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Worst-Case Analysis of Electrical and Electronic Equipment via Affine Arithmetic

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***Abstract*—This paper proposes to generate a smart tool that can inherently and effectively capture the results of parameter variations on the system responses of lumped and distributed electrical circuits. This methodology leverages the so-called affine arithmetic and represents parameter-dependent responses in terms of a multivariate polynomial. The affine representation is propagated from input parameters to circuit responses through a suitable redefinition of the basic operations, such as addition, multiplication or matrix inversion, that are involved in the circuit solution. The proposed framework is applied to the frequency-domain analysis of switching converters, and it turns out to be accurate and more efficient than traditional solutions based on Monte Carlo analysis.**

***Index Terms*—Affine arithmetic, circuit simulation, dc-dc switching converters, tolerance analysis, uncertainty, worst-case analysis.**

I. Introduction

In the design and fabrication process of electronic equipment, there have been a number of unknown paramters which significantly affect the product performance, such as the manufacturing process fluctuations and uncertain device characteristics. Due to these uncertainties, the assessment of device response is no longer regarded deterministic but as a random process. Traditional sampling-based methods such as Monte Carlo (MC) have been widely used for the analysis of circuit performance under variations [1], and the main drawback is the large number of required samples to converge, which is time-consuming for practical applications. Additionally, MC is unable to achieve the inherent worst-case (WC) responses [2], [3], and provides only a subset of the possible realizations.

As a more efficient and effective probabilistic technique, the Polynomial chaos (PC) theory has been successfully applied to the statistical simulation of circuits and systems in many domains of physics and engineering [4], [5], [6]. However, PC shares the drawback that the worst-case bounds arising from bounded uncertainties can be computed via post-processing and a large number of simulations only.

As an alternative, the inherent WC approaches such as interval analysis (IA), affine arithmetic (AA) and Taylor models (TM) are proposed during the 1960s [7] and the 1980s [8], respectively. Although all these methods directly provide an estimation of the true bounds of the responses, IA introduced relatively large over-estimation for not considering the correlation among variables, while AA and TM, as rigorous approaches combining the strength of IA, offers a reasonable inherent WC responses in a timely fashion [9], [10].

The key features of the state-of-the-art techniques for the computation of worst-case system responses are summarized as in Fig. 1. The figure provides a visualization of the intrinsic differences between the above mentioned techniques, where the upper bound of the response of an electronic equipment affected by parametric uncertainties (solid line) and the approximations achieved using MC, PC, IA, TM and AA (dashed lines) are illustrated in a preliminary and qualitative way.

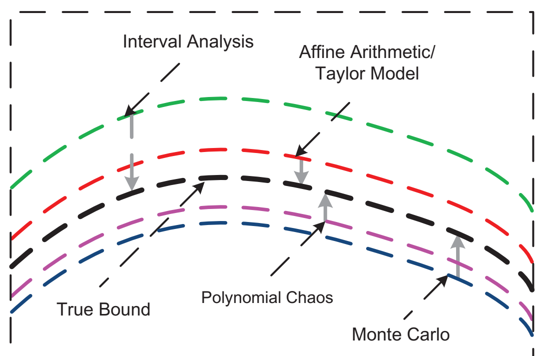

Figure 1. Approximations of the upper bound of a generic circuit or system response by means of different techniques. The solid black line is the "true" worst case, while the dashed lines are approximations obtained with MC, PC, IA, TM and AA.

In this paper, we focus on the affine arithmetic method, which provides a conservative estimation of the response bounds. The article is organized as follows. In Sec. II, the fundamental theory of AA is briefly summarized, and an extension of the AA theory to the basic matrix operations are subsequently introduced. In Sec. III, the enhanced AA technique is effectively applied to the simulation of a switching converter. Summary and conclusions are provided in Sec. IV.

II. Affine Arithmetic Fundamental and Extension

This section outlined the essential mathematical background of the so-called affine arithmetic, and details the extension of AA to matrix operations like inversion and matrix exponential. These operations are poorly documented in the literature, yet they are needed to successfully apply the AA technique to the solution of lumped and distributed circuits in the frequency domain.

