This is the author's version of the article that has been presented at IEEE ISCAS2016
The editorial version of the paper is available at http://dx.doi.org/10.1109/ISCAS.2016.7527219

Implicit Notch Filtering in Compressed Sensing by Spectral Shaping of Sensing Matrix

Mauro Mangia*, Fabio Pareschi†,‡, Riccardo Rovatti‡,*, Gianluca Setti†,*
* ARCES - University of Bologna - via Toffano 2/2 - Bologna - ITALY
† ENDIF - University of Ferrara - via Saragat 1 - Ferrara - ITALY
‡ DEI - University of Bologna - viale Risorgimeto 2 - Bologna - ITALY

***Abstract*—Compressed Sensing (CS) has recently emerged as an interesting and effective way to sample an input signal and at the same time compress it (i.e., reduce the number of measurements for the correct signal reconstruction with respect to the standard Nyquist approach). We show here that CS can be used also to exploit some operations typically performed by the preceding signal conditioning stage (sometimes, by a post-processing stage). In detail, we show that CS can be used to filter environmental disturbances exactly like a notch filter. Furthermore, this solution presents advantages in terms of input signal distortion with respect to the classical notch filter approach. An example on electrocardiographic signal is presented as case study.**

I. Introduction

Mixed mode circuits have emerged in last decades as the simplest, common and powerful way in which analog signals are processed. A typical signal processing chain is composed by an analog front-end followed by a digital block. The analog front-end is usually composed by a signal conditioning stage with the aim to filter and amplify the input signal, and an analog-to-digital converter capable of representing the signal with a set of digital words. The following digital block takes care of additional processing and of further operations such as compression, signal storing or transmission.

More recently, the Compressed Sensing (CS) paradigm [1] has been intensively investigated as a solution capable to add signal compression capabilities at an almost negligible cost directly into the analog-to-digital conversion stage [2]–[4], with an overall energy requirements reduction. Due to these peculiarity, CS has received particular attention for the acquisition of biological signals, with particular references to Body Area Sensor Networks (BASNs) [4], [5]. The aim of this paper is to show that the advantage of using CS in BASNs is actually twofold. On the one hand, CS can exploit sampling and compression features at the same time. On the other hand, we will show how CS can embed at no additional costs also others operations as the rejection of the environmental noise. This would simplify the overall architecture and increase the appealing of CS methodologies. This is also particularly important in the design of BASNs. The acquisition of both electrocardiography (ECG) end electroencephalography (EEG) signals is an example. Due to their small amplitudes, they may be strongly affected by an environmental noise, such as a $50\,\mathrm{Hz}$ or $60\,\mathrm{Hz}$ sinusoidal tone from a power line coupling. Here, system performance can be increased by *rejecting* this environmental noise, i.e., by filtering it out (either completely or only partially). The considered scenario is depicted in Fig. 1, where an input signal x, perturbed by a disturbance ν, is sampled and processed by an *encoder* stage, and then transmitted to the *decoder* stage. Here, we assume a CS-based system, exploiting the capability of the CS encoder

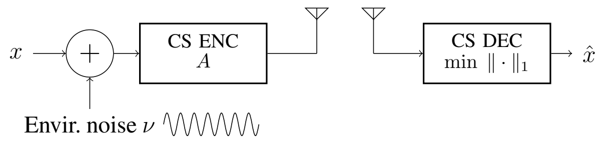


Fig. 1. Block diagram of the CS system including the effect of the environmental noise.

(characterized by the sensing matrix A) to simultaneously sense and compress the signal, that is reconstructed by solving the well-known CS convex optimization problem [6].

In this situation, independently of the encoding/decoding strategy, the most common and intuitive solution for interference rejection is to introduce a notch filtering in the signal conditioning stage (or, less frequently, as a post-processing stage) [7], [8]. The solution is effective, but has side effects. The system complexity may be increased and, more important, any signal information contained in the notched band is unavoidably lost. In this paper we will show that a new class of sensing matrix A with an appropriate statistical characterization may be extremely helpful in this situation. Note that the optimization of the CS by means of a proper characterization of the sensing process is not new in the literature. In [9], [10] approaches to reduce the CS reconstruction error are proposed, with [9] obtaining this by maximizing *rakeness*, i.e., the energy collected by CS samples, in a way similar to what a rake receiver does in a communication system [13]. In [11] authors showed how A can be exploited to add security related aspects to a CS system. Here we will show that a proper sensing matrix design procedure can achieve environmental noise rejection.

