

Correction to: Material description of fluxes in terms of differential forms

Original

Correction to: Material description of fluxes in terms of differential forms / Federico, Salvatore; Grillo, Alfio; Segev, Reuven. - In: CONTINUUM MECHANICS AND THERMODYNAMICS. - ISSN 0935-1175. - 31:(2019), pp. 361-362. [10.1007/s00161-018-0699-6]

Availability:

This version is available at: 11583/2831760 since: 2020-06-09T12:17:21Z

Publisher:

Springer

Published

DOI:10.1007/s00161-018-0699-6

Terms of use:

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

Springer postprint/Author's Accepted Manuscript

This version of the article has been accepted for publication, after peer review (when applicable) and is subject to Springer Nature's AM terms of use, but is not the Version of Record and does not reflect post-acceptance improvements, or any corrections. The Version of Record is available online at: <http://dx.doi.org/10.1007/s00161-018-0699-6>

(Article begins on next page)

Correction to:
“Material description of fluxes in terms of differential forms”

“Dedicated to Prof. David Steigmann in recognition of his contributions”

**Salvatore Federico · Alfio Grillo ·
Reuven Segev**

Received: date / Accepted: date

- 1 DOI: <https://doi.org/10.1007/s00161-018-0699-6>.
2 Available online: August 1, 2018
3 Journal: *Continuum Mechanics and Thermodynamics* (2019) 31: 361-362

- 4 **Correction to: Continuum Mech. Thermodyn. (2016) 28:379–390**
5 **<https://doi.org/10.1007/s00161-015-0437-2>**

6 Although the final result presented in Equation (34) of our work [?] is correct,
7 the proof of Equation (34) contains an error. The text starting immediately
8 after Equation (33) with “When a metric tensor \mathbf{g} ...” and ending immediately
9 before Equation (36) with “... in the alternative notation” should be replaced
10 with the text below.

11 **Correction to the Proof of Equation (34)**

12 Let us assume that the space \mathcal{S} is equipped with a metric tensor \mathbf{g} , i.e., a
13 symmetric and positive-definite tensor field valued in $[T\mathcal{S}]_2^0$, defining the scalar
14 product of two vectors \mathbf{u} and \mathbf{v} as $\mathbf{u}\cdot\mathbf{v} = \mathbf{g}(\mathbf{u}, \mathbf{v})$. The metric \mathbf{g} induces the

S. Federico
Department of Mechanical and Manufacturing Engineering, The University of Calgary, 2500
University Drive NW, Calgary, AB T2N1N4, Canada
E-mail: salvatore.federico@ucalgary.ca

A. Grillo
DISMA - Department of Mathematical Sciences “G. L. Lagrange”, Politecnico di Torino,
Corso Duca degli Abruzzi 24, 10129 Torino, Italy

R. Segev
Department of Mechanical Engineering, Ben Gurion University, P.O. Box 653, 84105 Beer-
Sheva, Israel

15 *musical isomorphisms* $\flat : TS \rightarrow T^*S : \mathbf{v} \mapsto \flat(\mathbf{v}) \equiv \mathbf{v}^\flat$, which maps a vector \mathbf{v}
 16 with components v^c to a covector \mathbf{v}^\flat with components $g_{ac}v^c$, and its inverse
 17 $\sharp : T^*S \rightarrow TS : \boldsymbol{\alpha} \mapsto \sharp(\boldsymbol{\alpha}) \equiv \boldsymbol{\alpha}^\sharp$, which maps a covector $\boldsymbol{\alpha}$ with components α_c
 18 to a vector $\boldsymbol{\alpha}^\sharp$ with components $g^{ac}\alpha_c$, where g^{ac} are the components of the
 19 inverse of the matrix $\llbracket g_{ab} \rrbracket$ of \mathbf{g} . The isomorphism \sharp and the metric tensor \mathbf{g}
 20 induce the scalar product of covectors $\boldsymbol{\alpha} \cdot \boldsymbol{\beta} = \mathbf{g}(\boldsymbol{\alpha}^\sharp, \boldsymbol{\beta}^\sharp) = \boldsymbol{\alpha}(\boldsymbol{\beta}^\sharp)$.

21 The $(n-1)$ -dimensional tangent bundle Ts of the hypersurface s determines
 22 a 1-dimensional sub-bundle of T^*S containing the annihilators of Ts , i.e., the
 23 covectors $\boldsymbol{\nu}$ such that $\boldsymbol{\nu}\mathbf{u} \equiv \boldsymbol{\nu}(\mathbf{u}) = 0$, for every $\mathbf{u} \in Ts$. Moreover, using the
 24 scalar product of covectors, we can define the *unit normal covector* \mathbf{n} to the
 25 hypersurface s as the annihilating covector such that $\|\mathbf{n}\|^2 = \mathbf{n} \cdot \mathbf{n} = 1$.

