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Fuzzy Based Tool to Measure the Resilience of Communities

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ABSTRACT: Last decades have been characterized by an increasing number of disasters all over the world. Therefore, the concept of community resilience has attracted the attention of the scientists who started to explore and assess the ability of communities to recover after undesirable events. In this work, a method for assessing the earthquake community resilience based on the PEOPLES framework is presented. PEOPLES is a framework that defines community resilience using seven dimensions. Each of the dimensions is defined through a set of indicators to describe the different aspects of resilience. The exact evaluation of the indicators is usually not possible due to the lack of deterministic data related to the damaged system after the disaster. The proposed method exploits a knowledge-based fuzzy modelling to allow the quantitative evaluation of the PEOPLES indicators taking into account uncertainties. The output of the implemented fuzzy method is a resilience index.

1 INTRODUCTION

According to “The International Disaster Dataset” (EM-DAT), the number of natural disasters and the corresponding number of people affected, with related economic losses, have shown an upward trend in the last years. This implies that communities are often not sufficiently resilient to natural catastrophes. Consequently, the concept of resilience has been deepened in the engineering field to assess the ability of a community to recover after an undesirable event. Indeed, since the adoption of the Hyogo framework in (Manyena 2006), strategies involved in hazard planning and disaster risk reduction have experienced a paradigm shift from a vulnerability assessment approach to a resilience-based approach (Mayunga 2007).

Since the concept of resilience is applicable in several disciplines, different definitions are available in the literature. Cimellaro et al. (2016) have conducted a comprehensive review on this topic. In their work, the resilience definition provided by Bruneau et al. (2003) emerges: resilience is “the ability of social units to mitigate hazards, contain the effects of disasters when they occur, and carry out recovery activities in ways to minimize social disruption”. This definition has been later improved by Cimellaro et al. (2010) who define resilience as: “a function indicating the capability to sustain a level of functionality or performance for a given building, bridge, life-line network, or community, over a period defined as the control time (T_C) that is usually decided by own-

ers, or society (usually is the life cycle, life span of the system etc.)”. Thus, resilience can be defined analytically as the area under the serviceability performance curve $Q(t)$ of a system, normalized accordingly to the considered control time (T_C):

$$R = \int_{t_1}^{t_r} \frac{Q(t)}{T_C} dt \quad (1)$$

where R is the resilience index; $Q(t)$ is the system functionality at time t ; t_1 is the moment when the disturbance occurs and the system functionality drops from its initial value q_0 to q_1 ; t_r is the moment when the initial serviceability is completely recovered and equal to q_r ; T_C is the control time. Figure 1 shows the performance curve derived using Equation 1. The serviceability $Q(t)$ ranges between 0% and 100% to indicate the complete absence of functionality of the service and its complete effectiveness, respectively.

Here, community resilience is evaluated exploiting a novel method, which benefits from the PEOPLES framework (Renschler et al. 2010, Cimellaro et al. 2016). PEOPLES is a layered framework: each dimension is divided into components, and each component is divided again into a set of indicators. (Kammouh et al. 2017b) conducted a comprehensive review on community resilience indicators that led to identifying 115 of them. They also proposed a method to transform the framework from a qualitative to a quantitative model through the measurement of indicator parameters. However, in specific scenarios,

some indicators may be difficult to obtain and quantify, as well as the interdependency among them. In order to track and represent such uncertainties, a method that enhances the one proposed by Kammouh et al. (2017b) is presented herein. The method introduces a fuzzy logic-based modelling of PEOPLES indicators that accounts for the uncertainties involved in the assessment process.

