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Mechanical behaviour of clastic formations at shallow – medium depths which bear hydrocarbon reservoirs could exhibit an important non-linear influence of the strain on the formation stiffness during depletion. Particularly in the early reservoir production stage, characterized by high uncertainty and little ground movement data for back-analysis, reliable determination of formation stiffness at very small strain and its degradation with increasing strain via experimental testing can play a key role in realistic subsidence predictions. The standard set of data acquisition by the oil industry represents a good starting point, but the information must still be corroborated and extended by dedicated lab tests analysis. The scope of this paper is to review a selection of the most used in situ data acquisition as well as laboratory techniques for the determination of the formation stiffness at (very) small strains and its non-linear degradation with increasing strain.
Keywords*: very small strain stiffness, degradation curve, in situ geophysics tests, laboratory tests.*

Moduli pseudo elastici dei geomateriali: un excursus. *La rigidezza delle formazioni clastiche a profondità medio/basse mineralizzate ad idrocarburi può mostrare una marcata dipendenza di tipo non lineare dalle deformazioni indotte dalla variazione di pressione dovuta all'estrazione dei fluidi. La possibilità di avvalersi di un'affidabile determinazione sperimentale della massima rigidezza delle formazioni (ovvero a deformazioni molto piccole) e di come essa varia all'aumentare delle deformazioni indotte risulta uno dei fattori critici in fase di studio previsionale della subsidenza potenzialmente indotta. Il set di dati sperimentali standard acquisito (sia in situ sia tramite analisi di laboratorio) dall'industri petrolifera è un buon punto di partenza ma queste informazioni devono essere corroborate e completate tramite mirate prove di laboratorio. Il presente articolo fornisce una panoramica delle tecniche sperimentali, sia di laboratorio sia di acquisizione sul campo, per la determinazioni della rigidezza al variare delle deformazioni indotte.*
Parole chiave*: rigidezza a piccolissime deformazioni, curva di decadimento del modulo elastico, indagini geofisiche in situ, test di laboratorio.*

1. Introduction

There is a good number of researchers (Burlan, 1979; Cole *et al.*, 1972; St John, 1975; Wroth, 1975; Arkinsos et a., 1991) who agree that "one of the major problems in ground engineering in the 1970s, and before, was the apparent difference between the stiffness of soils measured in laboratory tests and those back-calculated from observation of ground movements. These differences have now largely been reconciled through the understanding of the principal features of soil stiffness and, in particular, the very important influence of non-linearity". Nowadays, non-linear soil behaviour is a well-understood concept, widely applied in civil engineering. Conversely, the topic has had little attention in the hydrocarbon exploitation field, though mechanical behaviour of clastic formations at shallow – medium depths which bear hydrocarbon reservoirs could lie between rock mechanics and soil mechanics and, during depletion, they could exhibit a significant non-linear influence of the strain on the formation stiffness (Coti et al, 2018; Giani et al, 2018; Rocca, 2009). As a consequence, reliable determination of formation stiffness at very small strain and its degradation with increasing strain via experimental testing is critical if realistic predictions of ground movements due to hydrocarbon production are to be made. This is particularly remarkable at the early stages of reservoir production, when the uncertainty in all reservoir features and in the phenomenological evolution related to production is highest and few ground movement measurements are available for back-analysis. Nevertheless, how and how much the mechanical properties of the formations change due to pressure drop induced by hydrocarbon production is not a trivial task to address. A great many experimental techniques are available for stiffness parameter determination, both via lab test and in situ data acquisition.

The present paper reviews a selection of the most used data acquisition techniques for the determination of the formation stiffness at (very) small strains and its non-linear degradation with increasing strain.

2. Small-strain stiffness and degradation curve

As has been corroborated over the years (Jardine *et al.*, 1984; Jardine *et al.*, 1986; Jardine, 1992; Arkinsos *et al.*, 1991; Mair, 1993), the maximum strain at which soils exhibit almost fully recoverable deformation is found to be very small; then, with increasing strain, soil stiffness decays non-linearly, according to the well know semi-logarithmic reduction or decay curve (Figure 1).

