Fractional viscoelastic modeling of anti-rutting response of bituminous binders

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Abstract

A three-parameter fractional model, composed by a springpot in series with a dapshpot, was employed to describe the rheological behaviour of a set of polymer modified bituminous binders obtained by combining a single base bitumen with different types of Syrene-Butadiene-Styrene co-polymers at different concentrations. Due to their enhanced visco-elastic properties, polymer modified binders are widely used in asphalt pavements in order to prevent formation of ruts on road surfaces caused by accumulation of permanent deformation, especially at high in-service temperatures and under heavy traffic conditions.

The proposed fractional model was fitted to experimental data gathered from creep-recovery shear tests carried out by means of a Dynamic Shear Rheometer at various temperatures. The choice of adopting this type of test procedure was dictated by the fact that it has been proven to be effective in evaluating and ranking the anti-rutting response of bituminous binders.

Experimental strain-time curves were found to be in good agreement with model predictions. The fractional approach leads to a synthetic yet exhaustive description of the visco-elastic behavior of all materials. Variation of model parameters, each of which with a precise physical meaning, appears to be coherent with different binder compositions and testing conditions, reflecting the variations in terms of polymer concentration and temperature.

Keywords: asphalt pavements, fractional model, polymer modified binders, rutting, creep-recovery tests

1. Introduction

The term rutting is commonly used to indicate one of the main distress types affecting asphalt pavements, consisting in the accumulation of permanent deformation produced by repeated traffic loading (Figure 1). Formation of ruts on pavement surfaces produces low levels of comfort and safety for vehicles and users, thus reducing overall pavement serviceability. This failure mode is promoted by the presence of severe traffic and environmental conditions, such as those characterized by high percentages of heavy slow-moving trucks and high in service temperatures. For these reasons, rutting resistance of pavements is greatly influenced by the rheological properties of bituminous binders used in the top bound layers, due to their time and temperature-dependent nature (Monismith et al. 1985, Sousa et al. 1991, Delgadillo and Bahia 2010).

Different experimental approaches have been proposed to evaluate the anti-rutting potential of bituminous binders. The one originally introduced in the framework of the SUPERPAVE grading system is based on visco-elastic parameter $G^*/\text{sen}\delta$ (Harrigan et al. 1994), but its limits have been widely demonstrated by several studies (Bahia et al. 2001, D'Angelo and Dongre 2002). Other protocols currently adopted in binder characterization for the same purpose refer either to the zero shear viscosity concept (Sybilski 1996) or to parameters derived from material response under repeated creep loading (D'Angelo et al. 2007). In both cases, however, experimental studies have highlighted the existence of significant drawbacks related to the fact that they seem not to be fully adequate to capture actual properties of specific types of binders, such as polymer modified ones (Morea et al. 2010, Zoorob et al 2012).

More recently, Santagata et al. (2013, 2015) introduced a method based on single creep-recovery shear tests carried out at predefined loading and recovery times. Experimental findings showed the effectiveness of the method when applied to binders modified with polymers and crumb rubber since the high levels of induced strain allow to differentiate reversible and non-reversible components of response. Moreover, effects caused by variations of additive type, composition, structure and dosage can be clearly highlighted.

While creep-recovery tests are relatively simple and quick to perform, modeling of the experimental data gathered from measurements appears not to be a trivial task. This is due to the fact that binders, as many viscoelastic materials, exhibit a creep-relaxation behavior of the power-law type (Mainardi 1994, Hilfer 2000, Mainardi 2010), with the consequent need of introducing a great number of elements (and thus of parameters) in fitting operations when trying to describe such a response by means of classical exponential-type functions. Furthermore, since parameters are subjected to several restrictions, the employed numerical algorithm may revert to a complex constrained least squares problem (Sorvari and Malinen 2007).

Although power-law expressions for creep-recovery functions can be assumed *a priori*, without a physical sound derivation (Jäger et al. 2007, Füssl et al. 2014), they are implicitly generated by assuming a constitutive law of fractional type, i.e. involving time derivatives with non-integer order of strains and stresses (Carpinteri and Mainardi 1997). For this reason, approaches to visco-elasticity based on fractional calculus have attracted the attention of researchers for at least two decades, involving different materials and applications (Barpi and Valente 2003,2004 Atanckovic et al. 2013,2015, Deseri et al. 2014, Di Paola and Zingales 2012, Di Paola et al. 2013,2014, Paggi and Sapora 2015, Zopf et al. 2015).

