Optimization strategies for efficient antenna design

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Michele Beccaria[1], Ho Manh Linh[2], Andrea Massaccesi[1], Alessandro Niccolai[3], Nguyen Huu Trung[2], Nguyen Khac Kiem[2], Riccardo Zich[3], Paola Pirinoli[1]

[1]Dept. of Electronics and Telecomm. - Politecnico di Torino, Torino, Italy, [2]Dept. of Aerospace Electronics - Hanoi University of Science and Technology, Hanoi, Vietnam, [3]Dept. of Energy – Politecnico di Milano, Milano, Italy

Abstract

The conventional procedures for the design of antenna systems often yield a solution that is sub-optimal: this occurs especially when the configuration to be designed is complex, as could be a multi-beam antenna, since many parameters have to be managed and several goals, sometimes competing each other, have to be achieved or when the antenna constraints are not known “a priori” but could be just estimated during the optimization process itself. A possible solution to overcome these limitations consists in using a global optimizer. Here, several different approaches are considered and compared in terms of their performances when applied to different classes of problems.

1 Introduction

Optimization is a key aspect in the engineering system design. It can be faced with different methods: one of most basic and widely used method is the so-called *trial and error*. a typical approach to problem solving based on testing several system configurations until a reasonable solution is reached [1]. This method can be hardly applied to very complex problems because the solution space is too large.
Evolutionary Optimization Algorithms (EAs) are a very valid alternative because they do not require any special knowledge about the shape of the cost function, i.e. the function that mathematically models the problem to be solved, (on the other hand, this is required for linear and quadratic programming [2]), they do not need any initial guess close to the desired minimum (as is required by traditional non-linear optimizer like the simplex method [3]). Moreover, Evolutionary Algorithms do not require nor continuity nor derivability of the cost function [4].

Several evolutionary optimization algorithms are available in literature. The first one that was introduced is the Genetic Algorithm (GA) [5]. It was firstly implemented for binary problems, and then it has been also adapted successfully to real-valued problems. Another important EA is the Particle Swarm Optimization (PSO), an algorithm native for real-valued problems [6].
These two algorithms are the most established ones, but many others have been implemented and have shown very good optimization capabilities, finding a proper tradeoff between exploration and exploitation [4]. Some of them are the Differential Evolutionary (DE) [7], the Biogeography Based Optimization (BBO) [8], the Fireworks Algorithm (FA) [9].
Antenna optimization problems often involves many degrees of freedom, whose management becomes difficult when a deterministic procedure is adopted for its design and therefore the resulting configuration could be a sub-optimal solution. For this reason, EAs have been widely adopted to problems involving the design of an antenna, either of a single radiating element either of the entire system. The most widely EAs used for the optimization of antenna systems are undoubtedly the GA and the PSO. However, the increasing complexity of the problems to be optimized pushes the researchers to investigate and to develop new approaches with improved features, in terms of convergence, computational cost and reliability. In Section 2, the capabilities of two innovative algorithms, the Social Network Optimization (SNO) [10], [11] and the M_QC_{10}-BBO, that is an enhanced version of the BBO, [12], [13] are studied when applied to a complex antenna problem as the design of a scanning beam Reflectarray.
If global optimizers as the evolutionary algorithms are efficient tools for the design of a system that has to satisfy requirements that are known "a priori" and that are used to mathematically model the optimization problem itself, in some other case it occurs that not all the problem constraints are defined. This is for instance what happens in modern wireless communication systems such as fifth-generation (5G) mobile communication systems, that utilize massive MIMO (multiple input multiple output) configurations [14]. MIMOs consist in two sets of antennas, one placed at the base station and the other assembled in a small device, such as a mobile phone, where design space is very limited. Between these two sets of antennas there is a rich scattering multipath fading environment that can be modeled in the most accurate way, since it affects the system performance and in particular the definition of

the beamforming algorithm used by the antennas to generate the multibeams: as better as it can predict the behavior of the channel as higher is the performance of the entire system. However, the structure of the propagation environment is not known "a priori", but could only be communicated to the transmitter by the receiver, after its estimation, and this operation drastically increase the complexity of the algorithm; therefore, it becomes necessary to adopt an optimization process able to estimate in a sufficiently accurate way the propagation environment, trying at the same time to reduce its computational cost [15]. In section 3 some examples of the most commonly optimization techniques adopted in this contest are presented.

2 Evolutionary Optimization – based design of a scanning beam antenna

Optimizing the design of a system means select a proper set of physical variables (called design variables and represented by the vector $\boldsymbol{d}$) that can modify the performances of the system itself.
The design variables are represented in the EA by means of a set of optimization variables (vector $\boldsymbol{x}$) that can be mapped in a biunivocal manner to the design variables themselves.
The performances of the system should be mapped in a numerical value (the cost C) that should be minimized; this mapping procedure is performed by the *cost function*. The entire relation that occurs between the cost C and the optimization variables is called *objective function*.
The optimal set of parameters obtained after the optimization process is indicated with the symbol $\boldsymbol{b}$.
In the specific case of the antenna optimization, the objective function consists in most of the cases by the calculation of the antenna radiation pattern and the cost represents how far it is from the constraints it is asked to satisfy. At each iteration, the algorithm computes the cost relative to the different set of optimization variable that represents the EA population. This means that the objective function is computed, throughout the optimization process, thousands of times, and therefore it is necessary to describe the problem with a model sufficiently accurate but also not too much computationally expensive. In case of an antenna, this means that the use of a full-wave approach for its characterization is not feasible, since it would result in an unaffordable increase of the computational time. Therefore, other techniques, as the representation of the radiating elements with their equivalent circuital model or the use of approximated methods for the computation of the antenna radiating features are generally adopted.
Once the optimal solution is determined, at the end of the EA process, it is finally analyzed with a full-wave approach, to check that effective correctness of the optimization. In Figure 1 the entire optimization process is sketched. It is a general scheme, independent from the type of used algorithm and the type of antenna problem to be solved.

2.1 SNO

The Social Network Optimization (SNO) is an evolutionary algorithm based on the interaction and influence process that takes place in online social networks [10].
The algorithm is based on a population of individuals (users of the social network) and on the posts available online. These two data structures are the basis of the algorithm and they drive the information exchange process: users write posts with their opinions, these posts are read by other individuals and they are influenced and, thus, they change their opinions [11].