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A. Affine Arithmetic Fundamental

Affine arithmetic provided a representation of correlated interval variables. The general definition of the affine form for a given variable with n uncertainties writes:

$$\tilde{x} = x_0 + \sum_{i=1}^{n} x_i \epsilon_i, \tag{1}$$

where x_0 is the nominal or central value of the affine form, ϵ_i are symbolic real variables (noise symbols) whose values are unknown but assumed to lie in the interval $[-1, 1]$, and x_i are the corresponding coefficients. Based on representation (1) and on the definition of the noise symbols, the worst-case bounds are readily given as $x_0 \pm \sum_i |x_i|$.

Given two generic interval variables $\tilde{x}$ and $\tilde{y}$, the addition and subtraction are readily defined as $\tilde{x} \pm \tilde{y} = (x_0 \pm y_0) + \sum_{i=1}^{n}(x_i \pm y_i)\epsilon_i$. On the contrary, other operations, such as the multiplication, require special care, since the direct computation:

$$\begin{aligned} \tilde{z} &= \tilde{x} \cdot \tilde{y} \\ &= x_0 y_0 + \sum_{i=1}^{n}(x_0 y_i + y_0 x_i)\epsilon_i + \\ &+ \left(\sum_{i=1}^{n} x_i \epsilon_i\right)\left(\sum_{i=1}^{n} y_i \epsilon_i\right) \end{aligned} \tag{2}$$

yields an expression that does not belong to the standard affine form as addition or subtraction do. However, the last quadratic term in (2) is suitably replaced by an additional symbol, e.g., ζ, according to the following rule:

$$\left(\sum_{i=1}^{n} x_i \epsilon_i\right)\left(\sum_{i=1}^{n} y_i \epsilon_i\right) \approx \left(\sum_{i=1}^{n}|x_i| \sum_{i=1}^{n}|y_i|\right)\zeta = R(\tilde{x}\tilde{y})\zeta \tag{3}$$

This approximation allows to convert the new variable z into the affine form as follows:

$$\tilde{z} = z_0 + \sum_{i=1}^{n} z_i \epsilon_i + R(\tilde{x}\tilde{y})\zeta \tag{4}$$

where $z_0 = x_0 y_0$, $z_i = x_0 y_i + y_0 x_i$, and $R(\tilde{x}\tilde{y})$ is the new coefficient obtained from (3).

From (2)-(4), it is clear that a new term is added at each multiplication, leading to a growing number of variables in a chain of computations, with a detrimental impact on the compactness of the numerical representation and on the simulation efficiency. In order to solve this issue, Rutenbar proposed in [11] to distribute the additional quadratic terms into the existing uncertainty variables as follows:

$$\tilde{z} \approx z_0 + \sum_{i=1}^{n} z_i \left[1 + \left(1/\sum_{k=1}^{n}|z_k|\right) R(\tilde{x}\tilde{y})\right]\epsilon_i \tag{5}$$

This modification, which is used hereafter in this paper, has been proven to be a conservative approximation that bounds the true range of $\tilde{z}$ and at the same time successfully avoids the uncontrolled generation of new uncertainty symbols. Summarizing, the affine arithmetic introduces symbolic techniques into naive interval arithmetic to include the correlation among variables and to help to reduce the overestimation problem.

B. Matrix Inversion

The matrix inversion, which provides one of the key computational blocks for the frequency domain solution of a circuit, requires the joint application of the AA along with a suitable numerical technique or algorithm [9].

