

Influence of in-plane and out-of-plane stiffness on the stability of free-edge gridshells: A parametric analysis

Original

Influence of in-plane and out-of-plane stiffness on the stability of free-edge gridshells: A parametric analysis / Venuti, Fiammetta; Bruno, Luca. - In: THIN-WALLED STRUCTURES. - ISSN 0263-8231. - ELETTRONICO. - 131:(2018), pp. 755-768. [10.1016/j.tws.2018.07.019]

Availability:

This version is available at: 11583/2712019 since: 2018-08-28T08:35:18Z

Publisher:

Elsevier Ltd

Published

DOI:10.1016/j.tws.2018.07.019

Terms of use:

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

Elsevier postprint/Author's Accepted Manuscript

© 2018. This manuscript version is made available under the CC-BY-NC-ND 4.0 license
<http://creativecommons.org/licenses/by-nc-nd/4.0/>. The final authenticated version is available online at:
<http://dx.doi.org/10.1016/j.tws.2018.07.019>

(Article begins on next page)

Influence of in-plane and out-of-plane stiffness on the stability of free-edge gridshells: a parametric analysis

Fiammetta Venuti^{a,*}, Luca Bruno^a

^aPolitecnico di Torino, Department of Architecture and Design,
Viale Mattioli 39, I-10125, Torino, Italy

Abstract

Gridshells are form-resistant structures, which are suitable for covering large spans, especially when lightness and transparency are respectively relevant architectural and functional requirements. The majority of built gridshells are characterised by one or more free-edges, which derive from trimming the gridshell reference surface in order to provide building access or to integrate the gridshell within existing structures. Up to now, only few scientific systematic studies have been devoted to the effects of elastic boundary structures on the stability of gridshells. This study aims at filling some gaps about this issue. To do so, an ideal free-edge bending-inactive hybrid single-layer gridshell is analysed. The gridshell sensitivity to the flexural stiffness of the boundary arch and to the shear stiffness of the gridshell are investigated through an extensive parametric analysis, which was performed by means of numerical experiments. Results are first discussed in terms of the well-established load factor and buckling shape. Then a complementary mechanical reading is provided by introducing ad-hoc conceived local metrics of the in-plane and out-of-plane deformations at collapse. Three different mechanical regimes at collapse are outlined. In conclusion, a range-finding chart within the design parameter space is proposed to orient the structural analyst in the choice of the preferred regime.

Keywords: hybrid single-layer gridshell, instability, free-edge, boundary stiffness, numerical experiments

Nomenclature

GMNA	Geometrically and Materially Nonlinear Analysis
LBA	Linear Buckling Analysis
LF	Load Factor
CG	Complete Gridshell
PG	Partial Gridshell
E	modulus of elasticity
L	dome span length
f	dome rise length
l	characteristic length of quadrilateral face
p	subscript of the generic structural node
P	number of structural nodes
x	horizontal space coordinate
y	horizontal space coordinate
z	vertical space coordinate
K	discrete gaussian curvature
\mathbf{K}_e	elastic stiffness matrix

*Corresponding author. Tel: (+39) 011-090.4862
Email address: fiammetta.venuti@polito.it (Fiammetta Venuti)

\mathbf{K}_g	geometric stiffness matrix
I_b	moment of inertia of the boundary structure
I_e	moment of inertia of the grid elements
I_r	ratio of the boundary to grid moments of inertia
S_r	non-dimensional ratio between design parameters
Q	nodal resultant load
q	uniform load
s	uniform live (snow) load
g	dead load
A	quadrilateral surface
A_e	cross section area of grid elements
A_c	cross section area of diagonal cables
f_y	yield strength
ΔK	out-of-plane nodal deformation
γ_{xy}, γ_{yx}	components of the in-plane nodal deformation
Γ	in-plane nodal deformation
ν	nodal direction
μ	load multiplier
σ_0	cable initial prestressing
σ	stress field
ϕ	buckling mode shape
φ	generic buckling shape
λ	eigenvalue
δ	nodal displacement
δ_u	ultimate nodal displacement
κ	cotangent of the nodal angle

9 1. Introduction

10 Gridshells are form-resistant structures, which are designed to ideally bear the loads by means of in-plane internal
11 forces. Their geometry is generally defined, at least in steel gridshells, by approximating a reference continuous
12 surface through a discrete pattern of line-like structural members, which are mainly subjected to axial forces.

13 Gridshells find their natural application in large-span buildings, such as stadia, courtyards and expo pavilions,
14 where transparency and lightness might be relevant program requirements. Not surprisingly, the first known pioneer-
15 ing application of a doubly-curved gridshell, which dates back to the late 19th century and was designed by engineer
16 Vladimir Shukov, refers to the roof of a large-span Plate Rolling Workshop in Russia [1]. The geodesic dome of
17 the Zeiss-Planetarium in Germany, designed by Walther Bauersfeld and completed in 1926, is another example of
18 this kind. Throughout the second half of the 20th century, the milestone achievements of Buckminster Fuller [2],
19 Frei Otto [3] and Schlaich Bergermann und Partner [4, 5] helped defining the current technology and strategies for
20 gridshell design: on the one hand, the work of Frei Otto gave birth to what is currently known as a bending-active, or
21 post-formed, timber gridshell [6, 7]; on the other hand, Jörg Schlaich and Hans Schober focused on bending-inactive,
22 or pre-formed, steel/glass gridshells, most of which were derived from surfaces of translation and were based on the
23 use of quad patterns [8]. Since these pioneering structures, the gridshell structural concept has been widely applied to
24 a variety of buildings all over the world, both in traditional materials [5, 9] and innovative ones [10, 11, 12].

25 Gridshells are optimised and highly efficient structures, but this efficiency makes them highly prone to buckling
26 phenomena, which can lead to catastrophic collapse. Since the collapse of the Bucharest Exhibition Hall dome in
27 1963, a lot of research has been devoted to buckling and post-buckling behaviour of reticulated shells [13, 14, 15,
28 16, 17, 18, 19, 20]. The first approach to the problem was based on the continuum analogy, that is, the behaviour
29 of the gridshell is compared to the one of a shell, characterised by the same geometry of the gridshell and by an
30 equivalent thickness [21, 22, 23, 24]. The aim of this approach was to extend to gridshells the analytical expressions of
31 the buckling load that were available for continuous shells. This method, even though it can be useful in a preliminary

32 design phase, presents some drawbacks [13]: analytical solutions are available only for a class of continuous shells,
33 characterised by specific shape and boundary conditions; the continuum analogy does not allow to account for some
34 types of buckling that are peculiar of gridshell structures, such as nodal buckling and member buckling; the influence
35 of joint deformation cannot be taken into account. For all these reasons, the buckling behaviour of gridshells is usually
36 studied by means of non-linear numerical analyses of Finite Element Models of the whole structure.

37 The main factors, which influence the buckling behaviour of gridshells and that have been studied in the literature,
38 are the following: [13, 25]: i. *the Gaussian curvature* of the gridshell underlying surface [e.g. 13, 26, 16, 19];
39 ii. *the grid topology and spacing* [e.g. 27, 24, 20, 25]; iii. *the geometrical and mechanical imperfections* [e.g.
40 28, 29, 26, 30, 31, 32]; iv. *the joint stiffness* [e.g. 33, 29, 16, 34, 35]; v. *the boundary conditions*.

41 While the effects of the first four factors have been widely studied in the recent past, the last factor has been less
42 explored. Stability studies on gridshell domes are usually carried out by referring to a horizontal spring-plane and
43 rigid supports [29, 26, 15, 27, 36, 18, 37, 24, 38, 20, 39]. The effects of different rigid supports (pinned, roller or fixed)
44 have been comparatively studied in a few papers [40, 41, 13, 19]. Analogously, the gridshell barrel vaults studied in
45 the literature are usually delimited by a horizontal spring-plan and gable vertical plans, which are orthogonal to their
46 axes [26, 19, 39] and rigidly constrained at the spring-lines. To the authors' knowledge, the effects of different rigid
47 supports (fixed or pinned) along the spring-lines of a gridshell barrel vault have been discussed in [19] and [42] only.

48 Even fewer scientific systematic studies have been devoted up to now to the effects of elastic boundary structures
49 along the delimitation edges on the stability of gridshells, as recently highlighted in [42]: "This kind of elastic bound-
50 ary has not been extensively investigated, and studies are needed in each design to know how and if the supports
51 improve the buckling resistance.". This is even more surprising knowing that a horizontal spring line and/or infinitely
52 stiff perfect constraints only seldom occur in built gridshells. Usually and more and more frequently, gridshells are
53 trimmed by vertical or inclined planes (e.g. the Shukov gridshell in Vyksa [1]), or by curved surfaces (e.g. the Hippo
54 House at the Berlin Zoo [43]). As a consequence, they are constrained by elastic boundary structures along the de-
55 limitation edges. Master builders were certainly aware of the influence of elastic boundaries, as demonstrated by the
56 above cited built structures. However, to the authors' best knowledge, systematic studies applied to double curvature
57 single-layer gridshells are not available, and only a few number of studies analysed the effects of the stiffness of gable
58 boundary structures in barrel vault gridshells. Bulenda and Knippers [26] compared the barrel vault stability with
59 no stiffening boundary arches with the same structure stiffened by a boundary arch having 40 times the stiffness of
60 a IPE 360 profile. As expected, the differences in terms of mechanical behaviour are striking: the unstiffened vault
61 buckles analogously to a plane arch, while a spatial behaviour at collapse takes place in the stiffened vault, even if the
62 stiffening arch still shows not negligible ultimate displacements. Cai et al. [19] stiffen the vault with pre-tensioned
63 spoked wheels transverse diaphragms, analogously to the stiffening system adopted for the roof of the Museum of
64 Hamburg History courtyard [4]. In this case, the arch acts as a perfectly rigid support in its vertical plane.

65 This issue seems to be disregarded also in the case of continuous shells. A number of remarkable free-edge
66 concrete shells were conceived by master builders such as Heinz Isler [44] and Felix Candela [45]. In thiri works
67 several solutions were put in place at the shell free boundaries, ranging from unstiffened edge, to creases introducing
68 form stiffness, to edge beams providening inertial stiffness. However, once again "the stability of shells with free
69 edges is a rather unexplored field" [46]. In fact, even though single curvature cylindrical shells with flexible bottom
70 have been widely systematically studied over the past fifty years, and their buckling behaviour is well known [47, 48,
71 49, 50, 51], analytical solutions for double curvature shells with elastic boundaries are not available in literature.

72 In summary, a scatter exists between the design practice and the scientific literature in this field. In this framework,
73 the paper focuses on the delimitation of gridshells, and aims to fill in some of the gaps, which are currently evident in
74 the topic literature review. In terms of paper outline, Section 2 defines, classifies and reviews some relevant examples
75 of free-edge gridshells, both from the geometrical and mechanical points of view. The term Partial Gridshells (PG) will
76 be introduced here to denote free-edge gridshells as opposed to Complete Gridshells (CG). A new parametric study is
77 described in Section 3, in terms of geometrical setup, structural setup and design parameter space. Section 4 briefly
78 recalls gridshell structural modelling and computational approaches. The observables selected for post processing
79 are described in Section 5. Results and findings of the parametric analyses are described and discussed in Section 6,
80 in parallel with a resulting range-finding chart, developed in the design parameter space. Section 7 summarises the
81 conclusions and future works.

2. Partial Gridshells: definition and classification

Distinction between Complete Gridshells (CG) and Partial Gridshells (PG) is proposed in this paper. *Complete Gridshell* (CG) refers to gridshells whose shape results from trimming a reference gridshell geometry by a single surface, usually a horizontal or inclined plane. The structure spring lines, along which the gridshell is rigidly constrained, are given by the intersection between the reference geometry and the trimming surface (Figure 1a, red curve). By exclusion, *Partial Gridshell* (PG) denotes gridshell forms which are obtained through two or more geometrical cuts. PGs can be cut by vertical or inclined planes (Figure 1b) or curved surfaces (Figure 1c). PGs spring lines are given by the intersection between the reference geometry and one of the trimming surfaces - usually the one that lies horizontally. The intersection between the reference surface and the other trimming surfaces determines a number of PG free edges (blue dash-dot curves), which are not rigidly constrained.

