PEC Wedge Structures in Complex Environment using the Generalized Wiener-Hopf Technique

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Abstract— **In this work we present a new methodology to study complex canonical electromagnetic scattering problems constituted of perfectly electrically conducting (PEC) wedges immersed in complex environment. The method is based on the Generalized Wiener-Hopf Technique (GWHT) proposed by the authors. Engineering applications are considered in the field of electromagnetic compatibility and antenna technology.**

Keywords— Wedges, Inhomogeneous Material, Electromagnetic scattering, Generalized Wiener-Hopf technique, Electromagnetic Compatibility and Antenna technology

I. INTRODUCTION

Recently, the Generalized Wiener Hopf technique (GWHT) [1-2] has demonstrated its efficacy in studying complex diffraction problems (see for instance [3-9] and references therein).

In particular in this paper we consider wedges in complex environment. By slightly modifying the structure we present different electromagnetic practical problems.

For example, using the proposed method, the structures presented in Fig. 1 are modelled via similar mathematical equations although they represent different applications.

Fig. 1.a may be considered a wedge structure lying on a inhomogeneous material (half dielectric, half free space) while Fig. 1.b can be considered a waveguide structure filled by dielectric material and terminated by an infinite horn. Both, for instance, may be considered for modelling inlets in aerospace engineering.

The solution method based on the Generalized Wiener-Hopf Technique consists of the following steps:

1) For each canonical sub-region we deduce the Generalized Wiener-Hopf Equations (GWHEs) in spectral domain

2) We perform the Fredholm factorization technique [10-11] to reduce the GWHEs to integral representations by eliminating some of the WH unknowns (plus or minus)

3) By coupling the integral representations of the regions we obtain a system of Fredholm integral equations (FIEs) in terms of one kind of WH unknowns. The integral representations allow to obtain approximation of the other kind of unknown.

4) We perform analytical continuation of the approximate solution of the FIEs via difference equations obtained from the GWHEs

5) We evaluate physical and engineering parameters in terms of field components.

With steps 2-4 we substitute the fundamental mathematical procedure used in classical Wiener-Hopf (WH) technique [12] based on kernel factorization, decomposition and application of the Liouville's theorem.

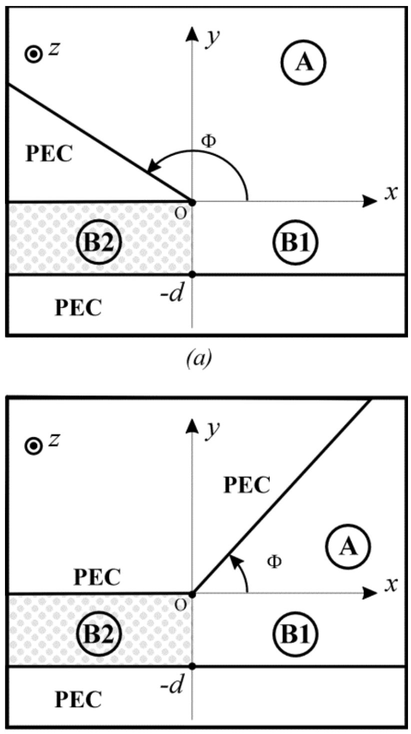

Fig. 1. Scattering by PEC Wedge structures in Complex Environment: a PEC wedge lying on a inhomogeneous material (half dielectric, half free space). Case (a) with $\Phi > \pi/2$, case (b) with $\Phi < \pi/2$ (the infinite horn).

With reference to Fig. 1, cartesian coordinates (x,y,z) as well as cylindrical coordinates (ρ,φ,z) centered in O are used to describe the problem. The geometry of the figure is basically a a PEC wedge lying on a inhomogeneous material.

With reference to Fig. 1, two regions are identified: the angular region A ($0<\varphi<\Phi, y>0$) and the inhomogeneous region B ($-d<y<0$) subdivided into two sub-regions: the free space rectangular region B1 ($x>0$, $-d<y<0$) and the rectangular dielectric region B2 ($x<0, -d<y<0$) characterized by the dielectric permittivity ε_r.

