

1 **Labyrinthine acoustic metamaterials with space-coiling channels for**
2 **low-frequency sound control**

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10 14 December 2017

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21 **Abstract**

22 We analyze the potential of fractal space-filling curves for the design of labyrinthine acoustic
23 metamaterials acting as perfect reflectors of low-frequency airborne sound.

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26 Keywords: low-frequency waves, labyrinthine acoustic metamaterial, space-coiling curve,
27 Fabry-Perot resonance, fractal organization, perfect reflector, tortuous porous material.

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30 1. Introduction

31 Acoustic metamaterials are composites
32 with an engineered structure governing
33 remarkable functionalities, e.g. acoustic
34 cloaking, transformation acoustics, and
35 subwavelength-resolution imaging [1,
36 2]. Apart from unusual effective
37 properties, the metamaterials offer
38 various possibilities to control
39 propagation of sound or elastic waves at
40 deep sub-wavelength scales [3, 4, 5]. This
41 can be achieved by incorporating heavy
42 resonators [3], Helmholtz resonators [6,
43 7], tensioned membranes [8, 9], or sub-
44 wavelength perforations or slits [10, 11,
45 12, 13] into a material structure. A class
46 of acoustic metamaterials with internal
47 slits is also known as “labyrinthine”.
48 They have recently attracted considerable
49 attention due to an exceptionally high
50 refractive index and the ability to
51 efficiently reflect sound waves, while
52 preserving light weight and compact
53 dimensions [13, 12, 14].

54 Labyrinthine metamaterials enable to
55 slow down an effective speed of acoustic
56 waves due to path elongation by means of
57 folded narrow channels [15, 13]. Their
58 high efficiency in manipulating low-
59 frequency sound has been experimentally
60 demonstrated for various channel
61 geometries. For example, Xie et al. [16]
62 have shown the negative effective
63 refractive index at broadband frequencies
64 for labyrinthine metastructures with zig-
65 zag-type channels. For the same
66 configuration, Liang et al [15] have
67 demonstrated extraordinary dispersion,
68 including negative refraction and conical
69 dispersion for low-frequency airborne
70 sound. Frenzel et al. have used the zig-
71 zag channels to achieve broadband sound
72 attenuation by means of three-
73 dimensional labyrinthine metastructures
74 [17, 18]. The issue of poor impedance
75 matching for labyrinthine metamaterials
76 has been addressed by exploiting tapered
77 and spiral channels [19] and
78 hierarchically structured walls [20].

79 Cheng et al. have proven almost perfect
80 reflection of low-frequency sound by
81 sparsely arranged unit cells with circular-
82 shaped channels that can induce artificial
83 subwavelength Mie resonances [12]. In
84 our previous work, we have proposed a
85 simple modification to the later design
86 (by adding a square frame) to achieve a
87 tunable functionality [14]. Moleron et al.
88 have emphasized the importance of
89 thermo-viscous effects on the
90 performance of labyrinthine structures
91 with sub-wavelength slits [21].

92 Most of the mentioned studies analyze
93 metamaterials with curved channels, in
94 which the direction of wave propagation
95 coincides with that for incident waves. In
96 this case, a folded channel behaves as a
97 straight slit of effective length L_{eff} ,
98 which approximately equals to the
99 shortest path taken by the wave to pass
100 through the structure [13, 21]. Thus, a
101 channel tortuosity appears to play no role.
102 Possible effects of the path tortuosity,
103 when a wave is allowed to propagate in
104 an opposite direction relative to that of
105 the incident field, remain not investigated
106 yet. A few papers have analyzed
107 labyrinthine metamaterials of such a type
108 of channels. In [19], Xie metamaterials
109 with spiral channels have been
110 investigated to enable tunability of
111 effective structural parameters, such as
112 refractive index and impedance.
113 Hierarchically organized channel walls
114 have shown to achieve a broadband wave
115 absorption [20]. These works are focused
116 on experimental validation of the
117 mentioned features, and lack a theoretical
118 analysis of wave behavior in a tortuous
119 channel.

120 The goal of this work is to investigate
121 numerically dispersion and propagation
122 properties of low-frequency airborne
123 sound in labyrinthine metamaterials with
124 channels, which allow changing the
125 direction of wave propagation. To this
126 purpose, we design sub-wavelength paths
127 in metamaterial unit cells along a fractal
128 curve of various iteration levels. In

129 particular, we consider a space-filling
130 curve for the channel design due to its
131 self-similar organization, clear algorithm
132 for the length elongation, and the ability
133 to fill in an occupied area. We provide a
134 complete theoretical analysis of the wave
135 dispersion in the designed metamaterials
136 complemented by the study of acoustic
137 transmission, reflection, and absorption
138 for a monoslab in the absence or presence
139 of thermos-viscous losses. Our results
140 demonstrate that, when a wave inside a
141 narrow channel is allowed to propagate in
142 the opposite (relative to the incident wave
143 front) direction, the wave dynamics is not
144 equivalent to that in a straight slit of an
145 effective length. The peculiar channel
146 tortuosity allows opening wide sub-
147 wavelength band gaps. At band gap
148 frequencies, total broadband wave
149 reflection occurs that is not influenced by
150 the presence of losses in air. Therefore,
151 the proposed labyrinthine metamaterials
152 have a great potential as efficient
153 reflectors for low-frequency airborne

154 sound. Moreover, to facilitate practical
155 exploitation of these metamaterials, we
156 propose to assemble reconfigurable
157 structures from equi-thickness thin
158 panels (sheets) that is a cheap alternative
159 to an additive manufacturing technique.

160 **2. Space-filling curves**

161 As mentioned above, the wave path is
162 elongated by exploiting the fractal
163 structure of space-filling curves [22].
164 First space-filling curves were
165 discovered by Peano [23] (later named
166 after him), and since then many other
167 curves were proposed [24]. An attractive
168 property of these curves is that they go
169 through every point of a bounding
170 domain for an unlimited number of
171 iterations. After initially being studied as
172 a curiosity, nowadays space-filling
173 curves are widely applied, e.g. for
174 indexing of multi-dimensional data [25],
175 transactions and disk scheduling in
176 advanced databases [26], building
177 routing systems [27], etc.

178 Among various space-filling curves, we
 179 have chosen the Wunderlich two-
 180 dimensional curve filling a square [22],
 181 which is constructed as follows. At the 1st
 182 iteration level, one draws an “S”-shaped
 183 curve starting at the bottom-left corner of
 184 a bounding square and ending at the top-
 185 right corner. At the n^{th} ($n \geq 2$) iteration
 186 level, 3 copies of the $(n-1)^{\text{th}}$ -level curve
 187 are arranged along each side of a square
 188 with every copy being rotated by 90°
 189 relative to the previous one. The curves
 190 are joined into an N-shaped route starting
 191 from the up-direction for the left column,
 192 then down for the middle column, and
 193 finally again up for the right column. At
 194 every iteration level N , the length of the
 195 Wunderlich curve is $(3^N - 1/3^N)$, while
 196 that of e.g. Hilbert’s curves is $(2^N -$
 197 $1/2^N)$ [22]. Faster length elongation
 198 enables more compact channel folding in
 199 a labyrinthine structure (and thus,
 200 increases the tortuosity effect, as will be
 201 shown later) that justifies the choice of
 202 the Wunderlich curve for this study.

203 3. Models and analysis

204 methods

205 Figure 1 presents square labyrinthine unit
 206 cells with an internal channel shaped
 207 along the Wunderlich curve of one of the
 208 three iteration levels, i.e “unit cell 1”
 209 (UC1), “unit cell 2” (UC2), and “unit cell
 210 3” (UC3). The structural material is
 211 aluminum with mass density $\rho_{Al} = 2700$
 212 kg/m^3 and speed of sound $c_{Al} =$
 213 5042 m/s . The thickness of bounding
 214 walls is fixed for all the unit cells and
 215 equals $d=0.5\text{mm}$.

216 The channel width is w , and the size of a
 217 square domain occupied by a single
 218 labyrinth is $a = 3^N \cdot (w + d) + d$,
 219 where N is the iteration level. We
 220 preserve an interconnecting cavity of
 221 width w between adjacent labyrinths.
 222 Thus, the metamaterials unit cell size is
 223 $a_{uc} = a + w$ (see Fig. 1a).

