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Level-crossing statistics of a passive scalar dispersed in a neutral boundary layer

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Abstract

The concentration of a passive scalar dispersed in a turbulent flow exhibits a complex stochastic dynamics. In this paper, we present a minimalist stochastic model that resembles the concentration statistics of a passive scalar emitted from a localized source in a neutral boundary layer. The model provides closed forms for the crossing rates and times – the mean frequency of exceeding a certain concentration level and the mean time above it–. Three concentration statistics are needed as model inputs: the mean, the standard deviation, and the integral scale. By giving analytical relationships also for these statistics, we provide a completely closed methodology that may serve as a rapid and practical tool to estimate the dynamics of a pollutant dispersed in the atmosphere. Results are validated against wind-tunnel measurements.

Keywords: Crossing rates, Crossing Times, Gamma distribution, Analytical relationships, Turbulent dispersion

1. Introduction

1 Turbulent flows are responsible for the chaotic mixing of many “sub-
2 stances” of natural and anthropic origins. Pollutants, heat, air moisture and
3 combustible chemicals are just some examples. In many cases, the substance
4 does not affect the fluid flow, so that it may be referred to as a *passive scalar*.
5 On the opposite, the fluid flow causes the passive scalar to exhibit a complex
6 turbulent dynamics (Fig. 1), whose many physical and statistical aspects
7

8 still need to be unveiled. For the wide-ranging implications of scalar turbu-
9 lence, many reviews have been dedicated to the subject in the last years, see
10 [1, 2, 3, 4, 5] and references therein.

11 In the atmospherical sciences, the crucial features of scalar turbulence re-
12 gard the statistics of pollutant and odour concentrations due to both natural
13 and anthropogenic releases. The knowledge of these statistics is necessary,
14 for instance, to determine the risk for human health generated by a toxic
15 substance [6, 7, 8] or the level of annoyance induced by a nuisance odor
16 [9, 10, 11].

17 Regarding the one-point statistics of the passive scalar concentration
18 $C[\mathbf{x}, t]$, several analytical models for the probability density function (PDF)
19 have been tested in the last decades against laboratory and field data [e.g.,
20 12, 13, 14]. The conclusion on which distribution better fits the data usually
21 depends on the experimental setup. Yet, recent results [e.g 15, 16, 17, 18]
22 have been converging on the choice of the Gamma distribution as the best
23 fit for the PDF of a passive scalar concentration released from a point source
24 in a neutral boundary layer

$$p_{\Gamma} = \frac{\lambda^{\lambda} C^{\lambda-1}}{\Gamma[\lambda] \mu^{\lambda}} e^{-\lambda C/\mu}, \quad (1)$$

25 where $\lambda = \mu^2/\sigma^2$, μ is the mean value, σ^2 the variance, and $\Gamma[\cdot]$ is the Gamma
26 special function [19]. Furthermore, the Gamma distribution has also been
27 observed to well fit the one-point PDF of concentration in confined turbulence
28 [20, 21]. For practical goals, the Gamma distribution is an encouraging result
29 as by just defining the first two statistical moments of C , all the one-point
30 statistics can be defined in an analytical and expeditious way.

31 Nonetheless, the knowledge of the PDF does not provide any information
32 on the temporal dynamics of the concentration, which is fundamental for
33 several purposes. For example, the exposure times are necessary to spec-
34 ify the risk for human health related to an airborne toxic substance –toxic
35 load=concentration×exposure time– [e.g. 7, 8]. Additionally, the annoyance
36 induced by nuisance odours, which are nowadays classified as atmospheric
37 pollutants by several jurisdictions [22], is controlled by the frequency of oc-
38 currence of whiffs. In fact, the human nose becomes insensitive to smells to
39 which is continuously subjected, so that low concentration smells at irreg-
40 ular intervals of time are actually more disturbing than a constant higher-
41 concentration smell [23].

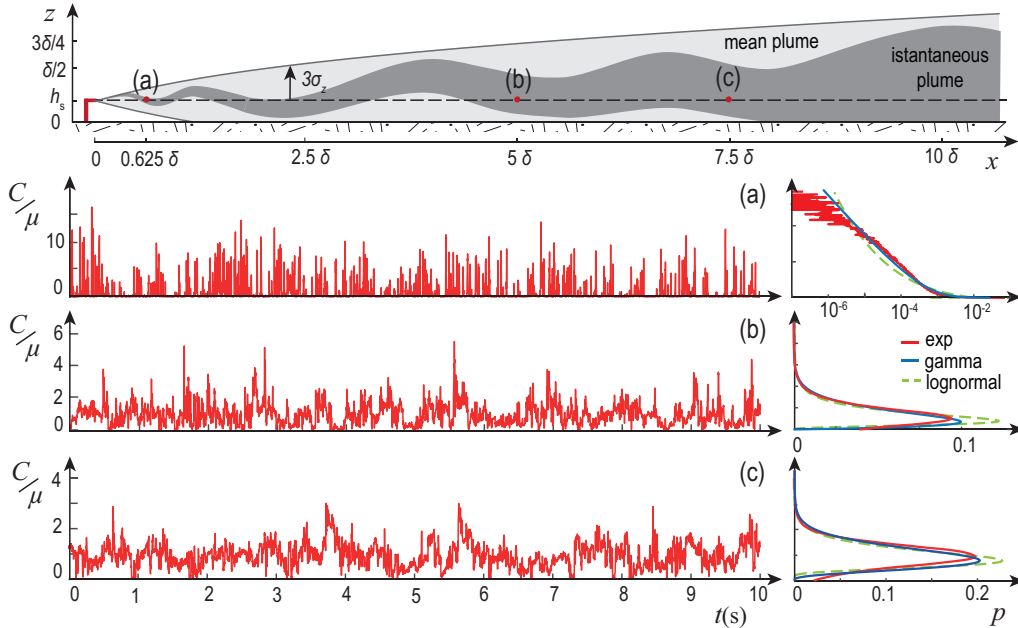


Figure 1: Sketch of the emission conditions and examples of experimental concentration time-series with their PDFs. The experimental PDFs are in solid red lines. The Gamma and Lognormal distributions are in solid-blue and dashed-green lines, respectively. $t(s)$ is time in seconds. Notice that only the first 10 seconds of the 15 minutes concentration series are shown (Elevated Source with 6 mm diameter from Bertagni et al. [18]). δ and h_s are the boundary-layer and source heights, respectively).

