A Framework for the Generation of Guaranteed Stable Small-Signal Bias-Dependent Behavioral Models

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***Abstract* — We present a numerical scheme for the identification of compact surrogate models of analog circuit blocks. The basic assumption is small signal operation, so that a local linearization can be applied around a given bias point, resulting in a bias-dependent linear state-space behavioral macromodel. The main novel contribution of this work is the ability to embed in the identification process a suitable set of constraints, that are able to guarantee the uniform stability of the model for any bias value within a prescribed design range.**

***Keywords* — Reduced order systems, stability, small-signal models**

I. Introduction

Modern mobile platforms include a huge number of highly integrated circuit blocks within small-volume Systems on Chip. Different Digital, Analog, Mixed/Signal (AMS) and Radio Frequency (RF) Circuit Blocks (CB) coexist on a single chip substrate, thus posing tremendous challenges for robust design. Different domains must be carefully isolated in order to properly operate without mutual interference.

Power distribution networks may provide undesired noise coupling paths, which must be carefully analyzed in a simulation-driven design flow, so that proper countermeasures can be put in place since early design stages. Power supply noise may be excited by any localized or distributed source that is connected to the power distribution network. Even few transistors switching simultaneously may absorb a large current that may excite global power distribution resonances, which will appear as voltage noise at any other device connected to the same power network. A comprehensive power noise analysis would require a combination of electromagnetic formulations for the characterization of the entire power distribution network (from chip to package, board up to the system voltage regulators), in addition to suitable models of the devices whose switching behavior is the main responsible for the power noise. Such devices may include a large number of individual transistors, so that a detailed electromagnetic/circuit co-simulation is clearly unfeasible.

The above discussion motivates several research efforts towards the generation of surrogate models that can accurately predict the dynamic response of complex systems and devices, in a much faster circuit simulation time with respect to detailed transistor-level models. Macromodeling is nowadays a common practice for complex linear electromagnetic structures [1]–[4]. For the specific application of this work, we will discuss a new approach for the generation of behavioral models of nonlinear devices, starting from input-output responses of the true transistor level circuit. Based on the consideration that in a well-designed system the power noise is much smaller in amplitude than the actual supply voltage, we assume a small-signal operation around a nominal bias-dependent operating point. Therefore, we cast our models in a bias-parameterized linear state-space form.

The proposed approach is not new [5]–[7]. The main novel contribution of this work is the formulation of the model identification algorithm. Based on a specialized adaptive sampling scheme, the model is constructed by embedding a set of constraints that guarantee its uniform stability for any bias value within its design range. As discussed in Sec. II, these constraints are based on the Positive Realness [8], [9] of the model denominator, assuming a parameterized Sanathanan-Koerner form for the model [6], [7], [10].

We illustrate our proposed algorithm on a selected set of numerical examples in Sec. III. This set is representative of all those potential application scenarios, consisting of those devices that are designed to operate as linearly as possible around a given operating point, including Low Noise Amplifiers (LNA), Operational Amplifiers (OA), Low Dropout regulators (LDO), programmable filters, and even individual components such as integrated inductors and capacitors whose inductance and capacitance can be tuned by changing bias conditions. We should remark that the proposed approach is not able to reproduce any nonlinear effect, with the exception of the bias dependence of the small-signal responses. In case the full (large-signal) characteristics need to be preserved, other approaches must be used.

II. Formulation

We consider a possibly complex P-port nonlinear CB described by the following nonlinear state-space equations

$$\dot{\mathbf{x}}(t;\vartheta) = \mathbf{f}(\mathbf{x}(t;\vartheta), \mathbf{u}(t); \vartheta) \tag{1}$$

$$\mathbf{y}(t;\vartheta) = \mathbf{g}(\mathbf{x}(t;\vartheta), \mathbf{u}(t); \vartheta) \tag{2}$$

where $\mathbf{u}, \mathbf{y} \in \mathbb{R}^P$ denote system inputs and outputs, and $\mathbf{x}, \dot{\mathbf{x}} \in \mathbb{R}^Q$ are an internal state vector and its time derivative. The additional variable $\vartheta \in \mathbb{R}$ defines an external parameter, that in this work we consider to be the nominal power supply

(bias) voltage. The model in (1)-(2) usually corresponds the transistor-level description of the CB, which may not be even available in full detail, since it may be embedded in some encrypted SPICE netlist for IP protection.