224 We analyze plane waves propagating in
 225 the plane of a unit cell cross-section. The
 226 metamaterial geometry is assumed to be

227 constant in the out-of-plane direction
 228 without a possibility to excite a
 229 momentum in this direction. Hence, the
 230 pressure field is always constant in the
 231 out-of-plane direction, and the wave
 232 dynamics can be analyzed by considering
 233 a two-dimensional (2D) geometry. The
 234 validity of this assumption is proven by a
 235 good agreement with the results of three-
 236 dimensional (3D) simulations given in
 237 the “Results and Discussion” section.

238 First, we analyze sound wave dispersion
 239 in the labyrinthine metamaterials that are
 240 infinite in both in-plane directions. By
 241 neglecting any losses in air, small-
 242 amplitude variations of harmonic
 243 pressure $p(\mathbf{x}, t) = p(\mathbf{x})e^{i\omega t}$ (with
 244 angular frequency $\omega = 2\pi f$, where f the
 245 frequency in Hz) are governed by the
 246 homogeneous Helmholtz equation:

$$247 \quad \nabla \cdot \left(-\frac{1}{\rho_0} \nabla p \right) - \frac{\omega^2 p}{\rho_0 c_0^2} = 0 \quad (1)$$

248 with air density $\rho_0 = 1.225 \text{ kg/m}^3$ and
 249 speed of sound $c_0 = 343 \text{ m/s}$ at
 250 temperature $T = 20^\circ\text{C}$. Since

251 characteristic acoustic impedance of
 252 aluminum is around $3e4$ times larger than
 253 that of air, we assume the structural walls
 254 being motionless and apply sound-hard
 255 boundary conditions at air-structure
 256 interfaces. The pressure distribution at
 257 opposite unit cell boundaries is constraint
 258 by the Floquet-Bloch periodic
 259 conditions:

$$260 \quad p(\mathbf{x} + \mathbf{a}) = p(\mathbf{x})e^{i\mathbf{k} \cdot \mathbf{a}} \quad (2)$$

261 with $\mathbf{a} = (a_{uc}, a_{uc}, 0)$ and wave vector
 262 $\mathbf{k} = (k_x, k_y, 0)$. More details about the
 263 dispersion analysis can be found in [14].

264 Next, we evaluate homogeneous wave
 265 propagation through a metamaterial
 266 monolayer. The analyzed model is given
 267 in Fig. 2. Plane wave radiation occurs at
 268 the left domain boundary at distance
 269 $10a_{uc}$ from the slab. At the right
 270 boundary, a perfectly matched layer of
 271 width $2a_{uc}$ is attached to eliminate
 272 unnecessary wave reflection. At the
 273 bottom and top boundaries, the Floquet-
 274 Bloch periodic boundary conditions (2)

275 enable to artificially extend the air
 276 domain in the vertical direction. The
 277 reflection $R = |p_r/p_i|^2$, transmission
 278 $T = |p_t/p_i|^2$, and absorption $A = 1 -$
 279 $R - T$ coefficients are evaluated by
 280 averaging incident p_i , reflected p_r , and
 281 transmitted p_t pressure fields along the
 282 lines located at distance a_{uc} from the
 283 metastructure.

284 In order to analyze how the length and
 285 tortuosity of a labyrinthine channel
 286 influences sound wave characteristics,
 287 we compare T and A values for the
 288 metastructures with those for straight slits
 289 of width w in solid blocks of length
 290 $L=L_{eff}$ or $L=a$, which are distributed at
 291 distances a along the vertical direction.

292 In the case of $L=a$, the blocks act as solid
 293 scatterers of the same size as labyrinthine
 294 structures, but without internal channels.
 295 The effective channel length L_{eff} is
 296 approximately equal to the shortest wave
 297 path from the input to the output through
 298 a labyrinthine channel (as shown e.g. by
 299 green line in Fig. 1a).

300 If a channel width is small compared to
 301 the wavelength of a propagating wave,
 302 thermal and viscous boundary layers near
 303 solid walls cause loss effects (*lossy air*).
 304 The thickness of these layers decreases
 305 with increase of frequency. The thickness
 306 of thermal boundary layer δ_{th} is
 307 evaluated as follows:

$$308 \quad \delta_{th} = \sqrt{\frac{k}{\pi f \rho_0 C_p}}, \quad (3)$$

309 where $k = 25.8 \text{ mW}/(\text{m}\cdot\text{K})$ is thermal
 310 conductivity, and $C_p = 1.005 \text{ kJ}/(\text{m}^3\cdot\text{K})$
 311 is heat capacity at constant pressure. The
 312 thickness of viscous boundary layer δ_{vis}
 313 is

$$314 \quad \delta_{vis} = \sqrt{\frac{\mu}{\pi f \rho_0}}, \quad (4)$$

315 with dynamic viscosity $\mu = 1.814\text{e-}5$
 316 Pa·s. The graphical representation of the
 317 relations (3), (4) is given in Fig. 3. At
 318 20°C and 1 atm, the viscous and thermal
 319 boundary layers' thicknesses are 0.22mm
 320 and 0.26mm at 100 Hz, respectively.

321 As the designed labyrinthine channel are
 322 relatively easy to model, we directly

323 include thermal conduction and viscous
 324 attenuation into the governing equations.
 325 Thus, the linearized system composed of
 326 Navier-Stokes equation, a continuity
 327 equation, and an energy equation is
 328 solved for acoustic pressure variations p ,
 329 the fluid velocity variations \mathbf{u} , and the
 330 acoustic temperature variations T . The
 331 mentioned variations describe small
 332 harmonic oscillations around a steady
 333 state. The corresponding equations are
 334 given in [28] and implemented in
 335 Thermoacoustic interface of Comsol
 336 Multiphysics [29].

337 The dispersion and transmission analyses
 338 are performed by means of the finite-
 339 element method as eigenvalue and
 340 frequency domain simulations in Comsol
 341 Multiphysics [29]. Acoustic domains are
 342 discretized with the maximum element
 343 size of $\lambda_{min}/12$, where $\lambda_{min} = c_0/f_{max}$,
 344 and f_{max} is the maximum analyzed
 345 frequency. This mesh resolves the
 346 smallest wavelength of the study with 12
 347 elements. To properly capture the wave

348 field variations within the viscous and
 349 thermal boundary layers, we
 350 implemented a frequency-varying mesh
 351 with 3-5 boundary layers along the
 352 thickness of a viscous layer.

353 **4. Results and discussion**

354 We consider the described labyrinthine
 355 metamaterials of two dimensions. In the
 356 first case, defined as a “fixed channel”
 357 case, we imply a constant channel width,
 358 $w = const$, at each iteration step.
 359 Thereby we aim at evaluating effects of
 360 the path tortuosity on sound propagation
 361 with the increase of the path length. For
 362 $w=4$ mm, the metamaterial unit cell sizes
 363 a_{uc} are 18 mm for UC1, 45 mm for UC2,
 364 and 126 mm for UC3. For another case,
 365 called “fixed unit cell” case, we assume a
 366 fixed unit cell size, $a_{uc} = const$, with the
 367 channel width becoming smaller at each
 368 iteration. In particular, we fix $a_{uc} = 14$
 369 mm that corresponds to the channel width
 370 $w = 3$ mm for UC1 and 0.9mm for UC2.
 371 For UC3, the internal channel is

372 eliminated for the wall thickness
 373 $d=0.5$ mm. For the specified value of a_{uc} ,
 374 the channel width in the “fixed unit cell”
 375 case is smaller than that in the “fixed
 376 channel” case for the same iteration level.
 377 Thus, by comparing wave propagation
 378 for these two cases, we can evaluate how
 379 different amount of thermo-viscous
 380 losses influences wave propagation in
 381 similar structured labyrinthine channels.

382 In the both cases, an internal unit cell
 383 channel is shaped along the Wunderlich
 384 curve. However, the channel length is
 385 different from the length of the
 386 Wunderlich curve due to deviations in the
 387 construction approaches. In particular,
 388 the algorithm of the Wunderlich curve
 389 construction assumes that the curve is a
 390 compressing mapping from a low-
 391 dimensional space into a 2D domain, the
 392 area of which is the same at each iteration
 393 level [22]. For our unit cells, we assume
 394 a constant thickness of the solid walls that
 395 incurs variations in the channel length
 396 relative to that of the Wunderlich curve.