42 The first attempt to address the temporal statistics of a passive scalar
 43 involved a fluctuating plume model calibrated with experiments [24]. Suc-
 44 cessively, some studies [25, 26] tried to use Rice’s theory [27] to relate the
 45 upcrossing rates to the joint PDF of the concentration and its time deriva-
 46 tive. However, this latter PDF is generally unknown, making Rice’s theory
 47 difficult to apply. A notable exception was provided by Yee [28], who derived
 48 closed relationships for the upcrossing rates (and times) by using Rice’s the-
 49 ory under the assumption of a Lognormal PDF for the in-plume concentration
 50 fluctuations (see Appendix A for further details). Yet, the Gamma distri-
 51 bution (1) is usually a better model than the Lognormal, as shown in Fig. 1
 52 and also pointed out in previous publications [e.g. 16, 29].

53 More recently, the research has focused on numerical stochastic models
 54 [30, 31, 32, 33], which nicely reproduce the concentration time-series, but
 55 offer no analytical solution for the level-crossing statistics. In general, these

56 stochastic models require a PDF and a time-scale to be set. These quantities
57 are usually evaluated from experiments, empirical relations or Lagrangian
58 micro-mixing models [33].

59 In this Paper, we provide a simple stochastic model for the concentration
60 dynamics in which the steady-state PDF is the Gamma distribution (1) and
61 the crossing rates and times, i.e., the mean frequency of exceeding a certain
62 concentration level and the mean time above it, are given in closed form.
63 Three one-point statistics need to be set in the model: the mean μ , the
64 standard deviation σ , and the temporal integral-scale τ . The latter is defined
65 as the integral of the autocorrelation function of C , and can be interpreted
66 as the temporal memory of the one-point concentration dynamics [34]. First,
67 we use wind-tunnel data [16, 18] to evaluate the triad (μ, σ, τ) and to verify
68 the analytical relationship for the crossing rates and times (see the Appendix
69 B for a brief description of the experimental setup). Second, we evaluate the
70 triad (μ, σ, τ) through analytical relationships, among which the one for the
71 Eulerian time-scale τ is a novelty. In this way, we provide a fully closed model
72 for the evaluation of the recurrence statistics of a passive scalar dispersed in
73 a turbulent flow.

74 2. The Stochastic Model

75 According to a well-established theoretical framework [35], the turbulent
76 dispersion of a fluctuating plume is phenomenologically driven by two main
77 physical processes: the transport by turbulent eddies of the plume centroid,
78 or centre of mass, and the relative dispersion around it. The former pro-
79 cess, also referred to as *meandering*, is fundamental in the proximity of the
80 source, where the plume has a small size and is transported as a whole by
81 turbulent eddies. The resulting one-point concentration time-series (Fig. 1a)
82 exhibits a very intermittent signal with random shots induced by the pas-
83 sage of the turbulent eddies transporting the passive scalar. Very far from
84 the source, the plume has spread enough to englobe these eddies, so that
85 the intermittent action of the meandering process becomes negligible with
86 respect to the homogenization induced by the *relative dispersion* (Fig. 1c).
87 In between the near and the far field, the intermediate plume size causes
88 both processes –meandering and relative dispersion– to be essential in the
89 concentration dynamics (see Fig. 1b, where the low-intensity shots induced
90 by the meandering are still recognizable).

91 From these considerations, we define a stochastic model for the concen-
 92 tration dynamics that takes into account the two physical processes and
 93 guarantees the Gamma distribution (1) as the steady-state PDF. This is the
 94 Compound Poisson Process (CPP) with linear losses

$$dC = -\frac{C}{\tau}dt + d\zeta, \quad (2)$$

95 where t is time and τ is the integral time-scale. The stochastic term $d\zeta$ is
 96 a white shot noise [e.g. 34] that represents the sequence of pulses at ran-
 97 dom times induced by the turbulent eddies (meandering). The shot intensity
 98 and the time interval between subsequent shots are extracted from space-
 99 dependent exponential PDFs with mean values σ^2/μ and $\tau\sigma^2/\mu^2$, respec-
 100 tively [e.g. 36]. The deterministic part of (2) recalls the relative-dispersion,
 101 or micro-mixing, models [e.g. 37, 38]), but without the relaxation of the
 102 concentration towards a mean value.

103 A crucial advantage of the CPP is analytical tractability. In particular,
 104 the upcrossing time T_ϕ^+ , which is the average time C stays above a certain
 105 threshold level ϕ , is known in closed form

$$T_\phi^+ = \tau e^{\phi/\mu} E[1 - \lambda, \lambda \phi/\mu], \quad (3)$$

106 where $E[n, m] = \int_1^\infty \exp[-m s]/s^n ds$ is the exponential integral function [19].
 107 The upcrossing rate N_ϕ^+ , which is the mean frequency of upcrossing the
 108 threshold level ϕ , can be readily obtained as

$$N_\phi^+ = \frac{P_\phi^+}{T_\phi^+} = \frac{(\lambda\phi/\mu)^\lambda \exp[-\lambda\phi/\mu]}{\tau \Gamma[\lambda]}, \quad (4)$$

109 where P_ϕ^+ is the probability of $C > \phi$, known from eq. (1). Equivalently,
 110 one could address the downcrossing rate N_ϕ^- and time T_ϕ^- , which are the
 111 mean frequency of downcrossing the level ϕ and the average time below it.
 112 In particular, $N_\phi^+ = N_\phi^-$, and thus $T_\phi^- = T_\phi^+ P_\phi^-/P_\phi^+$, where $P_\phi^- = 1 - P_\phi^+$ is the
 113 probability of $C < \phi$. However, for the more important practical purposes, we
 114 herein focus the analysis on the upcrossing statistics (in the paper we often
 115 use the term *crossing* in place of *upcrossing*).

116 Equations (3) and (4) provide an easy and ready-to-use tool to evaluate
 117 the upcrossing times and rates in every spatial point of interest starting from
 118 the triad (μ, σ, τ) .

119 **3. Analytical closures**

120 To provide a closed-methodology to evaluate the crossing times (3) and
 121 rates (4), we here give the analytical relationships for the triad (μ, σ, τ) .

