Improving efficiency of the shifted penalty method

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The method presented here is a variation of the classical penalty one, widely used in Contact Mechanics. The modification is suited to reduce the pathological penetration among the contacting surfaces, and could be used also to deal with the fictitious sliding in stick phase for frictional problems. The solution is achieved without introducing any additional set of forces or new unknowns into the global stiffness matrix. Also, the result is then achieved without any increase of the penalty parameters.

The slight but crucial modification of the original penalty contributions for the normal contact concerns the introduction of a shift parameter. Its role is that of moving the original penalty solution toward the exact (zero) value. With respect to the classical augmentation procedures, the solution improvement is not obtained with additional unknowns, but it is embedded within the original penalty contributions. In the current frictionless formulation, which has been proposed in [1] the problem is almost consistently linearized, and the shift update is carried out just before each Newton's iteration. Such update produces a minimal disturbance on the convergence rate. Hence, adding very few iterations with respect to the ones required from the consistent linearization of the original penalty method, a reduction of the penetration of several orders of magnitude can be achieved. The main drawback of the proposed strategy is due to an overdetermined system of equations that is used to compute the correct value of the shift.

Here a variation of the original formulation [1] that simplifies it and improves the algorithm performance is proposed.

Considering first a short review of the penalty formulation, we have to deal with the minimization of the total potential, Π, considering also the unilateral conditions

$$\Pi \rightarrow \min, \quad g_n \leq 0$$

where g_n is a signed measure of the distance among the bodies (gap), and the inequality constraint enforces the non-penetration of the solid bodies. Luenberger [2] states that the penalty approximation is accomplished by replacing the contact conditions with “a term that prescribes a high cost for violation of the constraints”. For two elastic bodies, the problem is then stated as

$$\Pi = \Pi_a(\mathbf{u}_a) + \Pi_b(\mathbf{u}_b) + \Pi_c(\mathbf{g}_n) \rightarrow \min$$

where Π_a and Π_b represent, respectively the total potential of body a and body b; Π_c deals with the contributions of the closed gaps and $\mathbf{g}_n$ represents its normal gap collection.

For the classical penalty contribution of a single point we have $\Pi_c = 1/2\,\varepsilon\, g_n^2$, where the penalty parameter, ε, determines the penetration error. Indeed, the main drawback of this method is due to the fact that the constraints can be exactly satisfied only for $\varepsilon \rightarrow \infty$. In the past several strategies have been proposed to deal with such a problem, see e.g., among many, to [3-7].

The enhancement of the method proposed in [1] is related to the concept of the shift, s, which is inserted into the penalty contribution of each contact point

$$\Pi_c = 1/2\,\varepsilon\left(g_n + s\right)^2$$

Let us consider it as an "unknown constant" In this way we introduce it first as a constant, hence we minimize the following problem

$$\Pi_a(\mathbf{u}_a) + \Pi_b(\mathbf{u}_b) + \Pi_c(\mathbf{g}_n, \mathbf{s})\Big|_{\mathbf{s}=\text{unknown constants}} \rightarrow \min$$

which results almost into the classical penalty minimization procedure

$$\delta\Pi_a(\mathbf{u}_a)\delta(\mathbf{u}_a) + \delta\Pi_b(\mathbf{u}_b)\delta(\mathbf{u}_b) + \delta\Pi_c(\mathbf{g}_n, \mathbf{s})\Big|_{\mathbf{s}=\text{unknown constants}}\delta(\mathbf{g}_n) = \mathbf{0}$$

Then the problem has to be linearized for a Newton's type iterative solution. However, an update of $\mathbf{s}$, which is initially assumed equal to zero, now can be considered, which gives

$$\Delta\delta\Pi_a(\mathbf{u}_a)\delta(\mathbf{u}_a)\Delta(\mathbf{u}_a) + \Delta\delta\Pi_b(\mathbf{u}_b)\delta(\mathbf{u}_b)\Delta(\mathbf{u}_b) + \Delta\delta\Pi_c(\mathbf{g}_n, \mathbf{s})\delta(\mathbf{g}_n)\Delta(\mathbf{s}) = \mathbf{0}$$

Hence, due to the presence of $\Delta(\mathbf{s})$ the global system of equations results into an underdetermined problem, that can be easily solved imposing

$$\Delta(\mathbf{s}) = \mathbf{g}_n$$

This permits to set an easy procedure with a continuous update of the shift. Preliminary results are very interesting, and have shown the nice characteristics of the method.

References

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