The paper is organized as follow. Sec. II briefly recaps the CS mathematical background, while Sec. III describes the proposed environmental noise rejection method. Results in processing artificial signals and ECG signals can be found respectively in Sec. IV and Sec. V. Finally, we draw the conclusion.

II. Compressed Sensing Framework

Let us focus on discrete-time representations of the input signal and the environmental noise, so that $x, \nu \in \mathbb{R}^n$ are composed by n samples with Nyquist frequency f_{Ny}. The entire CS framework is based on the *sparsity* assumption, i.e., each input signal instance x can be represented by a linear combination of few vectors of a proper basis $\Psi \in \mathbb{R}^{n\times n}$ such that $x = \Psi\alpha$, where $\alpha \in \mathbb{R}^n$ is a coefficients vector with no more than $\kappa \ll n$ non-zero elements. In this case we say that the class of input signals is κ-sparse. The CS aim is, under this assumption, to capture the signal information content by projecting x on a set of m sensing sequences (usually

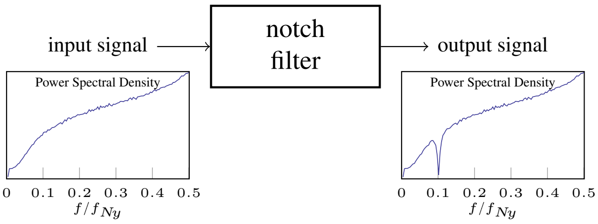


Fig. 2. Effect of environmental noise rejection by means of a notch filter on the input signal spectrum density.

randomly generated), arranged as the rows of a sensing matrix $A \in \mathbb{R}^{m \times n}$. Since $m < n$, we have a dimensionality reduction, and we can define the compression ratio as $\mathsf{CR} = n/m$.

In accordance to the notation of Fig. 1, the measurement vector $y \in \mathbb{R}^m$ is computed as

$$y = A(x + \nu) = A\Psi\alpha + A\nu \quad (1)$$

and then transferred to the decoder stage, whose output $\hat{x} = \Psi\hat{\alpha}$ is the reconstruction of the input signal x. This operation is possible due to the sparsity assumption: over the infinite set of vectors $\hat{\alpha}$ mapped by $A\Psi$ on y, we select the sparsest one by solving the following convex optimization problem [1], [6].

$$\begin{aligned} &\min_{\hat{\alpha}} \quad \|\hat{\alpha}\|_1 \\ &\text{s.t.} \quad \|A\Psi\hat{\alpha} - y\|_2^2 \leq \epsilon^2 \end{aligned} \quad (2)$$

where $\|\cdot\|_p$ stands for standard p-norm, and ϵ is used to take into account the effect of ν. Guarantees on the correct reconstruction of x are based on some properties of A. In particular, the CS theory states that the reconstruction error $\|\hat{x} - x\|_2$ is vanishing with probability one if A satisfies the so called Restricted Isometry Property (RIP) [12] and the number of measurements $m > O(\kappa \log(n/\kappa))$ [6]. The easiest way to ensure RIP is to generate the rows of A as instances of independent and identically distributed (i.i.d.) Gaussian (or Sub-Gaussian) random variables. Common hardware friendly choices are either to adopt i.i.d. antipodal random process, where the probability to have +1 or -1 is the same [4], or to adopt quantized instances of zero-mean independent Gaussian process [3].

III. Environmental Noise Rejection

Let us assume that the environmental noise ν is a sinusoidal tone at (known) frequency f_0, and let us introduce the signal-to-interference ratio (SIR) defined as

$$\mathsf{SIR} = \left(\frac{\|x\|_2}{\|\nu\|_2}\right)_{\text{dB}}$$

Of course, the lower the SIR, the higher the effect of ν on y in (1) and, consequently, the lower the quality of $\hat{x}$, that we can measure by defining the reconstructed signal-to-noise ratio (RSNR) as

$$\mathsf{RSNR} = \left(\frac{\|x\|_2}{\|x - \hat{x}\|_2}\right)_{\text{dB}}$$

A trivial solution for increasing RSNR is the rejection of the environmental noise by adopting a notch filter in the conditioning stage, attenuating all frequencies around f_0. The drawback is the effect sketched in Fig. 2, where we have assumed to remove an environmental noise at $f_0 = 0.1 f_{Ny}$. The filter is actually applied both to ν and to x; the effect on ν is to reduce its power, but the effect on x is that the output signal power spectral density (PSD) is substantially different from the PSD of the original input signal, i.e., there is a *signal distortion*. This can be easily understood when observing the system behaviour without any superimposed disturbance, as assumed in Fig. 2. Note that in some critical cases, as for clinical EEG, this is not acceptable, and the rejection of the environmental noise is made by post-processing the reconstructed signal with computationally expensive algorithm [8].