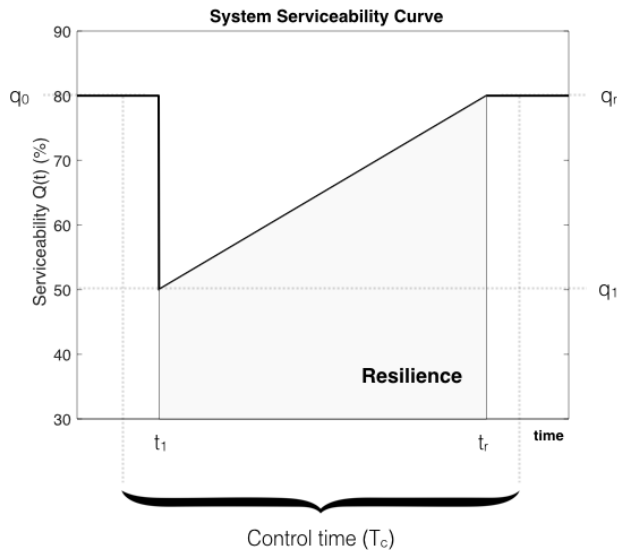


Figure 1. Example of a serviceability function and resilience evaluation.

2 PEOPLES FRAMEWORK

Challenges to make the concept of resilience operational have been addressed by Cimellaro et al. (2016) who defined the so called PEOPLES framework. The framework has been developed at the Multidisciplinary Center of Earthquake Engineering Research (MCEER) and has enhanced the initial research on resilience characteristics. By evaluating the functionality performance of community components (both in time and space), this framework guarantees a qualitative assessment of the community resilience. The acronym “PEOPLES” stands for seven community dimensions:

- 1 Population and demographics: it includes parameters that describe the social-economic composition of the community. This dimension measures the social vulnerability that could hinder the functionality of the emergency and recovery systems (e.g. population density, age distribution, presence and integration of minorities and socio-economic status.)
- 2 Environment and ecosystem: it estimates the capability of the environment and of the ecosystem to get back to its pre-hazard conditions. It includes water, air and soil assessments as well as a measure of the biodiversity and the sustainability relations.

- 3 Organized government services: it covers the services that the government guarantees before and after an extreme event. A great importance is given to the mitigation and recovery processes, which include the preparedness to hazards and all disaster risk reduction measures.
- 4 Physical infrastructure: it considers the buildings and facilities that are the prevalent interests of civil engineers and traditional resilience analysis. Particularly, two different aspects are analyzed in this dimension: facilities, which includes housing and services which are not crucial for the emergency response, and lifelines, which instead consists of the services that are of vital importance for the management of critical situations.
- 5 Lifestyle and community competence: This dimension takes into account the capability of a community to face problems by means of political partnerships. This includes both the abilities of a community (i.e. the skills of their components) and its perceptions (i.e. the judgements and feelings that a community has on itself.)
- 6 Economic development: it describes the economic situation of the community. It can be easily divided in two terms, a static component, which measures the present economic condition, and a dynamic one, which instead takes into account the development and economic growth of the community.
- 7 Social-cultural capital: This last dimension contains an evaluation of the community’s attitude to react to disasters and to return to the pre-event conditions. It includes a lot of subcategories that measure the people’s commitment in the community and the social-cultural heritage.

3 FUZZY LOGIC

Zadeh (1965) introduced the concept of fuzzy set and the theory behind it. This theory comes with the absence of any mathematical framework that is able to describe the complexity and vagueness included in processes where human intervention is significant. While in the crisp logic the variables belong only to one class, in the fuzzy logic a variable x can be a member of several classes (fuzzy sets) with different membership grades (μ). Thus, each fuzzy set is characterized by a membership function that associates to any input x a real number (μ) ranging between 0 (x does not belong to the fuzzy set) and 1 (x completely belongs to the fuzzy set) (Zadeh 1965). The strength of inference systems based on fuzzy logic relies on the following two main aspects:

- fuzzy inference systems can handle both descriptive (linguistic) knowledge and numerical data;
- fuzzy inference systems exploit approximate reasoning algorithm to formulate relationships between inputs by which uncertainties can be prop-

agated throughout the whole process (Tesfamariam & Saatcioglu 2008a)