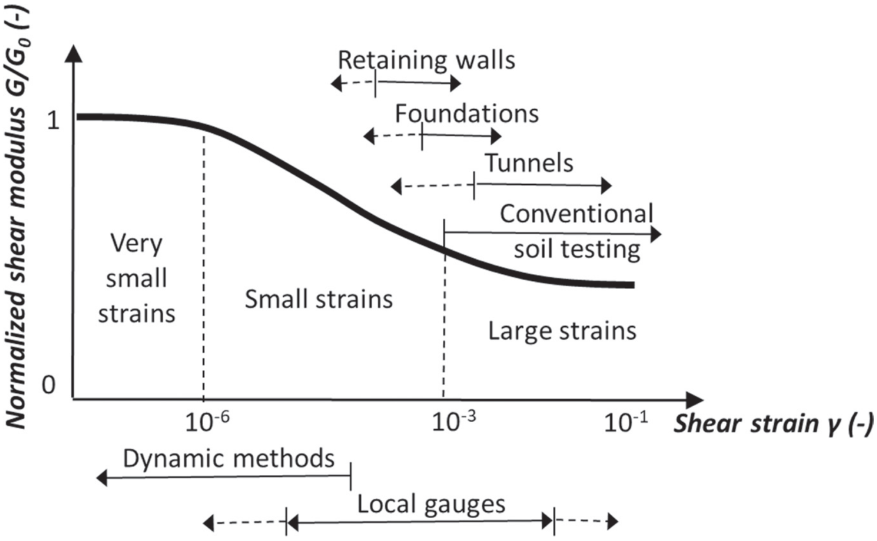

Fig. 1. Qualitative stiffness-strain decay curve of soils (Arkinsos *et al.*, 1991 mod; Mair, 1993 mod).
Andamento qualitative della curva di decadimento rigidezza-defromazioni per I terreni (Arkinsos et al., 1991 mod; Mair, 1993 mod).

The small strain stiffness and its degradation behaviour depend mainly on the status parameters and the physical properties of the material. In particular, strain amplitude, void ratio, fluid saturation, state of stress and over-consolidation ratio, plasticity index and inter particle bonding like cementation have been highlighted as the most effective factors in the soil field (Jardine *et al.*, 1986; Fjær *et al.*, 2008; Darendeli, 2001; Seed *et al.*, 1970; Vermeer, 1979). The results of the research carried out by Seed and Idris (1970) are shown in Figure 2 and are self-explanatory of the effects of some of the above-mentioned parameters on soil stiffness. Even if the research focused on sands, the derived macro considerations are still valid, at least under a qualitative viewpoint, even for soft/weak rocks hosting hydrocarbon reservoirs.

The very small strain stiffness, or initial shear G_0 or Young's modulus E_0, corresponds to the maximum values of reduction curve and defines elastic behaviour; it is believed to be a fundamental property of all types of geotechnical materials including clays, silts, sands, gravels and rocks (Tatsuoka, 2000). The transition between very small strain and small strains is difficult to quantify and it can be generally assumed in the range $10^{-6} \leq \gamma \leq 10^{-5}$. $\gamma \approx 10^{-3}$ is commonly assumed as the threshold value between small and large strains: it corresponds with the lower limit of classical laboratory testing (i.e. triaxial or oedometric tests with no special devices). Generally speaking, the range of deformation induced by hydrocarbon exploitation can be assumed within the range of very small to small strains (Benetatos et al, 2015; Benetatos et al, 2017; Giani et al, 2017).

Very small strain stiffness and its degradation curves can be determined in the laboratory and via in situ tests. Laboratory measurements are acquired via triaxial tests with strain measurements, benders elements, resonant column and torsional shear (Benz, 2000; Cadu *et al.*, 2012). In situ tests rely on seismic acquisition techniques that allow an indirect determination of the elastic stiffness in relation to wave propagation velocity measurement (Benz, 2000). Available techniques differ mainly on the basis of induced strain amplitude and frequency. Dynamic tests, based on wave propagation, involve either resonant and pulse measurement methods, and they impose high frequency and low strain amplitude to tested specimen (or formations). In field seismic measu-