In this work we consider time harmonic electromagnetic field with a time dependence specified by $e^{j\omega t}$ which is omitted.

For the sake of simplicity, in this work we present the problem only at Ez-polarization thus the field is independent from z and it has non-null $E_z(x,y,z)$, $H_x(x,y,z)$, $H_y(x,y,z)$.

The structure can be illuminated either by a modal field coming from the waveguide B2 or by a plane wave from region A with azimuthal direction φ_o and with propagation constant k. In this paper we focus the attention on the modal source.

For each region, taking inspiration from [13-15], we introduce the use of network representation of the relevant GWHEs and their integral representations. This framework allows to order and systematize the mathematical procedure avoiding redundancy.

II. Formulation

The formulation of the problem in spectral domain is defined in terms of Laplace transforms $V_+(\eta)$, $I_+(\eta)$, $I_{a+}(-m)$ respectively of the E_z at φ=0, H_x at φ=0 and H_ρ at φ=Φ:

$$
\begin{aligned}
V_+(\eta) &= \int_0^\infty E_z(x,0)e^{j\eta x}dx \\
I_+(\eta) &= \int_0^\infty H_x(x,0)e^{j\eta x}dx \qquad (1). \\
I_{a+}(-m) &= \int_0^\infty H_\rho(\rho,\Phi)e^{-jm\rho}d\rho
\end{aligned}
$$

With reference to [1-2] the GWHE of region A is

$$Y_\infty(\eta)V_+(\eta) - I_+(\eta) = -I_{a+}(-m) \qquad (2).$$

where $Y_\infty(\eta) = \xi(\eta)/kZ_o$, $\xi(\eta) = \sqrt{k^2-\eta^2}$, $m = -\eta\cos\Phi + \xi(\eta)\sin\Phi$.

We note that the unknowns defined in (1) and used in (2) are defined into different complex planes, i.e. η and m, thus (2) is a generalized version of Wiener-Hopf equation. The introduction of a special spectral mapping [1-2] and the application of Fredholm factorization [10-11] allows to eliminate the minus unknown $I_{a+}(-m)$ yielding an integral representation that relates the plus unknowns (1). Mapping back the integral representation to η plane we obtain [15]

$$I_+(\eta) = Y_\infty(\eta)V_+(\eta) + \mathscr{Y}[V_+(\eta')], \quad \eta\in\mathbb{R} \qquad (3).$$

where $\mathscr{Y}[\bullet] = \frac{1}{2\pi j}\int_{-\infty}^{\infty} y(\eta,\eta')[\bullet]d\eta$ with

$$y(\eta,\eta') = \frac{Y_\infty(\eta')}{\bar\eta(\eta')-\bar\eta(\eta)}\frac{d\bar\eta}{d\eta'} - \frac{Y_\infty(\eta)}{\eta'-\eta} + \sum_{n=1}^{n_{at}} \frac{q_{2n}^{\Phi_a}(\eta)}{\eta'-p_{2n}^{\Phi_a}(\eta)}u\left(\frac{\pi}{2}-n\Phi\right)$$

$$p_{2n}^{\Phi}[\eta] = \eta\cos 2n\Phi - \sqrt{k^2-\eta^2}\,\sin 2n\Phi$$

$$q_{2n}^{\Phi}[\eta] = \frac{1}{k Z_o}\left(\eta\sin 2n\Phi + \sqrt{k^2-\eta^2}\,\cos 2n\Phi\right)$$

and u(t) is the unit step function.

In (3) we have used an abstract notation that is associated to an equivalent network as in [15]. In particular region A is modelled via algebraic-integral operator admittance that relates the plus unknown $V_+(\eta)$ to $I_+(\eta)$ see Fig. 2.