Designing a fuzzy logic-based system follows two fundamental steps: 1) defining the membership functions and the fuzzification process; 2) designing the fuzzy inference system. Fuzzy methods have been widely developed and applied in several fields (Ross 2009). In the context of earthquake engineering, fuzzy methods have been exploited in different applications (e.g. (Tesfamariam & Saatcioglu 2008a, b, Tesfamariam & Saatcioglu 2010, Tesfamariam & Sanchez-Silva 2011, Tesfamariam & Wang 2011)). Fuzzy methods have been widely used also for developing structural control systems (Casciati et al. 2004). A clear procedure for the application of fuzzy logic can be found in (Tesfamariam & Saatcioglu 2008b).

A fuzzy logic system (FLS) can be defined as the nonlinear mapping of an input data set to a scalar output data. A FLS consists of four main parts: fuzzifier, rules, inference engine, and defuzzifier (Figure 2).

The process of fuzzy logic is: a crisp set of input data are gathered and converted to a fuzzy set using fuzzy linguistic variables, fuzzy linguistic terms and membership functions. This step is known as fuzzification. Afterwards, an inference is made based on a set of rules. Lastly, the resulting fuzzy output is mapped to a crisp output using the membership functions, in the defuzzification step.

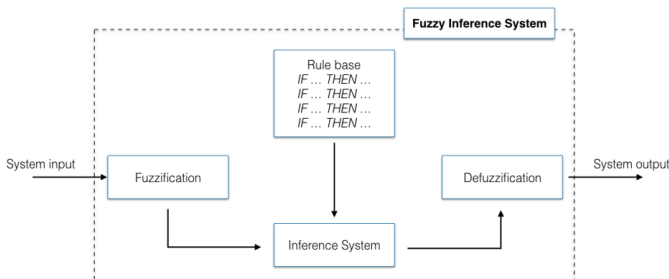


Figure 2. The Fuzzy inference system

3.1 Fuzzification

The basic input parameters have a range of values that can be clustered into linguistic quantifiers, for instance, very low (VL), medium low (ML), medium (M), medium high (MH) and very high (VH). The process of assigning linguistic values is a form of data compression and it is called *granulation*. The fuzzification step converts the input values into a homogeneous scale by assigning corresponding membership functions with respect to their specified granularities (Tesfamariam & Saatcioglu 2008b).

Membership functions are used in the fuzzification and defuzzification steps of a FLS, to map the non-fuzzy input values to fuzzy linguistic terms and vice

versa. A membership function is used to quantify a linguistic term. Note that, an important characteristic of fuzzy logic is that a numerical value does not have to be fuzzified using only one membership function. In other words, a value can belong to multiple sets at the same time. There are different forms of membership functions such as triangular, trapezoidal, piecewise linear, Gaussian, or singleton. The most common types of membership functions are triangular, trapezoidal, and Gaussian shapes. The type of the membership function can be context dependent and it is generally chosen arbitrarily according to the user experience (Mendel 1995).

3.2 Fuzzy Rules

The fuzzy rule base (FRB) is derived from heuristic knowledge of experts or historical data to define the relationships between inputs and outputs. The most common type is the *Mamdani* type, which is a simple IF-THEN rule with a condition and a conclusion. For instance, considering two inputs, the i^{th} rule has the following formulation:

$$R : IF x_1 \text{ is } A_1 \text{ AND } x_2 \text{ is } A_2 \text{ THEN } y \text{ is } B \quad (2)$$

where x_1 and x_2 are the inputs variable, A_1 and A_2 are input sets, y is the output, B is the output set. The completeness of a fuzzy model is determined by the description of the behaviour for all possible input values and requires a large number of rules. The rule base is the union of all the rules:

$$R = \bigcup_{i=1}^n R_i = R_1 \text{ ALSO } R_2 \text{ ALSO } \dots \text{ ALSO } R_n \quad (3)$$

In some cases it is possible to regulate the degree of influence of each rule on the final output. This can be done by adding weightings based on priority or consistency, in a static or in a dynamic way.

