the data with a suitable periodic function. Finally, combining the obtained results, we give an estimate of the amplitude of the J2 signal and we compare it to the theoretical value discussed in section II. In the conclusions, section IV, we summarize our results and their implications, and we describe the future developments of this work.

II. THE J2 EFFECT IN GALILEO CLOCKS

In this section we briefly summarize the theory of the J2 relativistic effect and the expected amplitude of its periodic component in the case of a Galileo satellite, according to [4].

In the framework of general relativity and in the weak field approximation, the ratio between the infinitesimal interval of proper time measured by a ground clock and the corresponding one measured by a satellite clock is

$$d\tau_G/d\tau_S = (1/c^2)(1 + \Phi_G - \Phi_S - v_G^2/2 + v_S^2/2) \quad (1)$$

where the subscript G stands for ground and S for satellite, c is the speed of light, Φ is the gravitational potential and v is the speed measured in the ECI (Earth Centered Inertial) frame. The gravitational potential at a distance r from the Earth's center is

$$\Phi(\boldsymbol{r}) = -GM/r + R(\boldsymbol{r}) \quad (2)$$

where G is the universal gravitational constant, M is the mass of the Earth, and $R(\boldsymbol{r})$ is a perturbing potential accounting for the fact that the Earth is not a perfect sphere and including the gravitational effect of other bodies, like the sun and the moon. The main contribution to the perturbing potential is given by the oblateness of the Earth and, in this work, we neglect the other, smaller contributions. With this approximation and by integrating (1), we obtain the time delay between a ground clock and a satellite clock, $\Delta t_{GS} = \tau_G - \tau_S$, accumulated during a time interval $\Delta\tau_G$ measured by the ground clock:

$$\Delta t_{GS} = (3GM/2a_0c^2 - L_G)\Delta\tau_G + 2\boldsymbol{r}\cdot\boldsymbol{v}/c^2 + \Delta t_{J2} \quad (3)$$

where a_0 is the mean semi-major axis of the satellite's orbit, $L_G = 6.969290134\times10^{-10}$ by definition, $\boldsymbol{r}$ and $\boldsymbol{v}$ are the position and velocity of the satellite, and Δt_{J2} is the J2 relativistic effect:

$$\Delta t_{J2} = (a_E^2 J_2/2a_0^2c^2)[(7GM/a_0)(1 - 3\sin^2 i/2)\Delta\tau_G + 3(GMa_0)^{1/2}\sin^2 i \sin(2u)] \quad (4)$$

where a_E is the Earth's equatorial radius, J_2 is the coefficient quantifying the Earth's oblateness, i is the inclination of the satellite's orbit and u is the satellite's position along the orbit. The first two terms in (3), i.e. the main part of the relativistic effect, are usually corrected at the user level, hence we focus on the J2 effect only. We note that (4) is the sum of two terms: the first one is the result of a constant frequency offset between the two clocks, whereas the second one is the periodic term we are interested in. Its period is one half of the orbital period.

The J2 periodic component can be rewritten as

$$\Delta t_{J2,per} = A\sin(2u) \quad (5)$$

where

$$A = 3a_E^2(GM)^{1/2}J_2\sin^2 i/2a_0^{3/2}c^2 \,. \quad (6)$$

Note that (5) is the J2 signal affecting the clock-to-clock time offset, whereas the J2 signal affecting the fractional frequency offset can be obtained from (5) by differentiation:

$$y_{J2,per}(t) = d\Delta t_{J2,per}(t)/dt = B\cos(2u) \quad (7)$$

where

$$B = 3GMJ_2a_E^2\sin^2 i/a_0^3c^2 \quad (8)$$

to the first-order approximation (see [4] for more details).

In principle, each satellite has its own orbital parameters and hence its own value of the orbital period, T, and of the amplitude of the J2 periodic frequency variation, B. However, according to [8, 9], all the Galileo satellites except GSAT0201 and GSAT0202 have the same semi-major axis and inclination of the orbit, and also the same eccentricity. This means that B and T are the same for all these Galileo satellites, up to the uncertainty in the declared orbital parameters. The theoretical value of B computed in [4] has a reported uncertainty that is erroneously too large. The correct value, computed according to the same assumptions made in [4], is reported in this paper. Summarizing, we have $B = (1.553 \pm 0.004)\times10^{-14}$. The orbital period is about 14 hours ($T = 50685$ s), and hence the expected period of the J2 signal is about 7 hours ($T_{J2} = 25342.5$ s).