The main assumption on which we base our work is our ability to extract small-signal responses through dedicated circuit simulations. Wihtout loss of generality, we consider small-signal scattering responses $\breve{\mathbf{H}}(\mathrm{j}\omega;\vartheta)$. A nested frequency and bias voltage sweep can thus be used to compute the $P\times P$ small-signal scattering matrix samples

$$\breve{\mathbf{H}}_{k;m}=\breve{\mathbf{H}}(\mathrm{j}\omega_k;\vartheta_m) \tag{3}$$

over a prescribed grid of frequency ω_k and bias voltage ϑ_m points. Our main objective is the construction of a surrogate model that approximates this data over the frequency band Ω and bias voltage range Θ of interest, and which can replace the original CB for running system-level circuit simulations in a much faster runtime.

We assume the small-signal model to be a bias-dependent Linear Time-Invariant (LTI) system. Our assumed model structure is the so-called Parameterized Sanathanan-Koerner form [6], [10]. Denoting with s the Laplace variable, we define

$$\mathbf{H}(s;\vartheta)=\frac{\mathbf{N}(s,\vartheta)}{\mathsf{D}(s,\vartheta)}=\frac{\sum_{n=0}^{\bar{n}}\sum_{\ell=1}^{\bar{\ell}}\mathbf{R}_{n,\ell}\,\xi_\ell(\vartheta)\,\varphi_n(s)}{\sum_{n=0}^{\bar{n}}\sum_{\ell=1}^{\bar{\ell}}r_{n,\ell}\,\xi_\ell(\vartheta)\,\varphi_n(s)}. \tag{4}$$

This structure casts the model as a ratio between a matrix-valued numerator $\mathbf{N}(s,\vartheta)$ and a scalar denominator $\mathsf{D}(s,\vartheta)$. As discussed in [8], [9], this structure guarantees that all model transfer matrix elements have common poles, which are implicitly parameterized by ϑ. Both numerator and denominator are cast as a superposition of basis functions, where $\varphi_0(s)=1$, $\varphi_{n>0}(s)=(s-q_n)^{-1}$ are elementary partial fractions associated to a set q_n of fixed poles [2], and $\xi_\ell(\vartheta)$ are Chebychev polynomials.

The model coefficients $\mathbf{R}_{n,\ell}\in\mathbb{R}^{P\times P}$ and $r_{n,\ell}\in\mathbb{R}$ can be determined through an iterative procedure called (Parameterized) Sanathanan-Koerner (PSK) iteration [6], [9], [10], which relaxes the nonlinear fitting problem that matches the model responses to the raw small-signal responses as a sequence of linear least squares problems of type

$$\min\left\|\frac{\mathbf{N}^{\mu}(\mathrm{j}\omega_k,\vartheta_m)-\mathsf{D}^{\mu}(\mathrm{j}\omega_k,\vartheta_m)\,\breve{\mathbf{H}}_{k,m}}{\mathsf{D}^{\mu-1}(\mathrm{j}\omega_k,\vartheta_m)}\right\| \tag{5}$$

where the denominator is initialized as $\mathsf{D}^0=1$, and the model coefficient estimates $\mathbf{R}^{\mu}_{n,\ell}$ and $r^{\mu}_{n,\ell}$ at iteration μ are computed by using an approximation of the denominator $\mathsf{D}^{\mu-1}$ available (known) from previous iteration. The iteration is stopped when coefficient estimates stabilize.

The above procedure is standard [6], [7]. However, with this basic formulation, it is not possible to guarantee that the model poles $p_n(\vartheta)$, i.e., the zeros of the denominator $\mathsf{D}(s,\vartheta)$, are uniformly stable for any value of the parameters ϑ. In order to enforce uniform stability, we consider that:

- only the denominator function $\mathsf{D}(s,\vartheta)$ is responsible for the stability of the model, since the "basis" poles q_n of numerator and denominator are only apparent singularities in (4);
- assume that $\mathsf{D}(s,\vartheta)$ is a passive immittance (Positive Real, PR) function: then, its inverse $\mathsf{D}^{-1}(s,\vartheta)$ is also a passive PR immittance function;
- any (rational) PR function has stable poles.

These facts lead us to the conclusion that, if the denominator $\mathsf{D}(s,\vartheta)$ is PR for any $\vartheta\in\Theta$, then its zeros $p_n(\vartheta)$, which are the parameter-dependent poles of our macromodel, will be stable irrespective of the value of ϑ.