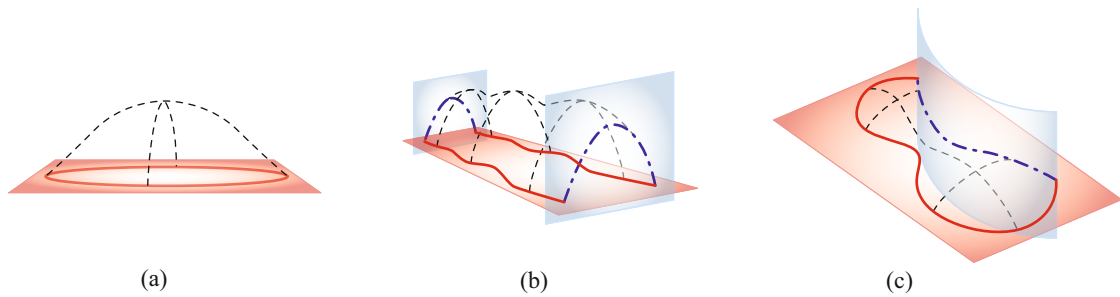


Figure 1: Reference and trimming surfaces of (a) a CG (after the Essen gridshell), (b) a PG cut by vertical planes (after the Downland gridshell) and (c) a PG cut by a curved surface (after the Hippo House)

CGs represent a minority and are mainly applicable when the structure spring lines are raised from the ground level or from extremely stiff substructures. To our knowledge, the only built projects of CG whose spring line lays on the ground level are two research pavilions: the timber bending-active Essen gridshell designed by Frei Otto in 1962 [3] (Figure 2a) and the experimental GFRP bending-active gridshell built at ENPC [52]. However, these prototypes did not have to deal with functionality issues and program requirements. Otherwise, other stiff structural elements are required and introduced to provide building access. The National Maritime Museum in Amsterdam, the Dresden Castle gridshell [53], the Neckarsulm dome [4], the Orangery gridshell at Chiddingstone Castle (Figure 2b), and the roof of the Great Court of the British Museum [54] are outstanding examples of this kind.

PGs include the majority of the built gridshells. In most cases the gridshell form needs to be trimmed due to functional or architectural requirements. When the gridshell spring lines lie on the ground level, the most common issue is to provide openings for building access, as in the Multihalle in Mannheim [55] (Figure 2c), the “Future of us” gridshell pavilion in Singapore [56] (Figure 2d) or the “Ephemeral cathedral” in Paris [10]. When the main gridshell function is to provide shelter, cuts might be necessary for other reasons: for instance, to array the base gridshell geometry, such as in the roof of the production hall in Vyksa [1] (Figure 2e) or in the West entrance of the Hannover Fair [57] (Figure 2f). In both cases, the roof is made of a modular repetition of the same base gridshell, which is trimmed by four vertical planes and, hence, bounded by as many parabolic arches. In other cases, the cuts are made to integrate the gridshell within existing buildings. Typical examples are gridshell roofs over courtyards, streets or squares: two outstanding examples are the Schlüterhof Roof at the German Historical Museum in Berlin (Figure 2g) and the Cabot Circus in Bristol [58] (Figure 2h). Besides functional needs, cuts can also identify architectural gestures, such as in the Hippo House at the Berlin Zoo [43] (Figure 2j) or in the Yas Island Marina Hotel in Abu Dhabi, UAE [59] (Figure 2k). Note that contemporary PGs are delimited by increasingly complex surfaces: vertical single-curvature concave ones (Figure 2j), combinations of planes and surfaces (Figure 2h), double-curvature surfaces (Figure 2k).

The CG mechanical behaviour under uniform loads mainly involves membrane stiffness. The same is expected not to hold for PGs. The structural design of PGs raises further issues with respect to CGs. Specifically, the mechanical behaviour of PGs is expected to be highly dependent on:

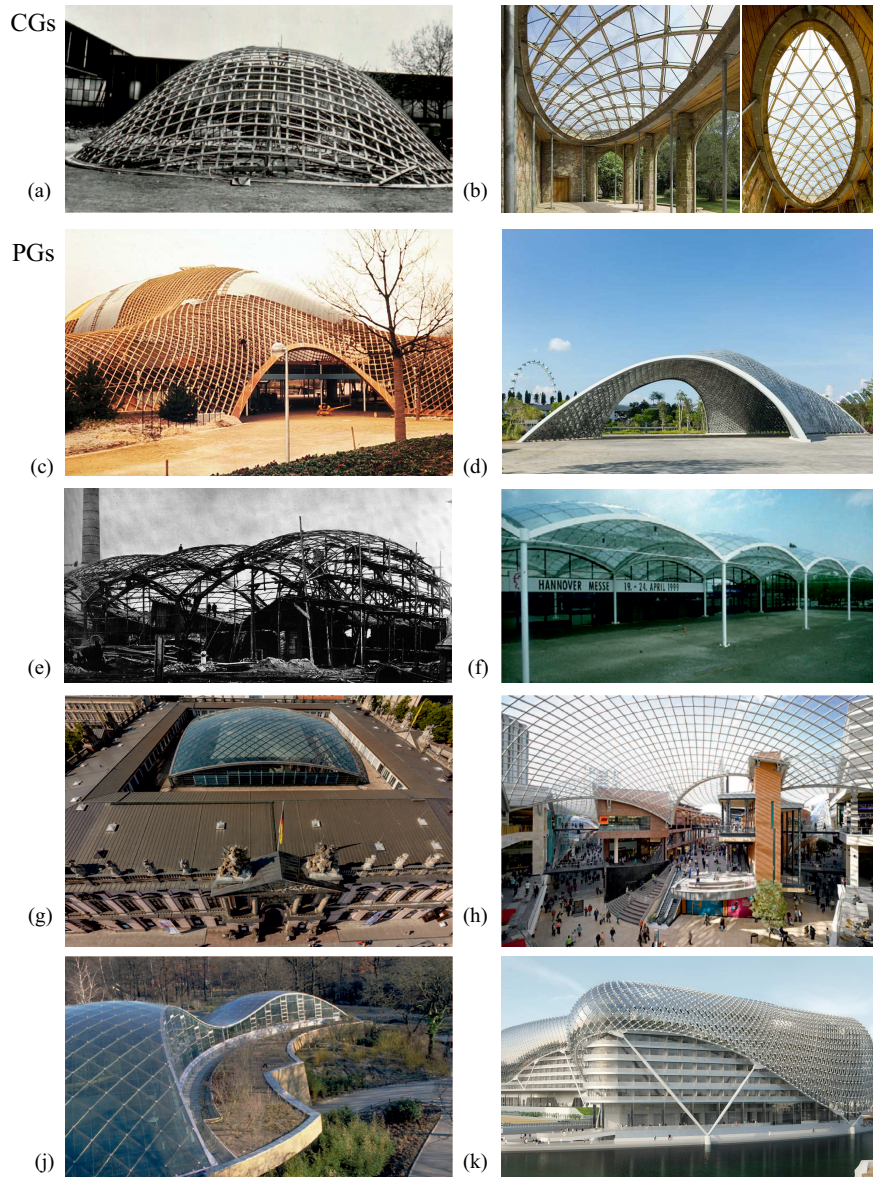


Figure 2: Survey of built CGs and PGs: (a) Essen gridshell [3], (b) Orangery gridshell, (c) Mannheim Multihalle [55], (d) “Future of us” Pavilion [56] (e) Vyksa gridshell [1], (f) Hannover Fair, West entrance [57], (g) Schlüterhof Roof (h) Cabot Circus [58] (i) Hippo House [43], (k) Yas Island Marina Hotel [59]

118 • *Geometrical factors*: the kind (either planar or curved) of the trimming surface, its orientation with respect to
 119 the grid direction, and the ratio between the gridshell and free-edge span (i.e. the location of the trimming
 120 surface);

121 • *Mechanical factors*: the stiffness of the boundaries and the in-plane shear stiffness of the gridshell.

122 This study focuses on the mechanical factors, while the influence of geometrical factors will be investigated in a
 123 subsequent work. The stiffness of the boundaries depends on the cross section of the boundary structure, its length
 124 and its constraints. A conceptual categorisation corresponding to increasing degree of stiffening can be outlined:

125 • the PG edges are not stiffened, that is, the boundaries are free and made of structural elements with the same

126 cross section as the grid elements. Examples of this kind are the Toledo I gridshell [60], a pavilion for the
127 Naples School of Architecture courtyard, and the Waitomo Caves Visitor Centre [61];

- 128 • the PG edges are stiffened by a *boundary structure*, whose axis coincides with the edge curve, and whose
129 flexural stiffness is greater than the flexural stiffness of the grid elements (e.g., boundary arches). The Mannheim
130 Multihalle [55] and Downland Museum [62] are examples of this kind;
- 131 • the PG boundary structures are further supported by vertical elements, such as columns, which reduce the length
132 of the free edge. The Hippo House [43] and the Savill Garden gridshell [63] are representative of this category.

133 The in-plane shear stiffness of the gridshell is mainly related to the kind of adopted mesh pattern. Generally
134 speaking, quad meshes have lower in-plane shear stiffness than triangular meshes. For this reason, quad meshes are
135 often stiffened by bracing cables [4], which have the effect of triangularizing the quad mesh. In this scenario, the
136 cross section of the diagonal elements is the structural parameter that mostly affects the in-plane shear stiffness of the
137 gridshell.

138 Here, a rough attempt is made to indirectly estimate some reference values of the boundary stiffness and grid-
139 shell in-plane shear stiffness of built PGs. Due to the lack of published data, the direct estimation of the stiffness
140 is not affordable in this study. Information about the section geometrical and material properties of the main struc-
141 tural elements can be found more easily in the literature. Figure 3 collects the section properties of the boundary
142 and diagonal elements for a small ensemble of built PGs, whose data were available. The survey includes differ-
143 ent gridshell layouts: unbraced and braced quad meshes, triangular meshes, bending-active and inactive, single and
144 double-layer. The selected section properties are the moment of inertia I_b of the boundary elements and the area A_c
145 of the diagonal elements, since they significantly contribute to the boundary stiffness and in-plane shear stiffness of the
146 gridshell, respectively. Both properties are expressed in dimensionless form, by scaling their value with respect to the
147 corresponding quantity of the grid element section:

$$\begin{aligned} I_r &= n I_b / I_e, \\ A_r &= n A_c / A_e \end{aligned} \quad (1)$$

148 where I_e and A_e are the moment of inertia and area of the grid elements, respectively, and n is the homogenization
149 coefficient given by the ratio between the moduli of elasticity of the boundary/diagonal element material and the grid
150 element material, if materials differ.

151 Overall, I_r varies in the range [1, 500], where the unit value refers to unstiffened boundary, and A_r in the range
152 [0, 0.5], where $A_r = 0$ refers to the unbraced case. In particular, hybrid bending-inactive single-layer steel gridshells
153 lie in a smaller interval of parameter values i.e. $0 < A_r < 0.1$, and $80 < I_r < 300$. Bending-active timber double-
154 layer gridshells have larger A_r values, i.e. $0.3 < A_r < 0.5$, since they are usually characterised by diagonal elements
155 inbetween the two laths, with the same cross section as the main grid elements in each lath. The Mannheim Multihalle
156 gridshell ($A_r \approx 0.3$) shows some differences among the collected cases, because the steel bracing cables spans over
157 about six gridshell quadrangles made of wood elements.

158 3. Parametric analysis set-up

159 The effects of the boundary stiffness and gridshell in-plane shear stiffness on the gridshell stability are investigated
160 through a parametric analysis by computational simulations.

161 A complete bending-inactive hybrid single-layer gridshell by Schlaich and Schober [4] is used as a reference
162 to derive the studied PG. This gridshell type is chosen because it permits to derive two limit cases: a quadrilateral
163 gridshell, where bracing cables are missing, and a triangular gridshell, in which the cables are replaced by trusses that
164 share the same cross section of the grid elements.