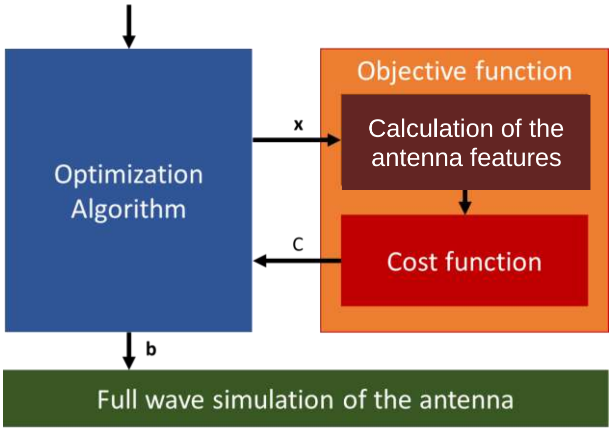


Figure 25: Summary of the optimization procedure adopted in the antenna optimization

Each user is characterized by a set of *opinions*. This is an array with the same size of the optimization variables vector. The interaction among users is driven by two kinds of networks: a friend network and a trust network. For defining these networks, a friend list and a reputation list are associated to each user. The friend list is the set of user ID of all the friends of the user, while the reputation list contains a reputation value for all the other users. All these user's information evolves during time thanks to the algorithm operators.
The interaction in network is a key aspect of this algorithm because it drives the tradeoff between exploration and exploitation. The two interaction networks are deeply different: the friend network is symmetric, the connections are particularly strong, and its evolution depends on events in the real word. On the other hand, the trust network is not symmetric, *i.e.* trust is not reciprocal, the connections are weaker, and its evolution depends only on online relations [16].
Also the posts are complex data structures: their main content is the status, *i.e.* the transposition of the opinion of the user. Other metadata are added to this content: the ID of the user that posted it, the posting time, and a visibility value. This is

very important because posts with high visibility can be read more often, and, thus, their impact on the other users is greater.
The post is the structure that interacts with the objective function: in fact, the status is the candidate solution (the vector of the optimization variables **x**), while the visibility value is the cost value *C* assigned by the objective function to the candidate solution. Figure 26 shows a summary of the data structures of SNO.
The evolution of these data structures is obtained by means of several algorithm operators. The main ones are the linguistic transposition, the reputation update, the trust network creation, the friend network evolution, the influencer selection, the crossover, and the idea contagion function.

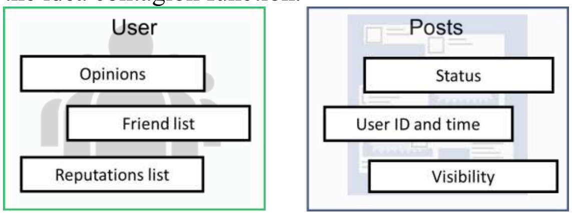


Figure 26: SNO data structures

Figure 27 shows all the operators in the loop of the algorithm. The red squares are the operators, while the blue and green rectangles represent respectively posts data structures and users' ones. The orange box is the optimization problem, and its interaction with the algorithm is underlined by the dashed lines.

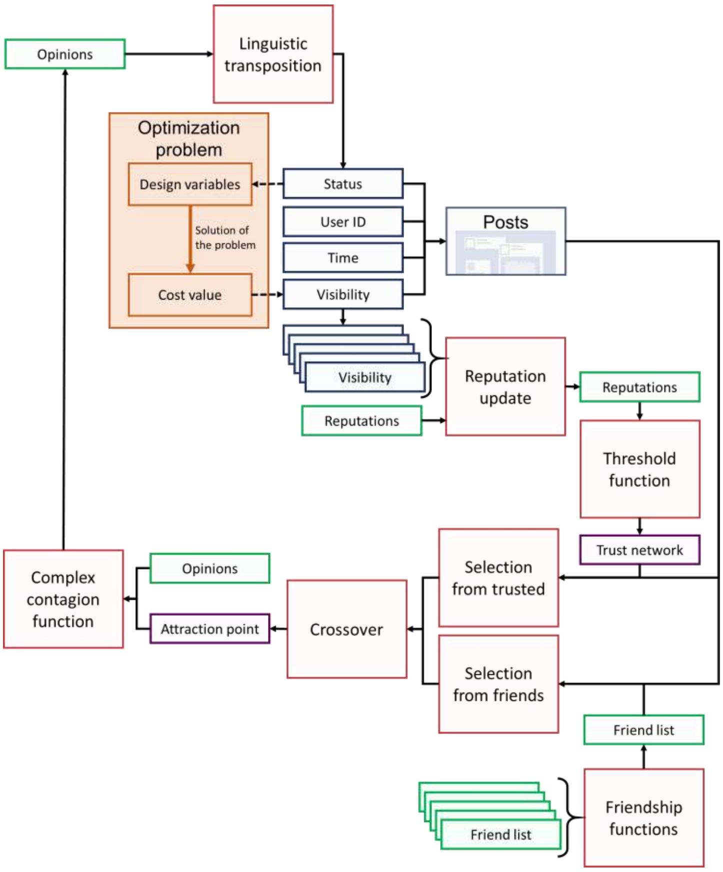


Figure 27: SNO evolution of data structures by means of the algorithm operators.

2.2 M_QC_{10}-BBO

The M_QC_{10}-BBO is a modified version of the BBO, aimed to improve its performance [12], [13]. As the original BBO, it takes inspiration from the migration process of the species among islands, also named habitats. Each of them represents a possible solution, while the species are the optimization variables and the habitat suitability index (HIS) represents the goodness of a solution, i.e. its fitness score.
The moving of species among islands is regulated by how good the habitat itself is, i.e. by its HIS, through the emigration and the immigration rates, that in the standard BBO depend linearly on the number of species present in an island. A high HIS characterized a crowded habitat and therefore the less performing species are pushed to migrate more favorable islands, i.e. where the number of species already present is small. On the other hand, a low performing habitat has a

low emigration rate and high immigration rate and therefore it receives species from better islands. The consequence of this mechanism is that the best habitats have low probability to share information with other high performing solutions and to converge to the optimum.
Despite of this consideration, the algorithm has been shown to perform well for some specific problems, like power distribution [13], while for antenna optimization it stagnates in local minima [14]. For this reason, some improvements have been introduced in BBO [12].
The first one is relative to the model that describes the relation between the number of species in a habitat and the emigration and immigration rates, not yet assumed to be linear but quadratic.
Moreover, it has been observed that the BBO is too much "deterministic" and therefore a novel implicit restart procedure, named "cataclysm" has been introduced: when the best among all the solutions did not improve in the last N iterations, all the solutions are destroyed (cataclysm) and new ones are randomly generated. In order to preserve the best solution, elitism applies and no cataclysm occurs again before at least N generations have passed.
The information about the use of the quadratic model as well as of the cataclysm and the number of iterations between two following events is codified in the name of the algorithm: "M_Q" indicates that the migration model is the quadratic one, "C" informs about the presence of the cataclysm while its numeric subscript *n* is related to the minimum number N of iterations between two cataclysms, being N=5n. Several tests have been done to fix the value of *n*: if it is too small the cataclysms are too close each other and the algorithm becomes a random search, while if too many iterations occur it falls back into the BBO. At the end, it was concluded that a reasonable value is n=10.