In the specific case of a CS system, some additional degrees of freedom can be used to attenuate the interfering signal while reducing the introduced signal distortion to a minimum level. For example, the value of ϵ in (2) can be tuned; its optimal value, however, depends on the interfering signal amplitude, that is unknown and may vary in an extremely wide range depending on the environment.

In this paper we propose an innovative approach that is based on the statistical characterization of the A matrix. The basic idea is to generate sensing sequences capable to reduce as much as possible the contribution of the term $A\nu$ in (1), without altering the randomness and so the RIP property of A, i.e., without altering the ability to correctly recover x. This can be simply achieved by generating the rows of A with a random process with prescribed statistical properties. More precisely, we design the rows of A whose PSD is flat everywhere, except around f_0 where it is attenuated.

In the following we will compare the performance of the CS-based system depicted in Fig. 1, assuming A is generated in four possible ways:

- the rows of A are generated accordingly to the standard CS as antipodal random sensing vectors with an i.i.d. process, i.e., with a flat PSD (AIID case);
- the rows of A are generated accordingly to the standard CS as Gaussian-distributed random sensing vectors, i.e., with flat PSD. To allow an easy hardware implementation, we limit to Gaussian sequences quantized with 6-bit as in [3] (QGIID case);
- the rows of A are antipodal random sensing vectors generated with a prescribed PSD. This can be obtained by using a Linear Probability Feedback Process [14] (ALPF case);
- the rows of A are quantized zero-mean Gaussian sequences with a prescribed PSD, obtained by 6 bit quantization of instances of multivariate zero-mean Gaussain process with a proper correlation profile (QMVG case).

The PSD of the sensing sequences obtained in the last two cases when $f_0 = 0.1 f_{Ny}$ is depicted in Fig. 3. Obviously, for ALPF since designing the PSD of sequences composed by just two symbols is a hard task, the attenuation obtain around f_0 is smaller than that obtained by QMVG Gaussian sequences.

IV. Simulation Results

The proposed method is first tested on artificial 10-sparse signals that exhibit the sparsity property on a wavelet basis[1] (the orthonormal Daubechies-4 wavelet basis [15]) with

[1]Biomedical signals usually exhibit the sparsity property on wavelet or similar basis

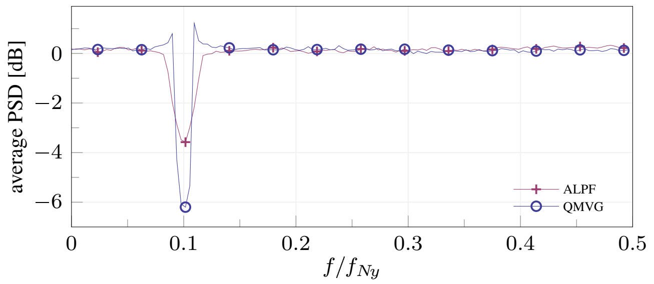


Fig. 3. PSD of the sensing sequences obtained in the ALPF case and in the QMVG case.

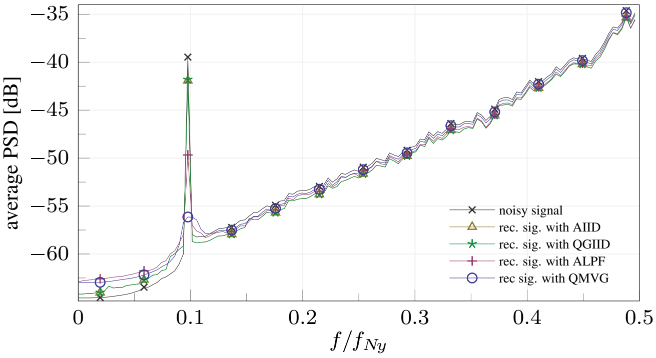


Fig. 4. PSD of the input (noisy) signal with SIR $= 14\,$dB and of the reconstructed ones for the different considered encoding strategies.