In the field of asphalt pavement engineering, few fractional approaches have been proposed for the study of the behavior of bituminous binders and mixtures (Oeser et al. 2008, Celauro et al. 2012, Fecarotti et al. 2012). In particular, a three-parameter model consisting in a dashpot in series with a springpot was recently introduced by Sapora et al. (in press) (see also Di Mino et al. in press), with the goal of reducing the number of fitting parameters (Oeser et al. 2008) and of increasing prediction accuracy (Celauro et al. 2012, Fecarotti et al. 2012). Such a model was successfully applied to creep-recovery test data obtained at various temperatures (ranging from 58 and 76°C) for two different modified binders (containing thermoplastic polymer and crumb rubber, respectively).

In the research work reported in this paper, the study of the anti-rutting response of modified bituminous binders by means of a fractional visco-elastic approach is extended to a wider array of materials, containing several polymers at different concentrations. Results are analyzed with the specific goal of highlighting the mechanical meaning of model parameters and of linking them to the characteristics of the considered binders.

Binders and testing procedures used in the experimental investigation are described in Section 2, whereas the implemented fractional approach is introduced in Section 3. Section 4 presents the comparison between test data and theoretical results, together with a thorough discussion of the mechanical meaning of model parameters; analogies and differences with the results presented in (Sapora et al. in press) are also addressed. Finally, Section 5 contains a synthesis of the main findings of the research work.

It is worth noting that the analysis presented herein lies in the framework of linear viscoelasticity of one-dimensional media, coherently with the experimental tests which were carried out: the solution is thus given in an analytical form, and the model parameters are obtained through a least-square fitting procedure. For the numerical handling of fractional viscoelastic material models through finite element analysis, see (Müller et al. (2013)).

Figure 1. Presence of ruts on pavement surface due to deformation in the top bound layers.

2. Materials and methods

Modification of bitumen by means of polymers has become very popular in the binder industry since it may provide a significant enhancement of performance-related properties (King et al. 1986, Collins et al. 1991, Wardlaw and Shuler 1991). For this reason, the use of polymer modified binders (PMBs) in road construction represents one of the preferred design options in the case of heavy duty pavements, subjected to intense traffic flow and severe environmental conditions.

Among the various types of polymers employed as chemical agents for the production of PMBs, Styrene-Butadiene-Styrene (SBS) synthetic rubbers are characterized by widespread diffusion: they are tri-block copolymers belonging to the category of the so-called thermoplastic elastomers that form, when adequately combined with base bitumen, a cross-linked network promoted by thermal and chemical bonding mechanisms. The elastomeric network consists of polystyrene domains connected by elastic butadiene threads which contribute to improve stiffness and elasticity of the material, especially at high temperatures. The ability of such polymers to form a completely cross-linked structure depends upon their concentration within the bituminous matrix and the thermodynamics of the combined system (Bahia et al. 2001).

Materials used in the experimental investigation described in this paper include several PMBs obtained from the factorial combination of a single base bitumen (70/100 pen grade), three different types of SBS and two concentrations. SBS polymers employed for modification were commercially available products differing from each other in terms of chemical composition (low and high styrene content) and molecular structure (linear and radial chains). Preparation of PMBs was carried out in the laboratory by making use of a bath-oil mixer, operated at 180°C for 60 min with a mixing speed of 800 rpm. Polymers were added to bitumen at two different dosages, equal to 3 and 6% by weight of base bitumen.

Description of the complete set of modified binders is given in Table 1. All of them were tested in short-term aged conditions simulated by means of the Rolling Thin Film Oven Test (RTFOT), according to the AASHTO procedure (2009). Examples of Scanning Electron Microscope (SEM) images illustrating the microstructure of PMBs with higher SBS concentration are reported in Figure 2.

PMB code	Description	
	Type of modifier	Dosage (%)
A-3	A - radial SBS with low Styrene content	3
A-6		6
B-3	B - radial SBS with high Styrene content	3
B-6		6
C-3	C - linear SBS with high Styrene content	3
C-6		6

Table 1. Description of modified bituminous binders used in the experimental investigation.