122 *The mean μ .* For a passive scalar released from a point source at $(x, y, z) =$
 123 $(0, 0, h_s)$, the mean field μ is well reproduced by the classical Gaussian model

$$\mu = c \exp \left[-\frac{y^2}{2\sigma_y^2} \right] \left(\exp \left[-\frac{(z - h_s)^2}{2\sigma_z^2} \right] + \exp \left[-\frac{(z + h_s)^2}{2\sigma_z^2} \right] \right), \quad (5)$$

where $c = \dot{M}/(2\pi\sigma_y\sigma_zU_s)$, U_s is the mean velocity at the source height, and \dot{M} is the passive scalar mass flux emitted at the source. The presence of the lower boundary has been included in (5) through a mirror imaginary source at $z = -h_s$ [e.g. 39]. σ_y and σ_z define the transversal and vertical mean plume spread, which, in the absence of experimental measurements, can be defined through the standard Taylor's approach [40]

$$\sigma_y^2 = \frac{d_s^2}{6} + 2\sigma_v^2 T_{L,v} \left[t_f - T_{L,v} \left(1 - \exp \left[-\frac{t_f}{T_{L,v}} \right] \right) \right], \quad (6)$$

$$\sigma_z^2 = \frac{d_s^2}{6} + 2\sigma_w^2 T_{L,w} \left[t_f - T_{L,w} \left(1 - \exp \left[-\frac{t_f}{T_{L,w}} \right] \right) \right], \quad (7)$$

124 where σ_v^2 and σ_w^2 are the variances of the transverse and vertical velocities,
 125 respectively, d_s is the source diameter, $t_f = x/U_s$ is the flight time, $T_{L,v} =$
 126 $2\sigma_v^2/(\varepsilon C_0)$ and $T_{L,w} = 2\sigma_w^2/(\varepsilon C_0)$ are the Lagrangian transverse and vertical
 127 time scales, being ε the turbulent kinetic energy dissipation rate and $C_0 = 4.5$
 128 the Kolmogorov constant [41, 16].

129 In Fig. 2, a graphical comparison between experimental and theoretical
 130 results for μ is reported (red lines and symbols).

131 *The variance σ^2 .* In a recent article [18], we have obtained an analytical
 132 solution for σ^2 from the transport equation of the PDF p of the passive-
 133 scalar concentration

$$U_s \partial_x p = (K_y \partial_y^2 + K_z \partial_z^2) p + \tau_m^{-1} \partial_\psi [p(\psi - \mu)], \quad (8)$$

134 where $K_y = d\sigma_y^2/2dt$ and $K_z = d\sigma_z^2/2dt$ are the transversal and vertical tur-
 135 bulent diffusivities, respectively, and τ_m is the mixing time-scale. ψ is the
 136 sample space variable of the concentration, i.e., the collection of all possible

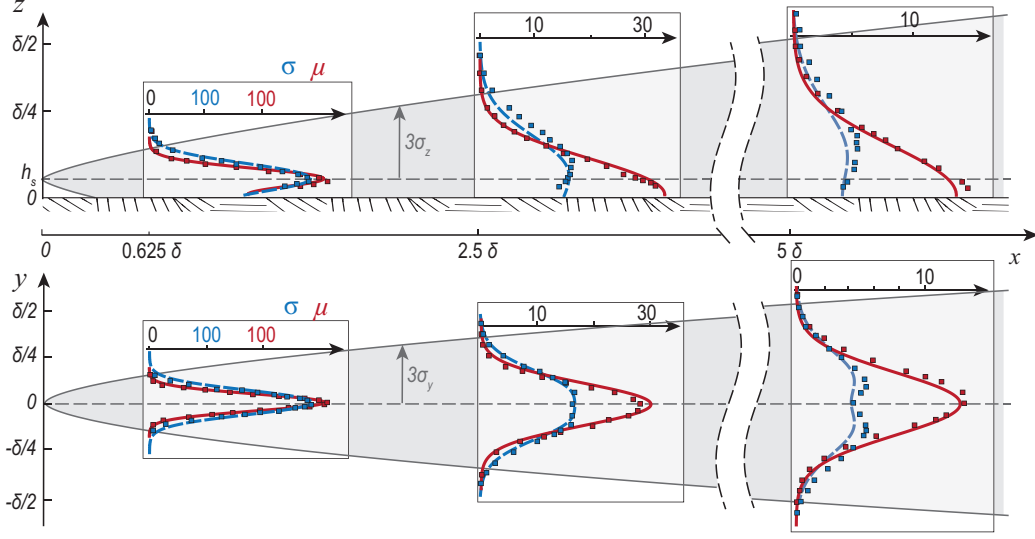


Figure 2: Vertical and transverse profiles of the mean μ (red) and the standard deviation σ (blue) of the concentration. Solid-red lines come from equation (5) for μ . Dashed-blue lines come from equation (9) for σ . Symbols correspond to experimental data LLS3 [16]. Concentration is here scaled with $\dot{M}U_s^{-1}\delta^{-2}$, being \dot{M} the mass flux emitted at the source.

137 outcomes of C . In eq. (8), the turbulent fluxes have been closed through a
 138 classical gradient-diffusion model, and the effect of molecular diffusion in the
 139 passive scalar mixing has been included through an Interaction by Exchange
 140 with the Mean (IEM) model [e.g. 42]. By solving the transport equation of
 141 the the statistical moments of concentration, derived from eq. (8), Bertagni
 142 et al. [18] obtained

$$\sigma^2 = \frac{2c^2x^2}{\tau_m U_s} \int_{\xi}^x \left(\frac{\exp \left[-2\frac{(x-x_0)}{\tau_m U_s} - \frac{x}{(2x-x_0)} \left(\frac{y^2}{\sigma_y^2} + \frac{(z-h_s)^2}{\sigma_z^2} \right) \right]}{x_0(2x-x_0)} + r_{\sigma} \right) dx_0 - \mu^2, \quad (9)$$

143 where ξ is the source parameter, and r_{σ} is the reflection term

$$r_{\sigma} = \frac{\exp \left[-2\frac{(x-x_0)}{U_s \tau_m} - \frac{x}{2x-x_0} \left(\frac{y^2}{\sigma_y^2} + \frac{(z+h_s)^2}{\sigma_z^2} \right) \right]}{x_0(2x-x_0)} \left(1 + 2 \exp \left[\frac{2h_s x (h_s x_0 + x_0 z - h_s x)}{x_0(2x-x_0)\sigma_z^2} \right] \right). \quad (10)$$

144 We invite the reader to refer to the original publication for further details
 145 on the derivation of (9). From dimensional analysis and best fitting with
 146 experiments, we found $\xi = \delta (d_s/h_s)^{10}$ for the source parameter [18]. Regard-

