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The Dynamic Behavior of the Basilica of San Francesco in Assisi Using Simplified Analytical Models

V. Arcidiacono¹, G.P. Cimellaro², E. Piermarini³, J. Ochsendorf⁴

ABSTRACT

The Basilica of San Francesco in Assisi endured stronger earthquakes for centuries before 1997 earthquake, which generated the collapse of the two vaults. Experts blame as possible reasons of collapse the damage cumulated from previous earthquakes and/or the retrofitting made to the structure over its lifetime. In this paper is presented the history of the retrofit interventions of the Basilica through the centuries, focusing mainly on the roof, which has been subjected to three major restorations through its life. It is shown using simple analytical models that the cumulative effects of the changes made to the roof of the Basilica affected the structure's dynamic behavior in a negative manner, increasing the seismic loads on the existing structural members. In particular, the numerical results show that the 1958 roof intervention has stiffened the structure,

¹ Trainee, Critical Infrastructure Protection, European Commission, Joint Research Centre, Via E. Fermi 2749, Ispra (VA), I-21027, Italy email: (vincenzo.arcidiacono@jrc.ec.europa.eu)

² Associate Professor, Department of Structural and Geotechnical Engineering (DISTR), Politecnico di Torino, 10129 Turin, Italy (gianpaolo.cimellaro@polito.it)

³ Graduate Student, Department of Civil and Environmental Engineering, Massachusetts Institute of Technology (MIT), 77 Massachusetts Avenue, Cambridge, MA, 02139, USA email: (vettore89@yahoo.com)

⁴ Professor, Department of Civil and Environmental Engineering, Massachusetts Institute of Technology (MIT), 77 Massachusetts Avenue, Cambridge, MA, 02139, USA email: (jao@mit.edu)

redistributing the seismic loads on the façade and the transept. This overload might explain the collapse of the two Gothic vaults during 1997 earthquake.

KEYWORDS: Basilica of San Francesco in Assisi, Historic Masonry Structures, Monumental Structures, Seismic Vulnerability, Seismic Retrofitting

INTRODUCTION

Seismic protection of historical heritage is a challenge that necessitate to know in-depth the characteristics of the monuments to mitigate the risks. Among these, special attention requires churches, which are located in areas where the seismic hazard is high such as Italy. Intensive research has been performed related to churches, which have been hit by earthquakes. For example, Betti and Vignoli (2008a) analyzed the actual efficiency of current techniques for repairing and strengthening existing historical churches by applying them to a specific case study: the Farneta abbey; and they also used a quasi-static approach (the seismic coefficient method) for the evaluation of the seismic loads applied to an historical Romanesque masonry church (Betti and Vignoli (2008b)). Brandonisio et al. (2013) analyzed the seismic behavior of masonry churches damaged during the 2009 L'Aquila earthquake by realizing that the ground motion has activated higher vibration modes with smaller participation factors. This activation of local modes could be avoided by adopting proper retrofit interventions, which would have tied up the structure, avoiding the local failures that are often observed.

In a more general way, Lagomarsino (2012) carried out a damage assessment on more than 700 churches following the 2009 L'Aquila earthquake. He has used a methodology aimed at

recognizing the collapse mechanisms in the different architectonic elements of the church which was developed after the 1997 earthquake in Umbria and the Marche. From the analysis it emerges that for a correct interpretation of damage and vulnerability, it is necessary a deep knowledge of local construction techniques and of the historic transformation sequence. A similar approach have been used by Leite et al. (2013) for the damage assessment of 112 churches which were affected by the 2010-2011 Canterbury earthquake sequence in New Zealand. Among the Churches that have been hit by earthquakes in Italy, the Basilica of San Francesco in Assisi, Italy is one of the most famous in the world and has a long history. In 1997, it was hit by a sequence of earthquakes. The main shocks were 5.6 and 5.8 magnitude respectively and they lead to the collapse of two Gothic vaults and widespread damage. The primary goal of this paper is to show how the cumulative effects of the changes made to the roof of the Basilica affected the structure's dynamic behavior in a negative manner increasing the seismic loads on the existing structural members. The stiffness and mass properties of each roof are determined to describe the modifications made to the structure in each restoration. Finally, an analytical model is used to quantify how much each roof intervention affected the dynamic behavior of the Basilica over the years. In particular, the seismic response to the 1997 earthquake has been analyzed.