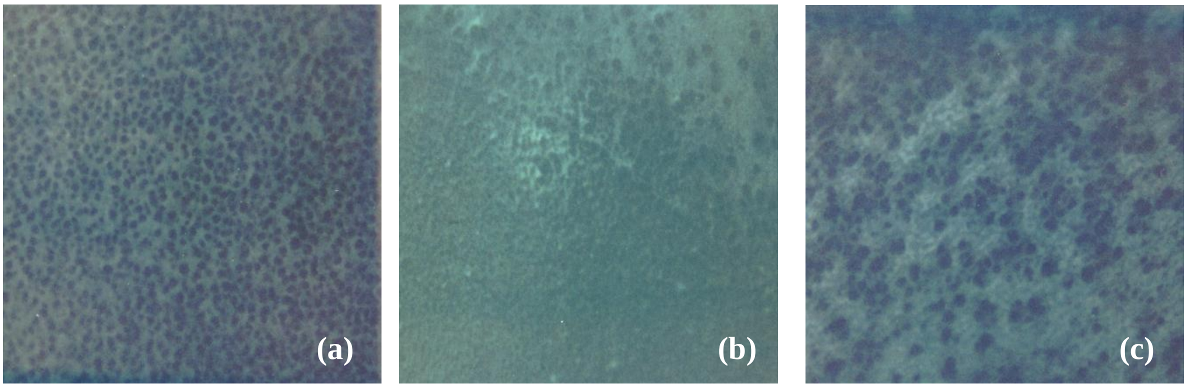

Figure 2. SEM images corresponding to binders A-6 (a), B-6 (b) and C-6 (c).

The protocol adopted for binder testing consists in a creep phase, during which a shear stress of 100 Pa is applied for a loading time of 900 s, followed by a recovery phase, in which load is removed and strain evolution is monitored for additional 900 s. Creep-recovery tests were carried out at different temperatures, selected depending upon the type of binder in order to emphasize its actual visco-elastic response. In particular, 40 and 60°C were adopted for PMBs containing 3% polymer

(A-3, B-3, C-3), whereas 60 and 80°C were used for those characterized by 6% concentration (A-6, B-6, C-6).

The testing device employed to carry out creep-recovery tests was a Dynamic Shear Rheometer (DSR), Physica MCR 301 DSR from Anton Paar Inc., an air bearing stress-controlled device equipped with a permanent magnet synchronous drive (minimum torque = 0.1 μNm, torque resolution < 0.1 μNm) and an optical incremental encoder for the measurement of angular rotation (resolution < 1 μrad). A cone-plate sensor system with 35 mm diameter and 4° cone angle was used for measurements.

Test specimens were prepared by pouring the binder (preheated at 150°C for the time needed to achieve a sufficient fluidity) on the lower plate of the sensor system and by thereafter sandwiching it between the plate and the upper cone. The amount of material placed on the plate was slightly overdosed, in order to allow the formation of a proper bulge at the periphery of the sample after reaching final measurement position. With the aim of preventing temperature gradients throughout the binder volume, specimens were conditioned at test temperature for 15 minutes before actual measurements.

The employed sensor system and the final test configuration reached after specimen preparation are displayed in Figure 3.

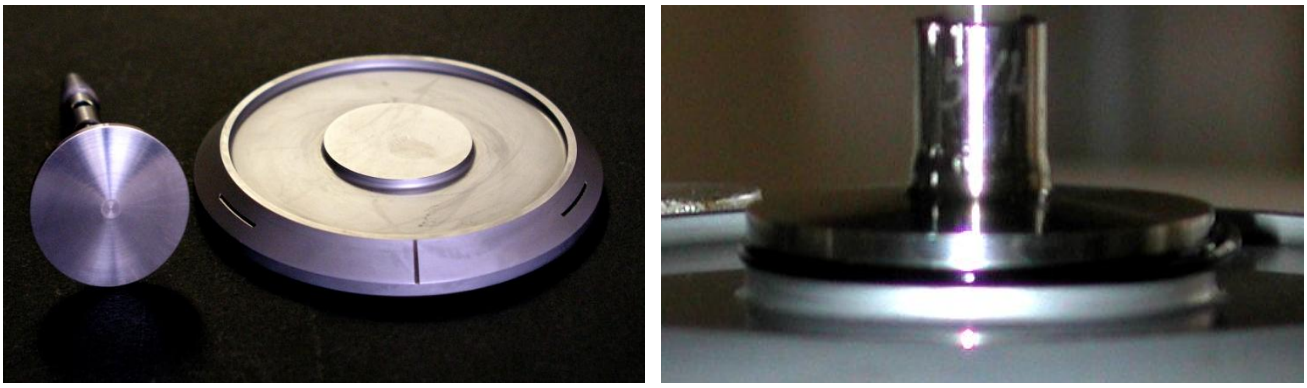

Figure 3. Sensor system and test configuration.

3. Implemented fractional model and parameter calibration

The fractional model implemented in the present study (Figure 4) includes a springpot connected in series with a dashpot (Sapora et al. in press). While the dashpot simply accounts for time-dependent viscous response under loading, the springpot is a structural element with a fractional order α governing the transition from elastic to viscous behavior.

The strain- stress (ε-σ) constitutive law of the model is the following:

$$\frac{d\varepsilon(t)}{dt}=\frac{1}{b_1}\frac{d^{1-\alpha}\sigma(t)}{dt^{1-\alpha}}+\frac{1}{b_2}\sigma(t) \tag{1}$$

where b_1 and $\alpha \in (0,1)$ are the springpot parameters, and b_2 is the dynamic viscosity related to the dashpot.