For the sake of illustration, the discussion starts from the inversion of a matrix given in affine form and where the variability is defined by one noise symbol only, i.e.,

$$\tilde{\boldsymbol{Y}} = \tilde{\boldsymbol{X}}^{-1} = (\boldsymbol{X}_0 + \boldsymbol{X}_1\epsilon_1)^{-1}, \tag{6}$$

where $\boldsymbol{X}_0$ and $\boldsymbol{X}_1$ play the same role of the scalar coefficients x_0 and x_1 in (1). In this simplified situation, among the algorithms available in the literature, the Sherman-Morrison formula provides a clever solution for the above problem when the matrix $\boldsymbol{X}_1$ has unitary rank. The resulting matrix $\tilde{\boldsymbol{Y}}$ writes:

$$\begin{aligned} \tilde{\boldsymbol{Y}} &= (\boldsymbol{X}_0 + \boldsymbol{X}_1\epsilon_1)^{-1} \\ &= \boldsymbol{X}_0^{-1} - \frac{1}{1+g}(\boldsymbol{X}_0^{-1}\boldsymbol{X}_1\epsilon_1\boldsymbol{X}_0^{-1}) \\ &= \boldsymbol{Y}_0 + \boldsymbol{Y}_1\epsilon_1, \end{aligned} \tag{7}$$

where $g = \mathrm{trace}(\boldsymbol{X}_0^{-1}\boldsymbol{X}_1)$. The above formula expresses the result in the standard affine form, with $\boldsymbol{Y}_0 = \boldsymbol{X}_0^{-1}$ and $\boldsymbol{Y}_1 = -\boldsymbol{X}_0^{-1}\boldsymbol{X}_1\boldsymbol{X}_0^{-1}/(1+g)$.

For a matrix $\tilde{\boldsymbol{X}}$ simultaneously affected by several noise symbols, i.e.,

$$\tilde{\boldsymbol{Y}} = \tilde{\boldsymbol{X}}^{-1} = (\boldsymbol{X}_0 + \boldsymbol{X}_1\epsilon_1 + \cdots + \boldsymbol{X}_n\epsilon_n)^{-1} \tag{8}$$

with matrices $\boldsymbol{X}_0, \ldots, \boldsymbol{X}_n$ of rank one, the following iterative procedure is established:

- Compute the initial rough estimate $\tilde{\boldsymbol{Y}} \approx \boldsymbol{X}_0^{-1}$;
- Carry out the update $\tilde{\boldsymbol{Y}} \approx (\boldsymbol{X}_0 + \boldsymbol{X}_1\epsilon_1)^{-1} = \boldsymbol{Y}_0 + \boldsymbol{Y}_1\epsilon_1$ where $\boldsymbol{Y}_0$ and $\boldsymbol{Y}_1$ are computed via (7);
- Solve the problem $\tilde{\boldsymbol{Y}} \approx (\boldsymbol{X}_0 + \boldsymbol{X}_1\epsilon_1 + \boldsymbol{X}_2\epsilon_2)^{-1}$ by wrapping the first two terms $[\boldsymbol{X}_0 + \boldsymbol{X}_1\epsilon_1]$ into a single matrix, for which the inversion is known from the previous step, and apply again (7);
- Repeat the previous step to progressively include the remaining terms $\boldsymbol{X}_3, \ldots, \boldsymbol{X}_n$.

For the general case of matrices with full rank, the proposed procedure is still valid and is applied without modifications by splitting each matrix into the sum of rank one matrices, e.g.,

$$\boldsymbol{X}_1\epsilon_1 = \boldsymbol{X}_{1,1}\epsilon_1 + \boldsymbol{X}_{1,2}\epsilon_1 + \ldots \tag{9}$$

where $\boldsymbol{X}_{1,k}$ is a matrix with null entries except for the kth column, which is the kth column of the original matrix $\boldsymbol{X}_1$.

III. Numerical Results of Boost dc-dc Converter

In this section, the feasibility and strength of the proposed method are demonstrated on the worst-case analysis of switching power converters.

The application example considered is shown in Fig. 2. It consists of a standard dc-dc switching converter in the boost configuration, where three design parameters, i.e., L, C and R, are assumed to have a large $\pm 50\,\%$ relative variation and are defined by means of three independent interval values.