165 The features of the case study are detailed below.

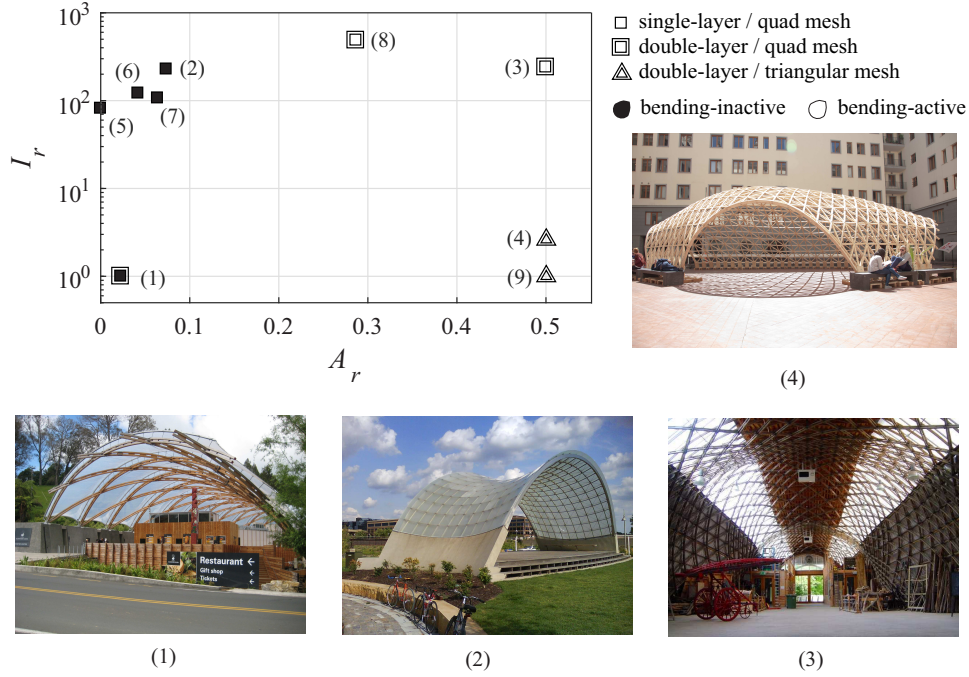


Figure 3: Survey of I_r - A_r values: (1) Waitomo Caves visitor centre [61], (2) Schubert Club Band Shell [64], (3) Downland Museum [62], (4) Toledo I gridshell (photocredit Gianluca Jodice) [60], (5) Cabot Circus [58], (6) courtyard roof at the Engineering Faculty at the University of Pisa [65], (7) Hippo House [43], (8) Mannheim Multihalle [55], (9) Toledo II gridshell [66].

3.1. Geometrical set-up

The geometry of the investigated partial single-layer gridshell is shown in Figure 4. The analytical form of the reference continuous dome (Fig. 4a) belongs to the translational surface type, having a parabola as both the directrix and generatrix, i.e. it is a paraboloid. The considered partial dome is obtained by sectioning the complete dome with a vertical plane corresponding to the $x-z$ plane. The obtained half dome is, therefore, bounded by a vertical parabolic arch, whose geometrical parameters are: span length $L = 30$ m, span to rise ratio $L/f = 8$ (Fig. 4d).

The discrete gridshell geometry results from the point wise sampling of the dome surface in P structural nodes ($p = 1, P$) along the directions of the directrix and generatrix having coordinates $\mathbf{X}(p) = \{x_p, y_p, z_p\}$. The nodes are connected along the directions of the directrix and generatrix by straight segments resulting in elemental planar quads [57], having a characteristic length $l \approx 1.5$ m (Fig. 4b). Each quad is braced by two diagonal cables, as shown in Figure 4c.

3.2. Structural set-up

The grid of quads is composed of steel beams. These are diagonally braced by prestressed cables. The structures are covered with glass panels, which do not contribute structurally but are taken into account as dead loads.

The structure is modelled by means of finite element software ANSYS[®] v17.2. Both the grid elements and the boundary arch are modelled in ANSYS using the BEAM188 finite element. The grid beam cross section mimics the one usually adopted by [4, 26]: a solid quadrilateral cross section with area $A_e = 2.5e - 3$ m² and moment of inertia $I_e = 5.2e - 7$ m⁴ is chosen. The 3D beam elements are based on Timoshenko's beam theory and adopts a cubic shape function. The cables are modelled by 3D tension-only trusses with circular cross section (LINK180 finite element in ANSYS).

The whole structure is assumed to be hinged at the boundaries, while the joints between steel bars are modelled as rigid, as usually done in the literature [26, 37, 27].

The constitutive model of the steel is linear elastic - perfect plastic, with a yield strength equal to $f_y = 355$ MPa, Young's modulus $E = 2.1e + 5$ MPa and Poisson's ratio $\nu = 0.3$. The diagonal cable prestressing is equal to $\sigma_0 = 100$

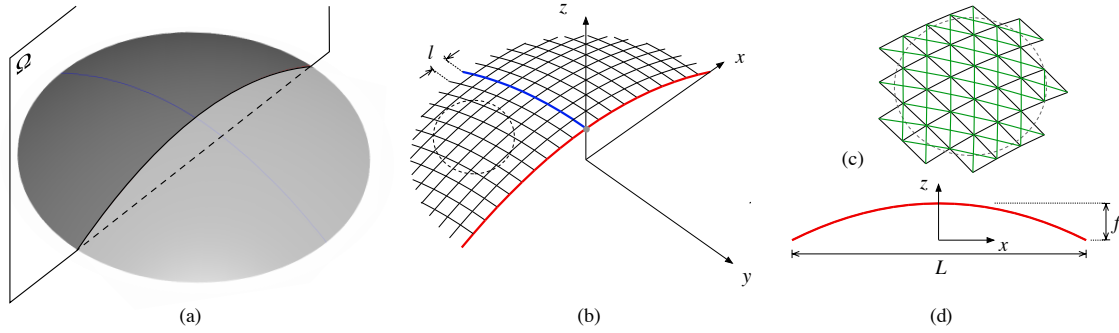


Figure 4: Set-up geometry: reference continuous surface (a), discrete gridshell surface (b, shown without cables), close-up view of a part of the dome including diagonal cables (c), section along the meridian vertical plane (d)

190 MPa, while steel bars are not subject to initial bending. The dead load g of structural steel members and of 20 mm-
 191 thick glass glazing is set equal to 0.5 kN/m^2 . As for the live load s , a uniform snow load $s = 0.75 \text{ kN/m}^2$ is applied on
 192 the structure. A uniform load case $q = 1.3g + 1.5s$ is applied to the structure. The resultant p -th nodal load is defined
 193 as $Q_p = \int_A q da \approx 4000 \text{ N}$, being A the quad surface.

194 3.3. Parametric analysis plan

195 For the adopted and fixed geometrical and structural set-ups, the boundary arch stiffness and the gridshell in-plane
 196 shear stiffness only depend on two design parameters, namely I_r and A_r , defined according to Eq.s (1). I_b is set equal
 197 for both the local axes of the arch cross section. Hence, it affects both the in-plane and out-of-plane flexural stiffness
 198 of the boundary arch. A_c affects the axial stiffness of the cables and therefore provides additional in-plane shear
 199 stiffness to the gridshell faces, with respect to the unbraced case. As a preliminary remark, it is worth noting that the
 200 out-of-plane behaviour of the boundary arch not only depends on its flexural stiffness, but also on the in-plane shear
 201 stiffness of the gridshell. This holds also when I_b tends to infinity and the arch behaves as a rigid body, being the arch
 202 hinged at its external boundaries. We also point out that the parameter I_b is expected to locally affect the stiffness of
 203 the gridshell boundary, while A_c uniformly affects the stiffness of the whole gridshell.

204 I_r varies in the range $[1, 10^6]$, where the lower and upper values correspond to unstiffened and infinitely stiffened
 205 boundary structure, respectively. A_r varies in the range $[0, 1]$. The lower bound corresponds to the limit case of
 206 unbraced quad gridshell, while the upper bound corresponds to the limit case where the cable cross section equals
 207 the grid element one. The boundary regions of the parametric analysis plan are not necessarily of practical interest
 208 in the real world design practice. Nevertheless, the computational testing allows to explore also uncommon, or even
 209 unphysical, scenarios to shed some light on the trend of the structural behaviour towards limit conditions.

210 Figure 5 illustrates the resulting $I_r - A_r$ parameter plan. Besides the limit values above, the values of both param-
 211 eters are selected in the light of the survey made in Section 2 (Figure 3) for bending-inactive hybrid gridshells. In
 212 particular, the parameter space is more densely sampled in the I_r range $[1, 10^3]$ and in the A_r range $[0, 0.1]$, which is
 213 the one of design interest for bending-inactive single-layer quad mesh gridshells. In total, 38 values of I_r and 7 values
 214 of A_r are explored, resulting in 266 simulations for each kind of structural analysis described in the following Section.
 215

216 4. Structural modelling

217 Two kinds of structural analysis are performed and briefly recalled below.

218 Linear Buckling Analysis (LBA)

219 LBA solves the eigenvalue problem:

$$\left[\mathbf{K}_e + \lambda \mathbf{K}_g(\sigma) \right] \Phi = 0, \quad (2)$$

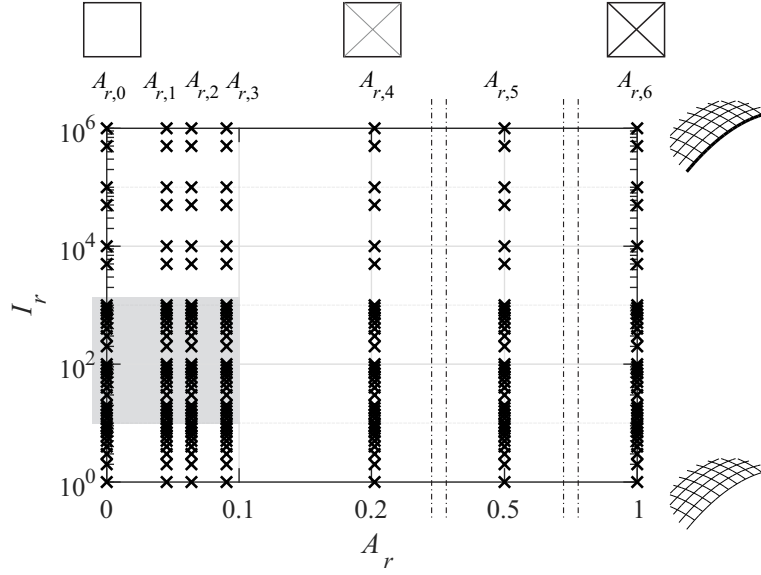


Figure 5: Discretisation of the I_r – A_r parameter plan. The grey area focuses on the values adopted in design practice of bending-inactive single-layer gridshells

220 where λ is the vector of load factors and Φ is the matrix of the associated buckling mode shapes. The geometric
 221 stiffness $\mathbf{K}_g(\sigma)$ is a function of the stress field σ . \mathbf{K}_g is evaluated through a static analysis that solves the
 222 equation

$$[\mathbf{K}_e + \mathbf{K}_g(\sigma_0)] \delta = \mathbf{Q} \quad (3)$$

223 where \mathbf{K}_e is the elastic stiffness, $\mathbf{K}_g(\sigma_0)$ accounts for the initial prestress σ_0 , if any, and \mathbf{Q} is the applied load.

224 *Geometrically and Materially Nonlinear Analysis (GMNA)*

$$[\mathbf{K}_e(\delta) + \mathbf{K}_g(\delta)] \delta = \mu \mathbf{Q} \quad (4)$$

225 where the geometric stiffness $\mathbf{K}_g(\delta)$ is a function of the displacement field $\delta(\sigma)$ and μ is the load multiplier
 226 that increases along the simulation. In the study, the steel nonlinear stress-strain relation is modelled by a bi-
 227 linear law (elastic-perfect plastic). Due to the progressive yielding of members under monotonically increasing
 228 load, the overall structural behaviour is described by a tangent stiffness matrix \mathbf{K}_e which softens for increasing
 229 displacement field $\delta(\sigma)$.