The next sections start with a brief description of the seismic and the structural history of the Basilica. Then, an analytical model based on the work of Arcidiacono et al. (2015) is described to show the trends in the structural seismic response. The numerical results of the model and the conclusions are discussed in the last two sections of the paper.

HISTORY OF THE BASILICA

The construction of the Monastery began two years after St. Francis of Assisi's death in 1226 and was completed in 1253 (Destinations, 2009). The Basilica (Figure 1) consists of two churches placed one on top of the other. The Lower Basilica was completed in 1230 and includes San Francesco's remains, while the Upper Basilica was completed in 1253 and consecrated by Pope Innocent IV. Both parts of the Basilica have similar architectural features: a one-aisle nave with four bays (Gothic vaults), a transept, and an apse. The interiors of both Basilicas are immediately striking due to the marvelous frescoes that were painted shortly after the completion of the Basilica. The paintings of the Upper Basilica began in the nave with a master Florentine painter Cenni di Pepi, known as Cimabue, in 1275. Cimabue was responsible for painting the apse, the walls and the vaults of the transept with important characters from the New Testament. The nave of the Basilica is decorated around the lower perimeter with frescoes depicting the life of San Francesco as well as stories of the life of Christ on the southern wall and other stories from the bible on the northern wall. The paintings of the nave are attributed to a variety of Roman master painters and another master Florentine painter Giotto di Bondone (Bonsanti et al., 1998).

The Lower Basilica has an Italian Romanesque style with barrel vaults that are stout and broad. It also contains the crypt of San Francesco located under the main altar as well as a large narthex, which serves as the entrance of the Lower Basilica. The Upper Basilica has a Gothic interior, which at the time in which it was constructed, was a very new style for Italy. The facade of Upper Basilica is Romanesque style with a large Gothic doorway, a rose stained glass window, and an oculus on the gable. A Romanesque bell tower is located on the South side of the church.

The dimensions of the Upper Basilica are about 73 m in the longitudinal direction, 33 m in the transverse direction, and approximately 21 m in height from the floor of the Upper Basilica to the roof (Figure 2). The nave is 50 m long and 15 m wide. The thicknesses of the masonry walls vary between 0.8 m to 3.2 m, near the cylindrical buttresses which in the XIV century were strengthened with arched flying buttresses (Bonsanti et al., 1998; Castex, 2008).

SEISMIC HISTORY OF ASSISI

As shown in Table 1, the historical earthquake data in Assisi are expressed in Mercalli-Cancani-Sieberg (MCS) scale, since this is one of the scales that allow to measure the seismic intensity of the historical earthquakes during most of the Basilica's lifetime. The data were taken from the Italian database DBMI11 (Locati et al., 2011) made by the Italian National Institute of Geophysics and Volcanology. Using the Italian Macroseismic Database (Locati et al., 2011) 75 earthquakes were identified in Assisi, 5 of which having a site macroseismic intensity equal to or greater than VI-VII MCS. They correspond to the earthquake of 1751, 1832, 1854 and 1915. However, none of them produced damage as great as the 1997 earthquake. For this reasons, the 1997 seismic event for its uniqueness is discussed in detail in the following section.