397 Hence, in the “fixed channel” case, when
 398 the area of a bounding square increases at
 399 each iteration step (in contrast to original
 400 construction approach of the Wunderlich
 401 curve), the channel length is elongated by
 402 a factor of 3^N relative to a . In the “fixed
 403 unit cell” case, the channel length
 404 increases as $3^N a - 1$.

405 **4.1 “Fixed-channel” case**

406 Figure 4 shows calculated dispersion
 407 relations for homogeneous waves
 408 propagating in UC1, UC2, and UC3
 409 along ΓX direction in the reciprocal k -
 410 space. The horizontal axis indicates
 411 normalized wavenumber $k^* = a_{uc}k$, and
 412 the vertical axes depict frequencies f in
 413 kHz and normalized frequencies $f^* =$
 414 $f a_{uc}/c_0$. Note different frequency
 415 ranges for each unit cell. The analyzed
 416 frequencies are limited to a sub-
 417 wavelength range, i.e. up to about
 418 $f a_{uc}/c_0 = 0.5$. For UC1, we consider
 419 modes forming the lowest band gap and
 420 extending up to 9 kHz; for the UC2 and

421 UC3, the frequency range is restricted to
 422 first 4 band gaps, and thus, it is limited to
 423 4 kHz and 500 Hz, respectively.

424 The dash-dot lines represent phase
 425 velocities of sound waves in lossless air
 426 for the lowest fundamental mode within
 427 a unit cell and in homogeneous air (when
 428 a unit cell is removed). As can be
 429 expected, the velocity slows down when
 430 a wave propagates through a labyrinthine
 431 channel. The velocity decreases by
 432 factors 1.63, 2.91, and 5.28, as compared
 433 to homogeneous air.

434 The dispersion relations in Fig. 2 are
 435 characterized by several frequency band
 436 gaps in the sub-wavelength region. All of
 437 them are located below $fa/c_0 = 0.45$.
 438 Hence, we conclude that the designed
 439 labyrinthine metamaterials can control
 440 sound waves at subwavelength scales. As
 441 N increases, the band gaps are shifted to
 442 lower frequencies. These shifts are
 443 directly related to the path elongation.
 444 For example, the 1st band gap starting
 445 from $fa/c_0 = 0.21$ for UC1, is shifted to

446 about 3 times lower frequency, $fa/c_0 =$
 447 0.069, for UC2, since the channel length
 448 in UC2 is 3 times longer than that in UC1.

449 The band gaps bounds are formed by flat
 450 parts of dispersion bands that describe
 451 localized modes. The corresponding
 452 pressure distribution are given in the 1st
 453 and 3rd columns of Table 1 for the 1st
 454 band gaps and Table 2 for the 2nd and 3rd
 455 band gaps. As green color indicates
 456 (almost) zero pressure, one can observe
 457 strong pressure localization within the
 458 labyrinthine channel. It is easy to find out
 459 that regardless of the iteration level, these
 460 localized modes correspond to Fabry-
 461 Perot resonances in a straight slit of width
 462 w and length L_{eff} [21, 13]:

$$463 \quad f_l^{FP} = lc_0/2L_{eff}, \quad (5)$$

464 where l is a positive integer. In the “fixed
 465 channel” case, L_{eff} equals $2.305d_{uc}$ for
 466 UC1; $L_{eff} = 5.667d_{uc}$ for UC2, and
 467 $L_{eff} = 16.642d_{uc}$ for UC3, where
 468 $d_{uc} = a_{uc}\sqrt{2}$. Note that odd values of l
 469 correspond to the lower band gap bounds,

470 while even values of l allow evaluating
471 the upper band gap bounds in Fig. 2.

472 The fact that multiple harmonics of the
473 Fabry-Perot resonances form the band
474 gap bounds, explains a similar structure
475 of the dispersion relation at various
476 frequencies with close values of phase
477 and group velocities for dispersion bands.

478 The pressure distributions in Tables 1 and
479 2 are also similar to those of the artificial
480 monopole, dipole and multipole
481 resonances [12]. For example, the
482 patterns at lower bound of the 1st band
483 gap (the 1st column in Table 1) resemble
484 that of a monopole, when the pressure is
485 concentrated in the central part of a
486 channel and equally radiates along two
487 propagation directions [12, 14]. Since an
488 effective dynamic bulk modulus (not
489 evaluated for our unit cells) is typically
490 negative in a limited frequency range
491 above the monopole resonance, one can
492 expect a high wave reflectance at these
493 frequencies [12]. The wave reflection and
494 transmission characteristics for the

495 labyrinthine structures are analyzed later
496 in this section.

497 Apart from the Fabry-Perot resonances,
498 wave dispersion in our labyrinthine
499 metamaterials is also characterized by the
500 presence of bands at the band gap
501 frequencies. These bands can be found
502 within each band gap for every analyzed
503 unit cell (see Fig. 4). The pressure
504 distributions at these modes (the 2nd
505 column in Tables 1 and 2) resemble those
506 for the dipole resonance and its higher
507 harmonics (compare to 3rd column of
508 Tables 1 and 2), but the pressure is not
509 localized inside a channel. Thus, these
510 modes do not represent standing waves of
511 localized pressure, but are propagating
512 ones with very slow (and often negative)
513 group velocities. They are analogous to
514 slow modes inside phononic band gaps
515 for elastic waves [30, 31]. The
516 mechanism of the slow mode excitation
517 and its dynamics will be investigated in
518 more detail in our future work. Here, we
519 leave these modes within the band gaps

520 (instead of separating a band gap into two
 521 parts), since we have not detected the
 522 presence of these modes in the
 523 frequency-domain simulations, even for a
 524 very fine frequency step (see Figs. 5 and
 525 7).

526 Frequency-domain simulation results are
 527 given in Figures 5 and 6 in terms of
 528 transmission and absorption coefficients
 529 for lossless and lossy air. We analyze
 530 wave propagation through a monolayer
 531 composed of the labyrinthine unit cells
 532 (Figs. 5a, 6a, 6c) and straight slits of
 533 length L_{eff} (Fig. 5b, 6b, 6d) and a_{uc} (Fig.
 534 5c). (Note that at certain very low
 535 frequencies in lossy air, the transmission
 536 and absorption coefficients appeared to
 537 be mesh-dependent, and hence, are not
 538 shown as unreliable.)

539 When losses in air are neglected, for all
 540 the structures, incoming waves are either
 541 transmitted or reflected, and thus, the
 542 absorption coefficient is zero (not shown
 543 in the graphs). Total transmission is
 544 achieved at frequencies of the Fabry-

545 Perot resonances (5). As can be seen, this
 546 effect is independent of the channel
 547 tortuosity and occurs in folded
 548 labyrinthine channels of any iteration
 549 level at almost the same frequencies as
 550 for straight slits. For the slit of length a_{uc} ,
 551 the first Fabry-Perot resonance appears to
 552 be higher than the analyzed frequency
 553 range. This structure acts as a rigid
 554 scatterer at sub-wavelength frequencies,
 555 and thus, it will be not considered further.
 556 When thermos-viscous losses are
 557 included, the transmission peaks decrease
 558 in magnitude and are shifted to lower
 559 frequencies compared to the lossless
 560 case. The later occurs due to slowing
 561 down of the wave velocity in dissipative
 562 air, as confirmed by experimental
 563 measurements in [21].

564 The striking differences in wave
 565 dynamics of straight and fractal-shaped
 566 channels occur between the frequencies
 567 of Fabry-Perot resonances. In case of
 568 straight slits, the main part of incoming
 569 waves is reflected, while about 15-20%

570 of the wave energy is transmitted through
571 a slit. For the labyrinthine metamaterials,
572 the same behavior is observed in the
573 propagating frequency range, while
574 within the band gaps total wave reflection
575 occurs with zero transmission coefficient.
576 If we remember, that the lower band gap
577 bounds are formed by the monopole
578 resonance and its higher harmonics, then
579 total reflectance is justified by a negative
580 values of effective dynamic bulk
581 modulus at the band gap frequencies.
582 Note that 100% wave reflection is
583 preserved even if thermos-viscous losses
584 are taken into account. In contrast to the
585 total transmission effect at Fabry-Perot
586 resonances, which is eliminated by the
587 loss mechanism, the total reflection is not
588 affected by dissipation. As the iteration
589 level increases, the band gaps, i.e. the
590 total reflection frequencies, are shifted to
591 lower frequencies and decrease in size
592 (compare Figs. 5a, 6a, and 6c). However,
593 the amount of transmitted energy at
594 frequencies of propagating modes also

595 decreases, that is not the case for the
596 straight slits (compare e.g. Figs. 6c and
597 6d).

