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Fatigue limit: Crack and notch sensitivity by Finite Fracture Mechanics

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Abstract

The paper investigates the fatigue feature sensitivity, considering the two standard cases of a center-through thickness sharp crack and a circular notch by the coupled criterion of Finite Fracture Mechanics (FFM). Similarly to other criteria based on a critical distance, the FFM approach involves two parameters: the plain specimen fatigue limit, and the threshold value of the stress intensity factor range for fatigue crack growth. On the contrary, the novelty of FFM is that the crack advance becomes a structural parameter, dependent on the geometry of the mechanical component. The accuracy of FFM is checked by considering experimental data available in the Literature, showing the potentiality of the coupled approach to predict size effects.

Keywords: **Finite Fracture Mechanics, fatigue limit, crack, defect sensitivity, hole, notch sensitivity**

1. Introduction

Cracks and notches represent the most common source of stress raisers in mechanical elements. At the design and inspection stages, an engineer must always be able to catch which size flaw can be tolerated and how a post-processing defect can reduce the fatigue strength of structural components. In this framework, the fatigue failure mechanisms of elements containing cracks or notches were traditionally treated by two different approaches. On one hand, the fatigue strength of cracked structures was generally addressed by Linear Elastic Fracture Mechanics (LEFM), involving the well-known concept of stress intensity factor (SIF) K_I (the analysis being here restricted to mode I loading conditions, for the sake of simplicity), whose effectiveness is unquestionable for sufficiently large features. Nevertheless, the LEFM approach has some limitations: i) it is not able to catch the transition from long to short cracks, overestimating the related fatigue strength: in the limit case of a plain specimen, an infinite strength is predicted; ii) it is unsuitable to deal with blunt notches, and their related stress capacity. On the other hand, the load carrying reduction of notched structural components was faced by some stress-based approaches, such as the local peak stress criterion, involving the stress concentration factor K_t. Of course, such approaches are not able to deal neither with flaws (i.e., small notches) nor with cracks, providing in the latter case a vanishing failure load.

Several attempts were made in the literature to link the linear elastic fracture mechanics and the notch mechanics, starting from the pioneering investigations carried out by Frost [1], Frost et al. [2] and Smith and Miller [3], which resulted in the Frost-Miller diagram reported in Fig. 1. The diagram consists in two curves and reports the fatigue limit of notched specimens $\Delta\sigma_f$, expressed in terms of stress range referred to the gross-section. The considered notches are characterized by a constant depth a, while the notch tip radius ρ and therefore the gross-section stress concentration factor $K_{tg} = \sigma_{max}/\sigma$ are varied. The two curves intersect at a particular value of K_{tg} named K_{tg}^*, which acts as a break point between the notch mechanics and the linear elastic fracture mechanics:

- for a notch with $K_{tg} < K_{tg}^*$ the fatigue limit is the critical stress for crack initiation and it can be estimated by the range of the peak stress at the notch tip $K_{tg}\Delta\sigma_f = \Delta\sigma_0$, where $\Delta\sigma_0$ is the fatigue limit of the plain material;

- for a notch with $K_{tg} > K_{tg}^*$, the fatigue limit is no longer governed by the crack initiation phenomenon, but it is the threshold stress at which a small crack, previously initiated at the notch tip, stops to propagate. A small non-propagating crack existing at the notch tip, the notch is equivalent to a crack characterized by the same size of the notch and, therefore, the fatigue limit can be estimated from the LEFM by applying the SIF-equality $\Delta K_I = \Delta K_{th}$, where ΔK_{th} is the threshold value of the SIF range for long cracks.

It is worth noting that Frost-Miller diagram in Fig. 1 rules out (i) short cracks, since LEFM overestimates their threshold stress, and (ii) sharp open V-notches, since their stress singularity is different from the square root valid for cracks, so that their fatigue behaviour cannot be described by an equivalent crack with the same depth of the notch.

Later on, a generalized diagram was proposed by Atzori and Lazzarin [4]. The diagram reported in Fig. 2 was obtained by analyzing the fatigue limit of centrally U-notched infinite plates under pure tensile loading, the notches having different sizes a, but constant acuity $\zeta = a/\rho$ and therefore also constant stress concentration factor K_{tg}.

The diagram of Fig. 2 consists of three curves illustrating the scale effect in the notch fatigue behaviour:

- For a sufficiently large notch size a also the notch tip radius ρ is large (the notch acuity $\zeta = a/\rho$ being constant), therefore the fatigue limit $\Delta\sigma_f$ can be estimated by the range of the peak stress at the notch tip, i.e. $K_{tg}\Delta\sigma_f = \Delta\sigma_0$. Accordingly, the material is fully notch sensitive.

- By reducing the notch size a (and the notch tip radius ρ as well), when $a < a^*$ the notch is equivalent to a crack of the same size, in agreement with the Frost-Miller diagram of Fig. 1. Precisely, when $a_0 < a < a^*$ the notch is equivalent to a long crack of the same size and, therefore, the fatigue limit is dictated by the SIF-equality $\Delta K_I = \Delta K_{th}$. It should be noted that a_0 is the well-known El Haddad-Smith-Topper parameter [5] $a_0 = 1/\pi(\Delta K_{th}/\Delta\sigma_0)^2$, while $a^* = K_{tg}^2 a_0$.