Given the adopted model structure, it can be shown that a sufficient condition for denominator passivity is

$$\mathrm{Re}\left\{\mathsf{D}(\mathrm{j}\omega,\vartheta)\right\}>0,\quad\forall\omega,\,\forall\vartheta\in\Theta. \tag{6}$$

This motivates our proposed modification to the GSK iteration by adding the following constraints

$$\mathrm{Re}\left\{\mathsf{D}(\mathrm{j}\omega_\nu,\vartheta_\nu)\right\}>0 \tag{7}$$

to the optimization problem (5), which thus becomes at each iteration a linear least squares problem with linear inequality constraints, whose numerical solution is simple due to its convexity. The main novel contribution in this work is the automatic determination of the sampling points ω_ν,ϑ_ν where the passivity constraints (7) should be enforced. These samples include both fixed and adaptively-determined sets:

- the fixed set comprises all frequency and parameter samples available as raw data;
- the adaptive set is determined by searching all the regions in the (ω,ϑ) plane where $\mathrm{Re}\left\{\mathsf{D}(\mathrm{j}\omega,\vartheta)\right\}<0$, and by performing a local search to determine the local (negative) minima. The latter points are added as samples in the adaptive set.

We now focus on the adaptive search process. We first note that the range where searching local minima is limited in the parameter space, since the possible bias voltage values are restricted to an interval Θ. Conversely, the frequency span where local real part minima can occur is unlimited and can be even outside the modeling bandwidth Ω. In fact, this is the most common situation. Fortunately, it is not necessary to adaptively sample along the frequency axis, since all frequencies $\bar{\omega}_i$ where $\mathrm{Re}\left\{\mathsf{D}(\mathrm{j}\omega,\vartheta)\right\}$ crosses the zero baseline are provided as the purely imaginary eigenvalues $\mathrm{j}\bar{\omega}_i$ of the Skew-Hamiltonian/Hamiltonian pencil [1]

$$\mathbf{M}(\vartheta)=\begin{bmatrix}\mathbf{A}&\mathbf{0}&\mathbf{B}\\ \mathbf{0}&-\mathbf{A}^{\mathsf{T}}&-\mathbf{C}^{\mathsf{T}}(\vartheta)\\ \mathbf{C}(\vartheta)&\mathbf{B}^{\mathsf{T}}&2D(\vartheta)\end{bmatrix},\quad\mathbf{K}=\begin{bmatrix}\mathbf{I}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{I}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}\end{bmatrix}$$

where $\mathbf{A},\mathbf{B},\mathbf{C}(\vartheta),D(\vartheta)$ provide a state-space realization of the (scalar) immittance $\mathsf{D}(s,\vartheta)$. Thus, for any prescribed parameter value ϑ, an algebraic eigenvalue calculation pinpoints the localized frequency bands that include a local negative minimum, which is thus easily find by a local descent-based search.

The main problem that remains is how to sample in the parameter range Θ. To this end, we define with $\psi(\vartheta)$ the

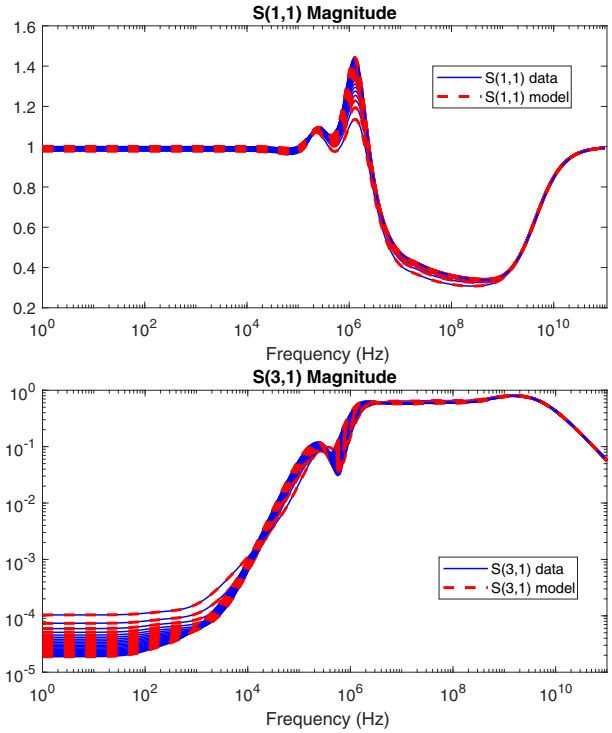

Fig. 1. Validation of two small-signal scattering responses of the LDO model, compared to raw data from transistor-level small-signal simulation for various bias voltage values.