598 To summarize the results, we can derive
599 two key conclusions. First, the wave
600 characteristics of labyrinthine meta-
601 materials with fractal-structured channels
602 differ from those for straight slits of the
603 effective length due to the tortuosity
604 effect, which becomes important, when a
605 wave is allowed to propagate in the
606 opposite (to the main wave field)
607 direction. Therefore, the derivation of
608 effective characteristics for this type of
609 metastructures should take into account
610 the tortuosity effects. Second, the
611 designed labyrinthine metamaterials can
612 be used as broadband low-frequency
613 sound reflectors of compact size, since
614 100% wave reflection can be achieved by
615 using a single unit cell.

616 The circular markers in Fig. 5a represent
617 the transmission values for the
618 corresponding 3D domain, which is
619 obtained by extruding the 2D model in

620 Fig.2 in the out-of-plane direction by
 621 height $4a_{uc}$. An excellent agreement
 622 between the 3D and 2D results justifies
 623 the introduced assumption on the two-
 624 dimensional character of the analyzed
 625 problem.

626 Finally, we note that the designed
 627 labyrinthine metamaterials can be
 628 considered as tortuous open-porous
 629 materials. The porosity level, evaluated
 630 as ratio of the area of air inside a unit cell
 631 to the total area of a unit cell, is 0.901 for
 632 UC1, 88 % for UC2, and 89 % for UC3,
 633 which is rather low compared to porosity
 634 of typical foams that is very close to 1
 635 [32]. However, the main difference
 636 between the porous materials and the
 637 designed labyrinthine metastructures is
 638 the physical mechanism of the wave
 639 control. Porous materials attenuate waves
 640 due to thermo-viscous losses with the
 641 absorption coefficient close to 1 for
 642 broadband frequencies. In contrast, the
 643 designed structures mainly reflect
 644 incident waves with absorption hardly

645 approaching 0.5 for single frequencies
 646 (see Figs. 6 a,c). In the next section, we
 647 evaluate the metamaterial performance
 648 for increased level of thermo-viscous
 649 losses.

650 **4.2 “Fixed-unit-cell” case**

651 Dispersion relations for UC1 and UC2
 652 for the “fixed unit cell” case, when $a_{uc} =$
 653 14 mm, are shown in Fig. 7 for
 654 homogeneous waves along ΓX direction.
 655 The dimensional frequency ranges are the
 656 same as those for the corresponding unit
 657 cells in the “fixed channel” case (see
 658 Figs. 4 a,b).

659 The structure of the dispersion relation in
 660 Fig. 7a is similar to that in Fig. 4a, except
 661 that the dispersion bands are shifted to
 662 higher frequencies due to a shorter length
 663 of the labyrinthine channel. From the first
 664 sight, more differences can be found by
 665 comparing the dispersion relations for
 666 UC2 in Fig. 4b and Fig. 7b. While in Fig.
 667 4b there are four band gaps, the relation
 668 in Fig. 7b is characterized by the presence

669 of a single wide band gap. This happens
 670 because the unit cell area, $A^{(fix_{uc})} = 14^2$
 671 mm^2 , in the second case is about 3 times
 672 smaller than that for the “fixed channel
 673 case”, $A^{(fix_w)} = 41^2 \text{mm}^2$. This causes
 674 shift of the monopole, dipole and
 675 multipole resonances, and thus, the
 676 related band gaps, to 3 times higher
 677 frequencies. However, for the non-
 678 dimensional frequencies, the band gaps
 679 remain unchanged. In general, it can be
 680 expected, that the dispersion relations for
 681 the two considered cases must be the
 682 same for non-dimensional frequencies,
 683 since the metamaterial structure is
 684 preserved. In contrast to this, we should
 685 observe differences in the transmission
 686 and absorption coefficients for lossy air
 687 in the two cases due to various amount of
 688 thermo-viscous losses in the channels of
 689 a different width.

690 Figure 8 shown the transmission and
 691 absorption coefficients for the labyrinthine
 692 monoslabs of “fixed unit cell” case and
 693 those for straight slits of the

694 corresponding length. In general, the
 695 features found by analyzing the “fixed
 696 channel” case are also observed in the
 697 present case. Namely, the wave
 698 propagation in the labyrinthine channels
 699 is not equivalent to that in the straight
 700 slits due to the presence of 100%
 701 reflection within band gap frequencies,
 702 which is independent of dissipation in air.
 703 However, as the channel in the “fixed unit
 704 cell” case is more than 4 times narrower
 705 relative to that in the “fixed channel”
 706 case, the influence of thermos-viscous
 707 losses is more pronounced that can be
 708 seen in larger absorption values at the
 709 Fabry-Perot resonant frequencies.

710 Therefore, wave attenuation within
 711 labyrinthine channels can be obviously
 712 increased by decreasing the channel
 713 width. In contrast to this, the porosity of
 714 the metamaterial decreases. Thus, for
 715 UC2, the structural porosity is 0.647 for
 716 the “fixed unit cell” case versus 0.88 for
 717 the “fixed channel” case. Thus, one can
 718 approach the functionality of the

719 designed labyrinthine metamaterials to
720 that of tortuous porous materials by
721 decreasing the structural external
722 dimensions and porosity.

723 **5. Conclusions**

724 In this work, we theoretically analyzed
725 the possibilities of labyrinthine acoustic
726 metamaterials with sub-wavelength
727 channels shaped along a space-filling
728 curve to control airborne homogeneous
729 sound waves. We demonstrated that, if an
730 internal channel allows wave propagation
731 in the opposite (to the incident pressure
732 field) direction, the dynamics of the
733 folded channel is not equivalent to that of
734 a straight slit of an effective length. In
735 particular, we found out that Fabry-Perot
736 resonances of a straight slit correspond to
737 the monopole, dipole and multipole
738 resonances in folded channels and govern
739 the generation of band gaps. Within the
740 band gaps, total wave reflection occurs
741 that is not influenced by the presence of
742 dissipation in air. Moreover, by
743 increasing the channel tortuosity and

744 further elongating a wave path, one can
745 achieve 100% reflection outside band
746 gaps. Despite the fact that for higher
747 iteration levels, the designed labyrinthine
748 metamaterials resemble tortuous porous
749 structure, they control wave propagation
750 due to wave interference effects, in
751 contrast to thermos-viscous dissipation in
752 porous structures. This results in a low
753 wave attenuation within a metastructure.
754 We show that the absorption can be
755 increased by decreasing the channel
756 width and the structural dimensions.
757 This is the first time that a fractal space-
758 filling curve has been considered for
759 designing wave paths in labyrinthine
760 metamaterials, and thus, further more in-
761 depth analysis is required to analyze the
762 influence of various factors, e.g. number
763 of turns or an angle of turn, as well as the
764 performance for inhomogeneous waves,
765 on wave dynamics in channels of such a
766 complex form. This studies will be
767 performed in our future work. In
768 conclusion, we believe that the proposed

769 structures could be of use as a new type
770 of broadband low-frequency sound
771 reflectors that can be inexpensively
772 assembled from thin equal sheets by
773 arranging them along indicated paths.