Damage effects recorded in the Assisi Monastery over the centuries including 1997 Umbria-Marche earthquake

The historical earthquakes in the region caused damage to the Basilica which had undoubtedly been weakened by ground movement over many centuries (Guidoboni et al., 2007). For

example, it is likely that the façade of the Upper Church was affected by the 1279 earthquake, such that it was partially re-built, as indicated by the different architectural styles observed in the building. According to the historical acts and documents of many towns in the region the 1279 earthquake with an inferred M_s of 6.7, caused destruction to properties and had many other ground effects over a wide area. Then after a period of moderate activity in the region, the 1703 earthquake occurred in Norcia, followed by six destructive events before the end of the century. Then in the following two centuries, the seismic activity in the region was high, but not as high as in the eighteen century.

Then in 1997, at about 2:30 AM, a 5.7 magnitude earthquake hit the Basilica. Coincidentally, Ghigo Roli, a photographer who had been shooting the Upper Basilica's frescoes for months, happened to be inside during the event. In his testimony (Bonsanti et al., 1998), he recalls that immediately after the first tremor, the air was filled with dust and the floor was littered with paint chips of frescoes. Upon his initial inspection of the church, Roli noted that a large crack had formed on the great arch along the south face of the transept and a large stone had fallen from the double lancet from a column on the North side of the Basilica (Figure 3). This was all of the damage witnessed by Roli that night, which by his testimony, seems to have been limited to non-structural damage.

The next morning at about 11:42 AM, a second tremor of magnitude 5.8 hit the Basilica. This time, Roli was just outside the doors of the Basilica and a camera crew from Umbria TV was at the front of the nave videotaping. Roli reports that “the jambs of the Basilica's doors rose, then fell while \lurching first forward then back...”. As seen in Figure 4, Umbria TV cameramen were

able to capture the collapse of the easternmost quadrant of the Gothic vault adjacent to the facade, which fell on individuals fleeing for the door and resulted in 4 deaths (Bonsanti, 1998). In addition to the vaults collapsing near the facade, those adjacent to the triumphal arch also collapsed.

The damaged areas of the Basilica are highlighted in Figure 3. The largest loss incurred due to the earthquake was the destruction of the precious frescoes that were painted on the portions of the vaults that collapsed, which included famous works by Cimabue and Giotto. Such works include Cimabue's fresco of San Matteo and Judea above the central nave and Giotto's vault of the Dottori della Chiesa near the facade, as seen in Figure 5. Immediate actions were taken after the earthquake to stabilize the surviving vaults and tympanum against other aftershocks. The ensuing restoration project involved preservation of the historic paintings as well as restoring structural integrity to the vaults and tympanum. Historians and preservationists sifted through the rubble in the nave looking for any surviving pieces of the vault rib or chips of fresco paint in order to rebuild the vaults with as much original material as possible. The restored works of San Matteo and Judea and Dottori della Chiesa can be seen in Figure 5.

HISTORY OF RETROFIT INTERVENTIONS

Throughout centuries, many structural interventions were carried out on the Basilica changing its structural behavior. As mentioned above the retrofit interventions involved the cylindrical buttresses (in the XIV century installing flying buttresses) and the Basilica's roof which had three major alterations during its life. In the literature, a comprehensive history of the different

roofs of the Basilica of San Francesco compiled in one source is not available. Thus, it is necessary to first summarize each of the different roofs the Basilica has had. Figure 6 shows the roof interventions (Crocì, 2001; Crocì et al., 2007; Rocchi, 2002).

The structure of the roof has changed three times since its completion in 1253, with the most recent intervention being after the 1997 earthquake. It is known that solely wood materials made the original roof of the Basilica; however, a scheme of the original roof design does not exist. For unknown reasons, the original roof was replaced in the 1475 by a new system of wood purlins and sheathing supported by large masonry arches above the vaults. This system remained in place for over 400 years until the 1958, when it was decided by the Genio Civile di Perugia to replace the wooden parts of the roof to protect against fire hazards (Rocchi, 2002). Thus, the wood purlins were replaced with new ones made of reinforced concrete and the wood sheathing was replaced with a concrete and masonry flooring system. This new roof was placed on top of a reinforced concrete ring beam that was installed along the perimeter of the nave and transept. Furthermore, during the 1958 intervention the builders also added ties between the masonry arches that run perpendicular to the nave above the ceiling to compensate for the added thrust loads, as seen in Figure 7. The ties were not applied at the nave opening of the transept and at the façade (Figure 8), which are the two locations where the vaults collapse during 1997 earthquake. The final modification to the roof occurred during the restoration of the Basilica after 1997 earthquake. In the last intervention, the original ties between the masonry arches, since they were making contact with the top of the vaults, were raised of about 0.6 m (Rocchi, 2002). The tympanum was rebuilt using the original stones that had fallen during the 1997 earthquake and grouted with a new mortar. The connection between the tympanum and the roof was modified