Definition of the fractional derivative of order α related to a generic function f can be expressed in the form (Samko et al. 1993):

$$\frac{d^{\alpha} f(x)}{dx^{\alpha}}=\frac{1}{\Gamma(1-\alpha)}\int_0^x \frac{f'(y)}{(x-y)^{\alpha}}dy \qquad \alpha \in (0,1) \tag{2}$$

where Γ is the Euler-Gamma function.

According to (2), the α-derivative of a function reverts to the function itself for $\alpha = 0$, and to the first order classical derivative for $\alpha = 1$. It should also be noted that as α varies, the mechanical meaning of b_1 in (1) obviously changes, passing from a stiffness ($Force \times Length^{-2}$) for $\alpha = 0$, to a viscosity ($Time \times Force \times Length^{-2}$) for $\alpha = 1$.

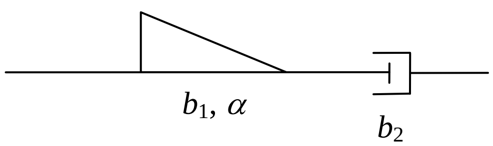

Figure 4. Fractional model implemented in the analysis.

The creep function $D(t)$ related to the constitutive law (1) can be derived as:

$$D(t)=\frac{t^{\alpha}}{b_1\Gamma(1+\alpha)}+\frac{t}{b_2}=\frac{t}{b_2}\left(1+\frac{b_2}{b_1\Gamma(1+\alpha)}t^{\alpha-1}\right) \tag{3}$$

whereas the creep-recovery constitutive relationship can be written as:

$$\varepsilon(t)=\left\{\left[\frac{t^{\alpha}}{b_1\Gamma(1+\alpha)}+\frac{t}{b_2}\right]U(t)-\left[\frac{(t-t^*)^{\alpha}}{b_1\Gamma(1+\alpha)}+\frac{(t-t^*)}{b_2}\right]U(t-t^*)\right\}\sigma_0 \tag{4}$$

being $U(t)$ the unit-step (or Heavisde) function, t^* the time at which unloading starts and σ_0 constant applied stress.

It is important to point out what are the limit cases of the considered fractional model. Elastic response is maximized for $\alpha = 0$, in which case the springpot reverts to a spring and b_1 plays the role of a stiffness: in this case the model coincides with the well-know Maxwell model consisting in a spring in series with a dashpot. On the other hand, viscous behaviour is amplified the most for $\alpha = 1$: the model then reverts to two dashpots in series, each with the same viscosity ($b_{1=} b_2$).

In order to fit the model to experimental data obtained from creep-recovery tests described in Section 2, the three model parameters α, b_1 and b_2 were determined by means of a numerical algorithm which evaluates the coefficients of a nonlinear regression function through a least squares estimate. Due to possible settlement phenomena observed in the initial phase of creep, data recorded in the first 1% of the loading history were neglected (Sapora et al. in press).

Values of α, b_1 and b_2 obtained from fitting, together with the corresponding absolute mean percentage error $\bar{\delta}$, are reported in Tables 1 and 2 for PMBs containing 3 and 6% polymer, respectively.

T [°C]	A-3				B-3				C-3			
	α	b_1	b_2	$\bar{\delta}$	α	b_1	b_2	$\bar{\delta}$	α	b_1	b_2	$\bar{\delta}$
40	0.570	0.966	2.14	1.44	0.567	0.596	3.12	1.37	0.731	0.0133	0.000	3.77
60	1.000	0.173	0.173	4.06	1.000	0.188	0.188	3.88	1.000	0.464	0.464	10.3

Table 2. Fitted parameters α, b_1 [Ns$^{\alpha}$/m^2], b_2[Ns/m^2] and mean percentage error $\bar{\delta}$ corresponding to PMBs containing 3% SBS tested at 40 and 60°C.

T [°C]	A-6				B-6				C-6			
	α	b_1	b_2	$\bar{\delta}$	α	b_1	b_2	$\bar{\delta}$	α	b_1	b_2	$\bar{\delta}$
60	0.259	0.044	6.41	1.75	0.104	0.063	61.4	0.69	0.156	0.0738	39.9	1.46
80	1.000	0.091	0.091	1.66	0.245	0.109	2.88	3.87	0.334	0.427	0.402	10.3

Table 3. Fitted parameters α, b_1 [Ns$^{\alpha}$/m^2], b_2[Ns/m^2] and mean percentage error $\bar{\delta}$ corresponding to PMBs containing 6% SBS tested at 60 and 80°C.