230 Buckling instability of SLGS is highly sensitive to imperfections, that should be taken into account in the structural
 231 design of gridshells. Despite this, in the present study both LBA and GMNA are performed on the perfect structure,
 232 i.e. neglecting the effects of equivalent nodal imperfections. This choice is due to four reasons: i. the choice of the
 233 imperfection shape to adopt in the design is still an object of debate in the civil engineering community [26, 38]; ii.
 234 the most popular imperfection models, e.g. [67], assume that the imperfection shape is equal to the first buckling
 235 mode shape, that depends in turn on the same parameters of the sensitivity study, i.e. I_r and A_r ; iii. the effects of
 236 the imperfection cumulate with, and are expected to prevail on, the parameter primary mechanical effects. It follows
 237 that the latter ones are noised and hidden by the former ones; iv. the computational testing, unlike testing on physical
 238 models, offers the opportunity to fully control or exclude some parameters and related effects (i.e. nodal imperfections
 239 in current case), to focus on the parameters of interest only, to isolate their effects, and finally to discuss results in a
 240 clearer and mechanically sound way.

241 The structural analysis is performed by means of the finite-element code ANSYS® v17.2. The Load Control
 242 procedure is applied within GMNA, where the iterative convergence is accomplished at each step by means of the
 243 standard Newton Raphson method [68].

244 **5. Definition of the observables**

245 To provide a synthetic description of the mechanical behaviour of the PG versus the selected design parameters,
 246 one bulk parameter and two nodal observables are adopted. They are defined in the following with reference to a PG
 247 with P structural nodes having coordinates $\mathbf{X}(p) = \{x_p, y_p, z_p\}$ ($p = 1, P$).

248 **5.1. Load Factor (LF)**

249 The load factor is the most relevant parameter usually considered at the Buckling Limit State. Its definition is
 250 herein recalled:

$$\mathbf{Q}_u = \text{LF } \mathbf{Q} \quad (5)$$

251 where \mathbf{Q}_u is the ultimate buckling load and \mathbf{Q} is the load condition as defined in Section 3.2. Within LBA, $\text{LF} = \lambda_1$,
 252 i.e. the lowest eigenvalue, which corresponds to the buckling mode shape ϕ_1 . Within GMNA, $\text{LF} = \mu_u$, being μ_u the
 253 load multiplier that induces the collapse ($dQ/d\delta = 0$ in a selected reference node), i.e. reaching a limit point, and δ_u
 254 the corresponding displacement field.

255 **5.2. Out-of-plane nodal deformation ΔK**

256 In order to locally evaluate the out-of-plane nodal deformation of the gridshell, the following nodal quantity is
 257 defined:

$$\Delta K(p) = K(p) - K_0(p), \quad (6)$$

258 where $K(p)$ and $K_0(p)$ are the discrete gaussian curvatures in the deformed and undeformed shapes, respectively.
 259 The discrete gaussian curvature $K(p)$ mimics at each node p the Gaussian curvature in the continuous analogy. With
 260 reference to Figure 6, it is defined as [69, 70]:

$$K(p) = 2\pi - \widehat{\mathcal{E}}_p \mathcal{N} - \mathcal{N} \widehat{\mathcal{W}}_p - \mathcal{W}_p \widehat{\mathcal{S}} - \widehat{\mathcal{S}}_p \mathcal{E} \quad (7)$$

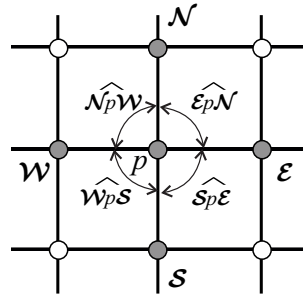


Figure 6: Definition of the discrete gaussian curvature $K(p)$

261 In the following, $\Delta K = K_u - K_0$ is adopted to mimic in discrete form the variation of the Gaussian curvature of a
 262 continuous shell between the undeformed geometry and the collapse deformed shape.
 263

264 **5.3. In-plane nodal deformation Γ**

265 To locally evaluate the in-plane deformation of each quadrilateral face, a nodal quantity equivalent to the shear
 266 deformation in continuum mechanics is defined in the following:

$$\Gamma(p) = \gamma_{xy}(p\mathcal{N}) + \gamma_{yx}(p\mathcal{E}), \quad (8)$$

267 where

$$\gamma_{xy}(p\mathcal{N}) = \frac{|\delta_{x,\mathcal{N}} - \delta_{x,p}|}{l_{p\mathcal{N}}}, \quad \gamma_{yx}(p\mathcal{E}) = \frac{|\delta_{y,\mathcal{E}} - \delta_{y,p}|}{l_{p\mathcal{E}}} \quad (9)$$

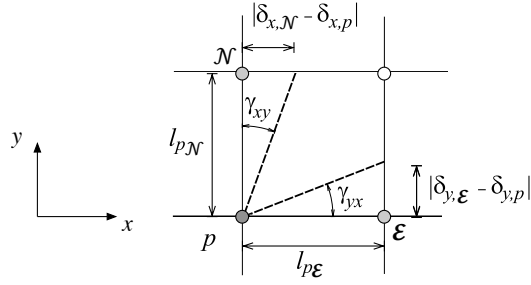


Figure 7: Definition of the equivalent shear deformation $\Gamma(p)$

and the displacement δ , the side of the quadrilateral face l and the node nomenclature are graphically defined in Figure 7 for the sake of clarity and conciseness.

In the following, $\Gamma = \Gamma_u$ refers to the collapse deformed shape, i.e. the displacements δ in Eq. 9 are the ultimate nodal displacements δ_u .

6. Results

This Section illustrates the results of the parametric analysis on the benchmark PG. First, the effects of the design parameters $A_r - I_r$ on Load Factor and buckling shapes are commented in 6.1. Then, the mechanical behaviour is rephrased in 6.2 by introducing a single synthetic design parameter and by referring to nodal observables. Different mechanical regimes are identified. Finally, a range-finding chart within the design parameter plan is presented in 6.3.

6.1. LF and buckling shapes versus design variables

Figures 8(a) and (b) show the LF versus I_r for each A_r obtained by LBA and GMNA, respectively.

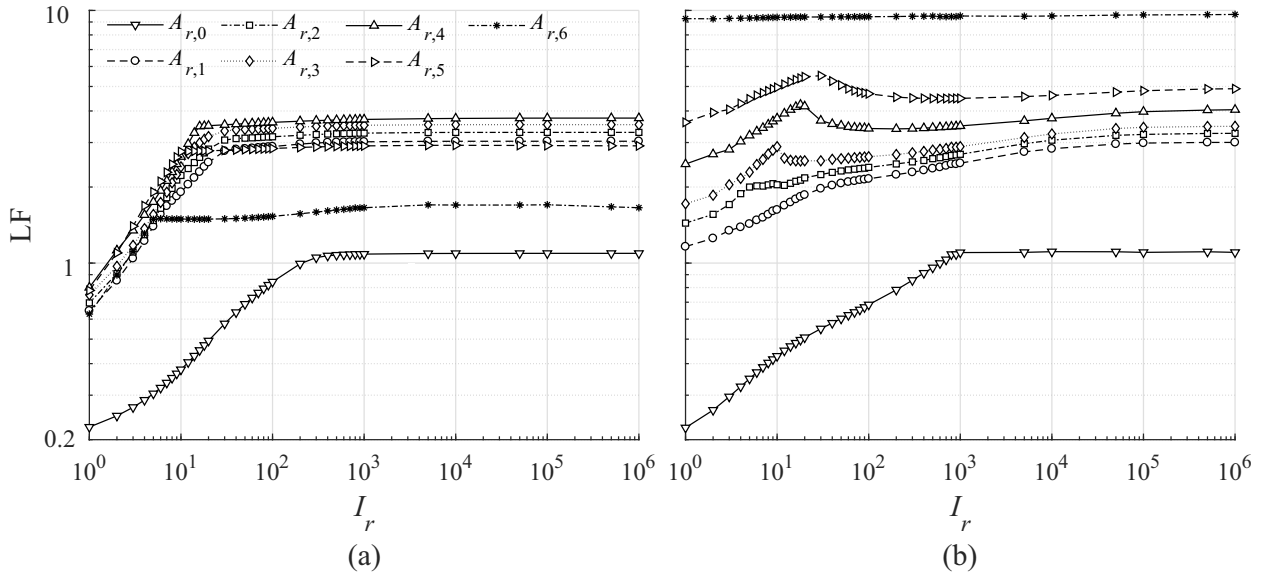


Figure 8: LF versus I_r for different values of A_r with LBA (a) and GMNA (b)

A general trend can be recognised for both kinds of analysis. LF increases for increasing stiffness of the boundary arch along two main branches: a first steeper branch and a second constant or slightly increasing one. However, the

281 value of I_r at which and the way in which the transition between the two branches occurs differ in the two models
 282 and for different values of A_r . In LBA the switch between branches is clearly recognisable, and regularly occurs for
 283 decreasing values of I_r as A_r increases. GMNA retains the same trend, and also almost the same LF values, only for
 284 the unbraced gridshell ($A_{r,0}$). Conversely, for the other values of A_r , a third branch can be observed in between the first
 285 two. In such a further branch, the LF trend versus I_r is still increasing for $A_{r,1}$, almost constant for $A_{r,2}$, and locally
 286 decreasing for $A_{r,3}$ to $A_{r,5}$. Furthermore, the third branch occurs at decreasing values of I_r up to $A_{r,2}$ and at increasing
 287 values of I_r from $A_{r,3}$. For $A_{r,6}$ the LF is nearly constant versus I_r .

288 For low to moderate values of $A_r \leq A_{r,4}$, the LF values predicted by LBA in the first steeper branch are lower than
 289 the ones obtained by GMNA, contrarily to what might be expected [42]. At the upper limit of the second constant
 290 or slightly increasing branch the two models predict comparable LF values. For high values of $A_r \geq A_{r,5}$, the LFs
 291 predicted by GMNA are higher than the ones obtained by LBA whichever the value of I_r . The obtained results show
 292 that LF is more sensitive to A_r than to I_r , and suggest that the non linear effects due to bracing cables in hybrid
 293 gridshells play a major role, which is necessarily underestimated by LBA.

294 We can expect that the above differences in LF reflect different simulated modes of instability. Hence the trends
 295 of LF can be better commented on by looking at the buckling shapes obtained via LBA and GMNA for some rep-
 296 resentative values of I_r and A_r (Figure 9). As regards A_r , three values are selected: $A_{r,2}$ is representative of built
 297 bending-inactive single-layer steel gridshells (see Figure 3, $A_{r,2} = 0.064$ as for the Hippo House); lower and upper
 298 limit cases ($A_{r,0} = 0$, $A_{r,5} = 0.5$) consider the unbraced cell and a very in-plane-rigid gridshell, respectively. The
 299 buckling shapes corresponding to $A_{r,6}$ are not plotted because the extreme stiffness of the gridshell involves almost
 300 constant values of LF versus I_r in GMNA, and it does not allow a sound kinematic reading of the buckling shapes.
 301 As regards I_r , six values are selected: besides the limit values $I_r = 1$ and 10^6 , four other values belong to the above
 302 described branches. An overall view of the buckling shapes allows to qualitatively identify some peculiar instability
 303 modes:

- 304 • the buckling shapes above the red dashed line are characterised by global instability mainly driven by the
 305 buckling of the boundary arch:
 - 306 – the buckling mode predicted by LBA is asymmetric in most cases, while GMNA always provides sym-
 307 metric buckling shapes due to the perfect geometry and symmetrical setup. This usually results in an even
 308 number n of antinodes in LBA arch buckling shape, and in $n + 1$ antinodes in GMNA;
 - 309 – LBA predicts a symmetric arch buckling shape for some pairs of $I_r - A_r$ values (highlighted in grey in Fig.
 310 9). It can be argued this is a consequence of the symmetric elastic constraint that cell bracing provides to
 311 the boundary arch. Such a constraint always holds for $A_r > 0$, but its effect depends on the ratio between
 312 the free edge bending stiffness and the cable axial stiffness;
 - 313 – the stiffening effect of the braced cells on the arch is higher in geometric non linear analysis than in linear
 314 analysis, because of the updated overall geometry of the braced cells;
 - 315 – in the light of the comments above, the higher LF values by GMNA look mainly due to the additional
 316 bracing stiffness accounted for by nonlinear analysis, while the switch from asymmetric to symmetric
 317 buckling shapes due to the GMNA symmetric setup seems to play a minor role, as testified by e.g. the
 318 minimal scatter of LF from LBA to GMNA for the unbraced PG, $A_{r,0}$;
- 319 • the buckling shapes below the blue dash-dot line are still characterised by global instability, but here the buckling
 320 mainly occurs on the gridshell, while the arch is almost undeformed. However, the two structural models
 321 provide different kinds of instability modes for the braced gridshells:
 - 322 – the shell-like buckling mode predicted by LBA involves the majority of nodes with a shorter wavelength
 323 than the cases above;
 - 324 – GMNA predicts a snap-through buckling of the gridshell portions with the lowest curvature, which are
 325 located near the spring line ("line or ring instability" according to the nomenclature in [13]);
- 326 • the buckling shapes in between the red and blue lines are characterized by a combined global instability, in
 327 which both the arch and the gridshell buckle. This combined instability is more evident for A_r values of design