147 ing the mixing time-scale τ_m , the IEM model is known to introduce spurious
148 fluxes that alter the concentration statistics [e.g. 37, 43]. Yet, Bertagni et al.
149 [18] have shown that this issue can be avoided for the present model of σ^2
150 by considering two formulations for the mixing time-scale. In the near field,
151 where meandering enhances concentration fluctuations ($\sigma/\mu > 1$), the mix-
152 ing time-scale may be considered constant and proportional to the turbulent
153 time-scale, i.e., $\tau_m \propto k/\varepsilon$, where k is the turbulent kinetic energy and ε
154 rate of dissipation. Instead, a more complicated model for τ_m , which ac-
155 counts for its spatial dependence, is needed in the far field, where relative
156 dispersion dampens the passive scalar fluctuations ($\sigma/\mu < 1$). Eventually, the
157 mixing time-scale is here evaluated as

$$\tau_m = \begin{cases} \alpha_1 k/\varepsilon, & \text{for } \sigma/\mu > 1, \\ \alpha_2 \sigma_r/\sigma_{ur}, & \text{for } \sigma/\mu < 1, \end{cases} \quad (11)$$

where the constants $\alpha_1 = 0.44$ and $\alpha_2 = 0.65$ have been obtained by a fit-
ting with the wind-tunnel experiments, σ_r is an isotropic length scale of the
plume spread, and σ_{ur} is the r.m.s. of the relative velocity fluctuations (the
difference between the turbulent velocity and the instantaneous velocity of the
plume centre of mass). The formulation for $\sigma/\mu < 1$ in (11) originally
comes from the work by Cassiani et al. [38] and has been later used also in
numerical simulations of dispersing plumes [e.g. 44]. The quantities involved
in (11) are modelled as

$$\sigma_{ur}^2 = \sigma_u^2 (\sigma_r/L_E)^{2/3}, \quad (12)$$

$$\sigma_r^2 = \frac{C_r \varepsilon (t_0 + t_f)^3}{1 + (C_r \varepsilon (t_0 + t_f)^3 - d_s^2)/(d_s^2 + 2\sigma_u T_L t_f)}, \quad (13)$$

158 where $L_E = (3\sigma_u/2)^{3/2} \varepsilon$ is the Eulerian integral length-scale, $t_0 = (d_s^2/C_r \varepsilon)^{1/3}$
159 is the inertial formulation for a dispersion from a finite source size [45], $C_r =$
160 0.3 is the Richardson constant [44], and σ_u^2 is calculated, because of the
161 inhomogeneity of the turbulent field, as the average of the variances of the
162 three velocity components. Notice that when the plume size reaches the
163 Eulerian integral length-scale, i.e., $\sigma_r = L_E$, meandering becomes negligible
164 with respect to relative dispersion in the plume spread, so that $\sigma_{ur} = \sigma_u$.

165 In Fig. 2, a graphical comparison between experimental and theoretical
166 results for σ is reported (blue lines and symbols).

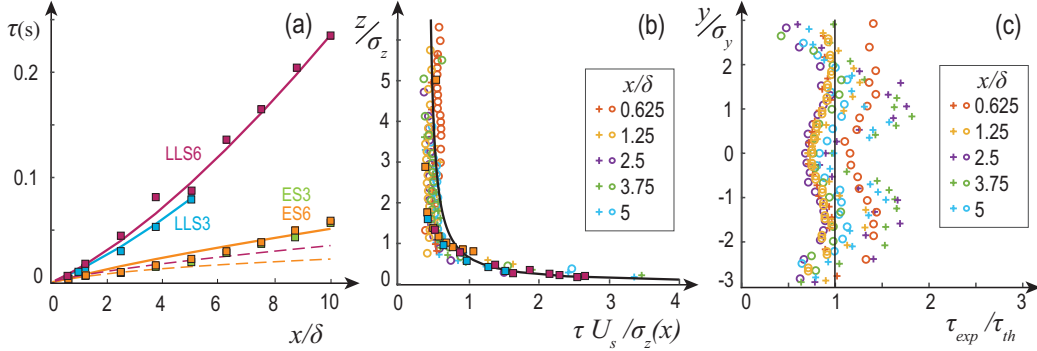


Figure 3: Integral time-scale τ . The solid lines come from eq. (14) and the symbols from several setups of the wind-tunnel experiments [16, 18]. (a) Integral time-scale τ on the plume axis ($y=0, z=h_s$) at increasing distances from the source. The dashed lines show τ from eq. (14) without the effect of the ground reflection ($r_\tau=0$). (b) Vertical profiles at several distances from the source of the scaled integral scale τ ($y=0$). The solid lines highlight the autosimilar trend $\alpha_3(1+r_\tau)$. Circles and crosses are from the ES6 and LLS3 cases by Nironi et al. [16], respectively. Filled squares are the experimental τ at the source height. (c) Transversal profiles at the source height ($z=h_s$) of the ratio between experimental and theoretical τ .

167 *The integral time-scale τ .* The third parameter, i.e., the integral of the auto-
 168 correlation function of C , can be interpreted as the temporal memory of the
 169 one-point concentration dynamics [34]. This Eulerian time-scale is usually
 170 defined through an empirical relationship that links it to the plume size and
 171 the mean velocity U [e.g. 46, 33, 47]. Indeed, the temporal correlation of
 172 the concentration series is crucially related to the plume spread. Near the
 173 source, in the meandering-dominated regime, the concentration signal is very
 174 low correlated (Fig. 1a). Further from the source, as the plume spreads and
 175 englobes the turbulent eddies, the one-point concentration signal increases
 176 its temporal correlation (Fig. 1b-c) [47]. This increasing trend of the tem-
 177 poral correlation with the distance from the source is also evident from the
 178 experiments (see symbols in Fig. 3a).

179 Here, we provide a novel model for τ that accounts for the presence of the
 180 lower boundary and the consequent vertical anisotropy of the turbulent field.
 181 For this reason, we adopt the vertical plume spread σ_z as the spatial scale
 182 of reference. Accordingly, the normalized integral scale $\tau U_s / \sigma_z$ is reported
 183 for several vertical profiles and the two experimental setups in Fig. 3b. The
 184 results show a self-similar behavior, which highlights the effect of the lower

185 boundary and the consequent anisotropy of the turbulent field. Notice that,
 186 because of the x -dependence of σ_z , the same z value corresponds to different
 187 positions in the axis z/σ_z when several x -profiles are reported (the filled
 188 squares in Fig. 3b are the integral scales at the source height h_s). Eventually,
 189 from Fig. 3b, we obtain to the following relationship for τ

$$\tau = \alpha_3 \frac{\sigma_z}{U_s} (1 + r_\tau), \quad (14)$$

190 where $\alpha_3 = 0.4$, and the term $r_\tau = (\sigma_z/z)$ stands for the reflection induced
 191 by the lower boundary, which smooths the concentration fluctuations thus
 192 increasing the temporal correlation of the concentration signal. Neglecting
 193 the lower boundary ($r_\tau = 0$) causes an high underestimation of the integral
 194 scale τ . This is evident in Fig. 3a, where τ at the source height h_s is reported
 195 for several experimental setups (solid lines for $r_\tau = (\sigma_z/z)$ and dashed lines
 196 for $r_\tau = 0$). For completeness, we also report the transversal dependency of
 197 τ in Fig. 3c. Most of the experimental data in the scaled coordinates are
 198 sparse around 1. Thus, for simplicity, the y -dependence of τ is neglected.