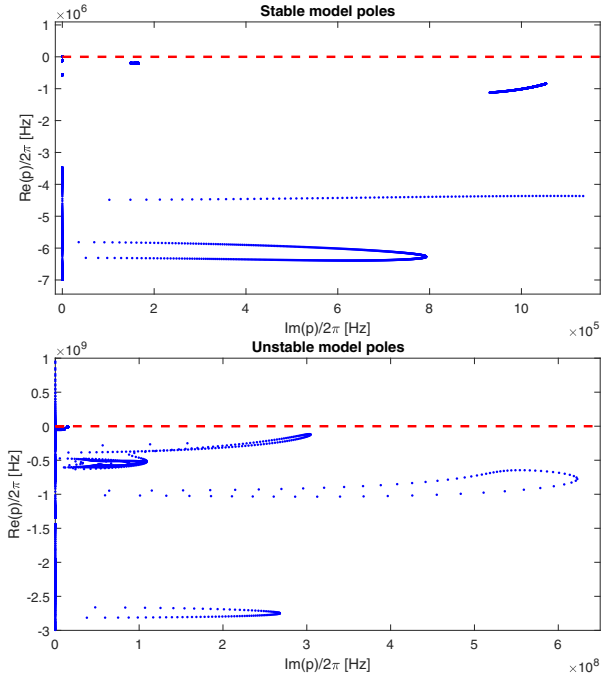

Fig. 2. Top panel: parameterized model poles for the uniformly stable model of the LDO. Bottom panel: poles of a standard model obtained without imposing denominator passivity.

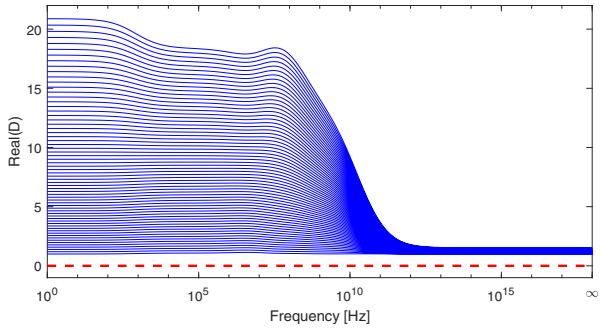

Fig. 3. Real part of LDO model denominator, computed over a fine sweep of the nominal bias voltage. Since $\mathrm{Re}\{\mathsf{D}(\mathrm{j}\omega,\vartheta)\} > 0$, it is guaranteed that the model poles are uniformly stable irrespective of the value of $\vartheta \in \Theta$.

distance of the corresponding Hamiltonian eigenvalue set from the imaginary axis. This distance is zero if the denominator is not passive and is larger than zero if the denominator is passive. It can be shown that $\psi(\vartheta)$ is piecewise regular and globally continuous. Starting from an initial coarse grid in Θ, a local (binary) refinement on $\psi(\vartheta)$ is then used to determine the parameter regions where passivity violations occur, i.e. where local negative minima of $\mathrm{Re}\{\mathsf{D}(\mathrm{j}\omega,\vartheta)\}$ exist.

The above adaptive sampling is performed at each iteration of the constrained PSK iteration. In order to guarantee the passivity of the denominator at the end of the identification process, we run a final passivity check on the stabilized $\mathsf{D}(s,\vartheta)$. If any passivity violation regions are still found, these are eliminated by a standard bivariate passivity enforcement scheme based on first-order perturbation, whose details are here omitted due to lack of space [8].

III. Examples

We illustrate our proposed modeling scheme on two examples taken from signal and power integrity applications. The first example is a Low Drop-Out (LDO) Voltage Regulator taken from a mobile platform design. This device has the purpose of reducing the power supply noise coming from the global power distribution network through a local feedback and filtering mechanism (see [5] for a more detailed description of the device). The device has three ports, of which port 1 is the "noisy" power supply port, port 2 is a reference voltage, and port 3 is where the supply voltage noise should be reduced as much as possible.

The results of proposed model generation are depicted in Fig. 1, which compares the model responses of selected small-signal scattering responses to the original responses used for model identification. A confirmation that the model is uniformly stable is provided by the top panel of Fig. 2, which depicts the model poles computed in a fine sweep of the bias voltage in the complex plane. The real part of all poles is negative. This is not true in the bottom panel of Fig. 2, which corresponds to the same poles obtained by a standard model identification based on (5), without imposing the passivity constraints (7). Figure 3 confirms that the real part of the model denominator is uniformly positive, thus implying uniform stability.