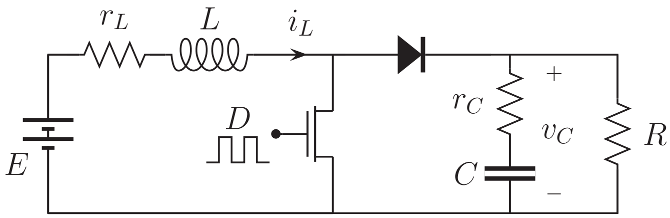

Figure 2. Dc-dc boost converter. $E = 20\,\mathrm{V}$, $r_L = 1\,\mathrm{m}\Omega$, $L = 5\,\mathrm{mH}$, $r_C = 5\,\mathrm{m}\Omega$, $C = 10\,\mu\mathrm{F}$ and $R = 20\,\Omega$. The switching frequency is $f_c = 10$ kHz.

The circuit of Fig. 2 is simulated in the frequency domain via the technique suggested in [12], [13]. The coefficients of the Fourier expansion of the steady-state circuit responses, like the inductor current $i_L(t)$, are obtained by solving an equivalent augmented circuit via the MNA tool and the corresponding matrix inversion.

Fig. 3 shows the the upper and the lower bounds of the harmonics of $i_L(t)$ computed via AA (black circles), compared with the fluctuation given by 10000 MC runs (gray bars). This comparison further establishes the accuracy of AA. As far as the efficiency is concerned, AA is 10 times faster than MC (4 min vs. 50 min).

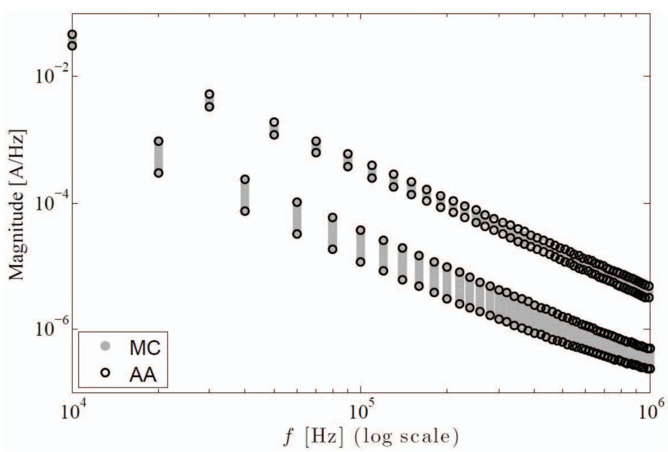

Figure 3. Spectrum of the current $i_L(t)$. Gray bars: spread of the harmonics obtained with MC; black circles: worst-case bounds given by AA.

From the frequency-domain spectrum, the steady-state time-domain responses are obtained in post processing. Fig. 4 provides the result for the inductor current.

IV. Conclusions

This paper provides an overview of the affine arithmetic as well as its application to the frequency domain worst-case analysis of electrical and electronic circuits. The principles of both scalar and matrix operations under the proposed affine framework have been detailed. The methodology is compared in terms of both simulation time and accuracy with the traditional Monte Carlo method and is successfully applied to a switching power converter example. Accurate results and remarkable speed-ups are achieved.

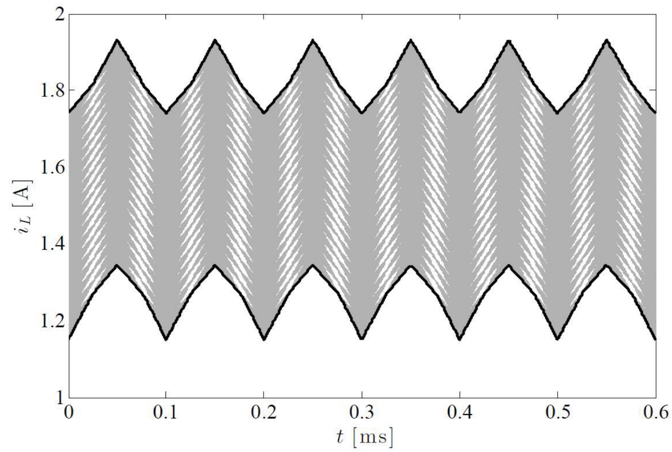

Figure 4. Steady-state time-domain current $i_L(t)$. Gray area: spread from MC analysis; solid black lines: worst-case bounds from AA-based simulation.

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