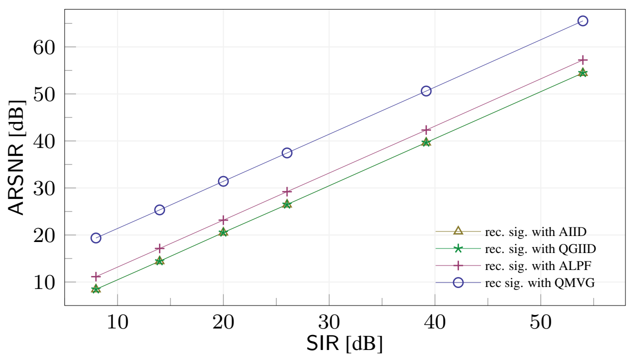


Fig. 5. ARSNR as a function of the SIR when $f_0 = 0.1 f_{Ny}$ in the Daubechies example.

$n = 256$, CR $= 2$ and a high-pass PSD profile. All results are obtained by Montecarlo simulation over 500 randomly generated instance of x with the corresponding vectors $\hat{x}$ obtained by solving (2) via the SPGL1 tool[2].

When considering an environmental noise with $f_0 = 0.1 f_{Ny}$ and SIR $= 14\,$dB, Fig. 4 shows the average PSD of the (corrupted) input signal and the average PSDs of the reconstructed signals when all aforementioned sensing strategies was employed. While both AIID and QGIID follow the input signal profile, i.e., they are unable to perform interference rejection, we have quite good results when using ALPF and even better ones when QMVG is considered.

To get a measurable quantity to evaluate the achieved results, we introduce the Average RSNR (ARSNR) as the mean value of the RSNR observed in all trials. Fig. 5 shows the ARSNR as a function of the SIR, where $f_0 = 0.1 f_{Ny}$. In the standard CS approach (both AIID and QGIID) it is clear that ARSNR $\approx$ SIR. Conversely, performance is increased by almost $3\,$dB and by almost $11\,$dB, respectively, in the ALPF and the QMVC case. Results with different values of f_0 are summarized in Tab. I and confirm the trend.

[2]online available at https://www.math.ucdavis.edu/ mpf/spgl1/

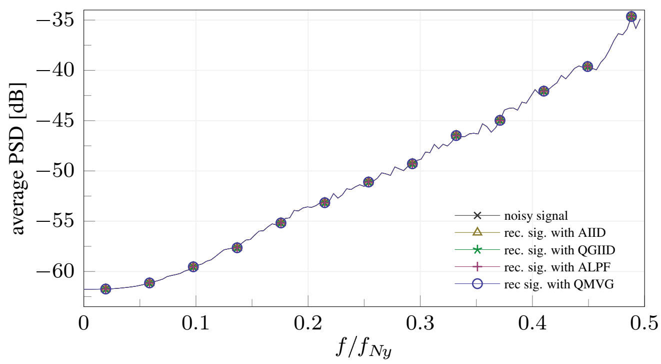


Fig. 6. PSD of noiseless input and of the reconstructed ones for the different considered encoding strategies.

		$f_0 = 0.2 f_{Ny}$			$f_0 = 0.3 f_{Ny}$			$f_0 = 0.4 f_{Ny}$		
SIR [dB]	∞	8	14	26	8	14	26	8	14	26
AIID	98.0	8.4	14.4	26.5	8.3	14.3	26.3	8.4	14.4	26.5
QGIID	98.0	8.4	14.4	26.5	8.2	14.3	26.3	8.4	14.4	26.5
ALPF	98.5	11.3	17.3	29.3	11.0	17.0	29.0	11.1	17.1	29.2
QMVG	98.3	19.8	25.7	37.8	19.1	25.0	37.0	19.4	25.4	37.4