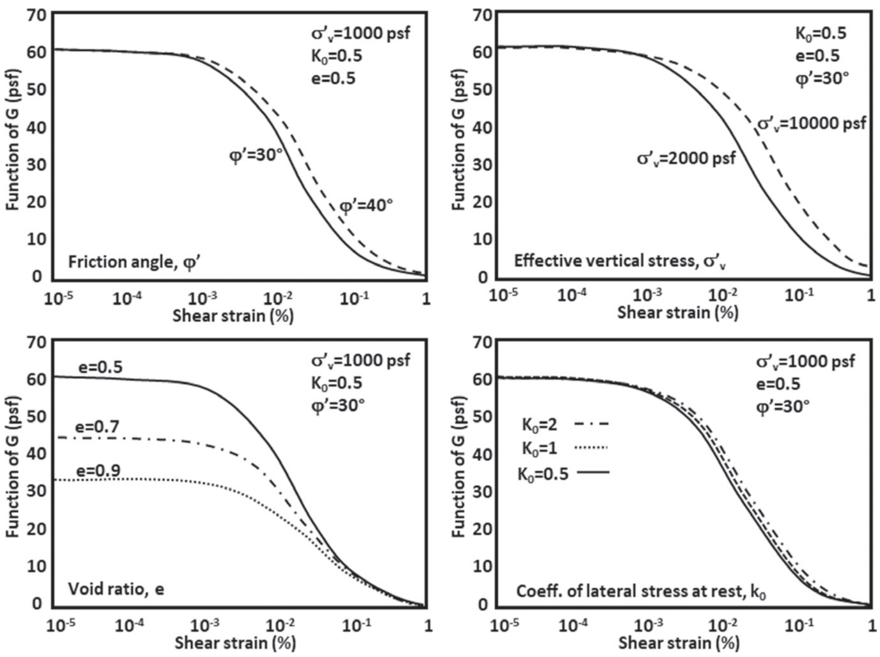

Fig. 2. Decay curve as a function of different parameters (Seed and Idris, 1970, mod).
Effetto di diversi paramentri sulla curva di decadimento (Seed and Idris, 1970, mod).

rements, the strain amplitude is typically 10^{6}-10^{-7} and frequencies can vary from 1-50 Hz for field seismic measurements to several KHz during laboratory analyses on specimens. Furthermore, cyclical tests (such as: cyclical triaxial tests and cyclical torsional shear test) are based on measurements of stress-strain relations at low frequency domain (usually below or in the order of 10 Hz) where inertia effects can be neglected. Typically, they involve strain amplitude in the order of small values. Deformation characteristics are typically evaluated by varying confinement states, cyclical loads and number of loading cycles.

Moreover, empirical correlations for decay curve definition have been developed over the last decades and they are available in the technical literature. The non-linear behaviour of soil at small to medium strains is mostly described via hyperbolic relationship, such as the one formulated by Hardin & Drnevich (1972):

$$\frac{G}{G_0} = \frac{1}{1+(\gamma/\gamma_r)} \quad (1)$$

where G is the secant shear modulus at any strain, G_0 is the elastic maximum shear modulus, which corresponds to G @ γ = 0.0001%, and γ_r is the reference shear strain defined by τ_{max}/G_0. Other similar hyperbolic models were proposed, among others, by Fahey & Carter (1993), Darendeli (2001) and Oztoprak and Bolton (2013). The models substantially differ for the introduction of best-fit parameters related to, for example, the curvature of the S-shape function and the elastic threshold strain beyond which the shear modulus falls below its maximum. It should be noted that each empirical correlation must somehow always rely on experimental data such as the maximum elastic shear modulus.

3. Data acquisition techniques

3.1. In situ data acquisition techniques

The heterogeneity of the ground materials and the fact that usually only a very small part of a given site is investigated can make the selection of an appropriate method for measuring stiffness parameters a challenging task. According to Clayton (2011) the appropriate choice of method is linked to different factors that include: the ground characteristics and the relative field experience, the specific advantages that each method can provide and the equipment and personnel available.

In many cases and in particular when the conditions of the material under investigation are poor, in situ testing is considered more appropriate since removal and transportation of the rock sample can eventually damage the sample itself and result in unreliable measurements at the laboratory. The two most common ways to perform in situ investigation are through the borehole of a well or by excavations. There are many in situ techniques from which dynamic elastic modules can be deduced.

Geophysical applications are frequently used for soil geomechanical characterisation because they are not destructive, the tested soil is in its initial field condition (e.g. initial stress, rock drainage) and the actual effective stress is preserved. Moreover, during the measurement a large portion of soil is tested, providing average values representative of the entire rock volume and not single-point measurements (Luna and Jadi, 2000). It should be noted that geophysical applications are applicable in low strain levels ($<10^{-6}$) where the correlation between the measured values and the mechanical rock properties are still linear ("dynamic range" Fig. 1) while for larger strains laboratory tests are more applicable. Some of the most common geophysical in situ techniques are presented below.

3.1.1. Surface seismic surveys

During seismic survey methods a seismic signal is generated on the Earth's surface, either by a weight drop for small scale surveys or by explosives / seismic vibrators for larger scale experiments, that propagate inside the earth's layers and is recorded back on the surface after been refracted on discontinuities in the subsurface. The measurement of the arrival times at the recording stations can be used to calculate the P-(compressional) and S-(shear) wave velocities. For geomechanical purposes the S-waves are more important since they produce shear torsion without change of the rock volume and thus are able to capture shear information connected solely to the soil particles and not their saturation fluids which cannot sustain shear stress, making them insensitive to the soil saturation.