- When $a < a_0$, the short crack (or defect) does not affect the plain material fatigue limit, so that the fatigue limit is given by $\Delta\sigma_f = \Delta\sigma_0$.

The diagram of Fig. 2 can be thought of as an extension of the well-known Kitagawa and Takahashi diagram [6]. The Atzori-Lazzarin diagram was validated in [7] by means of a large bulk of experimental results taken from the literature and relevant to different materials, notch geometries and loading conditions. The extension to open V-shaped notched was performed subsequently by Atzori et al. [8]. Furthermore, the method has been recently coupled with finite element simulation using coarse meshes [9]. It is worth noting that the fatigue behaviour of real components is smooth at the transition lengths $a = a_0$ and $a = a^*$, where the material exhibits the sensitivity to defects and to notches, respectively. Dealing with the defect-sensitivity region, El Haddad et al. [5] proposed to modify the SIF expression as $\Delta K_I = \Delta\sigma\sqrt{\pi(a+a_0)}$ to catch the behavior of short cracks.

The Theory of Critical Distances (TCD) by Taylor [10], see also the precursory work by Tanaka [11] who put forward a model based on LEFM concepts, allowed as well to encompass the two distinct areas of cracks and notches by using a unifying theoretical model accounting at the same time for size effects in damaged structures. The most simple criterion in the framework of TCD is the point stress one formalized by Taylor [10], also referred to as Point Method (PM).

Accordingly, the fatigue limit conditions are achieved when the range of the maximum principal stress at a distance $l_c = 1/(2\pi)(\Delta K_{th}/\Delta\sigma_0)^2 = a_0/2$ from the notch tip equals the plain fatigue limit $\Delta\sigma_0$. By referring to the frame of reference in Fig. 3, the PM criterion can be expressed as:

$$\Delta\sigma_y(x = a + l_c) = \Delta\sigma_0 \tag{1}$$

The other common stress criterion is the line stress one, also termed as Line Method (LM). It requires that the average stress upon the crack advance $l_c = 2/\pi(\Delta K_{th}/\Delta\sigma_0)^2 = 2a_0$ is higher than $\Delta\sigma_0$. In formulae:

$$\frac{1}{l_c}\int_a^{a+l_c} \Delta\sigma_y(x)\mathrm{d}x = \Delta\sigma_0 \tag{2}$$

Several studies have been carried out to establish which criterion between (1) and (2) provides the most accurate predictions [12, 13, 14, 15]. Indeed, the situation varies from case to case, and the best criterion depends on the particular geometry [16, 17, 18].

Stress based approaches have however some drawbacks, related to the fact that the critical distance l_c results a material constant: for very low structural sizes approaching l_c, the criteria fail in providing reasonable estimates. This is the case, for instance, of a trivial three point bending test on an un-notched element: if the height of the sample approaches l_c, the LM provides an infinite failure load since the stress resultant is zero. This is one of the reasons according to which the coupled FFM approaches by Leguillon [19] and Cornetti et al. [20] were introduced in the static framework.

The FFM criterion by Cornetti et al. [20] assumes a simultaneous fulfilment of two conditions. The former is the stress condition expressed by Eq. (2). The latter is the energy balance, and it provides the relationship between the average crack driving force and the fracture energy [21]. Under linear elastic assumptions, the discrete energy balance can be generalized to fatigue through the J-integral range formalism:

$$\frac{1}{l_c}\int_a^{a+l_c} \Delta J(c)\mathrm{d}c = \Delta J_{th} \tag{3}$$

where l_c is the length of a crack stemming from the feature tip. Recasting Eq.(3) in terms of the SIF range and the threshold value [22], yields:

$$\frac{1}{l_c}\int_a^{a+l_c} \Delta K_I^2(c)\mathrm{d}c = \Delta K_{th}^2 \tag{4}$$

At fatigue limit, the approach is thus expressed by a system of two equations, (2) and (4), in two unknowns: the critical crack advance l_c (which is no longer a mere material function) and the fatigue strength $\Delta\sigma_f$, implicitly embedded in the integrand functions in Eqs. (2) and (4). FFM has recently been proved to provide nearly identical predictions to the cohesive zone model for different geometries [23, 24]. By considering generic cohesive laws of power law type, even the trend of the crack advance is similar to that of the process zone [25]. **Note that FFM has been recently applied to assess the fatigue behavior of notched structures independently both in [26] and in [27]. Nevertheless, the expression of the energy condition through Eq.** (3) **represents a novelty to the authors' best knowledge.**

Finally, although not considered in the present work, it is worthwhile to mention the Strain Energy Density (SED) approach by Lazzarin and Zambardi [28], which assumes as a critical parameter the strain energy in a small region around the notch tip, and which has been proved to provide accurate results in different fatigue contexts [29, 30].