4. Discussion of results

Shear strain test data obtained for PMBs characterized by the lower SBS content (A-3, B-3, C-3) and theoretical results provided by Eq.(4) with fitted parameters reported in Table 2 are diagrammed as a function of time in Figs. 5-7. For each material and test condition, experimental and theoretical curves appear to be very close to each other, indicating that a good agreement was generally found. This is coherent with $\bar{\delta}$ values that are abundantly lower than 5%. The only exception is represented by binder C-3 tested at 60°C, for which the creep experimental curve deviates from the theoretical one, showing a trend characterized by the increase of strain rate with loading time; the corresponding mean percentage error is non negligible, reaching a value of the order of 10%.

In all cases, materials tested at 40°C exhibit a certain amount of delayed elastic strain which is partially recovered during the unloading phase. On the contrary, at 60°C the accumulated strain at the end of the loading phase is fully irreversible. This evidence can be explained by considering that the raise in temperature produces the breakdown of physical crosslinks within polymer styrene domains; in such conditions the SBS added to the bituminous matrix acts as a simple filler which contributes to increase viscosity of the binder without improving its elasticity.

Such a transition from visco-elastic to viscous behaviour is reflected in the variation of fitted fractional parameters. In particular, values of α determined at 40°C range, approximately, between 0.55 and 0.75 whereas at 60°C they become equal to 1 with the fractional model being reduced to two dashpots in series ($b_1 = b_2$). Anyhow, it must be pointed out that values of α equal to 0.55 or higher indicate the predominance of viscous components in material's response even at 40°C (i.e. the springpot is more similar to a dashpot than to a spring) suggesting the added amount of elastomer (3%) not to be sufficient to create a continuous polymeric network in accordance with results published elsewhere (Lu and Isacsson 1997). Moreover, the increase of the fractional exponent when passing from 40 to 60°C appears to be in contrast with findings of other studies (Di Mino et al. in press), in which α is showed to decrease with temperature (together with b_1 and b_2). Both trends can be considered acceptable from a practical point of view since the global response is described by the combined effects of α, b_1 and b_2 variations; however, the first one seems to be more logical from a theoretical point of view, matching the extreme cases described in Section 3. Similar considerations are valid for values reported in Table 3.

By comparing the materials to each other, it is clearly noticed that binder C-3 exhibits the most elastic response at T = 40°C, its behaviour being described by a single springpot. On the other hand, A-3 and B-3 present nearly the same value for α but different values of b_1 and b_2: the springpot contribution to overall deformation appears to be more significant for A-3, which presents a lower b_2/b_1 ratio (Eq. (3)), even though B-3 is less deformable in absolute terms. At 60°C, C-3 shows the highest value of b_2 that, as specified before, assumes the meaning of a dynamic viscosity, thus indicating the highest resistance to flow for such a material, followed, in order, by B-3 and A-3. This is coherent with the characteristics of the various SBSs used for modification and more specifically with the presence, in type C and type B, of a higher styrene content that contributes to increase the hardness of the polymers and, consequently, of the corresponding polymer-bitumen blends.

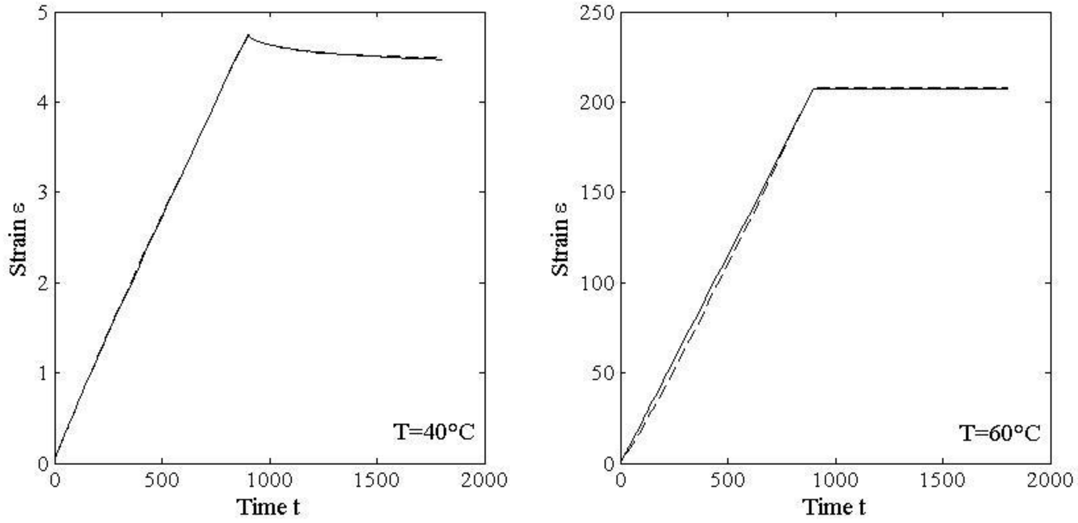

Figure 5. Creep-recovery curves of binder A-3 tested at 40°C (a) and 60°C (b): experimental data (dotted line) and theoretical results (solid line).