The second example we consider is an Operational Amplifier (OA) part of a base-band receiver chain. The

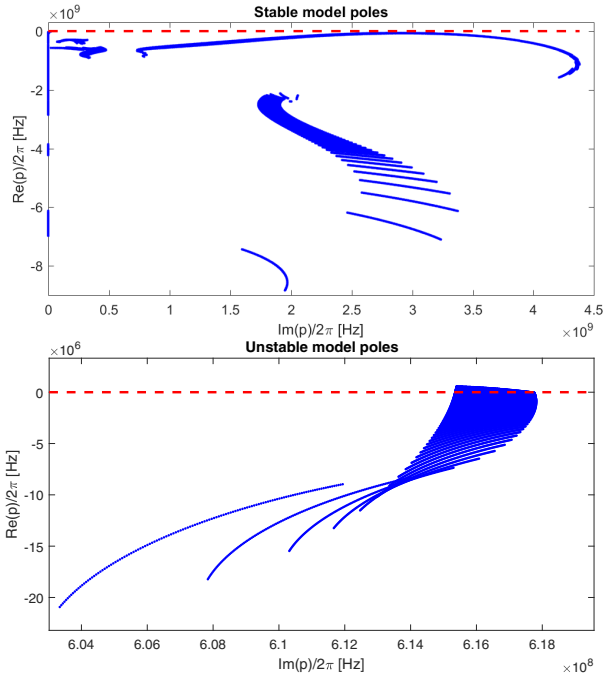

Fig. 4. Top panel: parameterized model poles for the uniformly stable model of the OA. Bottom panel: poles (zoom) of an OA model obtained without imposing denominator passivity. The plots accumulate all poles trajectories for a double sweep on both parameters (bias and gain).

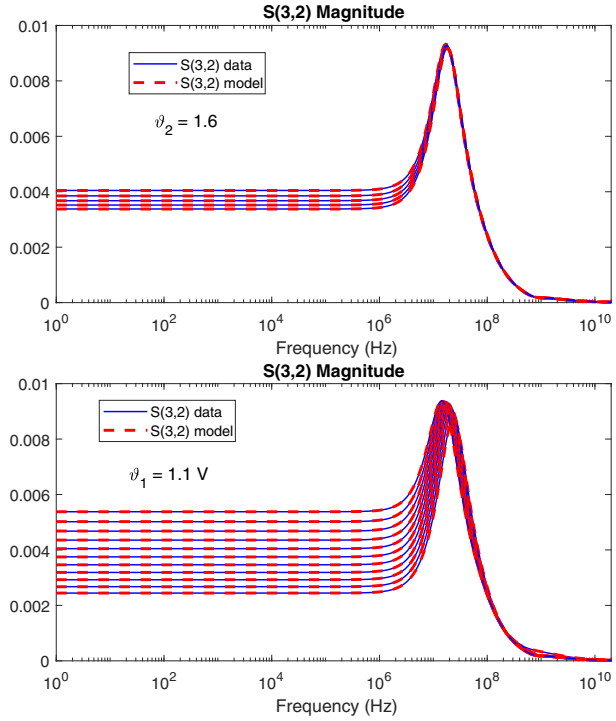

Fig. 5. Validation of a small-signal scattering response of the OA model, compared to raw data from transistor-level small-signal simulation for various bias voltage values and a fixed gain (top panel) and various gains with a fixed bias voltage (bottom panel).

OA is parameterized by its bias voltage ϑ_1 and its (programmable) gain ϑ_2, so we show here a generalization to a higher-dimensional parameter space with respect to previous example. Figure 4 compares the poles of our proposed guaranteed stable parameterized model to the poles of an unconstrained model, which are unstable for some parameter configurations. The top panel of Fig. 5 compares the model responses by fixing the gain ϑ_2 and sweeping the bias voltage ϑ_1, whereas the bottom panel fixes ϑ_1 and sweeps ϑ_2. A similar excellent accuracy was obtained for all other bias-gain combinations (not shown).

IV. Conclusions

We have presented an algorithm for the identification of guaranteed stable reduced-order behavioral macromodels of analog circuit blocks under small signal assumptions, including an explicit closed-form dependence on the bias level. The models are cast as compact behavioral SPICE netlists, which can be simulated using standard circuit solvers instead of the full transistor-level description, with major savings in computating times (speedup factors between 10X and 100X are typical, see [5]).

We remark that our approach is not directly comparable with the popular Vector Fitting (VF) scheme [2]–[4], which does not account for any dependence on external parameters. Rather, our approach generalizes VF to the multivariate case.

The proposed algorithm has been demonstrated on two relevant test cases, namely an LDO voltage regulator and a programmable base-band amplifier circuit.

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