We report here also the theoretical value of the amplitude of the J2 signal affecting the time offset, A. For the Galileo satellites on nominal orbit, we have $A = (6.26 \pm 0.02)\times10^{-11}$ s, i.e. about 0.06 ns, in agreement with the number declared in [5]. As a rule of thumb, an error of 1 ns in the estimation of signals' time of flights gives a position error at the level of 30 cm. Therefore, the J2 periodic signal may affect the navigation solution with an error at the level of about 2 cm, that could be relevant for high precision applications.

III. DATA ANALYSIS AND EXPERIMENTAL RESULTS

In this section we analyze Galileo space clock data looking for periodic frequency variations and in particular the J2 signal.

The data provided by ESA are the time offsets of the satellites' clocks with respect to a reference ground clock (typically an active hydrogen maser), that is not necessarily the same from day to day. Therefore, the first step of our preprocessing is to align the time offset data to a fixed, stable ground reference over the whole considered period. Note that periodic frequency variations correspond to periodic variations

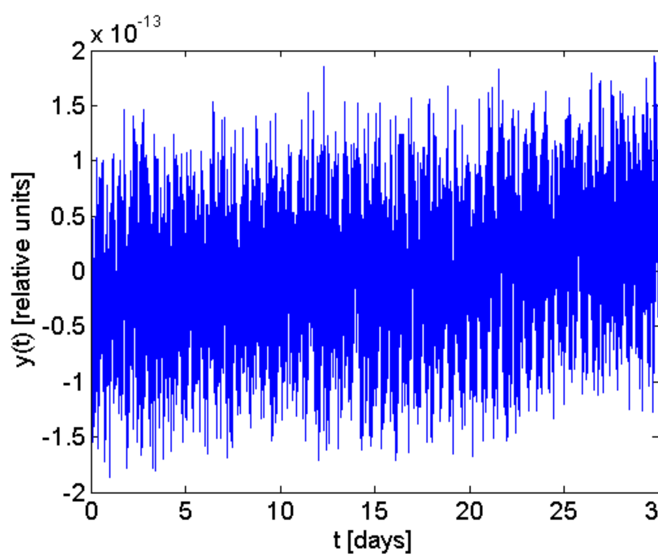

Fig. 1. Fractional frequency deviation of the passive hydrogen maser on board of the E24 Galileo satellite, for the month of January 2017 (outliers removed).

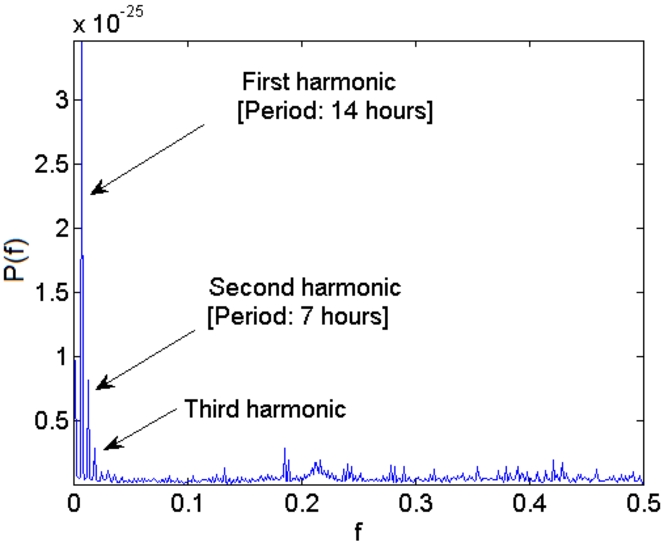

Fig. 2. Estimated power spectrum for the frequency deviation shown in Fig. 1. The peaks indicated by the arrows corresponds to the three harmonics discussed within the text.

