An Accurate Integration of Basis Functions with Corner Singularities for MoM Applications

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An efficient use of higher order models for the numerical solution of integral or differential equations requires the introduction and the use of singular basis functions each time edges or corners are encountered [1]-[5]. In fact, the behavior of the induced current (or field quantities) in the vicinity of edges or corners is not analytical, and electromagnetic vector quantities can be unbounded in the neighborhood of abrupt geometric discontinuities. Recently our research group has developed hierarchical high order bases and additive singular bases that have proved extremely useful for the numerical solution of the Electric Field Integral Equation (EFIE) for structures containing sharp edges [3]-[5]. In the case of corners, such as the tips of a square conducting plate, the singular bases to be used are more complex and may contain singularities stronger than those found for edges [2]. For brevity, this work considers quadrilateral cells of the type indicated in Fig. 1, that is a square of the x-y plane $\{0 \leq x \leq 1;\ 0 \leq y \leq 1\}$ with singular corner at x=y=0. Because of the complexity of the singular basis functions introduced for the treatment of corners in the Method of Moments (MoM) applications where one has also to deal with the Green's function singularity, it is necessary to develop new sophisticated integration techniques to deal with near-field and self-cell (source) integrals. The quadrilateral cell of Fig. 1 is subdivided into two sub-triangles which constitute the two integration domains we use to numerically evaluate the source integral in the case of observer located outside (far) from the cell. The precision that can be obtained using our new integration technique is for example illustrated by Fig. 2, which reports the error as the observation point varies along the bisector line that crosses the square cell of Fig. 1, for a square cell of side length equal to one wavelength, and for a singular basis function containing a singular corner factor of the form

$$S(x,y) = \frac{1}{(x^2+y^2)^{0.1}\sqrt{x\,y}} \qquad (1)$$

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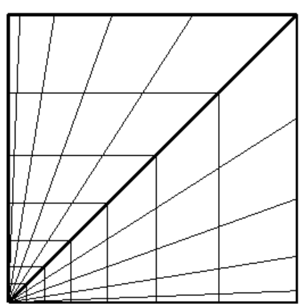

Figure 1.

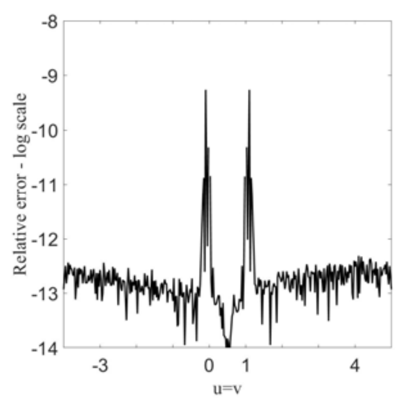

Figure 2.

The error shown in Fig. 2 is with respect to the result obtained by using an extremely large number of integration points. The new integration techniques will be discussed at the Conference together with several results obtained to validate them.

1. R.D. Graglia, and A.F. Peterson, ***Higher - order Techniques in Computational Electromagnetics***, SciTech Publishing/IET, Edison, NJ, 2016.
2. A.F. Peterson, R.D. Graglia, “Basis Functions for Vertex, Edge, and Corner Singularities: A Review,” invited paper, IEEE Journal on Multiscale and Multiphysics Computational Techniques, vol. 1, pp. 161- 175, 2016.
3. R.D. Graglia, A.F. Peterson, P. Petrini, ``Hierarchical singular vector bases for quadrilateral cell MoM applications," Proc. of the 2017 Int. Conf. on Electromagnetics in Advanced Applications (ICEAA), pp. 1929-1932, Verona, Italy, 2017.
4. R.D. Graglia, A.F. Peterson, P. Petrini, ``Hierarchical divergence conforming bases for edge singularities in quadrilateral cells,’’ IEEE Trans. Antennas Propagat., vol. 66 , no. 11, pp. 6191-6201, November 2018.
5. R.D. Graglia, A.F. Peterson, P. Petrini, ``Computation of EFIE Matrix Entries with Singular Basis Functions," IEEE Trans. Antennas Propagat., vol. 66 , no. 11, pp. 6217-6224, November 2018.