26 The integral (33) of an $(n-1)$ -form $\boldsymbol{\omega}$ on the hypersurface s can be ex-
 27 pressed in terms of the axial vector field \mathbf{w} of $\boldsymbol{\omega}$ with respect to the volume form
 28 $\boldsymbol{\mu}$, i.e., \mathbf{w} is such that $\iota_{\mathbf{w}}\boldsymbol{\mu} = \boldsymbol{\omega}$. If we introduce the *axial projector* $\mathbf{a} = \mathbf{n}^\sharp \otimes \mathbf{n}$
 29 (in components, $a^a_b = n^a n_b$) and the *transverse projector* $\mathbf{t} = \mathbf{i} - \mathbf{n}^\sharp \otimes \mathbf{n}$ (in
 30 components, $t^a_b = \delta^a_b - n^a n_b$, where \mathbf{i} is the spatial identity tensor, it holds
 31 that $\mathbf{i} = \mathbf{a} + \mathbf{t}$ and that any vector field \mathbf{w} can be decomposed as

$$\mathbf{w} = \mathbf{i}\mathbf{w} = (\mathbf{a} + \mathbf{t})\mathbf{w} = \mathbf{a}\mathbf{w} + \mathbf{t}\mathbf{w} = \mathbf{w}_a + \mathbf{w}_t, \quad (\text{C1})$$

32 where $\mathbf{w}_a = \mathbf{a}\mathbf{w} = (\mathbf{n}\mathbf{w})\mathbf{n}^\sharp$ and $\mathbf{w}_t = \mathbf{t}\mathbf{w} = \mathbf{w} - (\mathbf{n}\mathbf{w})\mathbf{n}^\sharp$ are the axial and the
 33 transverse component of \mathbf{w} , respectively. By construction, \mathbf{w}_t is an element of
 34 the tangent bundle of the $(n-1)$ -dimensional manifold $s \subset S$. Hence, due to
 35 linearity, the $(n-1)$ -form $\boldsymbol{\omega} = \iota_{\mathbf{w}}\boldsymbol{\mu}$ can be written as

$$\boldsymbol{\omega} = \iota_{\mathbf{w}}\boldsymbol{\mu} = \iota_{(\mathbf{w}_a + \mathbf{w}_t)}\boldsymbol{\mu} = \iota_{\mathbf{w}_a}\boldsymbol{\mu} + \iota_{\mathbf{w}_t}\boldsymbol{\mu}. \quad (\text{C2})$$

36 Let now $\{\mathbf{u}_1, \dots, \mathbf{u}_{n-1}\} \subset Ts$ be a set of linearly independent vectors span-
 37 ning Ts . Since \mathbf{w}_t can be expressed as a linear combination of $\mathbf{u}_1, \dots, \mathbf{u}_{n-1}$,
 38 we obtain

$$(\iota_{\mathbf{w}_t}\boldsymbol{\mu})(\mathbf{u}_1, \dots, \mathbf{u}_{n-1}) = \boldsymbol{\mu}(\mathbf{w}_t, \mathbf{u}_1, \dots, \mathbf{u}_{n-1}) = 0. \quad (\text{C3})$$

39 Comparing Eq. (C3) with the definition of $\boldsymbol{\omega}$ in Eq. (C2), we find

$$\boldsymbol{\omega}(\mathbf{u}_1, \dots, \mathbf{u}_{n-1}) = (\iota_{\mathbf{w}}\boldsymbol{\mu})(\mathbf{u}_1, \dots, \mathbf{u}_{n-1}) = (\iota_{\mathbf{w}_a}\boldsymbol{\mu})(\mathbf{u}_1, \dots, \mathbf{u}_{n-1}), \quad (\text{C4})$$

40 and, since (C4) must hold true for all $(n-1)$ -tuples $\{\mathbf{u}_1, \dots, \mathbf{u}_{n-1}\} \subset Ts$, we
 41 can write

$$\boldsymbol{\omega} = \iota_{\mathbf{w}}\boldsymbol{\mu} \equiv \iota_{\mathbf{w}_a}\boldsymbol{\mu}, \quad (\text{C5})$$

42 i.e., only the axial component of \mathbf{w} , which is the component parallel to the vector
 43 \mathbf{n}^\sharp associated with the normal covector \mathbf{n} to the hypersurface s , contributes to
 44 $\boldsymbol{\omega}$. Finally, by exploiting the result $\mathbf{w}_a = (\mathbf{n}\mathbf{w})\mathbf{n}^\sharp = (\mathbf{w}\mathbf{n})\mathbf{n}^\sharp$ and the linearity
 45 of the interior product, Eq. (C5) becomes

$$\boldsymbol{\omega} = \iota_{\mathbf{w}}\boldsymbol{\mu} \equiv \iota_{\mathbf{w}_a}\boldsymbol{\mu} = (\mathbf{w}\mathbf{n})\iota_{\mathbf{n}^\sharp}\boldsymbol{\mu} = (\mathbf{w}\mathbf{n})\boldsymbol{\alpha}, \quad (34 \text{ corr.})$$

46 where

$$\boldsymbol{\alpha} = \iota_{\mathbf{n}} \boldsymbol{\mu} \quad (1)$$

47 is the $(n - 1)$ -form induced on the hypersurface s by the volume form $\boldsymbol{\mu}$ and
48 the metric \mathbf{g} . Therefore, on the basis of these results, the flux of an exten-
49 sive quantity q across the hypersurface s can be expressed in the alternative
50 notation [...]

51 References

- 52 1. S. Federico, A. Grillo, R. Segev, Material description of fluxes in terms of differ-
53 ential forms, *Continuum Mechanics and Thermodynamics*, **28**(1-2), 379-390 (2016)
54 <https://doi.org/10.1007/s00161-015-0437-2>