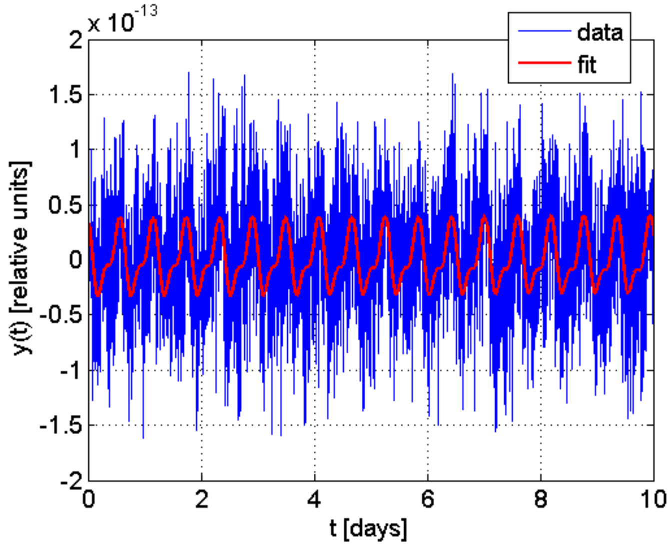

Fig. 3. Fit of the of the two periodic signals corresponding to the first two harmonics (peaks) of the power spectrum (red curve), along with the drift-removed fractional frequency deviation (blue curve).

of the time offset and vice versa, hence in principle the time offset data could be directly analyzed. However, time offset data are affected by a random walk due to the white frequency noise of the clocks, and also by day-boundary jumps due to the data processing applied by the analysis centers computing orbital and clock solutions. Therefore, the analyses performed in this work are based on the frequency offset data obtained by differentiating the time offset data. Indeed, the former are mainly affected by a white noise, and by outliers that can be easily identified an removed. In our preprocessing, the outliers are automatically detected and removed according to the approach proposed in [10]. The last step of the preprocessing stage is the estimation and removal of a linear frequency drift.

As a case study, in this work we focus on the Galileo satellite GSAT0205 (also known by its Satellite Vehicle ID, E24) and we analyze its frequency within the month of January 2017, when the satellite's master clock was a passive hydrogen maser. Fig. 1 shows the outliers-removed fractional frequency deviation of the E24 satellite's clock with respect to the Galileo Experimental Sensor Station (GESS) hosted at INRiM and connected to the local UTC time scale realization, namely UTC(IT), used as ground reference for the considered period. The data sampling time is $T_s = 300$ s.

We now analyze the preprocessed data with two different techniques. First, we compute the power spectral density to identify the main periodic signals affecting the data.

Fig. 2 shows the estimated power spectrum of the frequency deviation data represented in Fig. 1. We obtained this spectral estimate by using a Welch periodogram, computed with a rectangular window with $N_w = 840$ samples, and a 50% overlapping between consecutive windows. The horizontal axis is the normalized frequency, obtained by dividing the signal bandwidth by the sampling frequency $1/T_s$. Therefore, a sinusoidal component at the normalized frequency f_0, has a period of $T_0 = T_s/f_0$ seconds.

Aside from some background noise at medium and high frequencies, in the low frequency region the power spectrum reveals the presence of three sinusoidal components at the frequency values $f_1 = 0.006$ (first harmonic), $f_2 = 2f_1 = 0.012$ (second harmonic), and $f_3 = 3f_1 = 0.018$ (third harmonic). The period of the first harmonic is approximately 14 hours, and therefore corresponds to the satellite's orbital period. The period of the second harmonic is approximately 7 hours, and corresponds to the expected period of the J2 component. These three harmonics represent the spectral signature of a periodic component in the frequency deviation data, whose period corresponds to the period of the first harmonic (namely, 14 hours) and whose origin is possibly in the orbital estimation errors. The peak corresponding to the second harmonic is also due to the presence of the J2 periodic component.

The second technique is the one we already applied to the GPS clock data analysis described in [4]. We look for a period which is free from apparent clock non-stationarities, like frequency jumps or changes of frequency drift. Then, only if strictly necessary, we further elaborate the preprocessed data by filtering out possible stochastic long-term variations, using a moving average over a window of length N_{LT}. Note that, in order to avoid changing the amplitude of the periodic signals of interest, N_{LT} must be an integer multiple of the period of the first harmonic. Finally, we fit the data with a model function written as the sum of two independent sinusoids, with a total of six free parameters (i.e. amplitude, phase and frequency of each sinusoid). The aim is to fit the signals corresponding to the first two harmonics highlighted in Fig. 2 and to estimate their amplitudes, and in particular to give an estimate of the amplitude of the J2 signal.