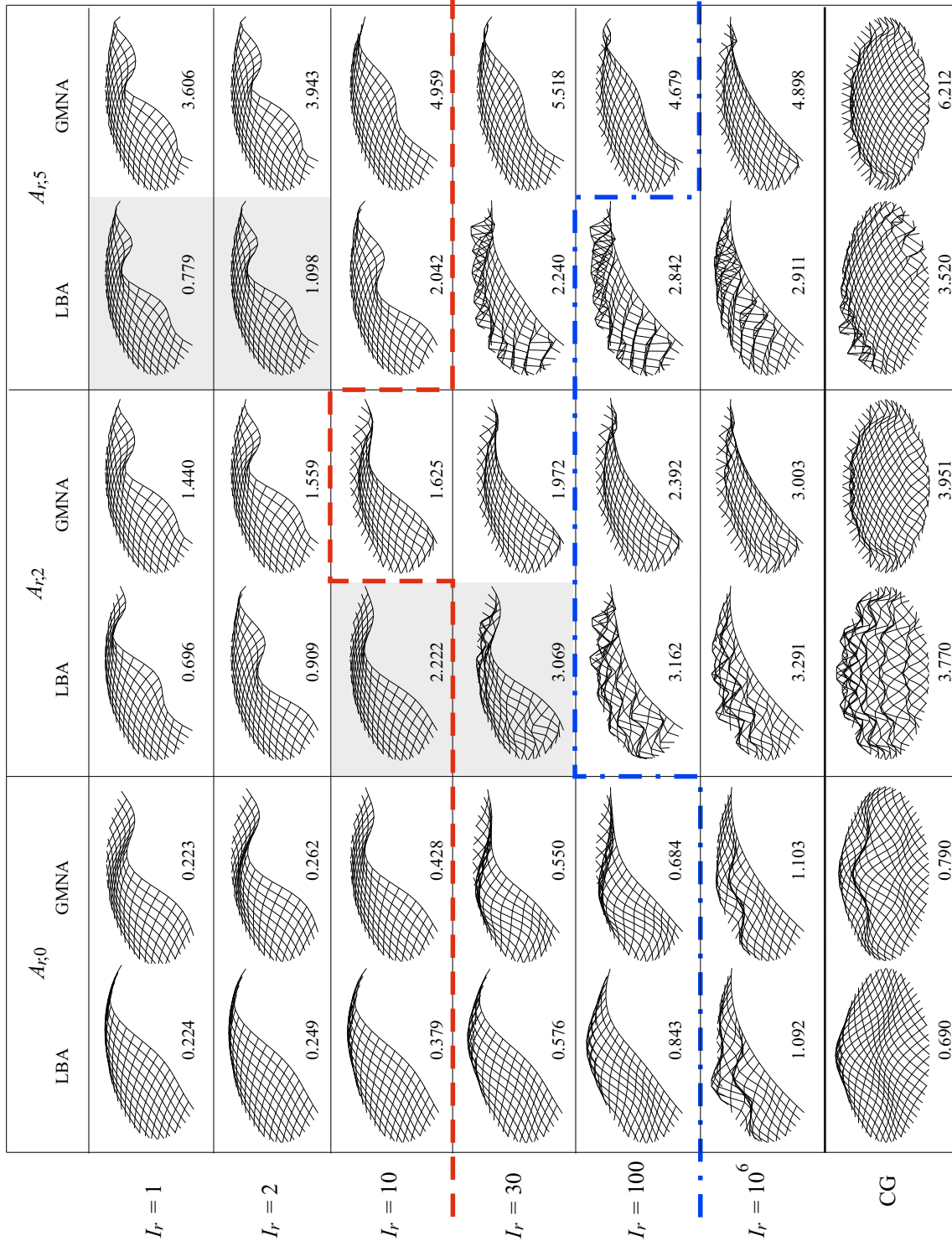


Figure 9: Buckling shapes and LF for selected values of I_r and A_r

328 interest ($A_{r,2}$) than for limit values. The pairs of $I_r - A_r$ in this region generally fall in the third intermediate
 329 branch recognized for GMNA in the $LF(I_r)$ curves (Fig. 8).

330 Some further comments cross the three instability modes outlined above for limit values of A_r . For $A_{r,0}$ the kind
 331 of instability predicted by both LBA and GMNA is almost the same for every I_r , except for the asymmetric and
 332 symmetric shapes, respectively, induced by the adopted model. This reflects in the close trend of the $LF(I_r)$ curves
 333 (Fig 8). For $A_r \geq A_{r,5}$ the bracing stiffening accounted for by GMNA progressively prevails on the flexural stiffness of
 334 the boundary arch for every value of I_r . This is the reason why $LF(I_r)$ is slightly varying for $A_{r,5}$ and nearly constant
 335 for $A_{r,6}$ in GMNA (Fig 8).

336 The effect of the cable stiffness on the LF is further investigated in Figure 10 for the limit case $I_r = 10^6$, i.e.
 337 extremely stiffen boundary arch (empty triangles). The $LF(A_r)$ trend by LBA (Fig. 10a) and GMNA (Fig. 10b) is
 338 qualitatively the same for $A_r \leq A_{r,4}$, i.e. LF increases for increasing A_r . Conversely, for $A_r \geq A_{r,5}$ the bracing stiff-
 339 ening accounted for by GMNA keep the monotonic increasing trend of $LF(A_r)$, while in LBA the shell-like buckling
 340 progressively localises, and the LF trend becomes decreasing in turn.

341 In the case $I_r = 10^6$ the behavior of the PG can be directly compared to the one of the corresponding Complete
 342 Gridshell. CG develops along the whole reference dome surface shown in Figure 4(a), it has the same cross section
 343 properties, the same internal and external constraints and it is loaded by the same uniform distributed load as the PG.
 344 The $LF(A_r)$ for both PG and CG are plotted in Figure 10 by empty and filled triangles, respectively, while buckling
 345 shapes of the CG for some selected values of A_r are included in Figure 9, last row. For all values of A_r , the bending
 346 stiffness of the boundary arch in its vertical plane is so high that it approximatively behaves as a rigid body, and vertical
 347 deflection at collapse are negligible. However, out-of-plane horizontal displacement of the free edge can result from
 348 the rigid rotation of the arch around the hinges at its footings, under the orthogonal horizontal thrust applied by the
 349 half-arches of the PG. The higher A_r , the higher the shear stiffness of the PG, the lower the amplitude of the boundary
 350 arch rigid body rotation outside its plan, the lower the resulting horizontal displacement at collapse. In the lower
 351 limit case of the unbraced structure ($A_{r,0}$), nil vertical displacements of the PG boundary involve LFs slightly higher
 352 than the ones of CG, whose buckling shape involves significant deflections along the central arch (See Fig. 9, first
 353 two columns, last two rows.) For low to moderate values of A_r , the not negligible shear deformability of the partial
 354 gridshell allows the arch to rigidly rotate out of its plane. It follows the LF of PG is always lower than the LF of
 355 CG. PG and CG LFs tend to the same value only for A_r upper limit value, where the extremely high shear stiffness
 356 of the PG does not allow significant rigid rotation of the boundary arch out of its plane. In other terms, free-edge
 357 displacements are almost nil both in the vertical and horizontal directions, and the same mode of instability occur for
 both PG and CG.

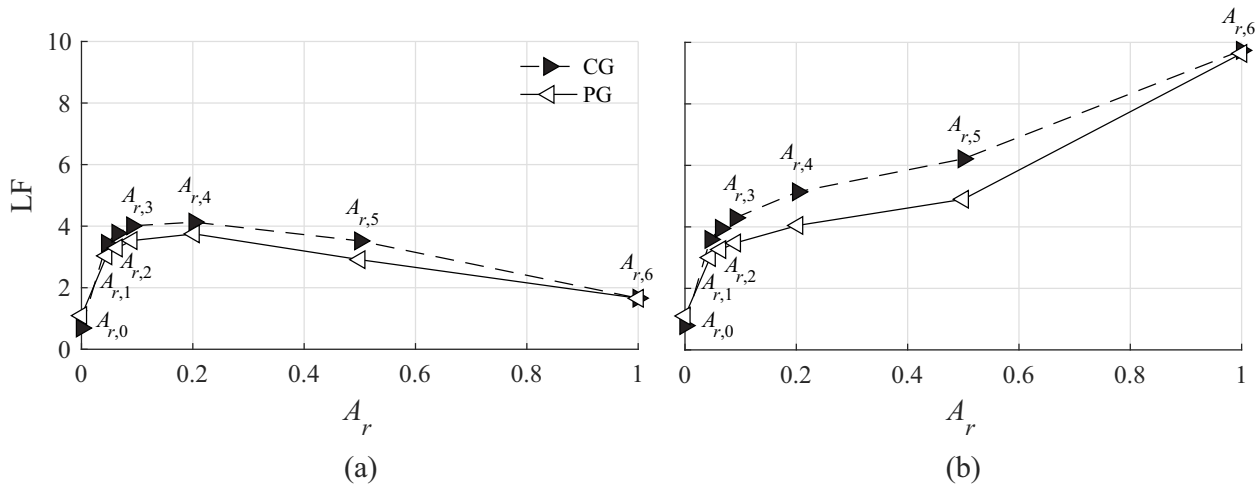


Figure 10: LF versus A_r ($I_r = 10^6$) for PG and CG with LBA(a) and GMNA (b)

359 In the next Section we propose a complementary, deeper insight of the results obtained by GMNA models. The
 360 analysis is focused on the PG with the values of A_r that are more relevant and recurrent in the design of bending-
 361 inactive hybrid single-layer free-edge gridshells ($A_{r,1}$, $A_{r,2}$, $A_{r,3}$, $A_{r,4}$). Therefore, the limit cases $A_{r,0}$, $A_{r,5}$ and $A_{r,6}$ are
 362 no longer considered.

363 6.2. Mechanical rephrasing

364 To comprehensively analyse the relative influence of the two design variables, the dimensionless ratio S_r is intro-
 365 duced:

$$S_r = \frac{I_r}{A_r}. \quad (10)$$

366 In words, such derived variable synthetically weights the flexural stiffness of the boundary arch to the shear stiffness
 367 of the braced gridshell. Figure 11 plots the LF versus S_r . This representation allows to recognise some common
 368 trends among the curves and related subranges of S_r , which were not clearly visible in Figure 8b. Specifically, three
 369 branches can be identified:

- 370 - for $S_r < 100$ the LF steeply increases for increasing S_r ;
- 371 - for about $100 < S_r < 400$ the LF trend changes with A_r , i.e. LF increases, remains constant or decreases for
 372 increasing value of A_r ;
- 373 - for $S_r > 400$ the LF moderately increases for increasing S_r . Moreover, for $S_r > 1000$ the curves have a
 374 common asymptotic trend.