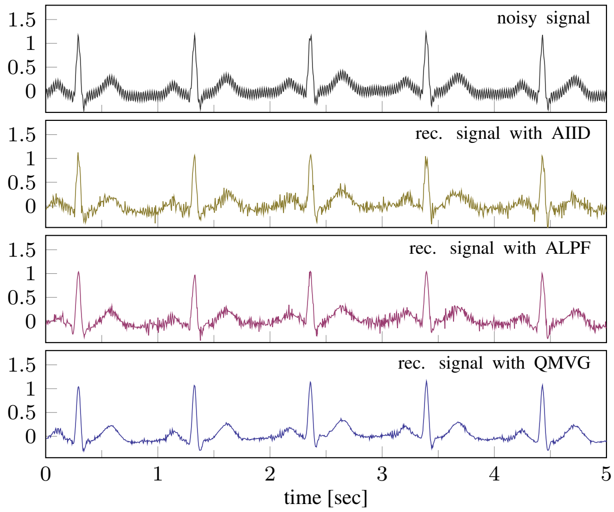
TABLE I. VALUES FOR THE ARSNR [dB] OBTAINED WITH DIFFERENT f_0 VALUES, DIFFERENT SIR VALUES AND FOR THE FOUR DIFFERENT SENSING STRATEGY CONSIDERED IN THE DAUBECHIES WAVELET BASIS EXAMPLE.

All aforementioned simulation results confirm the possibility of using sensing sequences with a prescribed PSD to achieve interference rejection at a given frequency. Results obtained with QMVG are better than that achieved with ALPF; however, we have also to consider that the hardware required to implement the ALPF strategy is much simpler with respect to that required for a QMVG. Yet, in both cases, there is a very important advantage with respect to standard solutions where a notch filter is added either before the signal acquisition [7] or as a post processing stage [8]. Fig. 6 compares the PSD of the input signal and the reconstructed signal using the considered sensing approaches when no interference perturbation is added. In other words, Fig. 6 shows the effect of the proposed filtering strategy on the input signal for the noiseless case. It is clear that, while a standard notch filter always generates a gap in the PSD as in Fig. 2, the proposed approach does not introduce any perturbation in the PSD of the reconstructed signals. The same conclusion can be inferred by looking at the first column of Tab. I in terms of ARSNR where SIR $= \infty$, i.e., where no interference is added to the input signal.

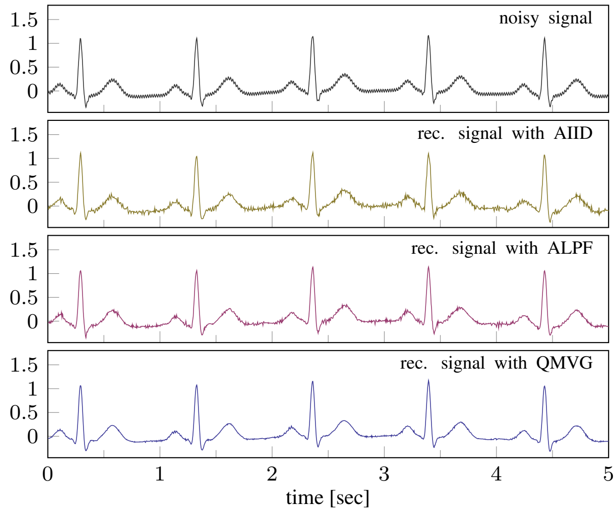
V. A REALISTIC CASE: ECG SIGNALS

To test the proposed approach in a more realistic setting, we consider here an ECG signal sampled at $256\,$Hz and perturbed by a $50\,$Hz power line interference. In order to compute ARSNR results not affected by unknown signal conditioning stage, we prefer to use here synthetic ECG signals generated

This is the author's version of the article that has been presented at IEEE ISCAS2016
The editorial version of the paper is available at http://dx.doi.org/10.1109/ISCAS.2016.7527219



(a) $\mathsf{SIR} = 10\mathrm{dB}$



(b) $\mathsf{SIR} = 20\mathrm{dB}$

Fig. 7. Plot (a) reports noisy ECG with reconstructed ECGs where $\mathsf{SIR} = 10\,\mathrm{dB}$ while (b) shows same profiles for $\mathsf{SIR} = 20\,\mathrm{dB}$.

SIR [dB]	0	5	10	15	20	25
AIID	0.5	5.0	9.4	13.7	17.9	22.1
QGIID	0.4	5.1	9.4	13.8	17.9	22.1
ALPF	3.2	7.6	12.1	16.4	20.6	24.7
QMVG	11.1	15.6	20.0	23.9	27.8	30.8

TABLE II. ARSNR [dB] IN THE RECONSTRUCTION OF ECGS BY CHANGING SIR WHEN DIFFERENT SENSING STRATEGY ARE EMPLOYED.

as in [9]. The simulation setting is equivalent to the previous one ($n = 256$ corresponding to 1 second time window, $\mathsf{CR} = 2$ and averages evaluated over 500 trials) except for the Ψ, ECG exhibit sparsity property when they are expressed on the orthonormal Symlet-6 wavelet basis [15].