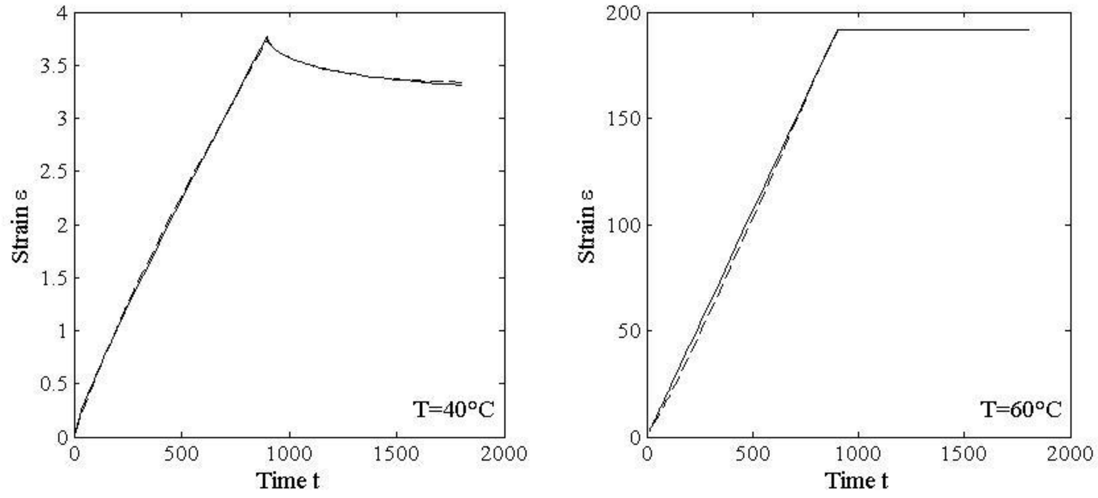

Figure 6. Creep-recovery curves of binder B-3 tested at 40°C (a) and 60°C (b): experimental data (dotted line) and theoretical results (solid line).

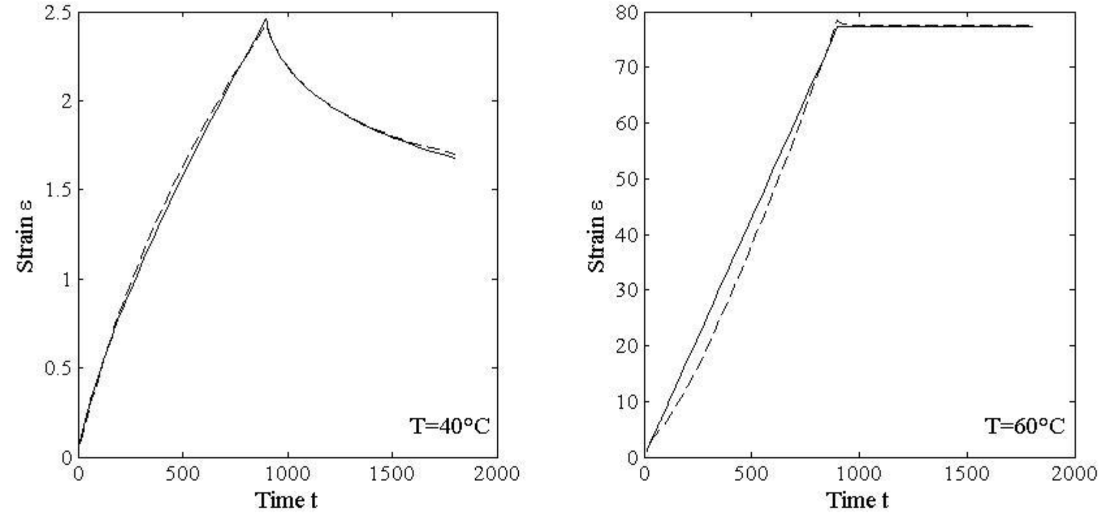

Figure 7. Creep-recovery curves of binder C-3 tested at 40°C (a) and 60°C (b): experimental data (dotted line) and theoretical results (solid line).

Diagrams reporting the results obtained for highly modified binders (A-6, B-6, C-6) are shown in Figs. 8-10. Also in this case a good fitting is observed ($\overline{\delta}$ < 5%) with the only exception, once again, given by binder containing type C SBS tested at the higher temperature. In fact, the creep curve at 80°C shows a trend similar to that displayed in Figure 7-b, which appears to be not well described by the proposed model (the mean absolute error exceeds 10%). This type of shear thinning behaviour can be therefore considered peculiar of the linear SBS used in this investigation when subjected to high levels of shear deformations (Stastna et al. 2003).