from a rigid connection by inserting shape memory alloy devices (SMAD) to reduce the out of plane seismic loadings on the tympanum (Figure 9). The Gothic vaults were strengthened putting aramid fibers to the extrados of the ribs of the vaults using an epoxy. These ribs were then connected to the reinforced concrete roof purlins via a system of tie rods, springs, and steel beams as shown in Figure 10. These were connected to steel beams installed between diagonal arches. Finally, the vaults themselves, which had been severely cracked during the earthquake, were strengthened using a special type of mortar. The masonry walls were strengthened with a steel truss that runs around the perimeter of the nave and transept (Figure 11) at about 7 m above the floor, i.e. where the flying arch buttresses intersect the cylindrical buttresses.

By observing the full seismic history of the Basilica and the history of roof interventions, it is interesting to note that not even 50 years after one of the most intrusive interventions in the structure's history, the Basilica sustained more damage than it ever had in its nearly 800 year history. The justification of this statement is discussed in detail in the following sections.

Literature review on different retrofit interventions

Numerous studies in literature show that these types of masonry structures have a very sensitive structural behavior. In fact, invasive or strengthening techniques can cause detrimental effects to their seismic behavior due to stiffness incompatibilities (Binda et al. 2003). Recent studies emphasize how invasive retrofits on historic masonry structures have played a role in the collapse of the Basilica of Santa Maria Di Collemaggio (Cimellaro et al., 2011; Cimellaro et al., 2012; Arcidiacono et al., 2014), the Basilica of San Bernardino, the Basilica of Santo Domenico, the Basilica of San Eusanio Martire (Lagomarsino et al., 2004; Lagomarsino, 2012), the Basilica

of San Marco (Modena et al., 2010), and the Basilica of Santa Maria Paganica (Carocci et al., 2010) during the 2009 L'Aquila earthquake. All these examples suggest, in a qualitative fashion, that the collapses of these Basilicas were due to incompatibility of modern materials with the existing structures as well as increased seismic loads due to the addition of mass and stiffness to the structure during the restoration. Therefore, the interventions on historic masonry structures, performed in order to protect the structures against hazardous events, can sometimes reduce the structure's seismic capacity. Often, with the intent to protect the structure by creating a box behavior, wooden roofs of Basilicas were replaced with stiffer and heavier roofs. Mele et al. have performed analyses of various Basilicas (Brandonisio et al., 2008; Mele et al., 2003) – assuming a macro-element structural behavior – to determine the effects of more rigid and heavy roofs. The results show as a trend that the additional mass and stiffness added to the system can sometimes change the global dynamic behavior in a negative manner.

Various studies, mainly by two authors Croci and Rocchi (Croci et al., 2007; Croci, 2001; Rocchi, 2002), have been written regarding the collapse of the two vaults of the Basilica of San Francesco. Croci studied the various methods that were used to strengthen and restore the Basilica, analyzing the structural behavior of the Basilica with a complex 3D finite element model. The first fundamental period of the Basilica with the roof 3 was about 0.3s. He said the modern roof with concrete and masonry flooring system installed in 1958 “did not involve any significant alternation of the building's dynamic behavior” (Croci et al., 2007). According to Croci, the collapse of vaults was due to both the negative effects from fill dust that was accumulated over the Gothic vaults during the centuries and the cumulative loss of curvature of

the vaults over the years. Moreover, Croci assumed that the lack of lateral stiffness at the roof level was one of the causes of the vault collapses in 1997.