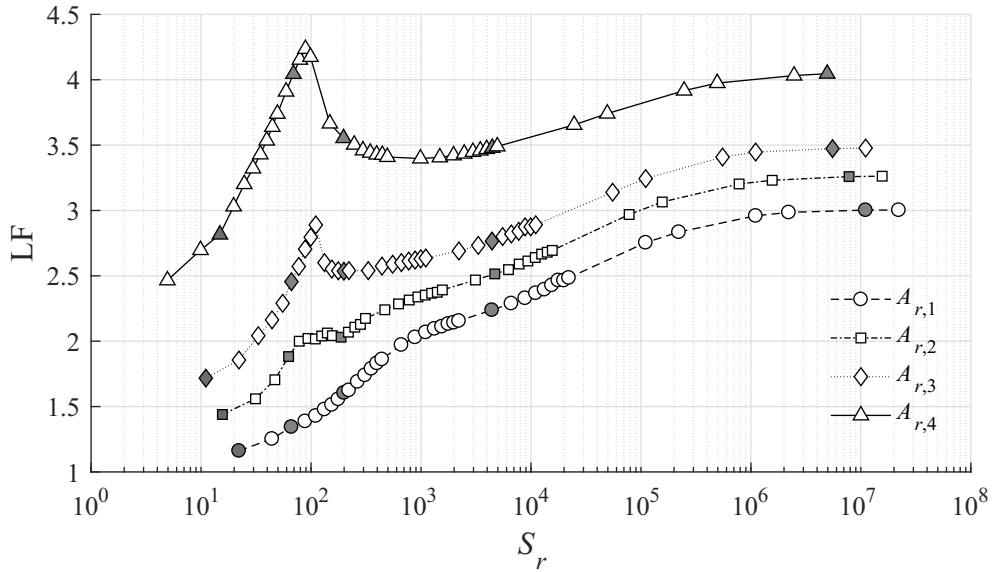


Figure 11: LF versus S_r for different values of A_r

375 A deeper insight into the PG mechanical behaviour corresponding to the above identified S_r ranges is provided in
 376 Figures 12-13. They plot in plan, for representative values of S_r (see filled markers in Figure 11), the nodal observables
 377 at collapse ΔK and Γ introduced in Section 5, scaled with respect to their maximum values $\hat{\Delta K}$ and $\hat{\Gamma}$, respectively.
 378 The visualisation of these normalised nodal quantities allows to discuss the topology of the buckling shapes more
 379 precisely and clearly than classic magnified axonometric deformed shapes do, e.g. in Figure 9. In particular, both
 380 Figures highlight three recurrent instability modes that are common to all A_r values. For the sake of clarity and by
 381 way of example, recognised node patterns are qualitatively clustered by dashed contours in the $A_{r,2}$ rows:

- 382 I. around low values of S_r (see first column), the highest deformations are concentrated along the boundary arch
 383 and adjacent nodes, and progressively diminish towards the spring line (red dashed contour);
- 384 II. as S_r increases (see second column), the highest deformations involve the nodes adjacent to both the boundary
 385 arch and the outer ring;
- 386 III. for high values of S_r (see third to fifth columns), the highest deformations are located in the nodes near the
 387 spring line (blue dashed contour), with a progressive involvement of a higher number of internal nodes as S_r
 388 increases.

389 In other words, when the boundary arch is relatively flexible, the collapse is *free-edge-driven* (I), i.e., it is mainly
 390 triggered by the buckling of the boundary arch. Conversely, when the boundary arch is relatively stiff, the collapse is
 391 *gridshell-driven* (III), that is, it is mainly initiated by the snap-through ring instability. In between, a transition mode
 392 (II) occurs, where both kind of instabilities are combined in the PG as a whole.

393 The spatial patterns briefly discussed above oriented us towards the heuristic definition of a further observable to
 394 identify more precisely the ranges of S_r in which the described instability modes take place:

$$\Gamma_r = \frac{\sum_{i=1}^{N_{\Omega_1}} \Gamma_i}{N_{\Omega_1}} \cdot \frac{N_{\Omega_2}}{\sum_{j=1}^{N_{\Omega_2}} \Gamma_j}, \quad (11)$$

395 where N_{Ω_1} and N_{Ω_2} are the number of nodes falling into the Ω_1 and Ω_2 regions, respectively (see scheme in Figure
 396 14 on the right). In words, the defined observable is the ratio between the average in-plane nodal deformation in
 397 two regions selected a priori on the basis of the spatial patterns in Figures 12-13. This observable is intended to
 398 provide a bulk quantitative metric that synthetically describes the in-plane deformation patterns discussed above (Fig.
 399 13). On the basis of the above equation, values of $\Gamma_r > 1$ describe the case where the in-plane deformations are
 400 mainly concentrated in the Ω_1 region (free-edge-driven deformation), while for $\Gamma_r < 1$ the deformations are mainly
 401 gridshell-driven. Figure 14 illustrates Γ_r versus S_r for the considered four values of A_r . The expected behaviour is
 402 generally confirmed and three main regimes corresponding to the above instability modes can now be clearly defined.
 403 Specifically:

- 404 - regime I corresponds to values of $\Gamma_r > 1$ for all three curves. Its upper bound $S_r = S_{r,I-II} \cong 50$ is given by the
 405 intersection between the $\Gamma_r = 1$ line and the $A_{r,2}$ curve;
- 406 - regime III corresponds to values of $\Gamma_r < 1$ for all three curves. Its lower bound $S_r = S_{r,II-III} \cong 120$ is given by
 407 the intersection between the $\Gamma_r = 1$ line and the $A_{r,3}$ curve;
- 408 - in between, a transition regime II is characterised by a sudden drop of Γ_r versus S_r . In this regime, different
 409 deformation patterns occur for different A_r values. In particular, in $A_{r,1}$ free-edge and shell-driven deformations
 410 are well balanced, in $A_{r,3}$ and $A_{r,4}$ the free-edge-driven deformation is more evident, while in $A_{r,2}$ the gridshell-
 411 driven behaviour prevails (compare Figure 13, second column). In other terms, the upper and lower bounds of
 412 regime II seem to depend on other parameters than S_r .

413 Moreover, regime III can be further divided into three sub-regimes on the basis of some common trends of the
 414 $\Gamma_r - S_r$ curves. In IIIa Γ_r varies non monotonically. Then, in IIIb, a monotonic decreasing trend can be recognised.
 415 Finally, in IIIc the three curves almost coincide and share a monotonic slightly increasing trend: in fact, as the
 416 arch stiffness tends to infinity, the collapse involves its out-of-plane rigid body rotation, which is retained by the
 417 gridshell in-plane shear stiffness. Therefore, an increasingly higher number of nodes in the Ω_1 region is involved in
 418 the deformation (see Fig. 13, third to fifth columns and related comments).

419 It can be observed that the general trend of $\Gamma_r(S_r)$ in Figure 14 and the features of the recognised regimes recall
 420 other transitional physical phenomena, such as the evolution of the mean drag aerodynamic coefficient with Reynolds
 421 number for a circular cylinder (compare [71], Fig. 4.5.2, and [72], Fig. 7.28).