In contrast with data illustrated before, α values are generally very low (except in the case of A-6 tested 80°C that will be commented later) with springpot elements approaching a condition corresponding to the spring case. This can be evidently attributed to the presence of a high amount of polymer that results in the formation of a diffused cross-linked structure with a pronounced rubber-like behaviour.

However, a distinction can be made between binder A-6 and the other two binders B-6 and C-6.

In fact, at 60°C the latter ones behave like viscoelastic solids which are able to recover most of the deformation experienced during the loading phase. This can be evidently attributed to the presence of a high styrene content which promotes the formation of rigid polystyrene domains within the bituminous matrix. If compared to each other, B-6 prevails on C-6 in terms of stiffness and degree of elasticity, probably due to its radial/branched molecular structure. At 80°, as obvious, both materials become less stiff (lower values of (b_2/b_1) and less elastic (higher values of α) even though they maintain a certain aptitude to recover deformation.

Binder A-6 at 60 °C shows a typical visco-elastic behaviour characterized by a lower stiffness and degree of elasticity with respect to that of abovementioned binders B-6 and C-6 (lower values of (b_2/b_1), higher value of α) and, as previously highlighted, at 80°C reaches viscous flow conditions (α = 1 with $b_1 = b_2$). Such an evidence can be linked with the low styrene content of the SBS used for the preparation of this type of PMB, that results in weak bonding between styrene blocks at very high temperatures thus overcoming polymer networking effects.

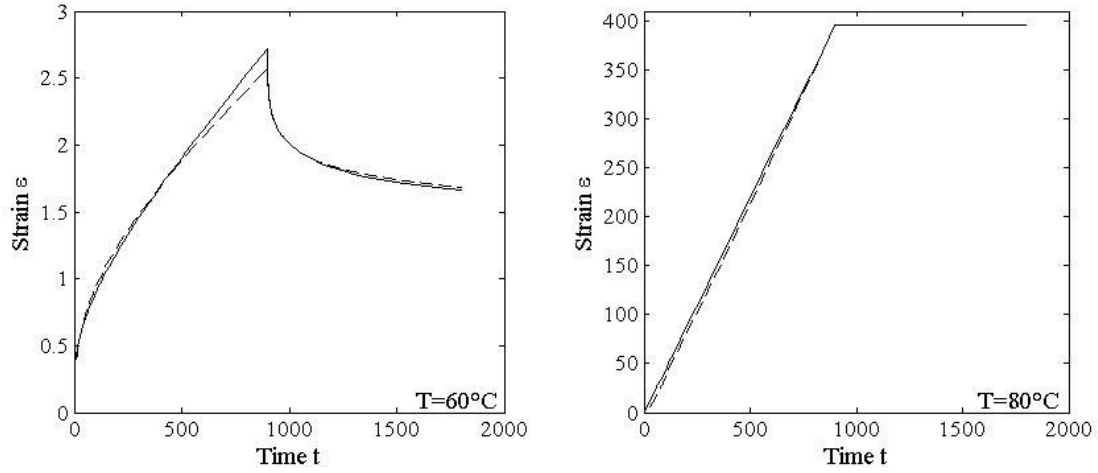

Figure 8. Creep-recovery curves of binder A-6 tested at 60°C (a) and 80°C (b): experimental data (dotted line) and theoretical results (solid line).