The aims of the present paper are essentially two: (i) to estimate the fatigue limit of a center-through thickness sharp crack and a circular notch by applying the coupled criterion of Finite Fracture Mechanics. In this framework, a will denote both the half crack length and the hole radius, without loss of generality; (ii) to verify the accuracy of Finite Fracture Mechanics by comparison with experimental fatigue limits taken from the literature. It is worth noting that fatigue failures of metallic materials have been observed well beyond 10^7 loading cycles in recent giga-cycle fatigue investigations [31, 32] when a stress range lower than the conventional fatigue limit was applied to the tested component, so demonstrating that the classical concept of fatigue limit might be questionable and some limitations and exceptions do exist. However, the fatigue limit still remains a useful material parameter for engineers engaged in the fatigue design of structural components.

2. Crack and notch sensitivity

Let us start by considering the case of a Griffith crack of length $2a$ in an infinite plate subjected to a remote uniaxial tension $\Delta\sigma$ (Fig. 3).

The stress field ahead of the crack tip, according to Westergaard's solution, writes:

$$\Delta\sigma_y(x) = \frac{x}{\sqrt{x^2 - a^2}}\Delta\sigma \tag{5}$$

whereas the SIF range can be expressed as

$$\Delta K_I(a) = \Delta\sigma\sqrt{\pi a} \tag{6}$$

Substituting Eqs.(5) and (6) into Eqs. (2) and (4), respectively, yields the same result:

$$\frac{\Delta\sigma_f}{\Delta\sigma_0} = \frac{1}{\sqrt{1+\pi(a/l_{th})}} \tag{7}$$

where $l_{th} = (\Delta K_{th}/\Delta\sigma_0)^2 = \pi a_0$. **This means that the fulfilment of the stress condition** (2) **implies automatically the fulfilment of the energy requirement** (4)**, and viceversa. Thus, in this case, there is no difference from the simple LM, and also the critical distance is the same, namely** $l_c = 2/\pi(l_{th})$. Results on the failure limit are reported in Fig.4, together with LEFM predictions: FFM allows to describe the behavior of short cracks, predicting a fatigue strength approaching the plain one as the crack length vanishes, and of long cracks, providing results converging to the LEFM asymptotic limit.

As concerns the case of a circular hole with radius a in an infinite plate subjected to a remote uniaxial tension (Fig. 3), the stress field is equal to [33]:

$$\Delta\sigma_y(x) = \frac{\Delta\sigma}{2}\left(2+\frac{a^2}{x^2}+3\frac{a^4}{x^4}\right) \tag{8}$$

whereas the SIF can be expressed as

$$\Delta K_I(c) = \Delta\sigma\sqrt{\pi c}F(s) \tag{9}$$

c being the length of a crack stemming from the hole edge. By considering a symmetrical crack propagation [34], the following relationship was proposed by Bowie [35]

$$F = 0.5(3-s)[1+1.243(1-s)^3] \tag{10}$$

and

$$s = \frac{c}{c+a} \tag{11}$$

The error introduced by Eqs. (10) and (11) was estimated to be less than 1% by Tada et al. [36].

Upon substitution of the stress field (8) into the stress condition (2), one gets:

$$\frac{\Delta\sigma_f}{\Delta\sigma_0} = \frac{2[1+(l_c/a)]^3}{2(l_c/a)^3+8(l_c/a)^2+11l_c/a+6} \tag{12}$$

On the other hand, inserting the SIF (9) into Eq. (4) (0 and l_c being now the integration extremes) yields

$$\begin{aligned}\frac{\Delta\sigma_f}{\Delta\sigma_0} &= \sqrt{l_{th}l_c/a^2}[3.447+\frac{\pi}{4}(\frac{0.2207}{(1+l_c/a)^7}+\frac{0.7725}{(1+l_c/a)^6}-\frac{0.9235}{(1+l_c/a)^4}+ \\ &+ \frac{2.486}{(1+l_c/a)^3}-\frac{0.8944}{1+l_c/a}+2(1+l_c/a)^2-3\mathrm{Log}(1+l_c/a))]^{-0.5} \quad (13)\end{aligned}$$

FFM predictions are obtained by equaling the right-hand sides of Eqs. (12) and (13) to get the dimensionless crack advance. This value should then be inserted into either Eq. (12) or Eq. (13) to obtain the dimensionless fatigue limit.

Results are presented in Fig. 4. As can be seen, FFM satisfies the two asymptotic limits of a vanishing radius ($\Delta\sigma_f/\Delta\sigma_0 \to 1$), and a sufficiently large hole when the strength is governed by the concentration factor $K_{tg}=3$ ($\Delta\sigma_f/\Delta\sigma_0 \to 1/3$). Furthermore, by introducing the notation $\tilde{a}=a/l_{th}$, a more detailed comparison between cracks and notches can be developed. Three ranges can be identified, as previously highlighted in the Atzori-Lazzarin diagram [4, 7]:

- $\tilde{a} \geq \tilde{a}_2$: in this case, the structure is *feature sensitive*: the differences of a notch from a crack are consistent, and they increase as the size increases. As a matter of fact, for sufficiently large crack lengths, the failure behavior of cracked elements is fully assessed by LEFM.

