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Analysis of Coupled Angular Regions in Spectral Domain

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Abstract—This paper analyses the problem of coupling multiple angular regions in spectral domain by using the generalized Wiener-Hopf technique. The paper introduces also the technique to obtain a solution of the problem by reducing the factorization problem to Fredholm integral equation. We present a test case constituted by two PEC wedges.

I. INTRODUCTION

Recently the authors of this paper has extended the Generalized Wiener-Hopf Technique (GWHT) to solve electromagnetic problems involving coupled angular and planar regions [1-8].

According to our opinion, the GWHT is able to contemporary handle planar stratified structures together with wedge structures [9].

This procedure allows to handle the problem of multiple wedges where angular regions and layers are alternating (see Fig. 1 for example).

In order to illustrate the formulation, we first recall how in literature canonical problems constituted of only planar regions or only angular regions are addressed.

Concerning the planar regions, there are two very excellent books on this topics [10, 11]. However in these books the presence of planar discontinuities is not systematically considered. An unified theory, that starts from the fundamental half plane problem, is presented in [12], where the stratified regions in presence of planar discontinuity is studied. The theory is based on the Wiener-Hopf (WH) technique where the unknowns are Laplace/Fourier transforms of the field components.

Concerning the angular regions, several methods have been proposed to solve the wedge problem.

For instance the canonical PEC wedge problem has been studied in the natural domain by separation of variables [11]. However, the natural domain limits the modelization to simple problems, and to overcome these difficulties, spectral representations have been developed in the past.

An exhaustive analysis of literature shows that the most important representations of angular region problems in spectral domain are:

1) The Sommerfeld Malyuzhinets functions
2) The Kontorovich Lebedev transform
3) The Laplace transform in radial direction.

Using Laplace transforms, the WH technique has been extended to angular regions too [13-16].

We note that for long time the WH technique was considered inapplicable to wedge problems of arbitrary aperture angle.

Since WH technique is now suitable to handle problem with rectangular and angular geometry, we state that GWHT is one of the best mathematical tool to handle coupled angular and planar regions problem since the Laplace transform of the field components can be defined in both kind of regions. Successful applications have already been reported in [1-9,16].
II. FORMULATION AND SOLUTION PROCEDURE

To briefly illustrate as the GWHT formulation works in the case of coupling multiple angular regions, let us consider the scattering problem constituted of two wedges in free space with E-polarized incident plane wave

\[ E_z = E_0 e^{jkr \cos(\theta - \phi)} \]  

where \( k \) is the free space propagation constant, see Fig. 1.

In this geometry we have three angular regions that couple with a planar layer. For each of these regions we can write a Generalized Wiener-Hopf Equation (GWHE) that involves plus and minus functions defined in suitable complex planes.

The complexity of the problem together with the arbitrariness of geometrical/material parameters do not permit a closed form factorization.

However, the solution of a GWHE problem can be obtained through Fredholm factorization [17-19]. It consists of the elimination of the minus functions in the WH equations through Cauchy decomposition formulas.

After several mathematical elaborations, see [4], the Fredholm factorization yields to the system of Fredholm integral equation of order three:

\[ V_+ (\eta) + \int_{-\infty}^{\infty} M(\eta, \eta') V_+ (\eta') d\eta' = N(\eta) \]  

where the three unknowns \( V_+ (\eta), V_2+ (\eta) \) and \( V_\pi+ (\eta) \) that define the vector

\[ V_+ (\eta) = \begin{bmatrix} V_1+ (\eta) \\ V_2+ (\eta) \\ V_\pi+ (\eta) \end{bmatrix} \]  

are the Laplace transforms of the electrical field \( E_z (x, y) \) in the three apertures where the angular regions couple with the layer region:

\[ V_1+ (\eta) = \int_0^\infty E_z(x, 0) e^{j\eta x} dx \]
\[ V_2+ (\eta) = \int_0^\infty E_z(x_2 + s, -d) e^{j\eta x_2} dx_2 \]
\[ V_\pi+ (\eta) = \int_0^\infty E_z(x, 0) e^{-j\eta s} dx \]  

The kernel \( M(\eta, \eta') \) is defined by:

\[ M(\eta, \eta') = \begin{bmatrix} Z+ (\eta) & Z_+ (\eta) e^{j\eta s} & 0 \\ Z_+ (\eta) e^{j\eta s} & Z+ (\eta) & 0 \\ 0 & 0 & Z_\pi (\eta) \end{bmatrix} \]  

where

The efficiency and the validity of the formulation of the two wedge problem has been ascertained in [5] for a particular selection of parameters (\( s = 0, \Phi = 0 \)).
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