3.3 Fuzzy Inference System (FIS)

After evaluating the result of each rule, the results are combined to obtain a final output. This process is called *inference*. Several accumulation methods can be used to combine the results of the individual rules. The maximum algorithm is generally used for accumulation. The evaluations of the fuzzy rules and the combination of the results of the individual rules are performed using fuzzy set operations. The operations on fuzzy sets are different with respect to the operations on non-fuzzy sets.

3.4 Defuzzification

After the inference step, the overall result is a fuzzy value. This result should be defuzzified to obtain a

final crisp output. This is the purpose of the defuzzifier component of an FLS. The defuzzification represents the inverse of the fuzzification process. It is performed according to the membership function of the output variable. There are several techniques to perform the defuzzification such as centre of gravity, centre of area, and mean of maximum methods.

4 METHODOLOGY

Recently, (Kammouh et al. 2017b) have developed a novel method to assess the resilience of communities based on the PEOPLES framework. The researchers provided a way to analytically quantify the resilience of communities through the use of indicators. A list of 115 resilience indicators representing the different aspects of a community have been identified. A full list of indicators can be found in (Kammouh et al., unpubl) while a part of the list can be found in (Kammouh et al. 2017b). Each indicator has been defined and computed using a set of parameters: the functionality before the disaster (q_0), the functionality after the disaster (q_1), the functionality after the restoration phase (q_r), and the recovery time (T_r) (see Figure 1). Moreover, a target value (TV) has been identified for each indicator in order to normalize q_0 , q_1 and q_r between 0 and 1 to have a computable and comparable dataset. Then, a serviceability function for the whole community is derived from the aggregation of these measurements, where the weighting scheme is defined according to the indicator importance. Finally, the community resilience is computed as the area under the community serviceability curve using Equation (1).

The methodology previously described represents a major break-through in the context of community resilience evaluation. In spite of being very simple, it can be applied to large communities and can serve as a powerful tool in preliminary decision-making processes related to natural catastrophic events. Nevertheless, this method is operable only if indicators can be numerically quantified, which may not be the case in some scenarios.

In this paper, a method that enhances the one proposed by (Kammouh et al. 2017b) is proposed. The method exploits a fuzzy logic-based modelling of PEOPLES indicators in order to deal with uncertainties and missing knowledge. In the following, the fuzzy modelling of PEOPLES indicators and the evaluation of community resilience using information gathered through the fuzzy inference system are discussed. Different approaches are proposed to match different levels of complexity, starting from a two-parameter approach then four-parameter approach and ending with a full translation of the PEOPLES framework. The proposed methodologies are not fully interchangeable and so only one of them

should be selected in accordance with the needed details level.

5 TWO-PARAMETER APPROACH

This approach adopts only two of the four serviceability parameters described before, namely serviceability initial drop q^* (previously referred to as q_0) and recovery time T^* (previously referred to as T_0). Fuzzy parameters have been chosen based on the research by (Bruneau et al. 2003) who describes the resilience of a system using the following three indicators: reduced failure probability; reduced consequences from failure; reduced time to recovery. The reduced failure probability has not been taken into account as it is not easily related to the herein adopted mathematical definition of resilience, which considers only the failure consequence q^* and the repair time T^* . Figure 3 presents a hierarchy of the two-parameter approach where both *time* and *initial drop* variables are used as inputs for the fuzzy system. The inputs are combined using a set of rules to obtain the output variable *fuzzy resilience*. The fuzzy output is defuzzified to get a crisp value that serves as a resilience index for the corresponding indicator.