2.3 Application of SNO and M_QC_{10}-BBO to the design of a scanning beam Reflectarray

The SNO and the M_QC_{10}-BBO have been applied to the design of a planar beam-scanning Reflectarray (RA) [17], [18]. RAs represent a good compromise between reflector antennas and arrays, and therefore they are suitable for high gain applications.
The antenna system is composed by a feed (usually a horn antenna) and a planar reflector. In order to compensate the absence of curvature present in a conventional reflector, the surface of the RA is divided in a proper number of square unit cell (with size lower than or equal to $\lambda_0/2$, being λ_0 the wavelength computed at the design frequency f_0). Each unit cell includes one or more re-radiating elements, whose selected geometrical parameters are varied to compensate the phase of the incident field and to obtain the desired radiation pattern.

The RA design procedure consists in a first step in which the unit cell behavior is analyzed and the curves representing the variation of the phase of the reflected field in correspondence of the unit cell, assumed embedded in a period structure, as a function of the selected geometrical parameters are obtained. Then the reflectarray is designed, considering the feed characteristics, the feed position and the desired features of the entire antenna: the proper value of the geometrical parameters of each cell is selected in such a way they provide the phase necessary to obtain the desired radiation pattern for a fixed position of the feed.

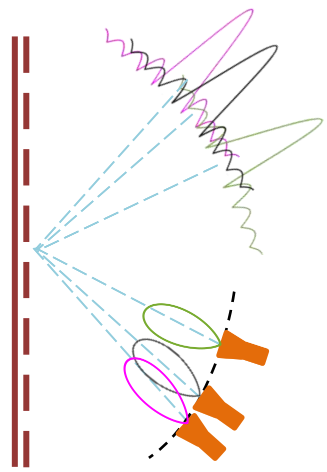


Figure 28: Side view of a scanning beam reflectarray: the feed moves along a circular arc, and changing its position also the direction of maximum radiation varies.

Things are different when a scanning beam antenna, based on the use of a passive RA, must be designed. In fact, in this case, the degrees of freedom of the unit cells are not enough to provide the proper value of the phase for each direction of maximum radiation, that is obtained for instance rotating the feed along a circular arc as sketched in Figure 4. It is therefore necessary to find the values that give the best trade off among the radiation patterns for all the considered pointing directions.

In view of the large number of unknowns whose values must be determined, this is a typical problem that could be conveniently solved with the aid of an evolutionary algorithm.

Here, a microstrip reflectarray is considered, made of a dielectric substrate with relative dielectric constant ε_r= 2.55 and height h=0.8 mm. The RA surface is discretized with 24x24 unit cells with size $\lambda_0/2$. The feed is a standard horn, located at a distance F=10.8λ_0 from the center of the reflectarray, and it can move along an arc, covering the angular range that corresponds to have a beam scanning between -40º and +40º.

Each unit cell includes a square patch (see inset in Figure 5), whose size d is used to control the phase and the amplitude of the reflection coefficient S_{11} provided by the cell itself. Their variation with d is plotted in Figure 5. S_{11} is computed with CST MWS®, carrying on a full-wave simulation of the unit-cell embedded in a periodic structure and for normal incidence.

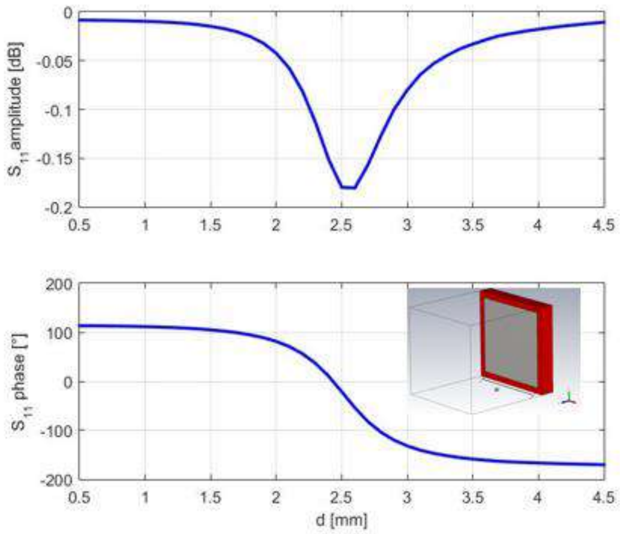


Figure 5: Variation of the phase (top) and amplitude (bottom) of the reflection coefficient provide by each unit cell, as a function of the size d of the square patch. Inset: sketch of the unit cell.

The total number of variables, i.e. the size d of the 24x24 square patches, thanks to the double symmetry, reduces to 148.

The estimation of the radiation pattern has been performed with the aperture field method [19], discretizing the space with 91 samples along the θ coordinate and 35 along ϕ. This choice is a very good tradeoff between the accuracy of the model and the computational time required by the optimization.

The aim of the optimization is to achieve the most constant gain for all the scan angles, controlling at the mean tine the Side Lobe Level (SLL). This has been codified imposing that in correspondence of four different directions of maximum radiation the antenna radiation pattern stays below a different mask, in which the SLL and the half beam width are limited. The values of the constraints have been selected for achieving a good behavior of the gain. It is important to notice that the numerical code used for the optimization is not able to calculate the gain, that can

be assessed only after the entire optimization process with the full wave simulation of the antenna itself.
The optimization process has been stopped after 50,000 objective function calls for both the algorithms. Moreover, since the EAs are stochastic techniques, 12 independent trials have been done to check their reliability.
Figure 6 shows the curves of convergences of the two algorithms. The thin lines represent the convergence of each independent trial, while the darker thick line is the average convergence. It is possible to show that the M_QC_{10}-BBO has a faster convergence in the first third of the optimization process, while SNO can keep a god convergence rate for all the time. Both the algorithms have a good reliability, as proved by the low dispersion of the curves corresponding to the single trials.