According to Rocchi instead, the cause of collapse is the roof modification during the years. Ancient structures have a monolithic static behavior, which can easily be disturbed by retrofit interventions such as the intervention carried out in 1958, where the wood roof system built by carpenters over 400 years ago was simply thrown away and replaced by modern materials. The next section describes the analytical model used to analyze the seismic structural behavior of the Basilica in the longitudinal direction.

ANALYTICAL MODEL OF THE BASILICA

It is accepted that the analysis of historic masonry structures is a very complex task (Branco et al., 2011; Casarin et al., 2008; Penna et al., 2010). Several studies exist in literature that regard the mechanical behavior of the masonry, the identification of the dynamic parameters, the design of adequate finite element models (Gizzi et al., 2014; Mistler et al., 2006; Ramos et al., 2004). Some of them focus on the global nonlinear finite element models comparing different repairing and strengthening techniques (Betti et al., 2008). Other authors developed both 3D and 2D nonlinear models, using an equivalent frame approach and finally a kinematic collapse analysis (Mallardo et al., 2008).

Implementation of traditional finite element procedures is difficult due to material nonlinearities, in-homogeneities, as well as complex connections and boundary conditions. Thus, when trying to predict the behavior of these types of structures using linear elastic models, it is very

important to decide before the analysis what types of results should be inferred from the output data. In fact, it is sometimes easier and more reliable to show trends in the behavior of these structures rather than trying to solve for specific values of the response. Therefore, in this paper a methodology is employed to determine how the roof interventions have changed the global dynamic behavior of the Basilica over the years by showing general dynamics trends. The primary indicator for monitoring the changes of the dynamic behavior of the structure is the fundamental period of the structure and the response spectra analysis. To show these trends, the structure has been simplified as a simple beam on elastic supports and solved for the fundamental period of the structure. The material properties for the historical masonry have been taken from Croci et al. (2007), Brandonisio et al. (2008), and Arcidiacono et al. (2014). As shown in Figure 12, the *analytical model* consists of a shear beam on an elastic support. The equation that governs its motion is the following (Arcidiacono et al., 2014):

$$\mu \cdot \left(\frac{\partial^2 \delta}{\partial t^2} + \ddot{u}_g \right) + C \cdot \frac{\partial \delta}{\partial t} + = K_{Beam} \cdot \frac{\partial^2 \delta}{\partial z^2} - K_{Supp} \cdot \delta \quad (1)$$

where μ is the participant mass per unit length of the Basilica; δ is the horizontal displacement of the nave axis; \ddot{u}_g is the ground acceleration; C is the damping coefficient; K_{Beam} and K_{Supp} are the shear stiffness of the beam and the longitudinal stiffness of the nave section; and z is the abscissa of the nave axis.

This model assumes that the façade and the transept are infinitely rigid in their planes. Hence, they are considered as rigid supports of the beam in the horizontal direction. The nave's windows are modeled assuming two section types of the nave's walls (1: bottom and 2: top). The stiffness

of the nave roof, the nave walls, and the cylindrical buttresses are consistent and symmetrical. The stiffnesses of the flying buttresses in the horizontal direction are not symmetrical because they react only in compression. With this assumption, it is possible to model each roof as shear beam with constant stiffness K_{Beam} and to assume that each adjacent section of nave wall, cylindrical buttress, and flying buttress will provide together the stiffness K_{Supp} . The participant mass μ is uniformly distributed on the beam length L . The natural frequencies and the effective modal masses of the *analytical model* can be expressed in a closed-form solution given by:

$$m_i = 2 \cdot \left(\frac{1 - \cos(i \cdot \pi)}{i \cdot \pi} \right)^2 \quad \omega_i = \sqrt{\frac{k_i}{\mu}} \quad \text{with} \quad k_i = K_{Beam} \cdot \left(\frac{i \cdot \pi}{L} \right)^2 + K_{Supp} \quad (2)$$