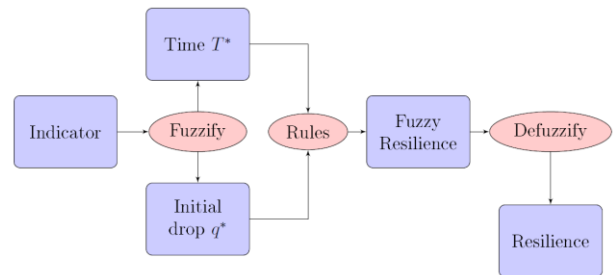


Figure 3. Schematic representation of the two-parameter approach.

5.1 Evaluating the initial serviceability drop q^* :

In the authors' opinion, two trapezoidal membership functions can be reasonably adopted in the present case. They are termed as "High" and "Low". The fuzzification used for q^* is [High; Low] \rightarrow [(0, 0, 0.35, 0.65); (0.35, 0.65, 1, 1)]. The membership functions are graphically shown in Figure 4.

5.2 Evaluating the recovery time T^* :

When speaking of recovery, the intent is the *full* recovery. Outperforming, or non-complete recovery, as indicated by (Cimellaro et al. 2010), are not generally predictable and therefore they are not included here. For the time variable T^* , three membership functions are suggested by the authors, namely: "short"; "long"; and "very long". The time variable is normalized based on a 3-year time span, which is normally the time reference for civil applications (i.e., 3 years corresponds to 1 on the horizontal axis).

Figure 5 shows the membership functions chosen by the authors. The membership functions are not symmetrical as they have been constructed to the favor of the “Long” and “Very Long” memberships. That is, high range of values of the restoration time T^* variable corresponds to the membership functions “Long” and “Very Long”.

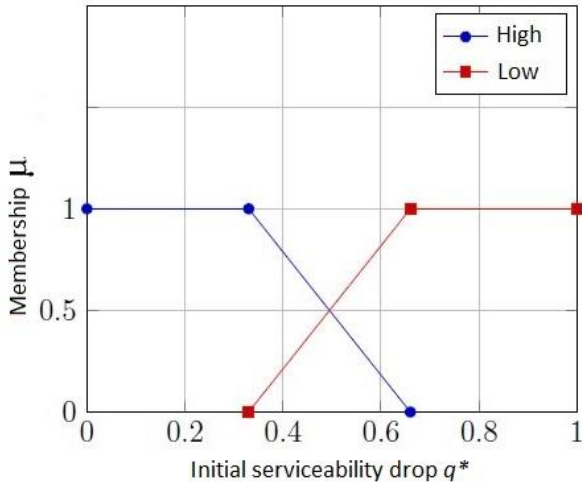


Figure 4. Membership functions for the serviceability variable q^* .

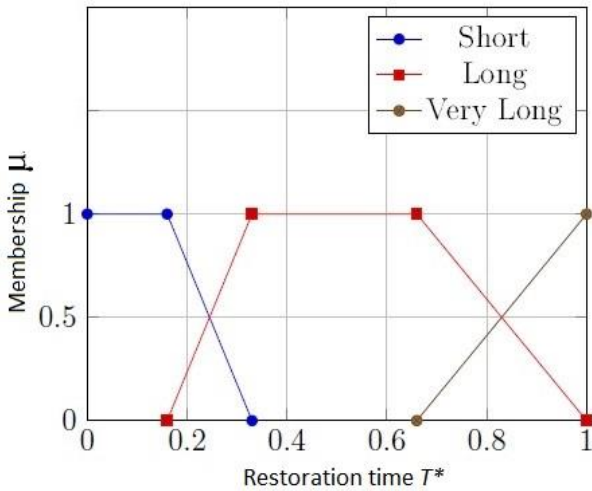


Figure 5. Membership functions for the downtime variable T^* .

The aim is to translate the given input variables (q^* and T^*) into one resilience measure R . This measure is itself fuzzy and so it is defined by a membership function. The chosen membership functions are depicted in Figure 6. Following the fuzzy approach, it is possible to define an output value calculated from the provided inputs on basis of a set of rules. The rules adopted in this study to relate the inputs and the output are shown in Table 1.