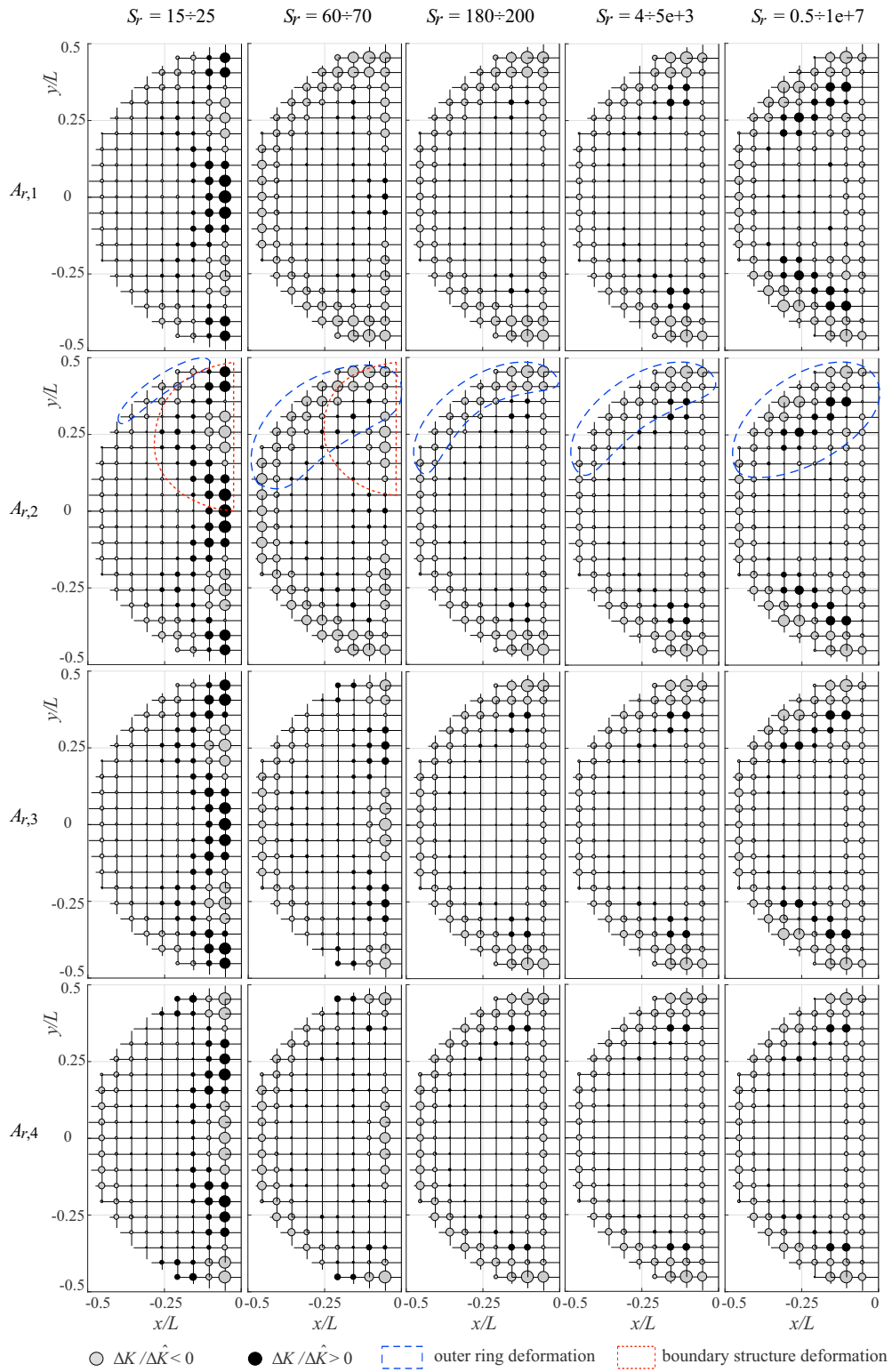


Figure 12: $\Delta K / \hat{\Delta K}$ for different S_r and A_r

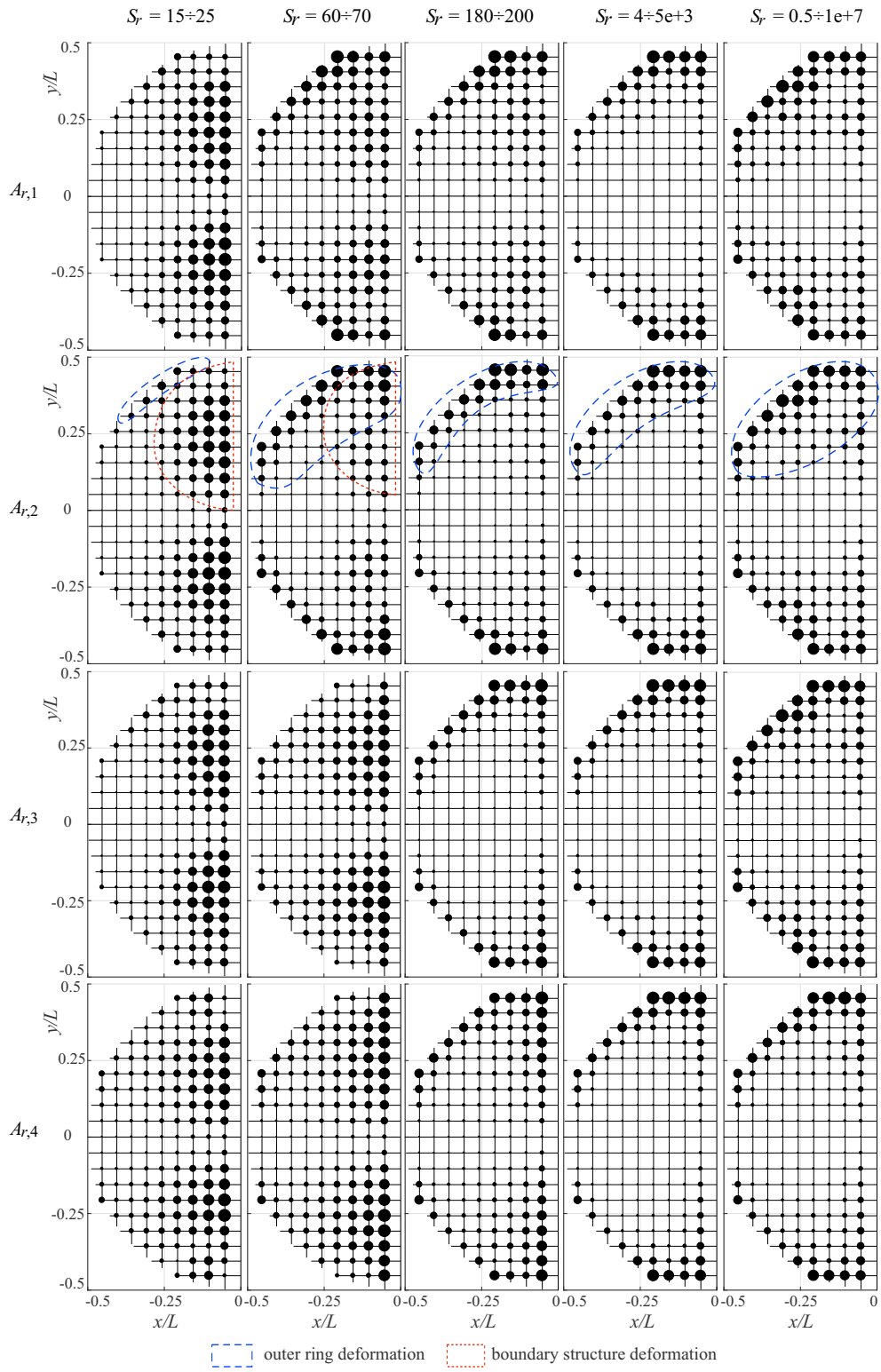


Figure 13: $\Gamma/\hat{\Gamma}$ for different S_r and A_r

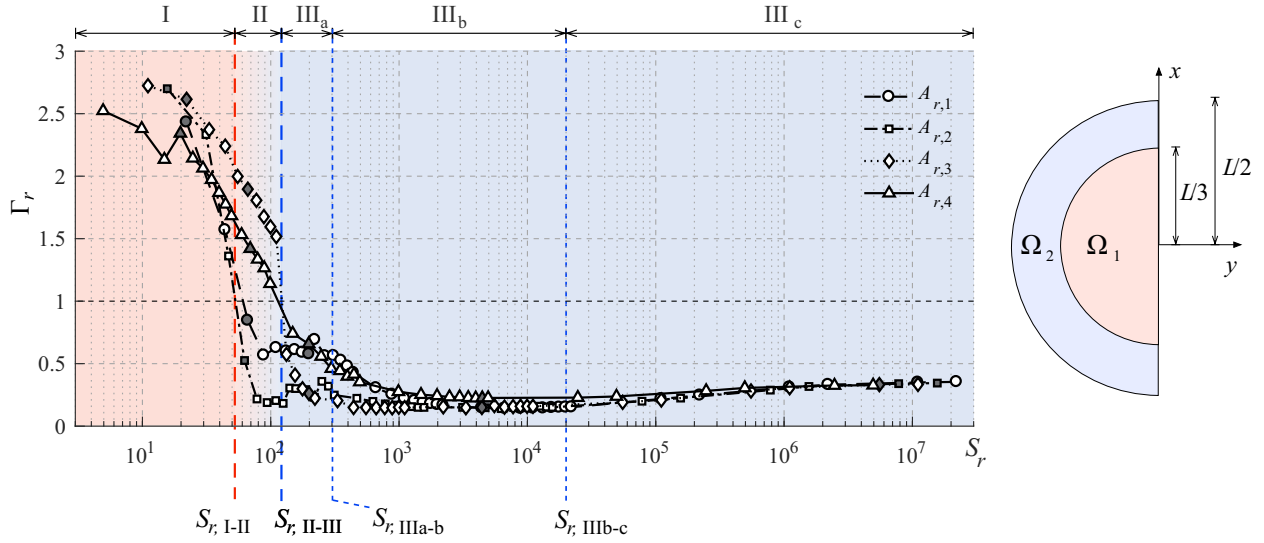


Figure 14: Γ_r versus S_r for different A_r (filled markers highlights the cases analysed in Figures 12 and 13)

422 6.3. Range-finding chart

423 On the basis of the analysis illustrated in the previous sub-Sections, a range-finding chart within the design parameter space is proposed in the following. The aim of the chart is to orient the structural analyst in identifying the preferred regime and, hence, possible ranges of the design parameters I_r and A_r .

424 The chart in Figure 15 plots the $I_r - A_r$ couples, which satisfy the following expression, obtained by substitution of Eq.s (1) in Eq. (10):

$$425 I_r = S_r A_r. \quad (12)$$

426 The straight lines delimiting the regime boundaries are obtained by substituting in Eq. (12) the values of S_r corresponding to the regime boundaries (see Fig. 14). In the chart, the design parameter space is limited to low values of both I_r and A_r in order to focus on the values of practical interest for bending-inactive hybrid single-layer steel gridshells. The $I_r - A_r$ couples of the built gridshells collected in Figure 3 are included for reference (black squares). It can be observed that the latter ones lie in Regime IIIb: this seems to prove that skilled designers size both grid and boundary structural elements such as to avoid boundary-driven instability (regimes I and II) and, in general, the regimes characterised by the high variability of LF.

427 Note that the chart in Figure 15 has been determined for the specific PG analysed in this study and can not therefore be considered as generally valid from a quantitative point of view. Nevertheless, the qualitative trend reported in the chart is expected to hold also for different gridshell set-ups.

438 7. Conclusions

439 The present study was conducted to define, classify and analyse, from a mechanical point of view, a specific kind of gridshell structures, here named “Partial Gridshell” as opposed to “Complete Gridshells”. PGs are gridshells whose form is delimited by two or more geometrical cuts. The intersections between the gridshell and the delimitation surfaces give rise to a spring line - along which the PG is rigidly constrained - and one or more curved free-edges. Despite the prevailing occurrence of this gridshell type in the building practice, the studies that systematically discuss the stability of gridshell domes always adopt CGs with horizontal spring lines as benchmarks.

440 An ideal hybrid PG is adopted as a benchmark to investigate its local and global stability. In particular, its sensitivity to the flexural stiffness of the boundary structure and to the shear stiffness of the gridshell are investigated through an extensive parametric analysis by numerical experiments. The adopted parameters are the ratio of the boundary arch and grid element moment of inertia I_r , and the ratio of the bracing cable and grid element cross section

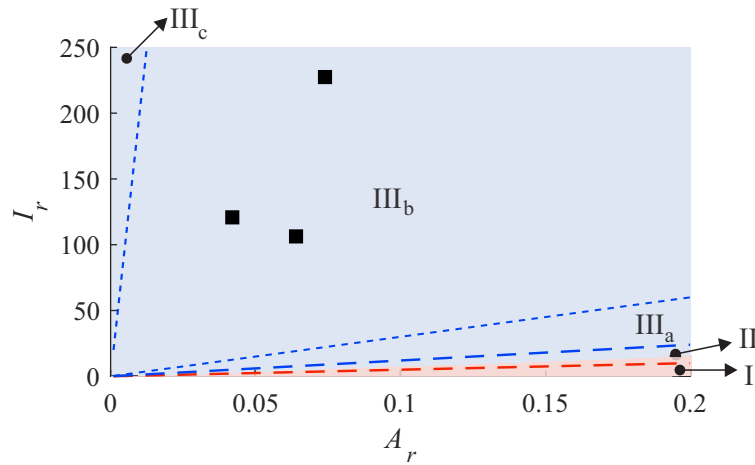


Figure 15: Range-finding chart in the I_r - A_r design plan

449 area A_r . The load factor and the collapse deformed shape under a uniform loading scenario are determined through
 450 geometrically and materially non-linear analysis.

451 The analysis of the results have highlighted the existence of three different mechanical regimes, depending on the
 452 value of the dimensionless parameter S_r . The latter was defined to evaluate the relative influence of the two design
 453 parameters I_r and A_r on the stability of the PG. Specifically:

- 454 • regime I ($S_r \approx < 50$) is characterised by a free-edge-driven collapse, i.e., in-plane and out-of-plane deformations
 455 mainly involve the boundary arch and adjacent gridshell nodes;
- 456 • regime II ($50 \approx S_r \approx < 120$) shows a combined behaviour, where both free-edge-driven and gridshell-driven
 457 instabilities occur;
- 458 • regime III ($S_r \approx > 120$) is characterised by a shell-driven collapse, where the main in-plane and out-of-plane
 459 deformations mostly involve the gridshell nodes in the outer ring adjacent to the spring line.

460 Finally, a range-finding chart in the design parameter plan is proposed as a qualitative tool for designers to identify
 461 the preferred regime and associated values of the design parameters. Remarkably, the built bending-inactive hybrid
 462 single-layer steel gridshells reviewed in the present survey lie in Regime IIIb: this seems to prove that skilled designers
 463 size both grid and boundary structural elements such as to avoid free-edge-driven instability (regimes I and II).

464 These results, despite relative to a specific geometrical and structural set-up, contribute to shed some light on
 465 the behaviour of bending-inactive free-edge gridshell structures. Although gridshell geometry, free-edge geometry
 466 and their stiffness greatly vary in design practice, the overall results of this study are likely to be general at least
 467 in a qualitative sense. In other words, the mechanical interaction between out-of-plane boundary stiffness and in-
 468 plane gridshell stiffness is expected to take place as a rule, and regimes I and III are likely to occur in general.
 469 Conversely, the watershed values of S_r at which transition between regimes occurs and the transition mode (including
 470 their combination in regime II) are expected to strongly depend on the specific features of each structure. For instance,
 471 subsequent studies could be devoted to the effect of geometrical factors, i.e. the kind and orientation of the trimming
 472 surface with respect to the grid direction, the ratio between the gridshell and free-edge span, and to the joint effects of
 473 imperfections. The generalisation of the study to bending-active gridshells is not straightforward and requires further
 474 investigations, in the wake of the few comparative studies between bending-active and inactive gridshells (e.g. [73]),
 475 and the preliminary evaluation of the free-edge role in the stability of bending-active gridshells [74].

476 Acknowledgements

477 The authors would like to thank Vittorio Nascé, Emeritus Professor at Department of Architecture and Design,
 478 Politecnico di Torino, for the fruitful discussions about the paper topics and challenges.