774

775 **Acknowledgement**

776 A.O.K. acknowledges Dr. Dmitry
777 Krushinsky (University of Wageningen,
778 the Netherlands) for the continuous
779 support of this work and fruitful
780 discussions. A.O.K. is supported by the
781 funding from the European Union's 7th
782 Framework programme for research and
783 innovation under the Marie Skłodowska-
784 Curie Grant Agreement No. 609402-
785 2020 researchers: Train to Move (T2M).
786 N.M.P. is supported by the European
787 Research Council (ERC PoC 2015
788 SILKENE No. 693670), and by the
789 European Commission under the
790 Graphene FET Flagship (WP14
791 "Polymer Nanocomposites" No. 604391)
792 and FET Proactive "Neurofibres" grant

793 No. 732344. FB is supported by
794 "Neurofibres" grant No. 732344.

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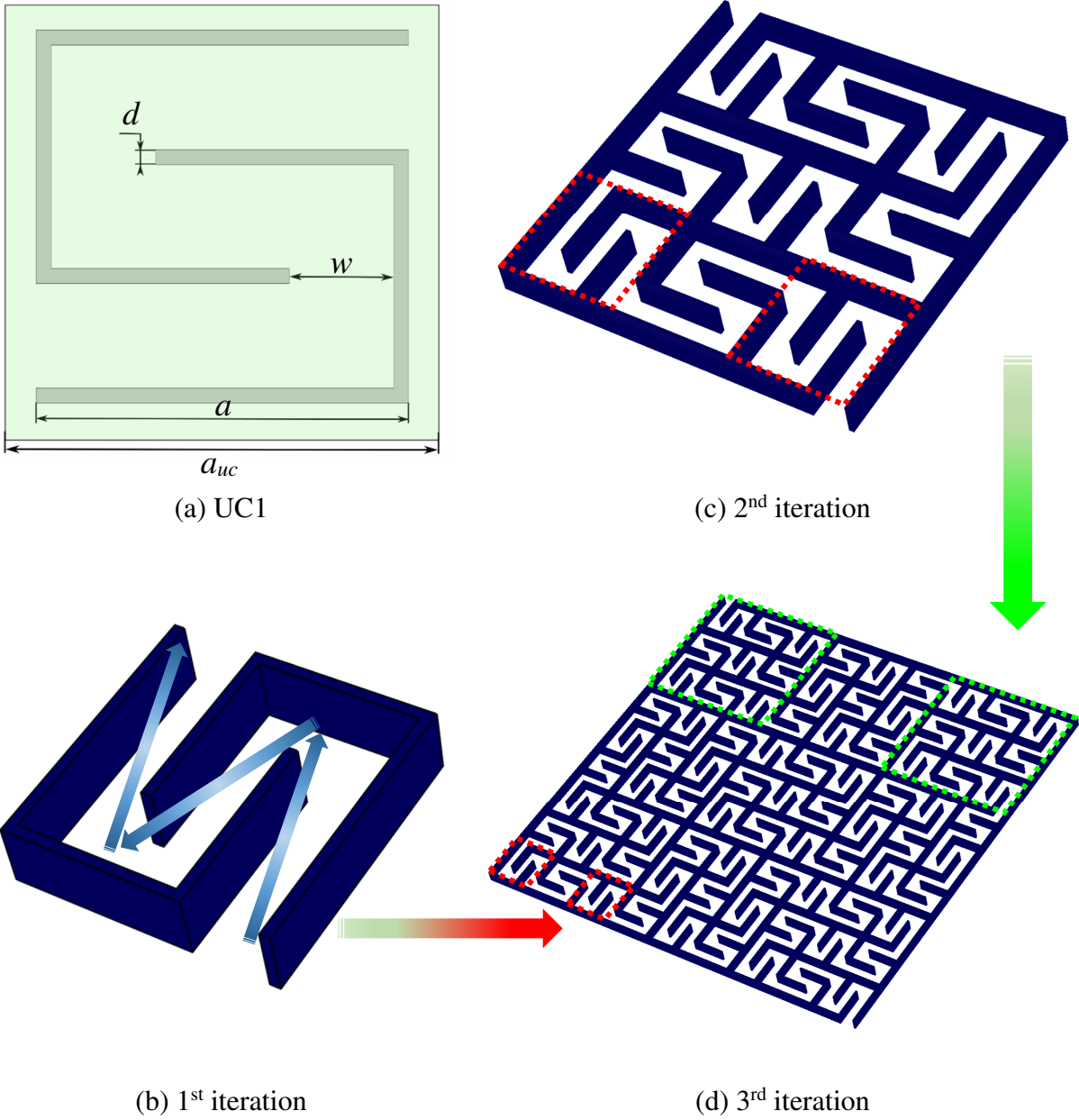


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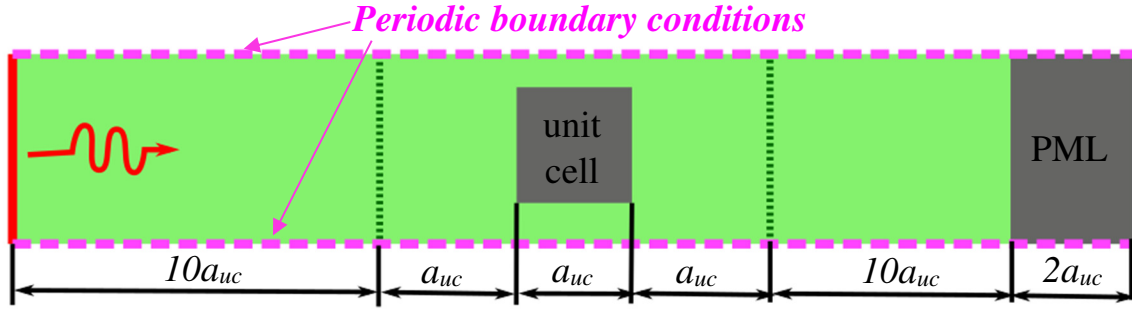


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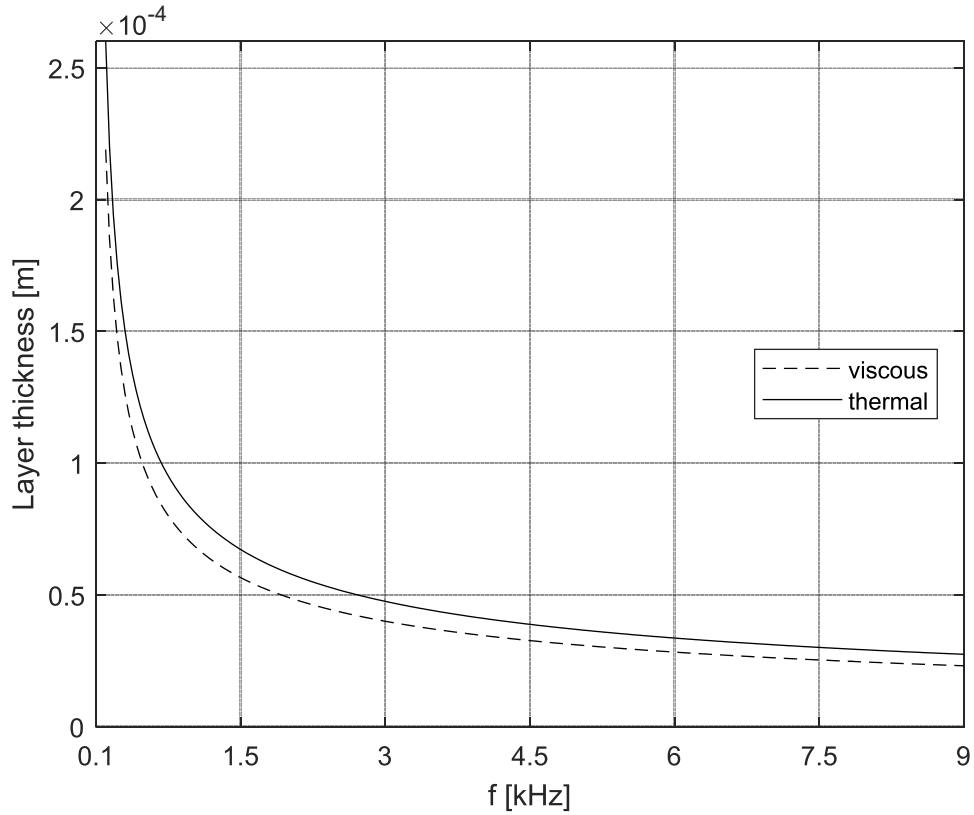