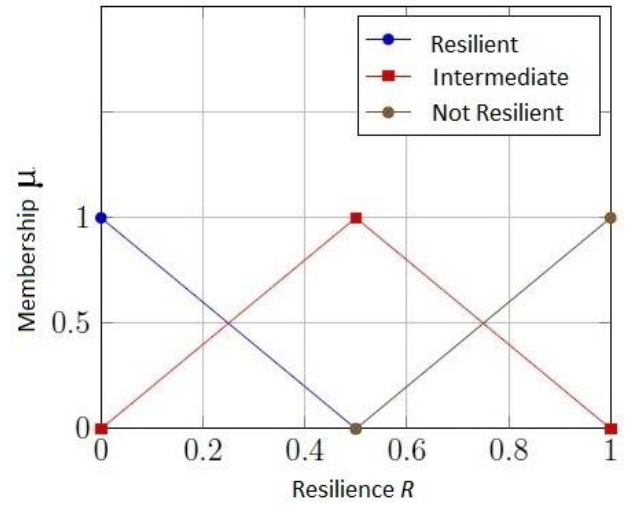


Figure 6. Membership functions for the resilience variable R .

Table 1. Fuzzy rule base for resilience.

T^*	q^*	R
short	high	resilient
Long	high	resilient
Very long	high	intermediate
Short	low	intermediate
long	low	not resilient
Very long	low	not resilient

5.3 Defuzzification

The fuzzy output variable is translated (defuzzified) into a numerical value that serves as a measure for resilience. Different methods for defuzzification can be used (Manyena 2006) such as center of gravity, weighted average, mean-max, center of largest area etc. The use of one method rather than another is dependent on the application. Here, the center of gravity method given in Equation 4 is used.

$$CoA = \frac{\int_{x_{min}}^{x_{max}} f(x) \cdot x \, dx}{\int_{x_{min}}^{x_{max}} f(x) \, dx} \quad (4)$$

where $f(x)$ is the function that shapes the output fuzzy set after the aggregation process and x stands for the real values inside the fuzzy set support ($[0, 1]$).

5.4 Weighting

The methodology introduced above applies to each indicator apart. It is often the case to aggregate different indicators into a single measure (i.e. community resilience) through a hierarchical structure. Usually, indicators contribute differently towards resilience and this necessitates weighting them according to their contribution. Different weighting schemes can be applied (Kammouh et al. 2017a, Kammouh et al., in press, Kammouh et al. 2017b). The one used in (Kammouh et al. 2017b) is here

adopted. This can be performed by simply allocating an importance factor (I) ranging between 1 and 3 to each indicator then applying the weighted average rule as follows:

$$R = \frac{\sum_{i=1}^N I_i R_i}{\sum_{i=1}^N I_i} = \sum_{i=1}^N w_i R_i \quad (5)$$

where R is the community resilience measure, R_i is the resilience measure of the i^{th} indicator, I_i is the corresponding importance factor (which can be scaled to preference), and w_i is the weighting factor of the i^{th} indicator. The difference with what was proposed in (Kammouh et al. 2017b) is that in this methodology the serviceability functions are translated into a resilience value before applying the weighting method (Figure 7). This simplifies the fuzzy system as it reduces the number of variables that need to be handled. Another approach that considers the weighting factors within the fuzzy system is introduced later in the paper.

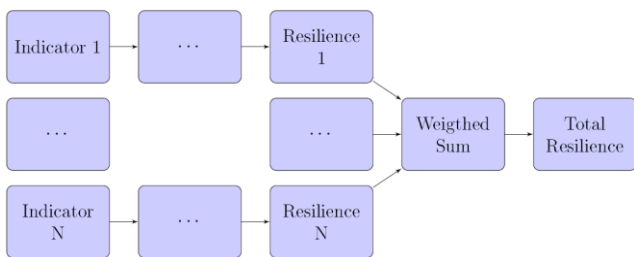


Figure 7. Hierarchical scheme of the fuzzy system with the weighting process.