199 4. Model application

200 The Compound Poisson Process (2) provides the analytical relationships (3)
 201 and (4) to evaluate the upcrossing times and rates. We compare the valid-
 202 ity of these relationships (lines) with wind-tunnel data (symbols) in Fig. 4
 203 and 5 (see the Appendix B for a brief description of the experimental setup
 204 and dataset). The only input required is the triad (μ, σ, τ) , which we de-
 205 fine through two strategies: i) the experimental values (solid blue lines), ii)
 206 the analytical closures (5)-(9)-(14) (black-dotted lines). The first strategy
 207 highlights the validity of the CPP model in reproducing the level-crossing
 208 statistics. The second strategy shows the efficiency of a completely analyt-
 209 ical approach. We may notice that, as the closed relationships (5)-(9)-(14)
 210 provide good estimates for the triad (see also Fig. 2 and 3), the dotted-black
 211 and solid-blue lines are very much alike.

212 Overall, the crossing times monotonically decrease with the concentration
 213 level. Instead, the crossing rates exhibit a maximum close to the mean con-
 214 centration value, as around it the concentration signal normally evolves. The
 215 agreement between model and experiment is good throughout the domain of
 216 plume dispersion for both the Elevated Source (ES) and the Low Level Source

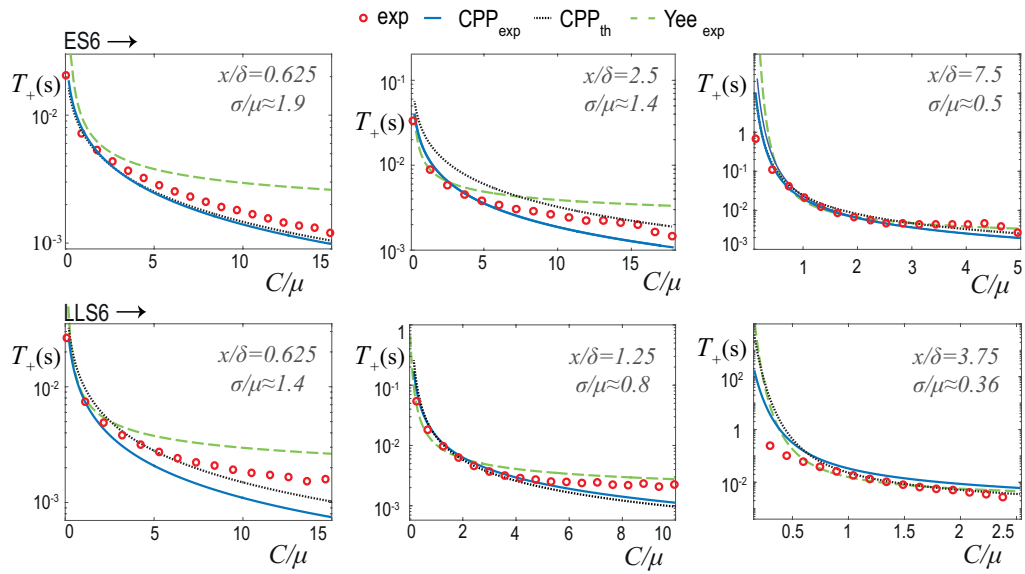


Figure 4: Comparison of experimental (symbols) and theoretical (lines) upcrossing times T_+ on the plume axis for two source configurations. The blue-solid lines (CPP_{exp}) come from eq. (3) with the experimental values for the triad (μ , σ , τ). The dotted-black lines (CPP_{th}) come from eq. (3) with the values for the triad (μ , σ , τ) obtained from the theoretical eqs. (5)-(9)-(14). The dashed-green lines come from the model by Yee [28] (Appendix A) with experimental values for the triad.

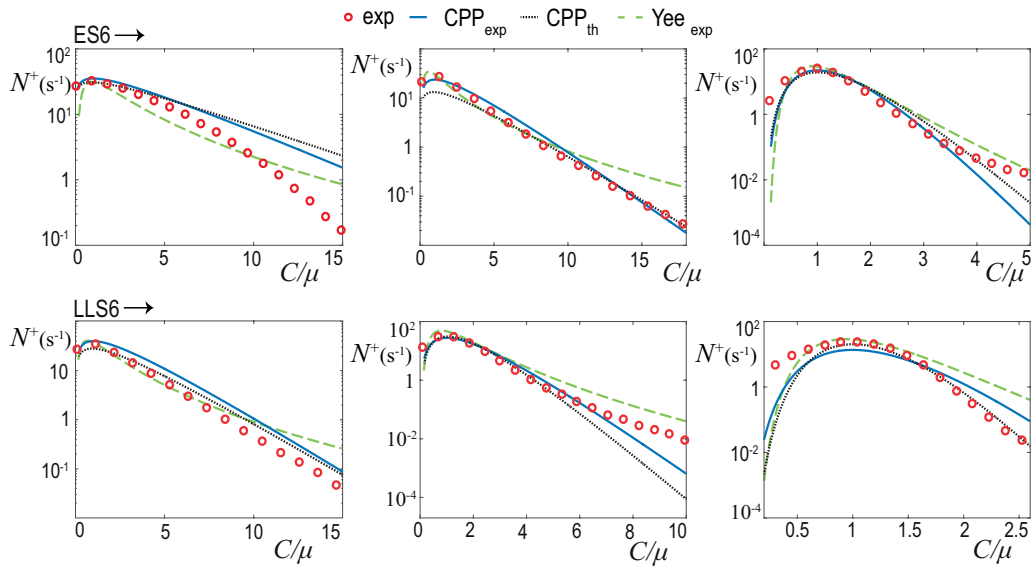


Figure 5: Comparison of experimental (symbols) and theoretical (lines) upcrossing rates N^+ for the same points of Fig. 4. The blue-solid lines (CPP_{exp}) come from eq. (4) with the experimental values for the triad (μ, σ, τ) . The dotted-black lines (CPP_{th}) come from eq. (4) with the values for the triad (μ, σ, τ) obtained from the theoretical eqs. (5)-(9)-(14). The dashed-green lines come from the model by Yee [28] (Appendix A) with experimental values for the triad.

217 (LLS), with numbering referring to the source diameter in mm. Some devi-
 218 ations in the comparison are visible for the peak concentration values in the
 219 close field ($\sigma/\mu \gg 1$). Nonetheless, the results are encouraging considered the
 220 simplicity of the stochastic model adopted and the approximations made to
 221 obtain the analytical relationships (5)-(9)-(14).