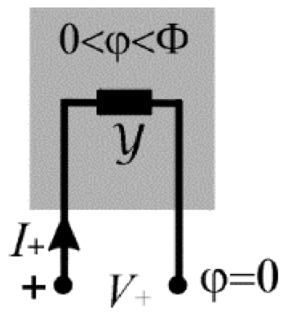

Fig. 2. Equivalent network model of region A corresponding to (3)

For what concerns region B, due to the presence of inhomogeneous sub-regions, the procedure to get an integral representation for $V_+(\eta)$, $I_+(\eta)$ presents some difficulties that were not present in a similar problem with a homogenous region B, see [6-7].

In this case, we need to start from the analysis of the wave equations with the help of the characteristic Green function procedure as applied in [13-14]. Continuity at the interface x=0 allows to obtain the final representation.

By applying the unilateral Fourier transform to the wave equation for the Ez component we get

$$\left(\frac{d^2}{dy^2}+\tau^2\right)\tilde{E}_z(\alpha,y) = f_\alpha(y) \qquad (4).$$

in α plane where the right-hand-side is related to the field components at the interface x=0 (initial conditions) and $\tau = \sqrt{k^2-\alpha^2}$ is the spectral propagation constant.

Focusing the attention on region B1, enforcing the boundary conditions we obtain:

$$\left(\frac{\int_{-d}^{0}\sin\left(\tau\left(y'+d\right)\right)f_{1\alpha}(y')dy}{jkZ_o\sin\left(\tau d\right)}\right)-I_+(\alpha)=Y_d(\alpha)V_+(\alpha) \quad (5).$$

with $Y_d(\alpha)=-j\dfrac{\sqrt{k^2-\alpha^2}}{k\,Z_o}\cot\left[\sqrt{k^2-\alpha^2}d\right]$,

To have a closed mathematical problem we represent $f_{1\alpha}(y)$ with a modal representation of the field obtained for x=0- thus in region B2:

$$E_z(y,x)=E_o\sin\left(\frac{\pi}{d}y\right)e^{-j\chi_1 x}+\sum_{n=1}^{\infty}C_n\sin\left(\frac{n\pi}{d}y\right)e^{j\chi_n x} \quad (6).$$

with $\chi_n=\sqrt{\varepsilon_r k^2-\left(\dfrac{n\pi}{d}\right)^2}$

Finally we obtain

$$\psi_-(\alpha)+\psi_+(\alpha)+\psi_+^i(\alpha)=I_+(\alpha)+Y_d(\alpha)V_+(\alpha) \quad (7).$$

where the Ψ functions depend on the source, the modal field expansion of region B2 (χ_n) and $V_+(\alpha)$ estimated for $\alpha=-\alpha_n=-\sqrt{k^2-\left(\dfrac{n\pi}{d}\right)^2}, n\in\mathbb{N}_0$.

Considering the regularity properties of $I_+(\alpha)$ and $V_+(\alpha)$ we obtain a representation of the coefficients C_n in terms of $V_+(-\alpha_n)$, thus (7) is a closed problem.

The application of Fredholm factorization to (7) allows to eliminate the minus function $\psi_-(\alpha)$ and it yields the integral representation

$$\psi_+^i(\alpha)+\psi_+(\alpha)-I_+(\alpha)=Y_d(\alpha)V_+(\alpha)+\mathcal{Y}_d[V_+(\alpha')] \quad (8).$$

As per (3), (8) can be interpreted as an equivalent network where the algebraic-integral operator admittance (right hand side) relates $V_+(\alpha)$ to $I_+(\alpha)$ with a source term $\psi_+^i(\alpha)$ and a dependent source term $\psi_+(\alpha)$.

After noting that η and α are the same spectral variable and considering (3) and (8), we eliminating by substitution $I_+(\alpha)$.

The result is a Fredholm integral equation of second kind in terms of $V_+(\alpha)$ where a limited number N of C_n can be considered. The equation is amenable of solution with simple discretization procedure [16].