Practical examples on the application of the fuzzy method to several case studies can be found in (Tsfamariam & Saatcioglu 2008a, b, Tsfamariam & Saatcioglu 2010). The same procedure is applied in this research but with the new variables and membership functions.

6 FOUR-PARAMETER APPROACH

Considering only two parameters to represent a resilience indicator may in some cases be insufficient, thus affecting the mentioned benefits of using the Fuzzy approach. Moreover, this may oversimplify the problem especially when specific information about the structure itself is available and to be added. For this reason, in certain cases it may be beneficial to build up the resilience curve from fuzzy parameters other than the recovery time and the initial drop. In fact, it has been pointed out by Comerio (2006) that further distinction in the repair time is possible. According to his work, the following parameters should be taken into account:

- Construction repair time;
- Mobilization time;

- Economic conditions of the interested region;

The mobilization time in particular, labelled as “irrational” in (Comerio 2006) (e.g. financing, workforce availability, relocation of functions or regulatory changes), is often not properly accounted for and therefore it should be given a special attention when evaluating downtime.

These three indicators may be fuzzyfied with a structure similar to the one adopted for the recovery time T^* . The result is similar to what shown previously with the only difference that new rules and membership functions are to be assigned to the new variables. When resilience measures are calculated, weighting is performed to obtain the system (community) resilience.

7 FULL PEOPLES

Most of the concepts described previously remain valid here. The only difference is that the approach introduced in this section includes the weighting of the variables within the fuzzy system. Normally, choosing adequate weighting factors is subjective and includes uncertainty. Although the inclusion of the weighting factors within the fuzzy system may add additional complexity as more variables are considered, it is certainly beneficial as it solves the uncertainty problem related to the weighting factors. To do that, two alternatives are proposed:

- Include the importance factor in the definition of the rules governing the fuzzy logic. In other words, assign rules such that the output is strongly related to the indicators with highest importance;
- Translate the importance factor into a fuzzy variable itself and include rules for it.

In both cases, rules have to be adapted to account for the importance factors. In the former case, rules are firmly tight to the particular application (i.e. hard to modify and not flexible); the latter case is, in this respect, more flexible but at the cost of additional complexity since additional rules have to be added to include the effect of the importance factor. This approach will be further developed and case studies will be added in future work. Figure 8 shows the logic flow where the weighting process is included as a separate variable in the first step before fuzzification:

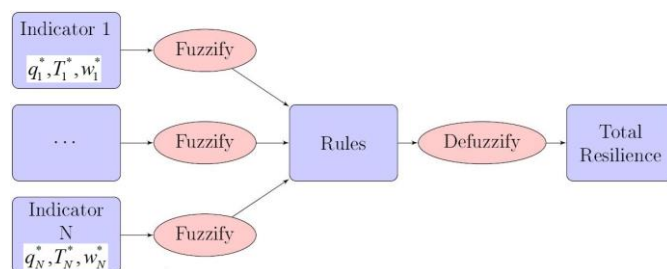


Figure 8. Full PEOPLES approach general hierarchical scheme with the weighting process included in the fuzzy system as a separate variable.

8 CONCLUSION

This paper extends the work previously done on the PEOPLES framework to evaluate the resilience of communities. It takes advantage of the fuzzy theory to account for the uncertainties involved in the evaluation of the PEOPLES' indicators. Different approaches are herein proposed depending on the level of complexity sought and the application type. The former considers only two fuzzy variables and it is intrinsically simple. However, oversimplification can affect the accuracy of the community resilience evaluation. A more comprehensive approach which includes a larger number of parameters with essential benefits in the resilience assessment at the cost of more demanding computational efforts is proposed. Possible developments in the application of weighting factors are also suggested for aggregating different indicators into a single community resilience measure.

9 ACKNOWLEDGMENT

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