- $\tilde{a}_1 < \tilde{a} < \tilde{a}_2$: in this case, the structure can be supposed to be *feature shape insensitive*: the strength is affected by the presence of a flaw, but regardless its type. In other words, a notch can be treated as a crack of the same size.

- $\tilde{a} \leq \tilde{a}_1$: in this case, the structure is *feature insensitive*: the fatigue limit is not affected by the presence of a feature, whatever this is, and which of course can be neglected.

By considering the cases in exam, i.e. a sharp crack and a circular hole, and fixing an engineering tolerance of 5%, the following estimations can be provided: $\tilde{a}_1 \simeq 0.04$ and $\tilde{a}_2 \simeq 1.19$. The two cases presented above refer to a sharp defect (i.e., the crack), and a blunt one (i.e, the circular hole); concerning sharp and blunt V-notches, a theoretical model based on non-conventional extensions of the LEFM is available in [8]. Furthermore, note that the present results refer to an infinite geometry and a central feature: in case of an edge one or of finite size geometries, the threshold values of $\tilde{a}_1$ and $\tilde{a}_2$ will be slightly different incorporating the shape factors adopted in LEFM studies [7].

Finally, it is important to underline that FFM predictions are very close to those provided by the following expression:

$$\frac{\Delta\sigma_f}{\Delta\sigma_0} = \sqrt[4]{\frac{1}{K_{tg}^4} + \left(\frac{a_0}{a+a_0}\right)^2} \tag{14}$$

Equation (14) was proposed by Atzori et al. [8], and it also reverts to Eq. (7) in case of a Griffith crack.

3. Comparison with experimental data

In order to verify the applicability of FFM in the fatigue framework a comparison with experimental data is invoked. The mechanical properties of the considered materials necessary for the FFM implementation are reported in Table 1 from the corresponding references. Note that the loading ratio R affects the values of both ΔK_{th} and $\Delta\sigma_0$, thus implicitly influencing the FFM analysis.

Data related to circular notches are firstly considered by referring to the experimental tests carried out in [37, 38, 39, 40]. By looking at the geometry of the samples ($K_{tg} \approx 3$), the size of the notch with respect to that of the sample is such to justify the use of the features of Eqs. (12) and (13). Results are presented in Fig.5, together with predictions by the PM (Eq.(1)). The FFM accuracy is relatively good (indeed, some data show a not negligible uncertainty, as observed in [10]), and the criterion is in tune with the PM: it is hard to say which criterion is the most accurate, depending the answer on the particular size a/l_{th}. By comparing Figs. 4 and 5 it should be observed, however, that almost all the data fall in the range $\tilde{a}_1 < \tilde{a} < \tilde{a}_2$, where the behavior at failure is insensitive to the notch shape. A more interesting analysis would be that to consider data falling in the range $\tilde{a} > \tilde{a}_2$, as recently performed in the static case by testing PMMA samples [34].

As concerns the case of center through thickness cracks, the experimental data reported in [5] are taken into account and the comparison is shown in Fig. 6. As already observed, FFM predictions coincide with those by the LM and they reveal to be extremely accurate, whereas the PM generally tends to overestimate the results.

Another kind of notch geometry is now considered, for the sake of completeness: the case of edge cracks of length a with a semi-circular crack front (see Fig.7), which were extensively studied experimentally in the past by [6, 39, 41]. In order to implement FFM, Eqs. (5) and (6) are obviously no longer valid.

Despite a complete precise investigation would require a 3D analysis, a simplifying approach is here considered. As concerns the stress field ahead of the crack tip, it can be approximated as follows:

$$\begin{cases} \Delta\sigma_y = \Delta K_I/\sqrt{2\pi(x-a)} & x/a \leq 1+Y^2/2 \\ \Delta\sigma_y = \Delta\sigma & x/a > 1+Y^2/2 \end{cases} \tag{15}$$

whereas the SIF can be expressed as

$$\Delta K_I(a) = Y\Delta\sigma\sqrt{\pi a} \tag{16}$$

with $Y \approx 0.73$ [16]. This value represents the maximum of the SIF, which varies along the crack front [42].

Inserting Eqs. (15) and (16) into Eqs. (2) and (4), respectively, yields:

$$\begin{cases} \dfrac{\Delta\sigma_f}{\Delta\sigma_0} = \dfrac{1}{Y}\sqrt{\dfrac{l_c}{2a}} & l_c/a \leq 1+Y^2/2 \\ \dfrac{\Delta\sigma_f}{\Delta\sigma_0} = \left(1+\dfrac{Y^2 a}{2l_c}\right)^{-1} & l_c/a > 1+Y^2/2 \end{cases} \tag{17}$$

and

$$\frac{\Delta\sigma_f}{\Delta\sigma_0} = \frac{\sqrt{2/\pi}}{Y\sqrt{l_c/l_{th}(1+2a/l_c)}} \tag{18}$$

Equations (17) and (18) represent the system to be solved to get the FFM solution. FFM predictions on the fatigue limit and experimental data are reported in Fig. (7), showing an excellent agreement.