Dynamic elastic modulus can be calculated in this way using the following formulas (SW-AJA, 1972) for the shear modulus (G), elastic modulus (E) and Poisson's ratio (ν):

$$G = V_s^2 \quad (2)$$

$$E = 2G(1 + \upsilon) \quad (3)$$

$$\nu = \frac{0.5\left(\frac{V_p}{V_s}\right)^2 - 1}{\left(\frac{V_p}{V_s}\right)^2 - 1} \quad (4)$$

where V_p is the P-wave velocity, V_s is the S-wave velocity and ρ is the density.

3.1.2. Down-hole/Cross-hole measurements

Seismic measurements are also performed inside boreholes. In the case of down-hole and up-hole measurements the seismic source can be positioned on the Earth's surface or inside a borehole respectively and the produced seismic waves are registered by an array of receivers which, depending on the experiment, can be installed inside the borehole or on the surface of the Earth (fig. 3). Similar to surface seismic surveys, the travel-times of the seismic waves are measured and are plotted versus depth. The maximum velocities of P- and S- waves (e.g. Woods, 1994, Gazetas, 1991) can be determined from graphs and through formulas 2,3 and 4 converted to mechanical properties.

The application of cross-hole measurements requires the presence of two or more wellbores equipped with geophones and a seismic source. Usually cross-hole experiments are more costly than the down-hole tests since more wellbores are needed, but the accuracy and resolution of the method at all depths means this method is optimal for calculating shear strain. Inside one of the boreholes the source of the seismic waves, which is specially designed to produce S-waves, is positioned while in the other borehole appropriately oriented geophones are coupled to the borehole walls to accurately register the P- and in particular S-wave arrival to be later used in the aforementioned formulas for the shear and elastic modulus calculation.

3.1.3. Sonic log measurements

The Sonic log tool provides the formation's transit travel time (Δt) by measuring the time seismic waves need to propagate between a transmitter and a receiver positioned on the tool. The log reports the transit time in microseconds per foot (μs/ft). Modern sonic log tools can be equipped with a different number of transmitters and receivers that in certain cases are also capable of reducing measurement errors due to poor borehole conditions or unwanted tool effects and provide quality results. The depth of investigation of the tool is generally between 2.5-25 cm from the borehole (Rider, 1999) while the one of the long-spaced sonic log tool can reach almost 50 cm. The range of frequencies used by the sonic tool are 10-40 kHz which are much higher than typical seismic frequencies (10-50Hz) so special care should be taken when comparing sonic log and seismic data. The values of the sonic log can be easily converted to seismic velocities and then to dynamic mechanical properties through appropriate formulas.

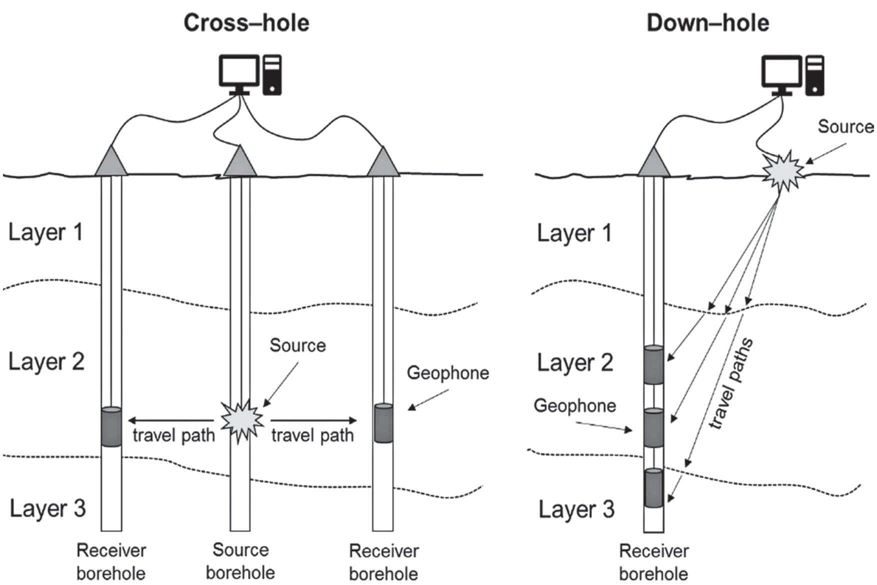

Fig. 3. Examples of cross-hole and down-hole seismic survey configuration.
Esempi di configurazioni di indagini sismiche cross-hole e down-hole.