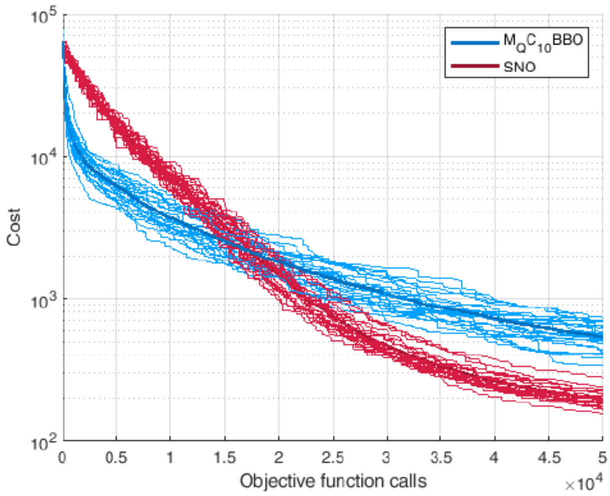
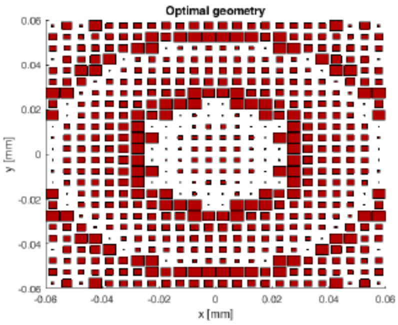


Figure 6: Curves of convergence of the two algorithms.

Figure 7 shows the optimal RA geometries obtained with the M_QC_{10}-BBO and the SNO.

To verify the effectiveness of the optimization process, these two configurations have been finally analyzed with the full wave method implemented in CST MWS® and their radiation patterns are computed for the feed four different positions that correspond to the values of the scanning angle equal to 10º, 20º, 30º and 40º. The radiation patterns relatively to the RA optimized with the M_QC_{10}-BBO are plotted in Figures 8 and 9, while in Figures 10 and 11 that for the SNO configuration are shown. In all the figures the masks used for the optimization are also represented.



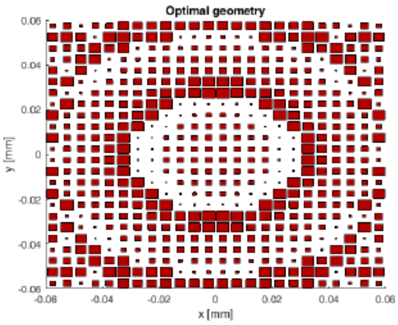


Figure 7: Optimal geometries obtained by (a) MQC10-BBO and (b) SNO.

It is possible to see that in all cases the radiation patterns well satisfy the con-

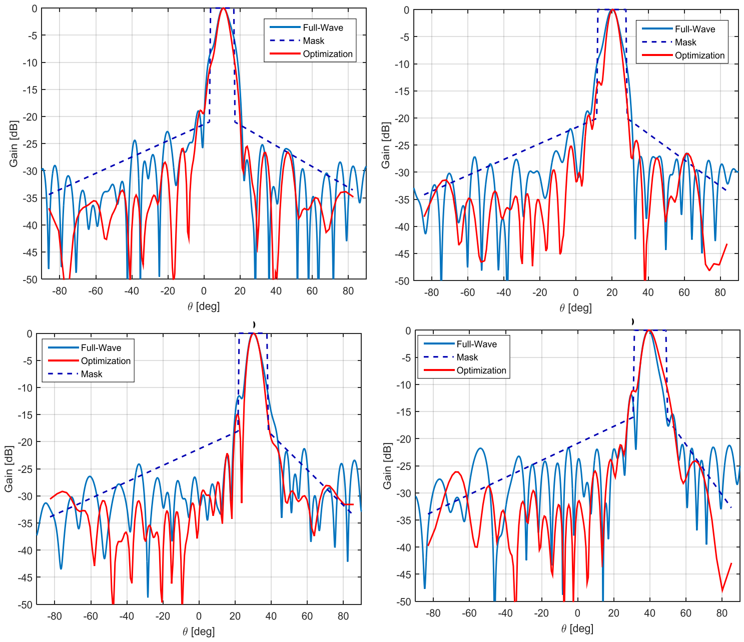


Figure 8: Radiation patterns for the MQC10-BBO solution, E-plane.

straints, since they are almost everywhere below the masks.

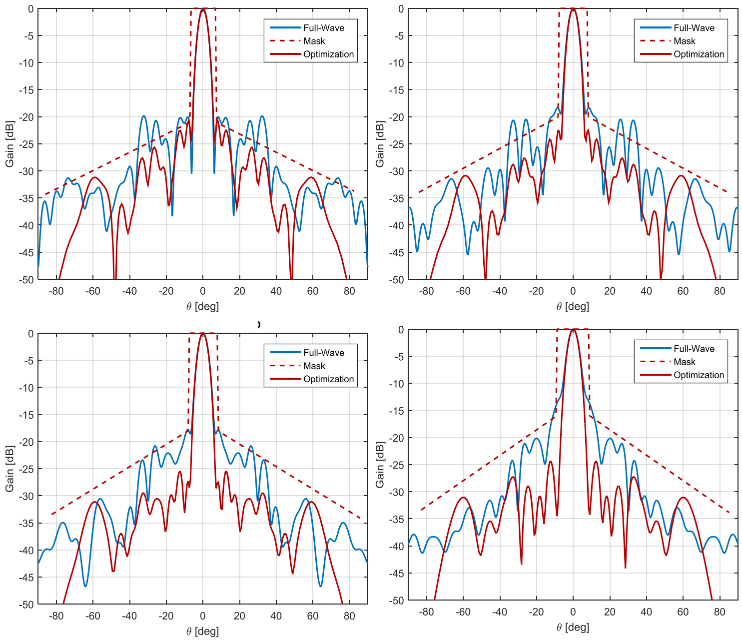


Figure 9: Radiation patterns for the MQC10-BBO solution, H-plane.

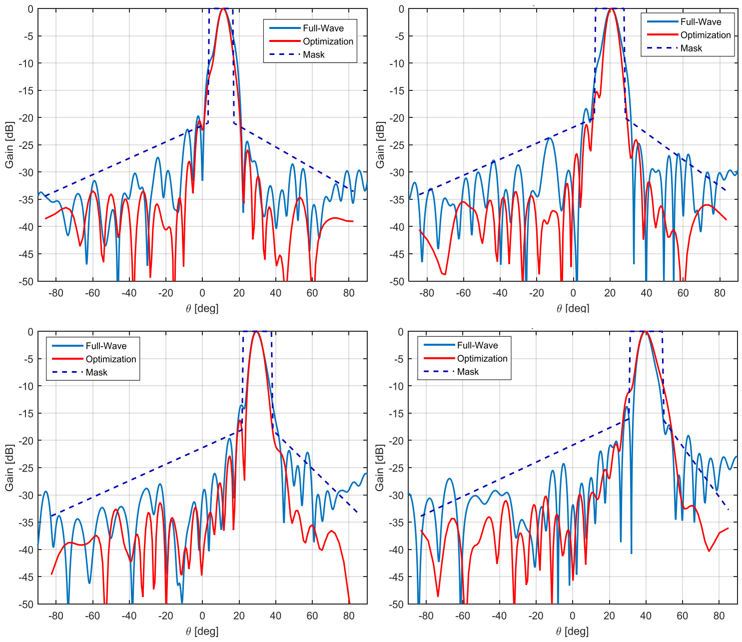


Figure 10: Radiation patterns for the SNO solution, E-plane.