Finally, the FFM crack advance for circular holes, center and surface cracks are reported in Fig. 8. In the first case, l_c/l_{th} varies between $2/\pi$ (un-notched samples) and $2/(\pi 1.12^2)$ (very large radii). For center cracks, $l_c/l_{th} = 2/\pi$. In the latter case, l_c/l_{th} changes from 1.19 to $2/\pi$ as the crack length increases.

4. Conclusions

The coupled criterion of FFM, already established in the framework of static fracture [21, 34], was applied to assess the fatigue limit of structures presenting defects and subjected to mode I loading conditions. The analysis was focused on the extreme cases of a sharp crack and a blunt notch, the effects of which decrease with the size. The shape of a feature does not affect the plain fatigue limit below a certain size $\tilde{a}_2$, but it is just below a lower value $\tilde{a}_1$ that the structure becomes feature insensitive. FFM estimations for $\tilde{a}_1$ and $\tilde{a}_2$ are provided, by presenting simple (semi-)analytical relationships. The present study constitutes a first important effort towards the FFM implementation to fatigue fracture: the following steps will include the investigation of welded joints [43] and other important fatigue defects, as well as fatigue time life of mechanical components on the basis of the idea presented by [44] (see also [45, 46]).

References

[1] N. Frost, Non-propagating cracks in V-notched specimens subjected to fatigue loading, Aeronaut. Q. 8 (1957) 1–20.

[2] N. Frost, K. Marsh, L. Pook, Metal Fatigue, Oxford University Press, 1974.

[3] R. Smith, K. Miller, Prediction of fatigue regimes in notched components, Int. J. Mech. Sci. 20 (1978) 201–206.

[4] B. Atzori, P. Lazzarin, Notch sensitivity and defect sensitivity under fatigue loading: Two sides of the same medal, International Journal of Fracture 107 (2001) 1–8.

[5] M. El Haddad, T. Topper, K. Smith, Prediction of nonpropagating cracks, Engineering Fracture Mechanics 11 (1979) 573–584.

[6] H. Kitagawa, S. Takahashi, Applicability of fracture mechanics to very small cracks or cracks in the early stage, in: Proceeding of the second international conference on mechanical behavior of materials, ASM, 1976, pp. 627–631.

[7] B. Atzori, P. Lazzarin, G. Meneghetti, Fracture mechanics and notch sensitivity, Fatigue & Fracture of Engineering Materials & Structures 26 (2003) 257–267.

[8] B. Atzori, P. Lazzarin, G. Meneghetti, A unified treatment of the mode I fatigue limit of components containing notches or defects, International Journal of Fracture 133 (2005) 61–87.

[9] G. Meneghetti, A. Campagnolo, F. Berto, Assessment of tensile fatigue limit of notches using sharp and coarse linear elastic finite element models, Theoretical and Applied Fracture Mechanics 84 (2016) 106–118.

[10] D. Taylor, Geometrical effects in fatigue: a unifying theoretical model, Int J Fatigue 21 (1999) 413–420.

[11] K. Tanaka, Engineering formulae for fatigue strenght reduction due to crack-like notches, Int J Fracture 22 (1983) 39–45.

[12] B. Atzori, P. Lazzarin, S. Filippi, Cracks and notches: analogies and differences of the relevant stress distributions and practical consequences in fatigue limit predictions, International Journal of Fatigue 23 (2001) 355–362.

[13] D. Taylor, The Theory of Critical Distances. A New Perspective in Fracture Mechanics., Elsevier Science. London, 2007.

[14] L. Susmel, The theory of critical distances: a review of its applications in fatigue, Engineering Fracture Mechanics 675 (2008) 1706– 1724.

[15] L. Susmel, D. Taylor, The Theory of Critical Distances to estimate lifetime of notched components subjected to variable amplitude uniaxial fatigue loading, International Journal of Fatigue 33 (2011) 900–911.

[16] P. Livieri, R. Tovo, Fatigue limit evaluation of notches, small cracks and defects: an engineering approach, Fatigue & Fracture of Engineering Materials & Structures 27 (2004) 1037–1049.

[17] B. Da Silva, J. Ferreira, J. Arajo, Influence of notch geometry on the estimation of the stress intensity factor threshold by considering the Theory of Critical Distances, International Journal of Fatigue 42 (2012) 258–270.

[18] V. Beber, B. Schneider, M. Brede, Efficient critical distance approach to predict the fatigue lifetime of structural adhesive joints, Engineering Fracture Mechanics (2019) 1–13.

[19] D. Leguillon, Strength or toughness? A criterion for crack onset at a notch, European Journal of Mechanics A/Solids 21 (2002) 61–72.

[20] P. Cornetti, N. Pugno, A. Carpinteri, D. Taylor, Finite fracture mechanics: a coupled stress and energy failure criterion, Engineering Fracture Mechanics 73 (2006) 2021–2033.