Approximate representation of $I_+(\alpha)$ are obtainable via (3) or (8).

To compute practical engineering parameters we need an analytical extension of the approximate solution, thus we resort to difference equations obtained from the GWHEs in the angular complex plane w (η=-k cos w).

Details on the implementation and numerical results in terms of physical and engineering parameters will be reported during the presentation at the conference.

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REFERENCES

[1] V.G. Daniele, “The Wiener-Hopf technique for impenetrable wedges having arbitrary aperture angle”, SIAM Journal of Applied Mathematics. Vol 63, pp. 1442-1460, 2003

[2] V.G. Daniele, G. Lombardi, “The Wiener-Hopf technique for impenetrable wedge problems,” in Proc. Days Diffraction Int. Conf., Saint Petersburg, Russia, pp. 50–61, Jun. 2005.

[3] V.G. Daniele, G. Lombardi G, “The Wiener-Hopf Solution of the Isotropic Penetrable Wedge Problem: Diffraction and Total Field,” IEEE Trans. Antennas Propagat., vol. 59, n. 10, pp. 3797-3818, 2011

[4] V.G. Daniele, “Electromagnetic fields for PEC wedge over stratified media: Part I,” Electromagnetics, vol. 33, n. 3, pp. 179-200, 2013.

[5] V.G. Daniele, G. Lombardi G, “Arbitrarily oriented perfectly conducting wedge over a dielectric half-space: diffraction and total far field,” IEEE Trans. Antennas Propagat., vol. 64, n. 4, pp. 1416-1433, 2016.

[6] V.G. Daniele, G. Lombardi, R.S. Zich, “The electromagnetic field for a PEC wedge over a grounded dielectric slab: 1. Formulation and validation,” Radio Science, 52, pp. 1-20, 2017.

[7] V.G. Daniele, G. Lombardi, R.S. Zich, “The electromagnetic field for a PEC wedge over a grounded dielectric slab: 2. Diffraction, Modal Field, SurfaceWaves, and Leaky Waves,” Radio Science, 52, pp. 1-18, 2017.

[8] V.G. Daniele, G. Lombardi, R.S. Zich, “The scattering of electromagnetic waves by two opposite staggered perfectly electrically conducting half-planes,” Wave Motion, vol. 83, pp. 241-263, 2018, doi:10.1016/j.wavemoti.2018.09.017

[9] V.G. Daniele, G. Lombardi, R.S. Zich, “The Double PEC Wedge Problem: Diffraction and Total Far Field,” Trans. Antennas Propagat., vol. 66, n. 12, pp. 6470-6673, 2018.

[10] V.G. Daniele, G. Lombardi, “Fredholm factorization of Wiener-Hopf scalar and matrix kernels,” Radio Sci., 42: RS6S01, pp. 1-19, 2007.

[11] V.G. Daniele, R.S. Zich, The Wiener Hopf method in Electromagnetics, Scitech Publishing, 2014

[12] B. Noble, Methods Based on the Wiener-Hopf Technique: For the Solution of Partial Differential Equations, London,UK:Pergamon, 1958.

[13] L. B. Felsen and N. Marcuvitz, Radiation and Scattering of Waves, Englewood Cliffs, NJ: Prentice-Hall, 1973

[14] V.G. Daniele, R.D. Graglia, G. Lombardi, P.L.E. Uslenghi, “Cylindrical resonator sectorally filled with DNG metamaterial and excited by a line source,” IEEE Antennas Wireless Propag. Lett., vol. 11, pp. 1060-1063, 2012.

[15] V.G. Daniele, G. Lombardi, R.S. Zich, “Network representations of angular regions for electromagnetic scattering,” Plos One, vol. 12, n. 8, e0182763, pp. 1-53, 2017, https://doi.org/10.1371/journal.pone.0182763.

[16] L.V. Kantorovich and V.I. Krylov, Approximate methods of higher analysis, Groningen, The Netherlands: Noordhoff, 1964