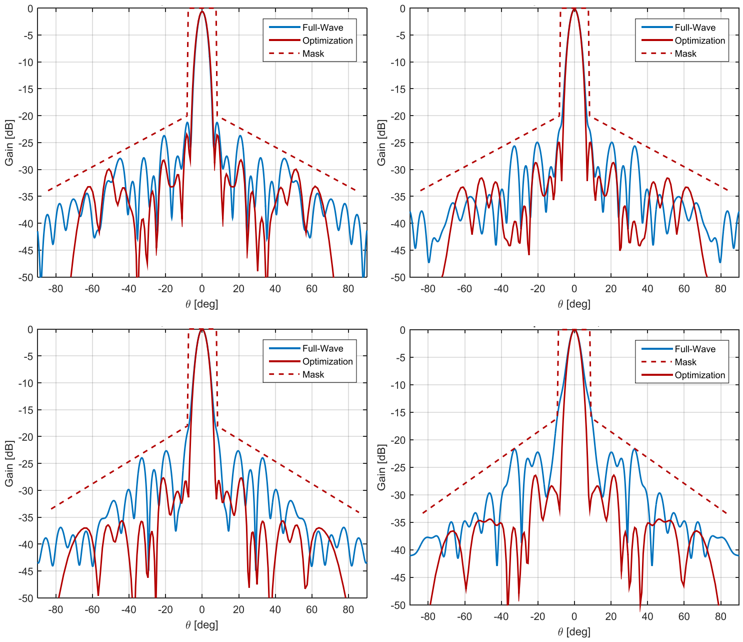


Figure 11: Radiation patterns for the SNO solution, H-plane.

Finally, in Figure 12 it is represented the gain as a function of the scan angle for the two antennas designed with the two algorithms. From this plot clearly emerges that both the RAs present a variation of the gain lower than 2 dB in the entire scanning range, and this definitively proves the effectiveness of the adopted optimization procedure, and of the two algorithms.

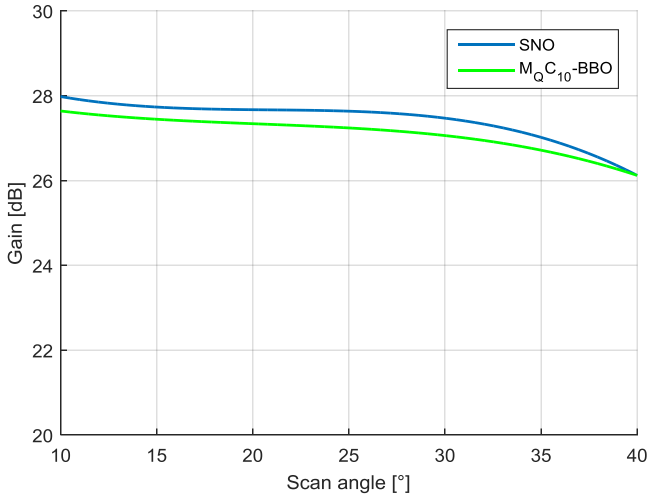


Figure 12: Gain as a function of the scan angle.

3 Optimization for MIMO antenna

Antennas for modern wireless communication systems such as 5G systems make use of the shorter element sizes at high frequencies to incorporate a larger count of radiating elements. The number of antenna elements in massive MIMO configurations is defined to be larger than 100 elements. These antenna arrays are essential for beamforming operations that play a vital role in modern wireless communication systems.

The beamforming system consists of an array processor and a linear or planar array of radiating elements. It is basically a spatial filter that is used to radiate or receive the maximum power in/from a predefined direction. Recently, two possible types of beamforming systems have been studied as candidates for next generation wireless networks: they are the digital beamformer and the hybrid analog-digital beamformer whose block diagrams are shown on the topo and the bottom of Figure 13 [20].

Digital beamforming allows multiple stream transmission and serves multiple users simultaneously. However, it may not always be ideally suited for practical implementations regarding 5G applications. The very high hardware complexity may significantly increase size, cost, energy consumption and complicated integration in mobile devices. On the other hand, it is well-suited for use in base stations. Hybrid beamforming has been proposed as a solution able to combine the advantages of both analog and digital beamforming architectures [20-22].

The criteria used to optimize beamforming systems include maximization of SNR, Minimum Mean Squared Error (MMSE), Linearly Constrained Minimum Variance (LCMV) and robust optimization by Min-max criteria [23].

Consider a beamforming system with M antenna elements. Denoting $s(t)$ the transmitted signal, the direction of arrival (DOA) angle of the wavefront plane associated with $s(t)$ is θ; therefore, the vector of array observation from M elements at time instant t is expressed as:

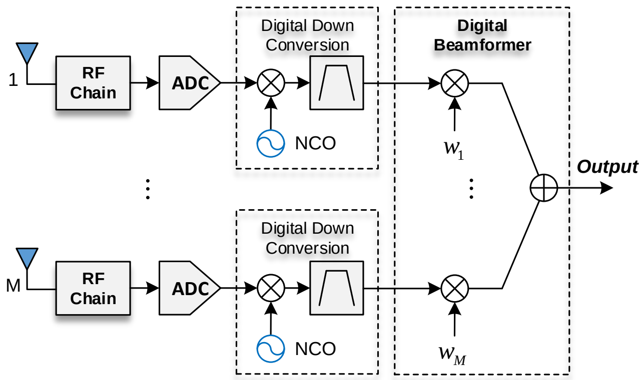
$$\mathbf{x}(t) = \boldsymbol{a}(\theta, \omega)s(t) + \mathbf{i}(t) + \mathbf{n}(t) \quad (1)$$

where $\boldsymbol{a}(\theta, \omega)$ is the steering vector, $\mathbf{i}(t)$ is the interference and $\mathbf{n}(t)$ is a Gaussian noise vector.