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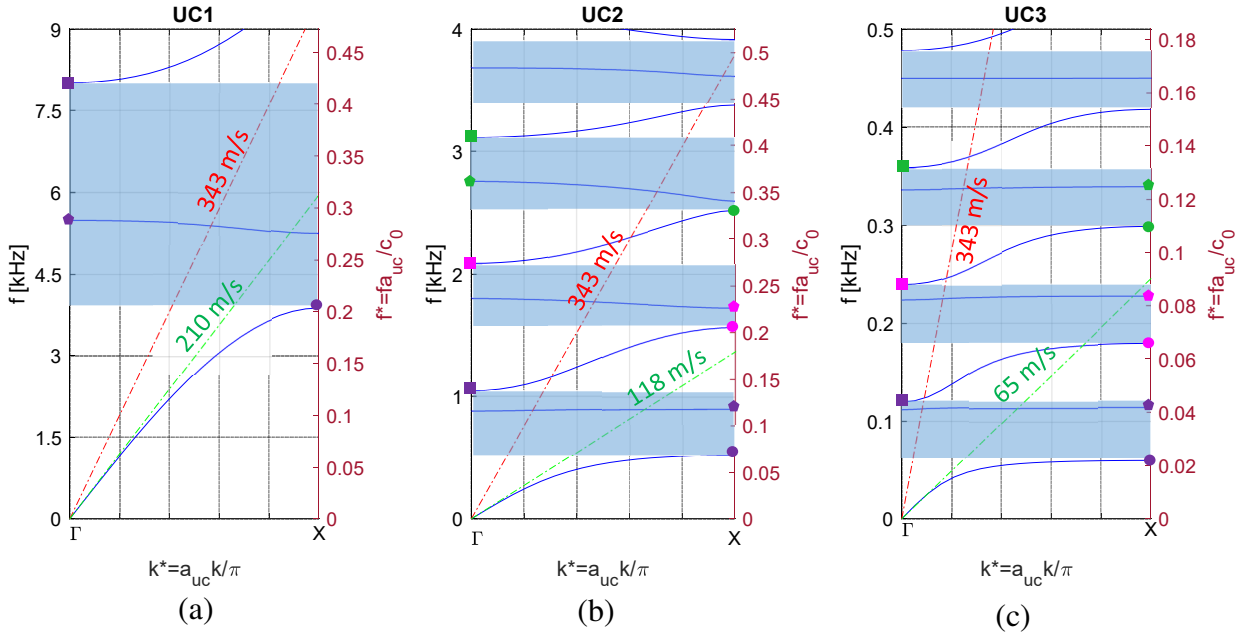


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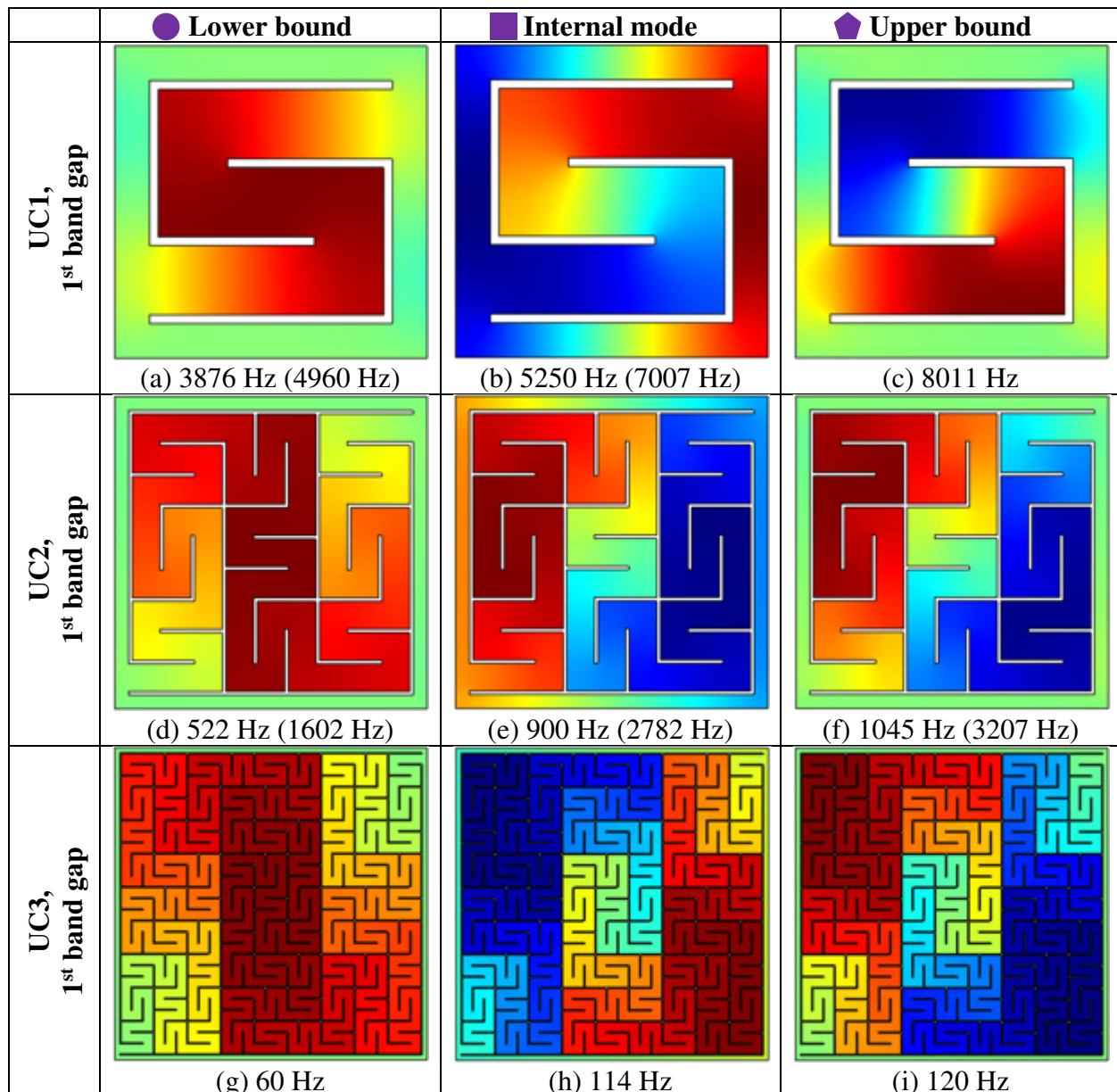
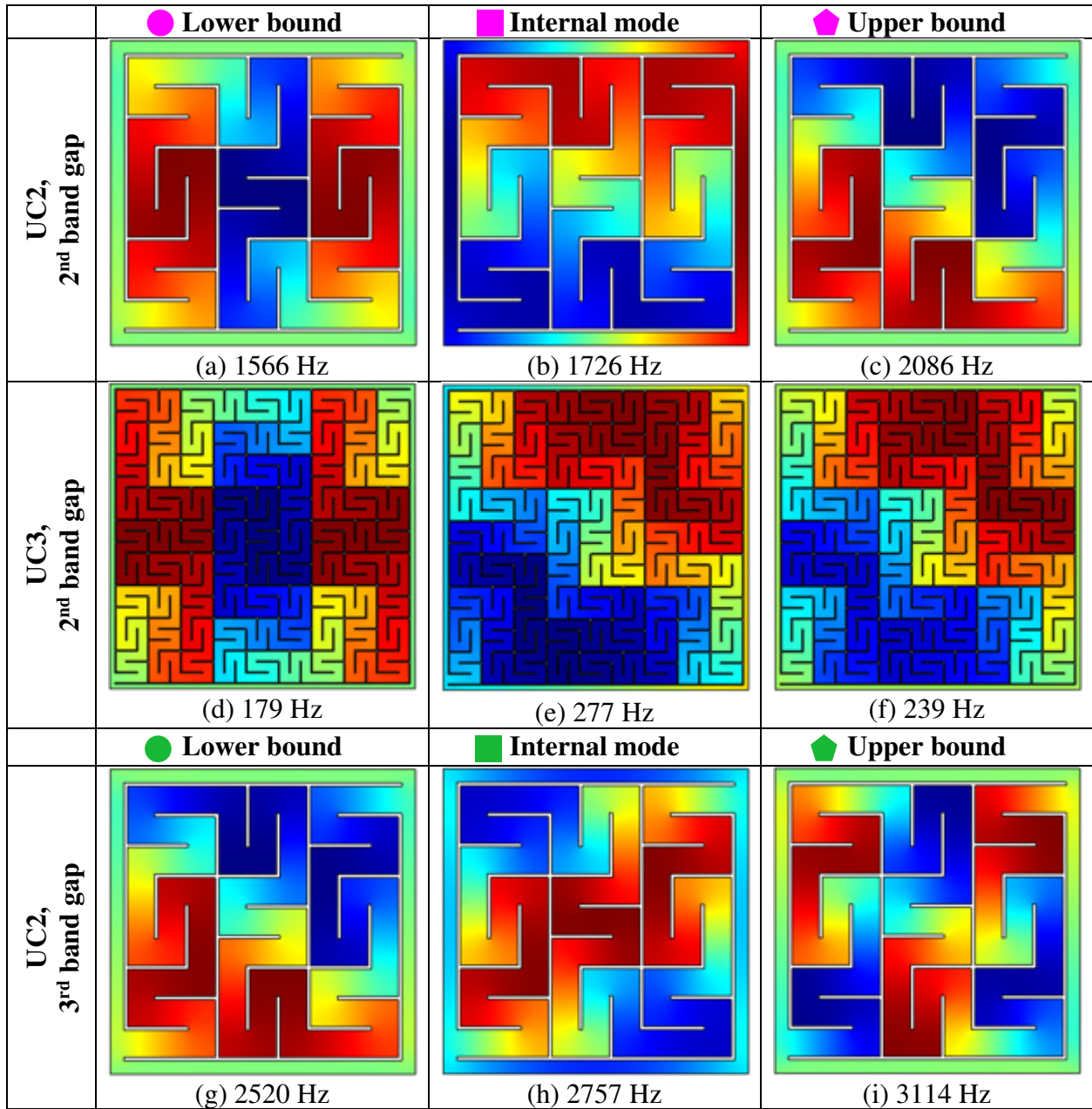
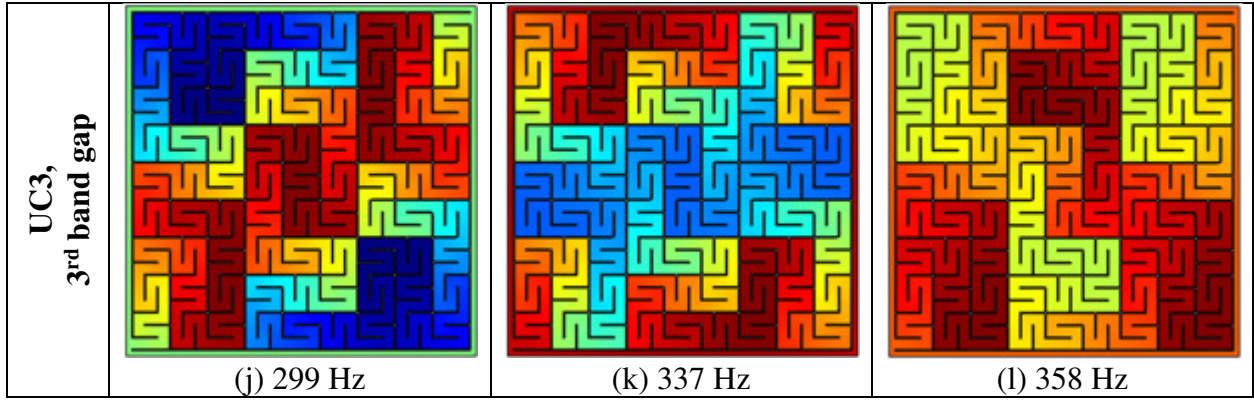