222 Additionally, we have included the results obtained through the model
 223 by Yee [28] (dashed-green lines). He achieved analytical relationships for
 224 the crossing times and rates by using Rice's theory under the assumption
 225 of a Lognormal distribution for the concentration. Also Yee's model needs
 226 three concentration statistics as input: the mean μ , the variance σ^2 , and a
 227 time-scale t_T (see Appendix A). We have used the experimental values for
 228 this triad, so that the dashed-green lines (Yee_{exp}) should be compared to
 229 the solid-blue lines (CPP_{exp}). Although both models show some inaccuracy,
 230 the CPP seems to yield better trends for the crossing rates and times. This
 231 is probably due to the better performance of the Gamma distribution with
 232 respect to the Lognormal one (see the panels in Fig. 1).

233 We wish to further add a comment about the role of intermittency. Near
 234 the source, the concentration signals show periods of zero concentration
 235 caused by the meandering motion of the plume. From a rigorous mathemat-
 236 ical point of view, the PDF of the intermittent concentration signal should
 237 be composed by a proper model (e.g., the Gamma p_{Γ}) for the distribution of
 238 the in-plume concentration fluctuations ($C > 0$) and an atom of probability
 239 in $C = 0$, i.e., $p = \Upsilon p_{\Gamma} + (1 - \Upsilon)\delta[C]$, where $\Upsilon = P_0^+$ is the intermittency
 240 factor and $\delta[\cdot]$ is the Dirac's delta. However, several reasons induced us to
 241 not formally include intermittency in our model: i) for the practical purposes
 242 of evaluating the probability of peak events and their average duration, it
 243 is indifferent if the probability of low values of concentration lies exactly in
 244 $C = 0$ or in a positive small interval of 0 (notice that $p_{\Gamma} \rightarrow \infty$ for $C \rightarrow 0$); ii)
 245 as Υ depends on the small-scale structures of turbulence [e.g. 48], its evalu-
 246 ation in laboratory and field experiments is strongly arbitrary (normally is
 247 defined as $\Upsilon = P_{\epsilon}^+$, where ϵ is an arbitrarily small value [16]) and, to the
 248 authors' knowledge, no reliable theoretical models are currently available;
 249 iii) we repeated the analysis including the experimental intermittency fac-
 250 tor (with $\epsilon = \mu/100$) and the so-obtained level-crossing statistics were within
 251 a relative difference of at maximum 30% (indeed the order of $1 - \Upsilon$). For
 252 these reasons and in favor of simplicity, we did not explicitly included inter-
 253 mittency in our mathematical formulation. However, we point out that we
 254 used our experimental results for the intermittency factor (with $\epsilon = \mu/100$)

255 in the analytical relationships by Yee. This was necessary, especially in the
256 meandering regime (first columns of panels in Figures 4 and 5), because of
257 an intrinsic limit of the the Lognormal distribution, which tends to 0 for
258 $C \rightarrow 0$ and partially loses the information about the probability of low values
259 of concentration.

260 5. Conclusions

261 In this paper, the Compound Poisson Process (2) is used to obtain analyt-
262 ical level-crossing statistics for a passive scalar released from a point source
263 in a neutral boundary-layer. Indeed, the minimalist model (2) provides the
264 Gamma distribution (1) as the steady-state PDF and the analytical relation-
265 ship (3) and (4) for the average crossing times T_ϕ^+ and rates N_ϕ^+ . The validity
266 of these results is verified by comparison with wind-tunnel data in Figs. 4
267 and 5.

268 Additionally, we have provided analytical relationships for the three input
269 parameters of the model: the mean μ , which is well resembled by the clas-
270 sical Gaussian model of plume dispersion (5); the variance σ^2 , determined
271 through the relationship (9) by Bertagni et al. [18]; and the integral scale τ ,
272 for which we propose the novel model (14). Clearly, more complicated nu-
273 merical approaches (e.g. Reynolds-averaged Navier-Stokes equations) could
274 be adopted to define the concentration statistics μ and σ to be used within
275 the model for T_ϕ^+ and N_ϕ^+ . Yet, we wished to propose a closed-methodology
276 to obtain the level-crossing statistics for the passive scalar dynamics by just
277 knowing the emission condition at the source and the velocity field.

278 The methodology here presented may serve as a rapid and practical tool
279 to estimate the dynamics of a substance dispersed in the atmosphere. A
280 possible application could be the extension of analytical operational models
281 (e.g., AERMOD or ADMS [49, 50]), which are currently used for the assess-
282 ment of chronic risks associated to the mean (time-averaged over an hourly
283 interval) concentration of exposure. Starting from the closed solutions for the
284 level-crossing statistics here proposed, the skills of these operational models
285 could be extended to the estimate of accidental risks, which are intimately
286 linked with the probability of exceeding a certain concentration threshold.
287 Furthermore, the present methodology could also benefit to the assessment of
288 nuisance odour dispersion, whose measurement in the field remains nowadays
289 a complicated task [e.g. 9].

290 Future research should possibly expand the present analysis of average
 291 level-crossing statistics to their probability distribution functions. Field mea-
 292 surements [51] suggested that a Lognormal distribution could be suitable for
 293 the purpose, but this would require an additional theoretical definition for
 294 the variance of level-crossing statistics. The same field-measurements also in-
 295 dicated that, when level-crossing statistics are considered, stable boundary-
 296 layers resemble neutral boundary-layers at further distance from the source.
 297 Yet, extensions of the present theory to non-neutral boundary-layers and dif-
 298 ferent emission conditions (e.g., line or distributed sources) remain an open
 299 challenge.

300 Appendix A. Resume of Yee's (2000) model

301 We here give the analytical results obtained by Yee [28] and used within
 302 this paper for a comparison with our model. We invite the reader to refer to
 303 the original publication for further details. Yee used Rice's theory [27] under
 304 the assumption of a Lognormal distribution for the in-plume concentration
 305 ($C > 0$)

$$p_{\log} = \frac{1}{C \sqrt{2\pi \log[\beta]}} \exp \left[-\frac{(\log[C] - \log[\mu/\sqrt{\beta}])^2}{2 \log[\beta]} \right], \quad (\text{A.1})$$

where $\beta = 1 + \sigma^2/\mu^2$. Starting from this assumption, Yee obtained a closed form for the joint PDF of the concentration C and its time derivative dC/dt , which is required by Rice's theory. Eventually, Yee provided the following analytical expressions for the crossing rates and times

$$N_{\phi}^{+} = \frac{\sigma}{2\pi \mu t_T} \frac{\exp \left[-\log^2 \left[\sqrt{\beta} \phi / \mu \right] / (2 \log[\beta]) \right]}{\sqrt{\beta \log[\beta]}}, \quad (\text{A.2})$$

$$T_{\phi}^{+} = P_{\phi}^{+} / N_{\phi}^{+}, \quad (\text{A.3})$$

where P_{ϕ}^{+} is the probability of $C > \phi$, defined from eq. (A.1). The time scale t_T , to which Yee referred to as Taylor micro-time scale, is defined as

$$t_T = \frac{\sigma}{\sigma_{C'}}, \quad (\text{A.4})$$

306 where $\sigma_{C'}$ is the r.m.s. of the concentration time derivative dC/dt , which
 307 requires experimental or field measurements. We stress out that these math-
 308 ematical results were originally derived just for in-plume concentration fluc-
 309 tuations ($C > 0$). However, they can be extended to an intermittent con-
 310 centration signal ($C \geq 0$) by considering the in-plume, instead of the total,

311 mean and variance, and the intermittency factor (as Yee suggested in the
312 conclusion of his paper). Accordingly, we have included the experimental
313 results for the intermittency in the evaluation of the level-crossing statistics
314 in the meandering regime (first columns of panels in Figures 4 and 5).