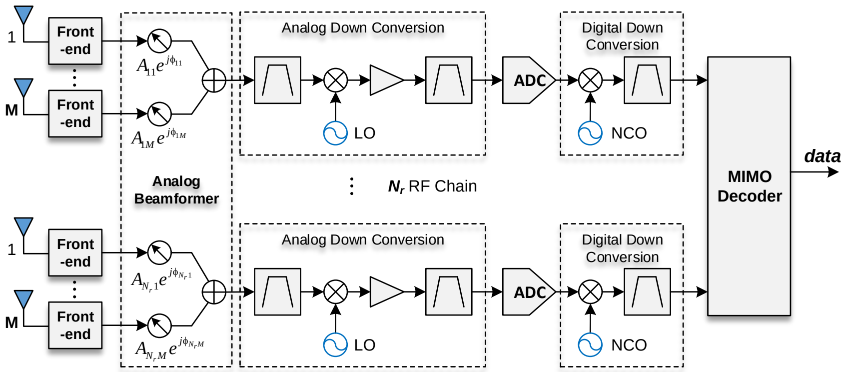
$$\boldsymbol{a}(\theta, \omega) = \left[1 \;\; e^{j\omega d sin(\theta)/c} e^{j\omega 2 d sin(\theta)/c} \ldots \;\; e^{j\omega(M-1) d sin(\theta)/c}\right]^H \quad (2)$$

representing d the distance between the two elements, ω is the carrier frequency and c the speed of light. The steering vector depends on the direction of arrival and the frequency. For simplicity, we denote $\boldsymbol{a}(\theta, \omega)$ with $\boldsymbol{a}$. The beamforming model is expressed as

$$\mathbf{x}(t) = \boldsymbol{a}s(t) + \mathbf{i}(t) + \mathbf{n}(t) \quad (3)$$



(a)



(b)

Figure 13: Digital (a) and hybrid (b) beamforming architecture at receiver.

There are two general beamforming systems, including narrowband beamforming and broadband beamforming. In the narrowband beamforming model, the output signal of beamformer at time instant t is $y(t)$ obtained by linear combination of signals of M elements as:

$$y(t) = \sum_{i=1}^{M} w_i^* \, x_i(t) \tag{4}$$

For broadband model, the output signal is expressed as:

$$y(t) = \sum_{i=1}^{M} \sum_{p=0}^{K-1} w_{i,p}^* \, x_i(t-p) \tag{5}$$

where $K-1$ is the number of delay stages at each channel of i^{th} element of the array. The output signal is expressed as:

$$y(t) = \boldsymbol{w}^H \mathbf{x}(t) \tag{6}$$

where $\mathbf{x}$ is the received signal vector. The vector $\mathbf{w}$ of length M represents the weights as:

$$\mathbf{w}^H = [w_0^*, w_1^*, \dots, w_{K-1}^*] = [\mathbf{w}^T]^* \tag{7}$$

The response of a single beamformer is therefore:

$$r(\theta, \omega) = \mathbf{w}^{\boldsymbol{H}} \mathbf{a} \tag{8}$$

The beampattern is defined as the squared magnitude of $r(\theta, \omega)$. Note that each of the weights in the vector $\mathbf{w}$ impacts to the response of the beamformer in terms of time and space. Output power or variance of estimated signal is determined as:

$$E\{|y|^2\} = \mathbf{w}^H E\{\mathbf{x}\,\mathbf{x}^H\}\mathbf{w} \quad (9)$$

where $E\{.\}$ denotes the mean value.
If the signal is wide sense stationary, the covariance matrix $\boldsymbol{R}_x = E\{\mathbf{x}\,\mathbf{x}^H\}$ is statistically independent over time. Although the signal statistic is not often stationary, the performance of the optimized beamforming is computed under the hypothesis that it is wide sense stationary.
The covariance matrix of the narrow band signal $s(t)$ at frequency ω_0 is:

$$\mathbf{R}_x = \sigma_s^2 \boldsymbol{a}(\theta, \omega_0)\boldsymbol{a}^{\boldsymbol{H}}(\theta, \omega_0) = \sigma_s^2 \boldsymbol{a}\boldsymbol{a}^{\boldsymbol{H}} \quad (10)$$

where σ_s^2 is the average signal power.
Beamforming is an important technique in array processing in order to optimize desired signal while minimizing interferences. The design of the beamformer under statistically optimal method requires statistical properties of the source and the statistical characteristics of the channel.

3.1 Maximization of SNR

The weight vector is the solution of the maximization of the SNR (Signal to Noise Ratio) problem:

$$\mathbf{w}_{\text{MaxSNR}} = \underset{\mathbf{w}}{\operatorname{argmax}} \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_n \mathbf{w}} \quad (11)$$

General solution $\mathbf{w}_{\text{MaxSNR}}$ requires both $\mathbf{R}_s = E\{\mathbf{s}\mathbf{s}^H\}$ and $\mathbf{R}_n = E\{\mathbf{n}\mathbf{n}^H\}$ are covariance matrices of signal and noise. Depending on applications, the calculation of $\mathbf{R}_s$ and $\mathbf{R}_n$ are different. For example, $\mathbf{R}_\mathbf{n}$ can be estimated during the absence of signal, while $\mathbf{R}_\mathbf{s}$ is estimated from signal and known DOA by equation (10). Note that the SNR does not change if the weight vector is multiplied by a scaling factor. Since the steering vector $\boldsymbol{a}(\theta, \omega)$ is fixed for a given signal, it is possible to choose a weight vector to satisfy $\mathbf{w}^H \boldsymbol{a}(\theta, \omega) = c$, where c is a constant. The problem of the SNR maximization can be rephrased in terms of minimizing the interference:

$$\mathbf{w}_{\text{MaxSNR}} = \underset{\mathbf{w}}{\operatorname{argmax}}\{SNR\} = \underset{\mathbf{w}}{\operatorname{argmin}}\,(\mathbf{w}^H \mathbf{R}_n \mathbf{w}), \text{ s.t.} \mathbf{w}^H \boldsymbol{a} = c \quad (12)$$

Using the method of Lagrange multipliers, the solution of the equation [8] is therefore:

$$\mathbf{w} = c \frac{\mathbf{R}_n^{-1} \boldsymbol{a}}{\boldsymbol{a}^H \mathbf{R}_n^{-1} \boldsymbol{a}} \quad (13)$$

3.2 Minimum Mean Squared Error

MMSE method minimizes the error signal between the transmitted signal and a reference signal $d(t)$. In this model, desired user assumes to transmit this reference signal, i.e. $s(t) = \alpha d(t)$ where α is the amplitude of the reference signal $d(t)$ and $d(t)$ is known at the receiver. The output signal of the beamformer is to track reference signal. MMSE method seeks the weight to minimize the average error signal power:

$$\mathbf{w}_{MMSE} = \underset{\mathbf{w}}{\operatorname{argmin}}\, E\{|e(t)|^2\} \tag{14}$$