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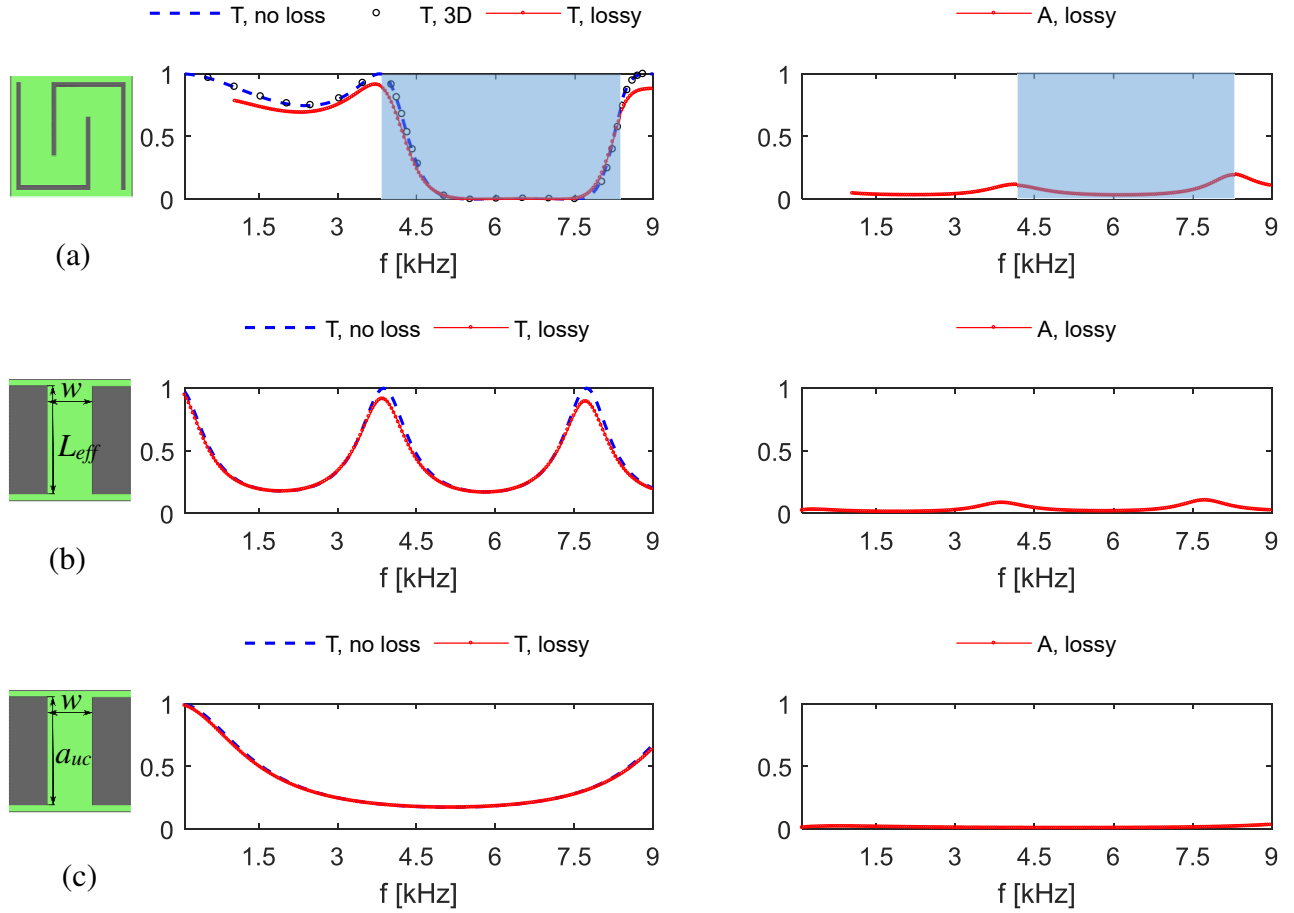