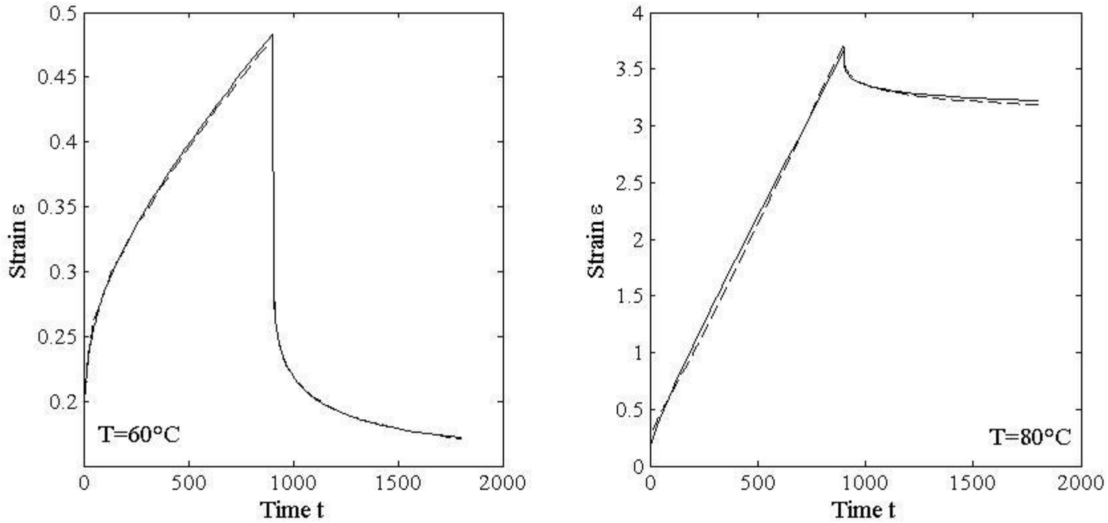

Figure 9. Creep-recovery curves of binder B-6 tested at 60°C (a) and 80°C (b): experimental data (dotted line) and theoretical results (solid line).

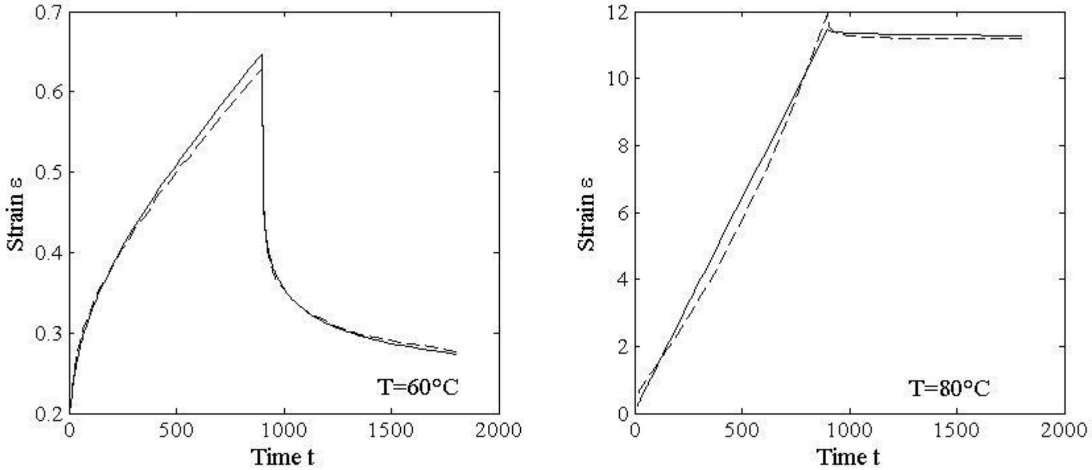

Figure 10. Creep-recovery curves of binder C-6 tested at 60°C (a) and 80°C (b): experimental data (dotted line) and theoretical results (solid line).

5. Conclusions

The three-parameter (α, b_1, b_2) fractional visco-elastic model consisting in a springpot in series with a dashpot was employed in the research study reported in this paper to model the creep-recovery experimental behaviour of several modified bituminous binders, differing in polymer type and concentration. The results were analyzed with the specific purpose of verifying the capability of the model of capturing peculiarities of material response and to link model parameter to variations in materials' composition and testing conditions.

From the findings of the study, the following main conclusions can be drawn.

Experimental strain-time curves obtained from testing were found in good agreement with model predictions, as indicated by absolute mean percentage error values always lower than 5%. The only exceptions are represented by binders containing the linear SBS polymer that showed a typical shear thinning behaviour when subjected to high levels of deformations. This confirms the effectiveness of the fractional approach that leads to a synthetic but exhaustive description of the visco-elastic behavior of materials with a precise physical meaning of model parameters: whereas b_1 and b_2 describe the stiffness and dynamic viscosity of the binder, α governs the global elastic-viscous transition.

Variation of abovementioned parameters appears to be coherent with the composition of different materials and testing conditions, reflecting the variations in terms of polymer concentrations and temperatures.

In particular, materials characterized by low polymer content exhibited high values of springpot parameter α, indicating materials' response to be significantly governed by viscous components.

These become even more predominant as temperature increases, with the concurrent decrease of b_2/b_1 ratio values.

In the case of highly modified materials, α values drop dramatically showing the evolution from a viscous to a pronounced visco-elastic behavior. This is especially true for binders containing SBS (both linear and radial) characterized by a high styrene content, the response of which can be assimilated to that of a visco-elastic solid.

Such results suggests SBS to be able to form a continuous cross-linked network only if added to the base bituminous matrix in a sufficiently high amount. In such conditions, binder containing radial SBS with high styrene content was found to prevail at both considered temperatures, as demonstrated by higher values of α and b_2/b_1 ratio.

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