In the case of E24, no major non-stationary behavior can be found within the analyzed period, so we fit the full one-month preprocessed data series without any further data processing. Fig. 3 shows the result of the fit (red curve) compared to the fractional frequency deviation of the clock (blue curve). For a better graphic rendering, it only shows the first ten days of the considered period.

The estimated periods of the fitted sinusoids are compatible with the true values of T and T_{J2}, hence we are actually fitting the signals corresponding to the first two peaks of Fig. 2. The estimated amplitudes are $A_1 = (2.82 \pm 0.08)\times10^{-14}$ for the first harmonic and $A_2 = (1.36 \pm 0.08)\times10^{-14}$ for the second harmonic plus the J2 signal. The ratio between the two is about 2, in agreement with the periodogram in Fig 2, where the ratio is approximately 4 because of the quadratic nature of the power spectrum. As a further confirmation that we are not just fitting the noise, we look also at the residuals: the standard deviation of the (zero-mean) fitted data is about 5.77×10^{-14}; the standard deviation of the residuals, when only the first harmonic is subtracted from the data, is reduced to about 5.41×10^{-14}; finally, when both the fitted sinusoids are subtracted from the data, the standard deviation of the residuals is further reduced to about 5.33×10^{-14}.

As already discussed in section I, A_2 should be considered as a biased estimate of the amplitude of the J2 signal, B. The bias is the amplitude of the second harmonic of the periodic signal due to the orbital estimation error, and the uncertainty of A_2 does not include it. Nonetheless, if we compare A_2 with B as obtained in section II, we note that the difference between the two is well below 3 sigma, hence the biased estimate is anyway in agreement with the theoretical value. As a final check, we fit the data with a model function written as a sinusoid with constrained amplitude and frequency, equal to the theoretical values computed for the J2 signal. We compare the residuals obtained in such case, to the residuals obtained when the signal with estimated amplitude A_2 is subtracted from the data: in both cases the standard deviation of the residual is practically the same, i.e. about 5.69×10^{-14}, less than the standard deviation of the fitted data, and confirming that the estimated signal is close to the expected J2 signal, despite the presence of a bias.

The estimation of such bias will be the subject of a future publication. In particular, as suggested in section I, we will try to minimize it by analyzing data from the period of the year in which the orbital estimation errors are smaller. This will be the period in which the first harmonic, as obtained from the power spectrum or other methods, is also minimized. Moreover, we foresee to extend the analysis also to other Galileo satellites, chosen among those using a passive hydrogen maser as master clock. We expect to observe a similar behavior (in terms of periodic variations) on all the satellites sharing the same orbital plane: for example, the period of the year in which the first harmonic is minimized, should be the same for all the satellites on the same orbital plane. If this is not the case, this could help in identifying other possible causes for the observed periodic variations, which could depend on the particular satellites.

IV. Conclusions

The power spectrum of the apparent clock frequency of the Galileo satellite E24 has been computed for the month of January 2017, based on the data kindly provided by ESA. It reveals the presence of three main sinusoidal components with periods of about 14, 7, and 4.7 hours. They are interpreted as the first, second, and third harmonic of a periodic signal with period of approximately 14 hours, mainly due to the orbital estimation errors. However, the peak visible at 7 hours is primarily due to the periodic component of the J2 relativistic effect, whose amplitude is estimated by fitting the data with a suitable periodic function. This estimate has a bias due to the presence of the second harmonic of the signal originated by the orbital estimation errors, but it is nonetheless compatible with the theoretical value. Indeed, the difference between the two is less than 3 sigma, where the estimated uncertainty does not include the bias.

The results obtained with this work represent a first validation of (5), which can be used as a time correction by the users who want to get rid of the J2 periodic effect and thus improve their positioning and timing solution. In particular, correcting the J2 effect would be useful for those applications requiring a positioning accuracy at the centimeter level, and also for the monitoring of GNSS on-board clocks, as already discussed in [4].

In our future works, we foresee to extend the analysis to other Galileo satellites, characterizing the variations of the power spectrum during a longer period, possibly one year. This would also allow to find a more precise estimate of the amplitude of the relativistic J2 periodic component, and to have a more comprehensive knowledge of the other observed periodic variations. Moreover, a separate study will be focused on the Galileo satellites GSAT0201 and GSAT0202, flying on a slightly different orbit with respect to the others and hence requiring a dedicated assessment of the J2 effect.

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