3.1.4. Continuous surface waves (CSW) analysis

The continuous surface wave method exploits the characteristics of the Rayleigh waves, a type of surface waves that propagate along the surface of solids and is characterised by both longitudinal and transverse motions with amplitudes that decrease exponentially with the distance from the surface. In this method, initially known as steady state vibration technique, a vibrator was used as a seismic source and a geophone as a receiver. The receiver was progressively moving away from the vibrator in order to map the wavelength for specific frequencies. High frequency excitation induces short wavelengths that penetrate only at shallow depth, while lower frequencies generate longer waves that sample deeper portions of the investigated rocks. In this way the phase velocity was calculated through the equation:

$$V_{ph} = \lambda f \tag{5}$$

where λ is the wavelength and f is the specific frequency. Using different frequencies was possible to create a Rayleigh wave dispersion curve and considering that the S-wave velocity at low strains is very similar to the one of the Rayleigh wave velocity, it was possible to calculate dynamic elastic modules for different depths.

3.2. Laboratory techniques

Laboratory techniques allow the determination of changes in stiffness under (quasi) static loading condition, via advanced triaxial testing, and under dynamic condition, via the resonant column apparatus, bender elements and cyclic triaxial testing.

3.2.1. Bender element testing

The bender elements consist in low voltage piezo-ceramic transducers for seismic wave velocity measurements in the specimen. Their frequency range is from 2 up to 10 KHz. The technique has become increasingly popular because compact bender elements are easily installable in standard geotechnical testing apparatus, such as oedometers, triaxial cells, resonant column. Furthermore, the test's value lies in its simplicity, its relatively low cost and its potential for determining anisotropy of shear modulus (Clayton, 2011). On the other hand, the accuracy in stiffness determination could be compromised by the uncertainty in the identification of the first break and therefore the travel time, even in case of negligible noise. Interpretation can become particularly challenging at lower frequencies in relation to which noise levels appear to increase.

3.2.2. Resonant column and torsional shear testing

Resonant column and torsional shear tests involve both triaxial and torsional load and today they are commonly performed within the same device. They mainly differ on the basis of the frequency and amplitude of the loading. Both solid and hollow specimens could be tested, in particular hollow specimens allows the reduction of the (radial) variability of applied strain within the specimen.

Resonant column tests are dynamic tests and they are based on the one-dimensional wave propagation theory. The test is performed setting an axially confined cylindrical specimen in a fundamental mode of vibration by means of torsional or longitudinal excitation of one end (Benz, 2007). The obtained strain levels range from very small (less than 10^{-5}) up to intermediate (10^{-3}) values; frequency is higher than 10 Hz. Resonant column tests have been used for 40 years to determine both elastic parameters (shear modulus and Young's modulus) of soils and weak rocks at very small strain levels and the rate of stiffness degradation with increasing strain (Clayton, 2011).

Torsional shear tests are static or quasi static cyclical tests where an axially confined cylindrical specimen is sheared through rotating one of the apparatus end plates (Benz, 2007). They are commonly applied for strains of 10^{-4} to 10^{-2} (Iwasaki et al, 1978).

4. Conclusions

Non-linear elasticity has proven to represent an effective and reliable basis for ground movement forecast in civil engineering; the same advantage could be potentially obtained for approaching the predictive analyses of subsidence induced by hydrocarbon production according to the non-linear elastic theory. In situ seismic surveys and sonic log along wellbores together with standard laboratory tests (such as oedometric and triaxial tests) still represent the standard set of data acquired by the oil industry; but a further effort in terms of more dedicated investigations is required. In fact, seismic geophysical methods can supply the very small strain stiffness at reservoir scale, but this data must still be corroborated and extended by dedicated lab tests analysis, using bender elements, resonant column and advance triaxial tests, for example. Laboratory testing, despite being complex, time consuming and sampling disturbance, can provide a greater range of stiffness data than field testing, and it becomes mandatory for the evaluation of stiffness degradation with strain (Clayton, 2011). Furthermore, very small strain stiffness values determined by different techniques could be seen differently, because of scale effects and heterogeneity, sampling disturbance, test development and interpretation uncertainty, among others. As a consequence, in case of systems with high uncertainty and/or significant effects on predictive scenarios, the need for data redundancy must also be taken into account.

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