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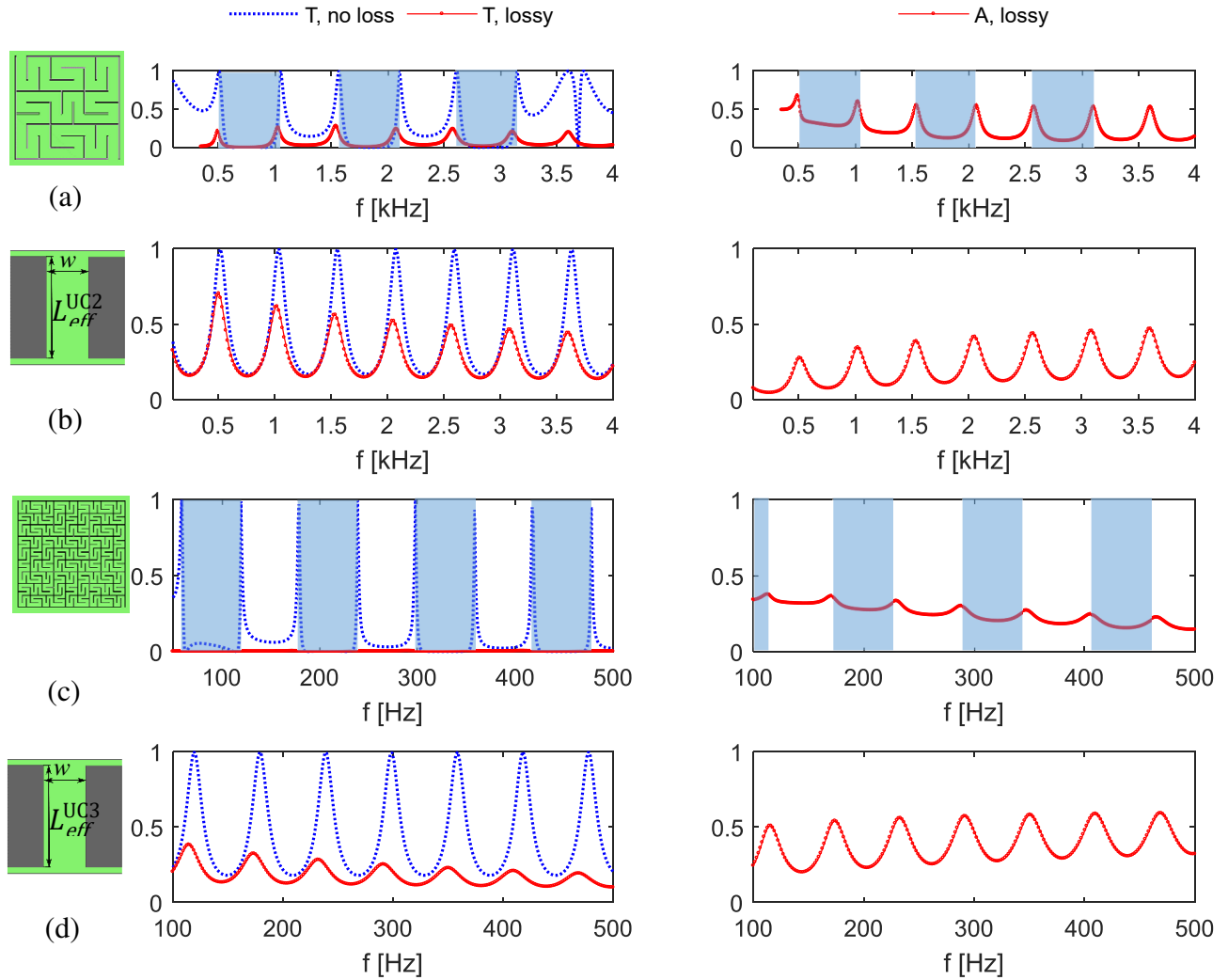


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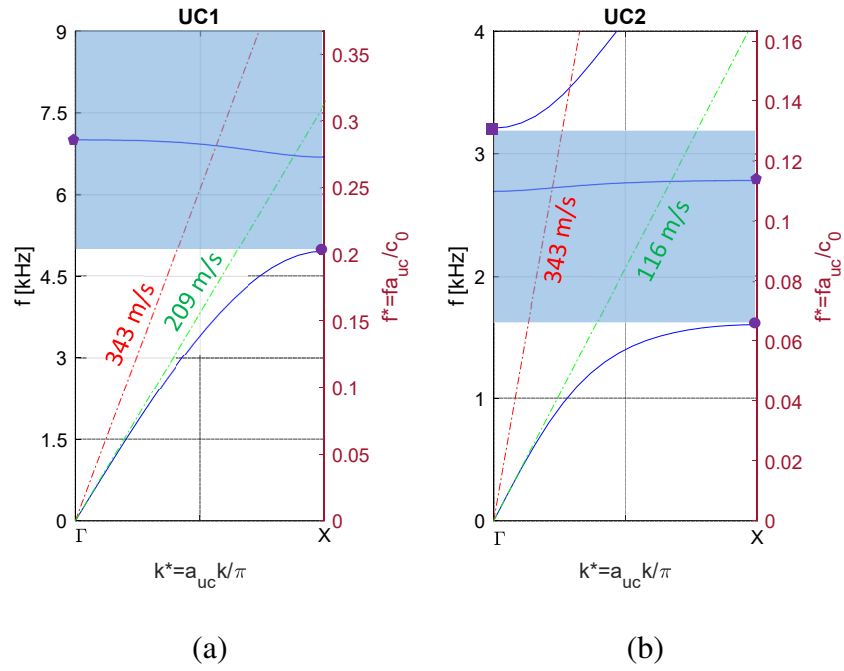


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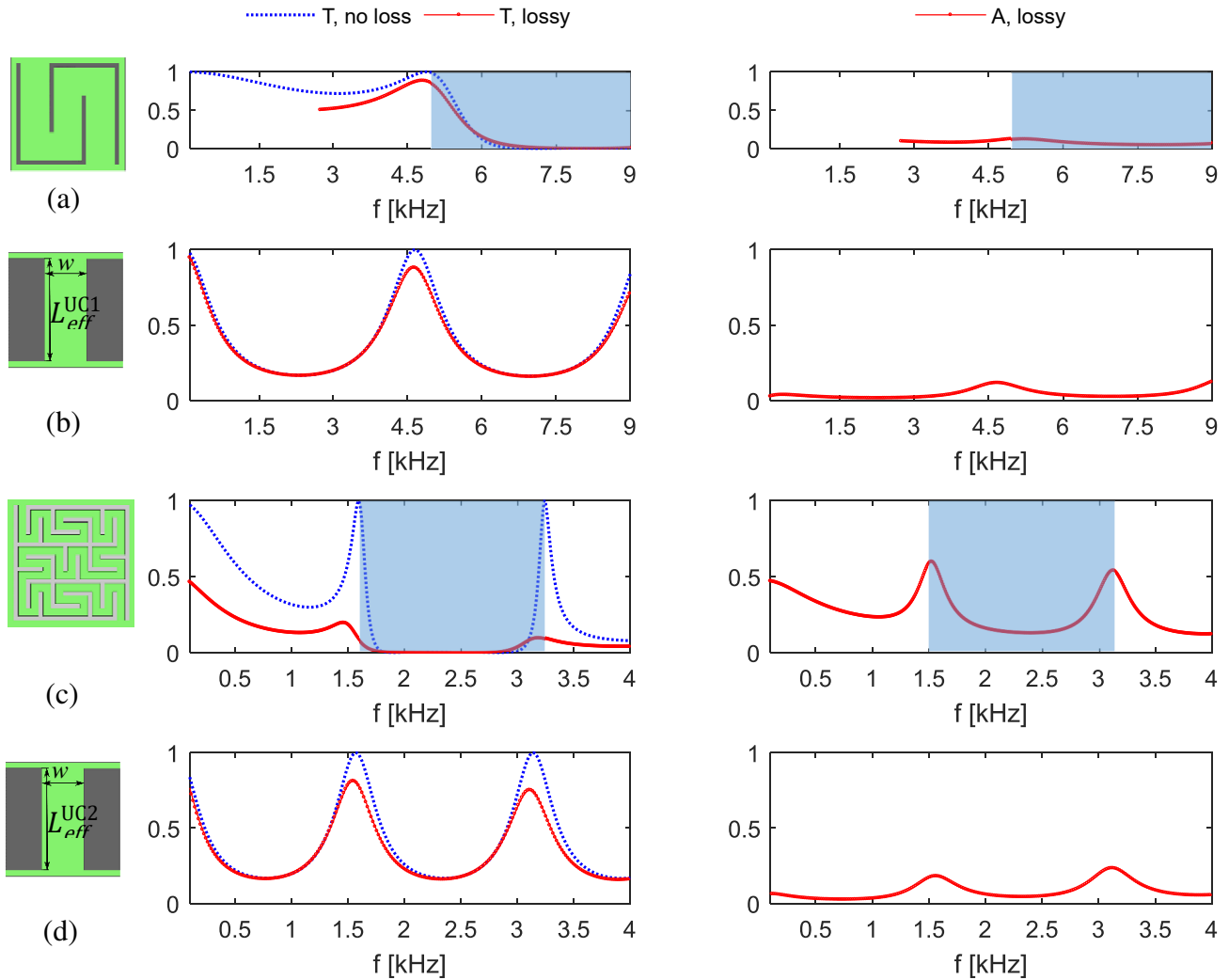


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