315 **Appendix B. Brief Description Of The Experimental Setup**

316 The experimental data used within this paper were collected and ana-
317 lyzed in Nironi et al. [16] and Bertagni et al. [18]. The experiments were
318 run in the atmospheric wind tunnel of the Laboratoire de Mécanique des
319 Fluides et d'Acoustique at the Ecole Centrale de Lyon, in France. This is a
320 recirculating wind tunnel 14 m long, 2.5 m high, and 3.7 m wide, in which a
321 neutrally-stratified boundary layer of height $\delta = 0.8$ m and free-stream veloc-
322 ity $U_\infty = 5 \text{ m s}^{-1}$ was generated. Ethane (C_2H_6) was used as a tracer in the
323 experiments, since it has a density similar to air, and was continuously dis-
324 charged from a source of varying diameter and elevation. As in Nironi et al.
325 [16], Bertagni et al. [18], the following notation is used for the source config-
326 uration: Elevated Source (ES3 and ES6, $h_s = 152$ mm), Lower Level Source
327 (LLS3, $h_s = 48$ mm, LLS6, $h_s = 40$ mm). The numbers in the acronyms stay
328 for the diameter in mm. We stress out that the concentration time-series
329 used to obtain Fig. 4 and 5 were measured on the plume axis for 15 minutes
330 with a sampling frequency of 1000 Hz, to assure statistical convergence to
331 the crossing times. Instead the time-series by Nironi et al. [16] are 5 minutes
332 long. The full experimental dataset is available at <http://air.ec-lyon.fr/>.

333 **References**

- 334 [1] B. Shraiman, E. Siggia, Scalar turbulence, *Nature* 405 (2000) 639.
- 335 [2] Z. Warhaft, Passive scalars in turbulent flows, *Ann. Rev. Fluid Mech.*
336 32 (2000) 203–240.
- 337 [3] P. E. Dimotakis, Turbulent mixing, *Annu. Rev. Fluid Mech.* 37 (2005)
338 329–356.
- 339 [4] K. R. Sreenivasan, Turbulent mixing: A perspective, *Proc. Nat. Acad.*
340 *Science* (2018) 201800463.
- 341 [5] E. Villiermaux, Mixing Versus Stirring, *Ann. Rev. Fluid Mech.* 51 (2019)
342 245–273.

- 343 [6] M. Kampa, E. Castanas, Human health effects of air pollution, *Environ.*
344 *Pollut.* 151 (2008) 362–367.
- 345 [7] D. R. Sommerville, K. H. Park, M. O. Kierzewski, M. D. Dunkel, M. I.
346 Hutton, N. A. Pinto, Toxic load modeling, *Inhalation Toxicology* (2006)
347 137–158.
- 348 [8] A. Gunatilaka, A. Skvortsov, R. Gailis, A review of toxicity models for
349 realistic atmospheric applications, *Atmospheric environment* 84 (2014)
350 230–243.
- 351 [9] L. Capelli, S. Sironi, R. Del Rosso, J.-M. Guillot, Measuring odours in
352 the environment vs. dispersion modelling: A review, *Atmos. Environ.*
353 79 (2013) 731–743.
- 354 [10] D. Oetl, E. Ferrero, A simple model to assess odour hours for regulatory
355 purposes, *Atmos. Environ.* 155 (2017) 162–173.
- 356 [11] M. Ravina, D. Panepinto, J. M. Estrada, L. De Giorgio, P. Salizzoni,
357 M. Zanetti, L. Meucci, Integrated model for estimating odor emissions
358 from civil wastewater treatment plants, *Environ. Sci. Pollut. R.* (2019)
359 1–16.
- 360 [12] B. Sawford, Conditional concentration statistics for surface plumes in
361 the atmospheric boundary layer, *Boundary-Layer Meteorol.* 38 (1987)
362 209–223.
- 363 [13] E. Yee, D. Wilson, B. Zelt, Probability distributions of concentration
364 fluctuations of a weakly diffusive passive plume in a turbulent boundary
365 layer, *Boundary-Layer Meteorol.* 64 (1993) 321–354.
- 366 [14] R. M. Gailis, A. Hill, E. Yee, T. Hilderman, Extension of a fluctuating
367 plume model of tracer dispersion to a sheared boundary layer and to a
368 large array of obstacles, *Bound.-Lay. Meteorol.* 122 (2007) 577–607.
- 369 [15] E. Yee, A. Skvortsov, Scalar fluctuations from a point source in a tur-
370 bulent boundary layer, *Phys. Rev. E* 84 (2011) 036306.
- 371 [16] C. Nironi, P. Salizzoni, M. Marro, P. Mejean, N. Grosjean, L. Soulhac,
372 Dispersion of a passive scalar fluctuating plume in a turbulent boundary
373 layer. Part I: Velocity and concentration measurements, *Bound.-lay.*
374 *Meteorol.* 156 (2015) 415–446.