The average error signal power:

$$\begin{aligned} E\{|e(t)|^2\} &= E\{|\mathbf{w}^H\mathbf{x}(t) - d(t)|^2\} = \\ &= E\{|\mathbf{w}^H\mathbf{x}\mathbf{x}^H\mathbf{w} - \mathbf{w}^H\mathbf{x}d^* - \mathbf{x}^H\mathbf{w}d + dd^*|^2\} \\ &= \mathbf{w}^H\mathbf{R}\mathbf{w} - \mathbf{x}^H\mathbf{r}_{xd} - \mathbf{r}_{xd}^H\mathbf{w} + dd^* \end{aligned} \tag{15}$$

where $\mathbf{r}_{xd} = E\{\mathbf{x}d^*\}$.
Computing the derivative of (15) with respect to $\mathbf{w}^H$ and setting it to zero it is possible to obtain:

$$\frac{\partial E\{|e(t)|^2\}}{\partial \mathbf{w}^H} = \mathbf{R}\mathbf{w} - \mathbf{r}_{xd} = 0 \tag{16}$$

whose solution is:

$$\mathbf{w}_{MMSE} = \mathbf{R}^{-1}\mathbf{r}_{xd} \tag{17}$$

known as optimal Wiener filter. This method requires reference signal to train the beamformer.

3.3 Linearly Constrained Minimum Variance

LCMV method consists in minimizing the output power of the beamformer methods. It keeps the response according to the direction of arrival of the desired signal fixed in order to preserve the desired signal while minimizing the impact of the undesired components, including noise and interference that come from other directions other than desired direction.

The output response of the signal source with direction of arrival θ and frequency ω is determined by $\mathbf{w}^{\boldsymbol{H}}\boldsymbol{a}(\theta, \omega)$. Linear constraint for the weights satisfies the constraint $\mathbf{w}^{\boldsymbol{H}}\boldsymbol{a}(\theta, \omega) = c$, where c is a constant to ensure that all the signals with frequency ω come from the direction of arrival θ are passed with response c.

The minimization of output due to interferences is equivalent to minimizing the output power (minimum output power):

$$\mathbf{w}_{MoP} = \arg\min_{\mathbf{w}} E\{|y|^2\} = \arg\min_{\mathbf{w}}\{\mathbf{w}^{\boldsymbol{H}}\mathbf{R}_{\mathbf{x}}\mathbf{w}\}, \text{ s.t. } \mathbf{w}^{\boldsymbol{H}}\boldsymbol{a}(\theta,\omega) = c \quad (18)$$

Using the method of Lagrange multipliers, find $\min_{\mathbf{w}}[\boldsymbol{\mathcal{L}}(\mathbf{w};\lambda)]$, where:

$$\boldsymbol{\mathcal{L}}(\mathbf{w};\lambda) = E\{|\mathbf{w}^{\boldsymbol{H}}\mathbf{x}|^2\} + \lambda(\mathbf{w}^{\boldsymbol{H}}\boldsymbol{a} - c) = \mathbf{w}^{\boldsymbol{H}}\mathbf{R}_{\mathbf{x}}\mathbf{w} + \lambda(\mathbf{w}^{\boldsymbol{H}}\boldsymbol{a} - c) \quad (19)$$

$$\frac{\partial\mathcal{L}}{\partial\mathbf{w}^H} = \mathbf{R}_{\mathbf{x}}\mathbf{w} + \lambda\boldsymbol{a} \quad (20)$$

The solution of the equation is:

$$\mathbf{w}_{LCMV} = -\lambda\mathbf{R}_x^{-1}\boldsymbol{a} = c\frac{\mathbf{R}_x^{-1}\boldsymbol{a}}{\boldsymbol{a}^{\boldsymbol{H}}\mathbf{R}_x^{-1}\boldsymbol{a}} \quad (21)$$

In practice, the uncorrelated noise component ensures $\boldsymbol{R_x}$ is invertible. If $c = 1$, the beamformer is called Minimum Variance Distortionless Response (MVDR) beamformer. The solution of the MVDR beamformer is equivalent to maximizing the SNR solution by replacing $\sigma^2\boldsymbol{a}(\theta,\omega)\boldsymbol{a}^H(\theta,\omega) + \mathbf{R_n}$ by $\boldsymbol{R_x}$ and applying the inverse matrix lemma $[\mathbf{A} + \mathbf{BCD}]^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}[\mathbf{DA}^{-1}\mathbf{B} + \mathbf{C}^{-1}]^{-1}\mathbf{DA}^{-1}$, we have:

$$\mathbf{R}_x^{-1} = [\mathbf{R}_n + \sigma_s^2\boldsymbol{a}\boldsymbol{a}^{\boldsymbol{H}}]^{-1} = \mathbf{R}_n^{-1} - \frac{\mathbf{R}_n^{-1}\boldsymbol{a}\boldsymbol{a}^{\boldsymbol{H}}\mathbf{R}_n^{-1}}{\boldsymbol{a}^{\boldsymbol{H}}\mathbf{R}_n^{-1}\boldsymbol{a} + \sigma_s^{-2}} \quad (22)$$

$$\Rightarrow \mathbf{R}_x^{-1}\boldsymbol{a} = \mathbf{R}_n^{-1}\boldsymbol{a} - \frac{\mathbf{R}_n^{-1}\boldsymbol{a}\boldsymbol{a}^{\boldsymbol{H}}\mathbf{R}_n^{-1}\boldsymbol{a}}{\boldsymbol{a}^{\boldsymbol{H}}\mathbf{R}_n^{-1}\boldsymbol{a} + \sigma_s^{-2}}$$

$$= \mathbf{R}_n^{-1}\boldsymbol{a} - \frac{(\boldsymbol{a}^{\boldsymbol{H}}\mathbf{R}_n^{-1}\boldsymbol{a})\mathbf{R}_n^{-1}\boldsymbol{a}}{\boldsymbol{a}^{\boldsymbol{H}}\mathbf{R}_n^{-1}\boldsymbol{a} + \sigma_s^{-2}} = \left(\frac{\sigma_s^{-2}}{\boldsymbol{a}^{\boldsymbol{H}}\mathbf{R}_n^{-1}\boldsymbol{a} + \sigma_s^{-2}}\right)\mathbf{R}_n^{-1}\boldsymbol{a}$$

$$= c\mathbf{R}_n^{-1}\boldsymbol{a} \quad (23)$$

3.4 Robust optimization by Min-max criteria

A robust optimization algorithm finds the beamforming weight solution that minimizes the worst case (the best of the worst conditions) on a set of signals $\mathbf{r}_f$ (in time domain or frequency domain for frequency beamformers) and by $MSE(\mathbf{r}_f, \hat{\mathbf{r}}_f)$ criteria, with a constant $q > 0$ and a positive define matrix Q [24]. The problem is stated in min-max optimization as:

$$\mathbf{w}_{MNM} = arg\min_{\mathbf{w}_r}\max_{\mathbf{r}_f:\,\mathbf{r}_f^H\mathbf{Q}\mathbf{r}_f\le q^2} MSE(\mathbf{r}_f, \hat{\mathbf{r}}_f)$$

$$= arg\min_{\mathbf{w}_r}\max_{\mathbf{r}_f:\,\mathbf{r}_f^H\mathbf{Q}\mathbf{r}_f\le q^2} E\left\{\left|\hat{\mathbf{r}}_f - \mathbf{r}_f\right|^2\right\}$$

$$= arg\, \min_{\mathbf{w}_r} \max_{\mathbf{r}_f:\, \mathbf{r}_f^H \mathbf{Q} \mathbf{r}_f \le q^2} \left\{ \mathbf{w}_r^H \overline{\mathbf{R}}_x \mathbf{w}_r + \left|\mathbf{r}_f\right|^2 |1 - \mathbf{w}_r^H \mathbf{A}_r|^2 \right\} \tag{24}$$

where the covariance matrix of observation vector $\overline{\mathbf{R}}_x = E\{(\mathbf{r}_f \mathbf{r}_f^H)\}$.

$$\overline{\mathbf{w}}_{MNM} = arg\, \min_{\mathbf{w}_r} \max_{\substack{\mathbf{r}_f:\, \mathbf{r}_f^H \mathbf{Q} \mathbf{r}_f \le q^2 \\ \overline{\mathbf{R}}_x: \sum \max\{tr(\overline{\mathbf{R}}_x \overline{\mathbf{R}}_x^H)\}}} MSE(\mathbf{r}_f, \hat{\mathbf{r}}_f) \tag{25}$$

The problem solution is determined by the method of Lagrange multipliers:

$$\overline{\mathbf{w}}_{MNM} = q^2 \frac{\overline{\mathbf{R}}_x^{-1} \mathbf{A}_r}{1 + q^2 \mathbf{A}_r^H \overline{\mathbf{R}}_x^{-1} \mathbf{A}_r} \tag{26}$$

Approximate solution and weight vector can be found by adaptive methods such as steepest descent, conjugate direction, gradient, conjugate LMS (Least Mean Squares) and interactive LMS [25].

The performance of the beamforming systems under various optimization criteria is analyzed by means of the Monte-Carlo simulations. The simulations estimate the influence of some parameters on the performance of the system including SNR (Signal to Noise Ratio) and SIR (Signal to Interference Ratio). Array configuration is ULA, number of antennas M = 64, difference DOA angle between transmitted signal and interference $\Delta\theta = 10^o$.

The system performance is evaluated with the Normalized Root Mean Square Error (NRMSE) and the final value is the average value of all Q values after each simulation:

$$NRMSE = \mathrm{mean}_{1 \le q \le Q} \left\{ \frac{\sqrt{\frac{1}{N} \sum_{k=1}^{N} |\hat{x}_q(k) - x_q(k)|^2}}{\left|\max\left(x_q(k)\right) - \min(x_q(k))\right|} \right\} \tag{27}$$

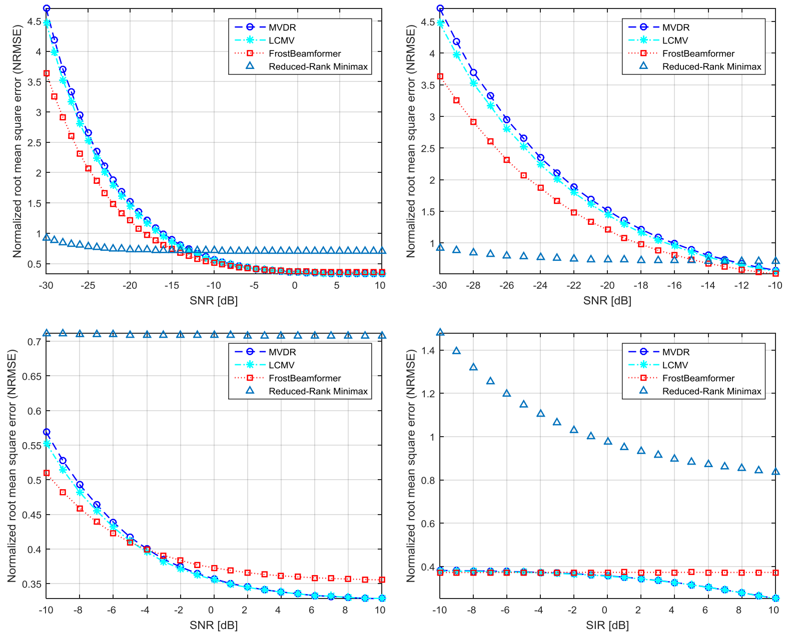


Figure 14. NRMSE according to SNR and SIR.

The simulation results are presented in Figure 14 according to the SNR ranges. It is possible to see that when SNR assumes a low level, in weak signal range, the beamforming based on min-max optimization yields significant result. That is, the system is more robust. But, when the signal level is stronger, MVDR provides a better result against interference. For signal estimation when SIR varies, we see that the performance of the system according to MVDR and LCMV method give best result, meanwhile the min-max optimization yields bad result when SIR changes.

Figure 15 shows the beampattern of the hybrid beamformer with four RF chains and a number of the antennas of 64 (a) and 100 (b). We can see that the optimized beamformer has about four dominant beams. This beampattern means that the data streams can be successfully transmitted through those beams.

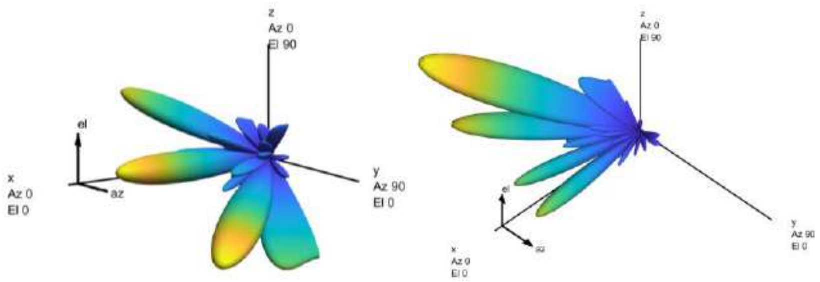


Figure 15. Beampattern of the hybrid beamformer with the number of the antenna elements is 64 (a), 100 (b)

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