Results for two different values of SIR are shown in Fig. 7 and clearly indicates that, while standard CS techniques are not effective in rejecting the $50\,\mathrm{Hz}$ interference (only the AIID case is considered as a standard CS case), both ALPF and QMVG are successful in this task. This is confirmed by results in Tab. II where ARSNR values for all aforementioned decoding strategy are reported by changing the amount of injected disturbance.

VI. Conclusion

An innovative sensing matrix design procedure to achieve environmental noise rejection was presented with simulation on both artificial sparse signals and ECGs. Remarkably, the ARSNR increases up to 11 dB with respect to standard CS approaches in the noisy cases. Furthermore, no signal distortion is observed when the environmental noise is not present. This makes this approach a good candidate in the rejection of the effects of power line interference with an unknown SIR.

References

[1] D.L.Donoho, "Compressed Sensing," IEEE Transactions on Information Theory, vol. 54, n. 4, pp. 1289–1306, 2006

[2] J. Yoo, S. Becker, M. Loh, M. Monge, E. Candes, and A. Emami-Neyestanak, "A 100MHz-2GHz 12.5x sub-Nyquist Rate Receiver in 90nm CMOS," *2012 IEEE Radio Frequency Integrated Circ. Symp.*, pp. 31–34, 2012.

[3] D. Gangopadhyay, E. Allstot, A. Dixon, K. Natarajan, S. Gupta, and D. Allstot, "Compressed Sensing Analog Front-End for Bio-Sensor Applications," *IEEE Journal of Solid-State Circuits*, vol. 49, no. 2, pp. 426–438, Feb. 2014.

[4] F. Pareschi, P. Albertini, G. Frattini, M. Mangia, R. Rovatti, G. Setti, "Hardware-Algorithms Co-design and Implementation of an Analog-to-Information Converter for Biosignals based on Compressed Sensing," *IEEE Trans. on Biomed. Circuits Syst.*, 2016. DOI: 10.1109/TBCAS.2015.2444276.

[5] A.M.R. Dixon, E.G. Allstot, D. Gangopadhyay, D.J. Allstot, "Compressed Sensing System Considerations for ECG and EMG Wireless Biosensors", *IEEE Transactions on Biomedical Circuits and Systems*, vol. 6, no. 2, pp. 156–166, April 2012.

[6] E.J. Candes, T. Tao, "Decoding by linear programming," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4203–4215, Dec. 2005

[7] L. Yong, H. Y. Jiang, "The reduction of noises in ECG signal using a frequency response masking based FIR filter," *Biomedical Circuits and Systems, 2004 IEEE International Workshop on* pp. S2/4-17-20, Dec. 2004

[8] A. de Cheveign, J.Z. Simon, "Denoising based on time-shift PCA",*Journal of Neuroscience Methods*, v. 165, no. 2, pp. 297–305, Sep. 2007

[9] M. Mangia, R. Rovatti, G. Setti, "Rakeness in the Design of Analog-to-Information Conversion of Sparse and Localized Signals," *IEEE Transactions on Circuits and Systems - Part I*, vol. 59, no. 5, pp. 1001-1014, May 2012

[10] M. Elad, " Optimized projections for compressed sensing", *IEEE Transactions on Signal Processing*, pp. 5695–5702, Dec. 2007.

[11] V. Cambareri, M. Mangia, F. Pareschi, R. Rovatti, and G. Setti, "Low-complexity multiclass encryption by compressed sensing," *IEEE Trans. on Signal Processing*, vol. 63, no. 9, pp. 2183–2195, 2015.

[12] R. Baraniuk, M. Davenport, R. DeVore, M. Wakin, "A Simple Proof of the Restricted Isometry Property for Random Matrices," Constructive Approximation, vol. 28, no. 3, pp. 253-263, 2008

[13] R. Rovatti, G. Mazzini, G. Setti, "Enhanced rake receivers for chaos-based DS-CDMA", *IEEE Transactions on Circuits and Systems - Part I*, vol. 48, n. 7, pp. 818-829, 2001

[14] R. Rovatti, G. Mazzini, G. Setti, "Memory-m Antipodal Processes: Spectral Analysis and Synthesis", IEEE Transactions on Circuits and Systems - I, vol. 56, n. 1, pp. 156-167, 2009

[15] S. Mallat, *A wavelet tour of signal processing: the sparse way*, Elsevier, 2008