- 375 [17] G. Efthimiou, S. Andronopoulos, I. Tolia, A. Venetsanos, Prediction
376 of the upper tail of concentration distributions of a continuous point
377 source release in urban environments, *Environ. Fluid Mech.* 16 (2016)
378 899–921.
- 379 [18] M. Bertagni, M. Marro, P. Salizzoni, C. Camporeale, Solution for the
380 statistical moments of scalar turbulence, *Phys. Rev. Fluids* (2019).
- 381 [19] M. Abramowitz, I. Stegun, Handbook of mathematical functions: with
382 formulas, graphs, and mathematical tables, volume 55, Courier Corpo-
383 ration, 1965.
- 384 [20] E. Villermaux, J. Duplat, Mixing as an aggregation process, *Phys. Rev.*
385 *Lett.* 91 (2003) 184501.
- 386 [21] J. Duplat, E. Villermaux, Mixing by random stirring in confined mix-
387 tures, *J. Fluid Mech.* 617 (2008) 51–86.
- 388 [22] M. Brancher, K. D. Griffiths, D. Franco, H. de Melo Lisboa, A review
389 of odour impact criteria in selected countries around the world, *Chemo-*
390 *sphere* 168 (2017) 1531–1570.
- 391 [23] R. Scorer, *Air Pollution*, Pergamon Press, 1972.
- 392 [24] U. Högström, A method for predicting odour frequencies from a point
393 source, *Atmos. Environ.* 6 (1972) 103–121.
- 394 [25] L. Kristensen, J. Weil, J. Wyngaard, Recurrence of high concentration
395 values in a diffusing, fluctuating scalar field, in: *Boundary Layer Studies*
396 *and Applications*, Springer, 1989, pp. 263–276.
- 397 [26] E. Yee, P. Kosteniuk, G. Chandler, C. Biltoft, J. Bowers, Recurrence
398 statistics of concentration fluctuations in plumes within a near-neutral
399 atmospheric surface layer, *Boundary-Layer Meteorology* 66 (1993) 127–
400 153.
- 401 [27] S. O. Rice, Mathematical analysis of random noise, *Bell System Tech-*
402 *nical Journal* 23 (1944) 282–332.
- 403 [28] E. Yee, An analytical model for threshold crossing rates of concentration
404 fluctuations in dispersing plumes, *Bound.-lay. Meteorol.* 98 (2000) 517–
405 527.

- 406 [29] E. Yee, P. Kosteniuk, G. Chandler, C. Biltoft, J. Bowers, Statistical
407 characteristics of concentration fluctuations in dispersing plumes in the
408 atmospheric surface layer, *Bound.-lay. Meteorol.* 65 (1993) 69–109.
- 409 [30] S. Du, D. J. Wilson, E. Yee, A stochastic time series model for thresh-
410 old crossing statistics of concentration fluctuations in non-intermittent
411 plumes, *Bound.-lay. Meteorol.* 92 (1999) 229–241.
- 412 [31] T. Hilderman, D. Wilson, Simulating concentration fluctuation time
413 series with intermittent zero periods and level dependent derivatives,
414 *Bound.-lay. Meteorol.* 91 (1999) 451–482.
- 415 [32] A. R. Jones, D. J. Thomson, Simulation of time series of concentra-
416 tion fluctuations in atmospheric dispersion using a correlation-distortion
417 technique, *Bound.-lay. Meteorol.* 118 (2006) 25–54.
- 418 [33] M. Cassiani, P. Franzese, J. Albertson, A coupled Eulerian and La-
419 grangian mixing model for intermittent concentration time series, *Phys.*
420 *Fluids* 21 (2009) 085105.
- 421 [34] L. Ridolfi, P. D’Odorico, F. Laio, *Noise-induced phenomena in the en-*
422 *vironmental sciences*, Cambridge University Press, 2011.
- 423 [35] F. Gifford, Statistical properties of a fluctuating plume dispersion model,
424 in: *Adv Geophys*, volume 6, Elsevier, 1959, pp. 117–137.
- 425 [36] F. Laio, A. Porporato, L. Ridolfi, I. Rodriguez-Iturbe, Mean first passage
426 times of processes driven by white shot noise, *Phys. Rev. E* 63 (2001)
427 036105.
- 428 [37] B. Sawford, Micro-mixing modelling of scalar fluctuations for plumes in
429 homogeneous turbulence, *Flow, Turbul. Combust.* 72 (2004) 133–160.
- 430 [38] M. Cassiani, P. Franzese, U. Giostra, A PDF micromixing model of
431 dispersion for atmospheric flow. Part I: development of the model, ap-
432 plication to homogeneous turbulence and to neutral boundary layer,
433 *Atmos. Environ.* 39 (2005) 1457–1469.
- 434 [39] S. Arya, *Air pollution meteorology and dispersion*, volume 6, Oxford
435 University Press New York, 1999.

- 436 [40] G. I. Taylor, Diffusion by continuous movements, *P. Lond. Math. Soc.*
437 2 (1922) 196–212.
- 438 [41] H. Tennekes, J. L. Lumley, *First Course in Turbulence*, Cambridge,
439 Mass. MIT Press, 1972.
- 440 [42] S. Pope, *Turbulent flows*, Cambridge university press, 2000.
- 441 [43] M. Cassiani, A. Radicchi, J. Albertson, U. Giostra, An efficient algo-
442 rithm for scalar PDF modelling in incompressible turbulent flow; numer-
443 ical analysis with evaluation of IEM and IECM micro-mixing models,
444 *J. Comput. Phys.* 223 (2007) 519–550.
- 445 [44] M. Marro, P. Salizzoni, L. Soulhac, M. Cassiani, Dispersion of a pas-
446 sive scalar fluctuating plume in a turbulent boundary layer. Part III:
447 Stochastic modelling, *Bound.-lay. Meteorol.* 167 (2018) 349–369.
- 448 [45] P. Franzese, Lagrangian stochastic modeling of a fluctuating plume in
449 the convective boundary layer, *Atmos. Environ.* 37 (2003) 1691–1701.
- 450 [46] G. L. Iacono, A. M. Reynolds, Modelling of concentrations along a
451 moving observer in an inhomogeneous plume. biological application:
452 model of odour-mediated insect flights, *Environmental Fluid Mechanics*
453 8 (2008) 147–168.
- 454 [47] D. J. Wilson, *Concentration fluctuations and averaging time in vapor*
455 *clouds*, John Wiley & Sons, 2010.
- 456 [48] P. Chatwin, P. J. Sullivan, The intermittency factor of scalars in turbu-
457 lence, *Phys. Fluids A-Fluid* 1 (1989) 761–763.
- 458 [49] C. McHugh, D. Carruthers, H. Edmunds, Adms–urban: an air quality
459 management system for traffic, domestic and industrial pollution, *Int.*
460 *J. Environ. Pollut.* 8 (1997) 666–674.
- 461 [50] A. J. Cimorelli, S. G. Perry, A. Venkatram, J. C. Weil, R. J. Paine, R. B.
462 Wilson, R. F. Lee, W. D. Peters, R. W. Brode, Aermol: A dispersion
463 model for industrial source applications. part i: General model formula-
464 tion and boundary layer characterization, *J. App. Meteorol.* 44 (2005)
465 682–693.

- 466 [51] E. Yee, R. Chan, P. Kosteniuk, G. Chandler, C. Biltoft, J. Bowers,
467 Measurements of level-crossing statistics of concentration fluctuations
468 in plumes dispersing in the atmospheric surface layer, *Bound.-layer*
469 *Meteor.* 73 (1995) 53–90.