[21] A. Carpinteri, P. Cornetti, N. Pugno, A. Sapora, D. Taylor, A finite fracture mechanics approach to structures with sharp V-notches, Engineering Fracture Mechanics 75 (2008) 1736–1752.

[22] T. Anderson, Fracture Mechanics: Fundamentals and Applications, Taylor & Francis, 2017.

[23] P. Cornetti, A. Sapora, A. Carpinteri, Short cracks and V-notches: Finite Fracture Mechanics vs. Cohesive Crack Model, Engineering Fracture Mechanics 168 (2016) 12–16.

[24] P. Cornetti, M. Munoz-Reja, A. Sapora, A. Carpinteri, Finite fracture mechanics and cohesive crack model: Weight functions vs. cohesive laws, International Journal of Solids and Structures 156 (2019) 126–136.

[25] P. Cornetti, A. Sapora, Penny-shaped cracks by Finite Fracture Mechanics, International Journal of Fracture 214 (2019) 97–104.

[26] A. Sapora, P. Cornetti, A. Campagnolo, G. Meneghetti, Fatigue crack onset by finite fracture mechanics, Procedia Structural Integrity 18 (2019) 501–506.

[27] Y. Liu, C. Deng, B. Gong, Discussion on equivalence of the theory of critical distances and the coupled stress and energy criterion for fatigue limit prediction of notched specimens, International Journal of Fatigue 131 (2020) 105326.

[28] P. Lazzarin, R. Zambardi, A finite-volume-energy based approach to predict the static and fatigue behavior of components with sharp V-shaped notches, International Journal of Fracture 112 (2001) 275–298.

[29] F. Berto, P. Lazzarin, A review of the volume-based strain energy density approach applied to V-notches and welded structures, Theor. Appl. Fract. Mech. 52 (2009) 183–194.

[30] F. Berto, P. Lazzarin, Fatigue strength of structural components under multiaxial loading in terms of local energy density averaged on a control volume, International Journal of Fatigue 33 (2011) 1055–1065.

[31] C. Bathias, P. Paris, Gigacycle fatigue in mechanical practice, Marcel Dekker, 2005.

[32] T. Sakai, Review and prospects for current studies on very high cycle fatigue of metallic materials for machine structural use, J. Solid Mech. Mater. Eng. 3 (2009) 425–439.

[33] E. Kirsch, Die theorie der elastizitat und die bedurfnisse der festigkeitslehre, Z Ver Dtsch Ing 42 (1898) 797–807.

[34] A. Sapora, P. Cornetti, Crack onset and propagation stability from a circular hole under biaxial loading, International Journal of Fracture 214 (2018) 97–104.

[35] O. Bowie, Analysis of an infinite plate containing radial cracks originating at the boundary of an internal circular hole, J. Math. Phys. 35 (1956) 60–71.

[36] H. Tada, P. C. Paris, G. R. Irwin, The Stress Analysis of Cracks Handbook, Paris Productions Incorporated: Third Edition. St Louis, MO, USA, 2000.

[37] P. Lukás, L. Kunz, B. Weiss, Non-damaging notches in fatigue, Fatigue & Fracture of Engineering Materials & Structures 9 (1986) 195–204.

[38] D. Du Quesnay, M. Yu, T. Topper, An analysis of notch-size effects at the fatigue limit, Journal of Testing and Evaluation 16 (1988) 375–385.

[39] P. Lukás, L. Kunz, B. Weiss, R. Stickler, Notch size effect in fatigue, Fatigue & Fracture of Engineering Materials & Structures 12 (1989) 175–186.

[40] M. T. Yu, D. L. Du Quesnay, T. H. Topper, Notched fatigue behaviour of two cold rolled steels, Fatigue & Fracture of Engineering Materials & Structures 14 (1991) 89–101.

[41] P. Lukás, L. Kunz, Effect of mean stress on short crack threshold, Short fatigue cracks, in: ESIS, 1992, pp. 265–275.

[42] Y. Murakami, M. Endo, Quantitative evaluation of fatigue strength of metals containing various small defects or cracks, Engineering Fracture Mechanics 17 (1983) 1–15.

[43] D. Wang, H. Zhang, B. Gong, C. Deng, Residual stress effects on fatigue behaviour of welded T-joint: A finite fracture mechanics approach, Materials and Design 91 (2016) 211–217.

[44] N. Pugno, P. Cornetti, A. Carpinteri, New unified laws in fatigue: From the Wohler's to the Paris' regime, Engineering Fracture Mechanics 74 (2007) 595–601.

[45] L. Susmel, D. Taylor, A novel formulation of the theory of critical distances to estimate lifetime of notched components in the medium-cycle fatigue regime, Fatigue Fract Eng Mater Struct 30 (2007) 567–581.

[46] M. Ciavarella, P. Antuono, G. Demelio, Generalized definition of crack-like notches to finite life and SN curve transition from crack-like to blunt notch behavior, Engineering Fracture Mechanics 179 (2017) 154–164.