479 **References**

- 480 [1] M. Beckh, R. Barthel, The first doubly curved gridshell structure - Shukhovs building for the plate rolling workshop in Vyksa, in: Proceedings
 481 of the Third International Congress on Construction History, Cottbus, 2009.
- 482 [2] R. Fuller, J. Ward, The artifacts of R. Buckminster Fuller: a comprehensive collection of his designs and drawings, Garland, 1985.
- 483 [3] I. Liddell, Frei otto and the development of gridshells, *Case Studies in Structural Engineering* 4 (2015) 39–49.
- 484 [4] J. Schlaich, H. Schober, Glass-covered grid-shells, *Structural Engineering International* 6 (2) (1996) 88–90.
- 485 [5] H. Schober, *Transparent Shells: Form, Topology, Structure*, John Wiley & Sons, 2015.
- 486 [6] F. Otto, E. Shauer, J. Hennicke, IL 10 Gitterschalen / Grid Shells, Stuttgart, 1974.
- 487 [7] B. Burkhardt, O. Frei, IL 13 Multihalle Mannheim, Stuttgart, 1978.
- 488 [8] A. Holgate, *Glass Grid Roofs. The Art of Structural Engineering: The Work of Jörg Schlaich and His Team*, 1997.
- 489 [9] J. Chilton, G. Tang, *Timber Gridshells: architecture, structure and craft*, Routledge, 2017.
- 490 [10] L. du Peloux, F. Tayeb, O. Baverel, J. Caron, Construction of a large composite gridshell structure: A lightweight structure made with
 491 pultruded glass fibre reinforced polymer tubes, *Structural Engineering International* 26 (2) (2016) 160–167.
- 492 [11] D. Rockwood, *Bamboo Gridshells*, 2015.
- 493 [12] P. Davey, *Engineering for a Finite Planet: Sustainable solutions by Buro Happold*, 2009.
- 494 [13] V. Gioncu, Buckling of reticulated shells: State-of-the-art, *International Journal of Space Structures* 10 (1) (1995) 1–46.
- 495 [14] K. Ishikawa, S. Kato, Elastic-plastic dynamic buckling analysis of reticular domes subjected to earthquake motion, *International Journal of*
 496 *Space Structures* 12 (3-4) (1997) 205–215.
- 497 [15] S. Kato, M. Fujimoto, T. Ogawa, Buckling load of steel single-layer reticulated domes of circular plan, *Journal of the International Association*
 498 *for Shell and Spatial Structures* 46 (147) (2005) 41–63.
- 499 [16] A. López, I. i. Puente, M. A. Serna, Direct evaluation of the buckling loads of semi-rigidly jointed single-layer latticed domes under symmetric
 500 loading, *Engineering Structures* 29 (2007) 101–109.
- 501 [17] S. Yamada, Buckling load evaluation method for single layer cylindrical lattice shells, *Journal of Civil Engineering and Architecture* 6 (3)
 502 (2012) 268–279.
- 503 [18] F. Fan, J. Yan, Z. Cao, Stability of reticulated shells considering member buckling, *Journal of Constructional Steel Research* 77 (2012) 32–42.
- 504 [19] J. Cai, L. Gu, Y. Xu, J. Feng, J. Zhang, Nonlinear stability analysis of hybrid grid shells, *International Journal of Structural Stability and*
 505 *Dynamics* 13 (1) (2013) 1–16.
- 506 [20] J. Yan, F. Qin, Z. Cao, F. Fan, Y. Mo, Mechanism of coupled instability of single-layer reticulated domes, *Engineering Structures* 114 (2016)
 507 158–170.
- 508 [21] D. Wright, Membrane forces and buckling in reticulated shells, *Journal of the Structural Division, ASCE* 91 (1965) 173–201.
- 509 [22] K. Heki, The continuum treatment of discrete systems and its application to the analysis of lattice structures, *Bulletin of the IASS XXVI-*
 510 *3* (89) (1985) 17–24.
- 511 [23] S. J., General stability analysis of lattice shells by continuum modeling, *International Journal of Space Structures* 7 (4) (1992) 275–284.
- 512 [24] S. Malek, T. Wierzbicki, J. Ochsendorf, Buckling of spherical cap gridshells: A numerical and analytical study revisiting the concept of the
 513 equivalent continuum, *Engineering Structures* 75 (15) (2014) 288–298.
- 514 [25] D. Tonelli, N. Pietroni, E. Puppo, M. Froli, P. Cignoni, G. Amendola, R. Scopigno, Stability of static aware Voronoi grid-shells, *Engineering*
 515 *Structures* 116 (2016) 70–82.
- 516 [26] T. Bulenda, J. Knippers, Stability of grid shells, *Computer & Structures* 79 (2001) 1161–1174.
- 517 [27] F. Fan, Z. Cao, S. Shen, Elasto-plastic stability of single-layer reticulated shells, *Thin-Walled Structures* 48 (2010) 827–836.
- 518 [28] C. Borri, P. Spinelli, Buckling and post-buckling behavior of reticulated shells affected by random imperfections, *Computer & Structures*
 519 *30* (4) (1988) 937–943.
- 520 [29] S. Kato, I. Mutoh, M. Shomura, Collapse of semi-rigidly jointed reticulated domes with initial geometric imperfections, *Journal Construc-*
 521 *tional Steel Research* 48 (1998) 145–168.
- 522 [30] S. Yamada, A. Takeuchi, Y. Tada, K. Tsutsumi, Imperfection-sensitive overall buckling of single-layer lattice domes, *Journal of Engineering*
 523 *Mechanics* 127 (2001) 382–386.
- 524 [31] A. Zhang, X. Zhang, J. Ge, Influence of initial geometrical imperfections on stability of a suspended dome for Badminton Arena for 2008 Olympic
 525 Games, *Spatial Structures* 12 (4) (2006) 8–12, in Chinese.
- 526 [32] J. Guo, Research on distribution and magnitude of initial geometrical imperfection affecting stability of suspen-dome, *Advanced Steel Con-*
 527 *struction* 7 (4) (2011) 344–358.
- 528 [33] F. Fathelbab, The effect of joints on the stability of shallow single layer lattice domes, Ph.D. thesis, University of Cambridge (1987).
- 529 [34] K.-J. Hwang, J. Knippers, S.-W. Park, Influence of various types node connectors on the buckling loads of grid shells, in: *IASS Symposium,*
 530 *2009*.
- 531 [35] H. Ma, F. Fan, P. Wen, H. Zhang, S. Shen, Experimental and numerical studies on a single-layer cylindrical reticulated shell with semi-rigid
 532 joints, *Thin-Walled Structures* 86 (2015) 19.
- 533 [36] F. Fan, J. Yan, Z. Cao, Elasto-plastic stability of single-layer reticulated domes with initial curvature of members, *Thin-Walled Structures* 60
 534 (2012) 239–246.
- 535 [37] J. Cai, Y. Xu, J. Feng, J. Zhang, Nonlinear stability of a single-layer hybrid grid shell, *Journal of Civil Engineering and Management* 18 (5).
- 536 [38] L. Bruno, M. Sassone, F. Venuti, Effects of the equivalent geometric nodal imperfections on the stability of single layer grid shells, *Engineering*
 537 *Structures* 12 (2016) 184–199.
- 538 [39] R. Mesnil, C. Douthe, O. Baverel, B. Léger, Linear buckling of quadrangular and kagome gridshells: A comparative assessment, *Engineering*
 539 *Structures* 132 (2017) 337–348.
- 540 [40] T. Ueki, S. Kato, I. Kubodera, Y. Mukaiyama, Study on the elastic and elasto-plastic buckling behaviour of single layered domes composed
 541 of members having axial and bending springs at the ends, in: *IASS Symposium, 1991*.

- 542 [41] S. Kato, S. Yamada, H. Takashima, R. Shibata, Buckling stress of a member in a rigidly jointed single-layer reticular dome, in: IASS
543 Symposium, 1991.
- 544 [42] I. for Metal Spatial Structures, (Draft) Guide to buckling load evaluation of metal reticulated roof structures, Tech. rep., International Asso-
545 ciation for Shell and Spatial Structures (2014).
- 546 [43] J. Schlaich, H. Schober, Glass roof for the Hippo House at the Berlin Zoo, *Structural Engineering International* 7 (4) (1997) 252–254.
- 547 [44] J. Abel, J. Chilton, Heinz isler - 50 years of "new shapes for shells", *Journal of the International Association for Shell and Spatial Structures*
548 52 (3).
- 549 [45] M. Garlock, D. Billington, Felix Candela: Engineer, Builder, Structural Artist, Yale University Press, 2008.
- 550 [46] L. Kollár, E. Dulácska, Buckling of shells for engineers, Wiley, Chichester, 1984.
- 551 [47] S. Timoshenko, J. Gere, Theory of elastic stability, McGraw Hill, New York, 1961.
- 552 [48] L. Donnell, Beams, plates and shells, McGraw-Hill, New York, 1976.
- 553 [49] W. Flugge, Stress in shells, Springer-Verlag, Berlin, 1960.
- 554 [50] A. Zingoni, Liquid-containment shells of revolution: A review of recent studies on strength, stability and dynamics, *Thin-Walled Structures*
555 87 (2015) 102–114.
- 556 [51] L. Godoy, Buckling of vertical oil storage steel tanks: Review of static buckling studies, *Thin-Walled Structures* 103 (2016) 1–21.
- 557 [52] C. Douthe, O. Baverel, J. Caron, Gridshell in composite materials: towards wide span shelters, *Journal of the International Association for*
558 *Shell and Spatial Structures* 48 (3) (2007) 175–180.
- 559 [53] M. Kahl, T. Mansperger, Überdachung des kleinen Schlosshofes des Residenzschlosses Dresden, *Stahlbau* 82 (7) (2013) 531–536.
- 560 [54] C. Williams, The analytical and numerical definition of the geometry of the British Museum Great Court roof, in: *Mathematics and Design*
561 2001, 2001.
- 562 [55] E. Happold, W. Liddell, Timber lattice roof for the Mannheim Bundesgartenschau, *The Structural Engineer* 53 (3) (1975) 99–135.
- 563 [56] C. Poirriez, T. Wortmann, R. Hudson, Y. Bouzida, From complex shape to simple construction: fast track design of the "Future of us
564 gridshell" in Singapore, in: T. T. K. Kawaguchi, M. Ohsaki (Ed.), *Proceedings of the IASS Annual Symposium 2016 Spatial Structures in*
565 *the 21st Century*, 2016.
- 566 [57] J. Glymph, D. Shelden, C. Ceccato, J. Mussel, H. Schober, A parametric strategy for free-form glass structures using quadrilateral planar
567 facets, *Automation in Construction* 13 (2) (2004) 187–202.
- 568 [58] H. Schober, S. Justi, Cabot Circus, Bristol - Ebene Vierecknetze für freigeformte Glasdächer, *Stahlbau* 81 (Supplement S1) (2012) 28–42.
- 569 [59] A. Pena de Leon, Two case-studies of freeform-facade rationalization, in: *Proceedings of the 30th International Conference on Education and*
570 *research in Computer Aided Architectural Design in Europe*, Vol. Modes of Production - Volume 2, 2012, pp. 491–500.
- 571 [60] Toledo gridshell, http://www.gridshell.it/gridshell_napoli/.
- 572 [61] A. Cattnach, Waitomo Caves Visitor Centre, New Zeland Timber Design Journal 18 (3) (2010) 7–12.
- 573 [62] O. Kelly, R. Harris, M. Dickson, The construction of the Downland gridshell, *The Structural Engineer* 79 (17) (2005) 25–33.
- 574 [63] R. Harris, S. Haskins, J. Roynon, The Savill Garden gridshell: design and construction, *The Structural Engineer* 86 (17) (2008) 27–34.
- 575 [64] SCI Steel Knowledge, Schubert Club Band, Structural Stainless Steel Case Study 10 (2012) 1–4.
- 576 [65] G. Del Guerra, M. Froli, Proposal to roof the courtyards of an historical building in Pisa with glass and steel grid shells: Form finding and
577 stability problems, *Journal of Architectural Engineering ASCE* 15 (2) (2009) 62–66.
- 578 [66] B. D'Amico, A. Kermani, H. Zhang, A. Pugnale, S. Colabella, S. Pone, Timber gridshell: numerical simulation, design and construction of a
579 full scale structure, *Structures* 3 (2015) 227–235.
- 580 [67] Eurocode 3 UNI EN 1993-1-6:2007: Eurocode 3: Design of steel structures - Part 1-6: Strength and stability of shell structures (2007 2004).
- 581 [68] SAS IP, Inc., ANSYS, Inc. Theory Manual. (2011).
- 582 [69] C. R. Calladine, Theory of Shell Structures, 1983.
- 583 [70] L. Najman, P. Romon, Modern approaches to discrete curvature, Springer International Publishing, 2017.
- 584 [71] E. Simiu, R. Scanlan, Wind Effects on Structures: Fundamentals and Applications to Design, John Wiley & Sons Inc, 1996.
- 585 [72] Eurocode 1 UNI EN 1991-1-4:2005: Actions on structures - Part 1-4: General actions - Wind actions.
- 586 [73] S. Kato, M. Fujimoto, T. Ogawa, Reliability analysis of buckling of reticulated single layer roofs of general form under vertical loads, *Journal*
587 *of the International Association for Shell and Spatial Structures* 57 (3) (2016) 227–244.
- 588 [74] F. Tayeb, J.-F. Caron, O. Baverel, L. D. Peloux, Stability and robustness of a 300m² composite gridshell structure, *Construction and Building*
589 *Materials* 49 (2013) 926–938.