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Material	Loading ratio R	ΔK_{th} (MPa$\sqrt{\text{m}}$)	$\Delta\sigma_0$ (MPa)	l_{th} (mm)
G40.11 steel [5]	-1	16	550	0.85
Steel [6]	0	5.5	549	0.10
2.25 Cr-1 Mo steel [37]	-1	12	440	0.74
Copper [37]	-1	5.0	146	1.20
SAE1045 steel [38]	0	6.9	448	0.24
2.25 Cr-1 Mo steel [39]	-1	10	500	0.40
SAE1010 steel, CR 22 [40]	-1	10	410	0.59
SAE945X steel, CR 61 [40]	-1	12	630	0.36
2.25 Cr-1 Mo steel [41]	0	5.7	340	0.28

Table 1: Mechanical properties of the considered materials.

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Figure 1: Frost-Miller diagram: fatigue limit in terms of nominal gross-section stress range $\Delta\sigma_f$ as a function of the theoretical stress concentration factor K_{tg} for notches of constant depth a.

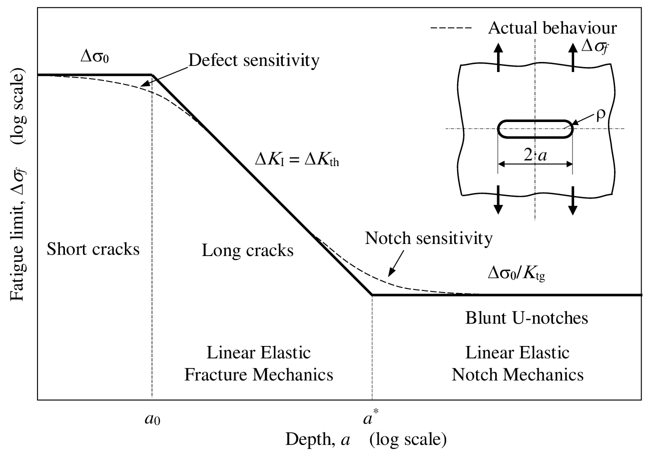

Figure 2: Atzori-Lazzarin diagram: scale effect in the fatigue behaviour of a crack or a U-notch.

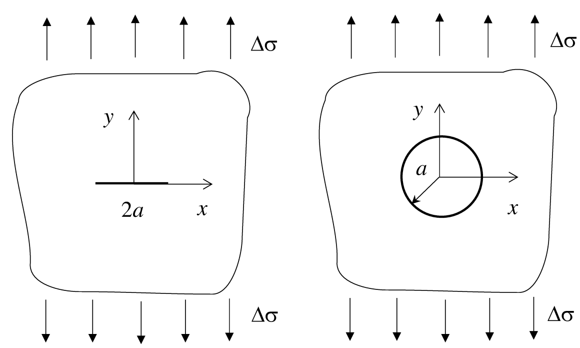

Figure 3: Infinite tensile plate: center crack of length $2a$ and a circular hole with radius a.

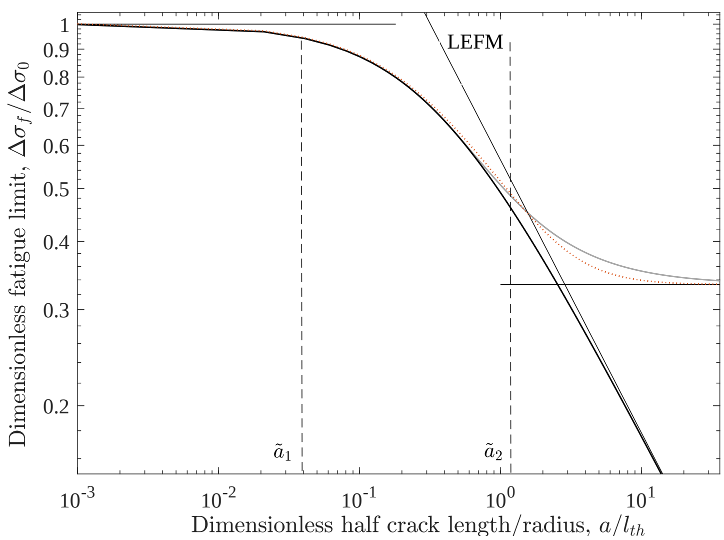

Figure 4: FFM size effects on the fatigue limit related to cracks (thick black line) and notch-like-holes (gray line). The two horizontal asymptotes for small and large sizes, and the LEFM prediction (with the typical slope equal to 1/2) are also depicted. The dotted line refers to results by Eq. (14).

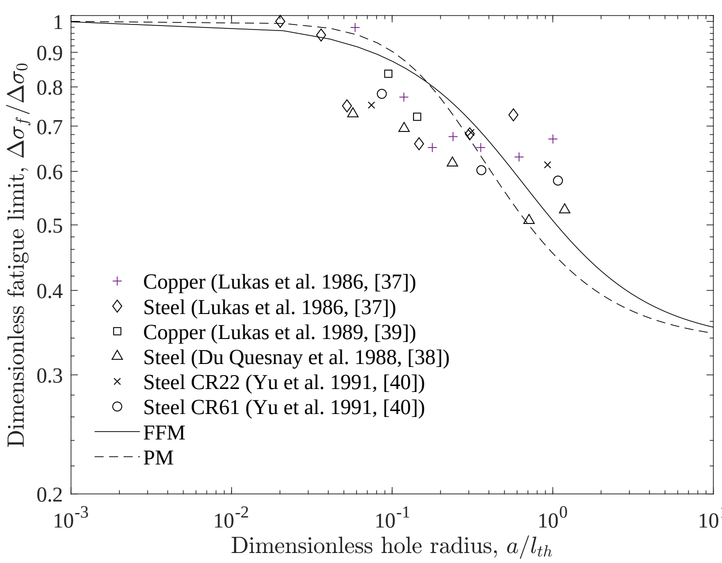

Figure 5: Fatigue limit for elements containing a circular notch: predictions by FFM, PM, and experimental data.

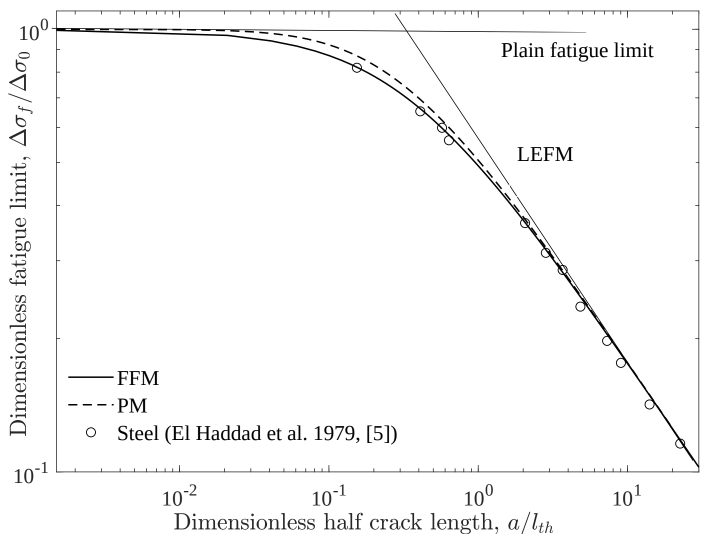

Figure 6: Fatigue limit for elements containing a center through thickness crack: predictions by FFM, PM, and experimental data.

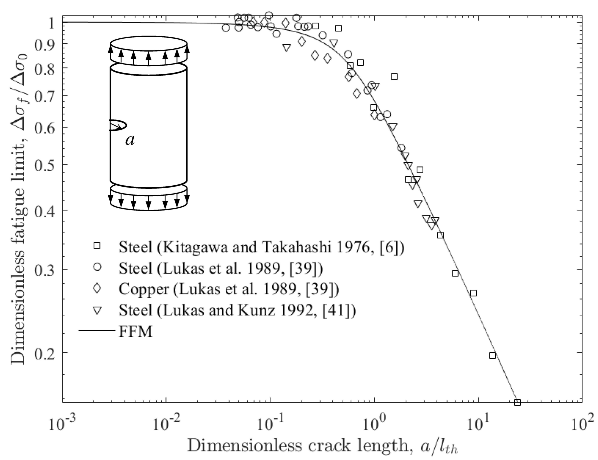

Figure 7: Fatigue limit for elements containing semi-circular surface cracks: FFM predictions and experimental data.

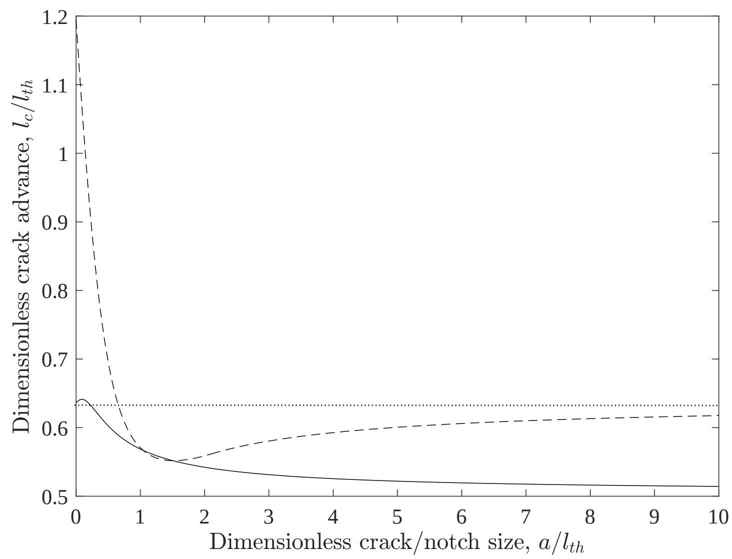

Figure 8: FFM crack advance for a central through thickness crack (dotted line), a circular notch (continuous line) and a semicircular surface crack (dashed line).