Note that the effective modal masses are greater than zero only for odd indices. Hence, the response spectrum analysis should be made with only odd modes. The model in Equation (2) can be used to analyze the general trends in the dynamic behavior of the church, but the mass and the stiffness associated to each different roof should be estimated. While the mass and the stiffness of the supporting members can be evaluated using the material characteristics and member sizes, the stiffness of the horizontal beam is more difficult to quantify. Thus, 3D finite element models have been used to determine the relative stiffness of each roof model. The masonry slab is modeled as a shell with stiffness properties of concrete, but with a typical density of 640.74 kg/m³. The roof stiffness at every section is determined applying a shear load and measuring the relative displacement. Instead, the support stiffness K_{Supp} – i.e., the horizontal stiffness of the nave section (shown in Figure 13a) – is given by:

$$K_{supp} \cdot L_1 = \frac{1}{(C_1 + C_2) \cdot \frac{1 - k_B \cdot C_2}{1 + k_B \cdot C_1} + C_2 + C_3} + \frac{1}{C_1 + 2 \cdot C_2 + C_3} \quad (3)$$

where k_B is the stiffness of the flying buttress and C_x are the coefficients defined by the following equations:

$$\begin{aligned} C_1 &= \frac{H_1^3}{3 \cdot E_W \cdot I_1} & C_3 &= \left(\frac{H_1}{2 \cdot I_1} + \frac{H_2 - d}{3 \cdot I_2} \right) \cdot \frac{(H_2 - d)^2}{E_W} \\ C_2 &= \frac{(H_2 - d) \cdot H_1^2}{2 \cdot E_W \cdot I_1} & k_B &= \frac{E_B \cdot A_B}{L_B} \cdot \cos^2 \alpha \end{aligned} \quad (4)$$

where H_1 , H_2 , d , α , and L_B are the geometrical characteristics of the nave section defined in Figure 13b; E_W and E_B are, respectively, the elastic modules of nave's walls and of the flying buttress; I_1 , I_2 , and A_B are the properties of the element sections described in Figure 13c. The mass per unit length μ is evaluated summing the contribution of the roof and of the nave's walls. Analytically, it is given by:

$$\mu = \mu_{Beam} + \mu_{supp} = \mu_{Beam} + \frac{\sum_{j=1}^2 \int_{H_{j-1}}^{H_j} [f(v_w(x)) + f(v_{w/o}(x))] \cdot \rho_w \cdot (A_j + L_j \cdot W \cdot \zeta) \cdot dx}{L_1} \quad (5)$$

where v_w and $v_{w/o}$ are the horizontal displacements of the nave's walls with and without the flying buttress that are used as mass participation factors (see Figure 13a, deformed shape); x is the vertical axis of the nave section; ρ_w is the unit weight of the masonry walls; A_j , L_j , and W are the properties of the element sections described in Figure 13c; ζ is a factor that considers the weight of the Gothic vaults; and $f(\cdot)$ is a function that is defined as follows:

$$f(x) = \text{sign}(x) \cdot \min(|x|, 1) \quad (6)$$

Finally, the modal displacements and rotations of the nave's axis and the modal load – acting on the supports – and shear – acting on the beam – are given by:

$$\begin{cases} F_{\text{Supp},i}(z) = K_{\text{Supp}} \cdot \delta_i(z) \\ S_{\text{Beam},i}(z) = K_{\text{Beam}} \cdot g_i(z) \end{cases} \quad \begin{cases} \delta_i(z) = \sin\left(\frac{i \cdot \pi}{L} \cdot z\right) \cdot \bar{q} \\ g_i(z) = \frac{i \cdot \pi}{L} \cdot \cos\left(\frac{i \cdot \pi}{L} \cdot z\right) \cdot \bar{q} \end{cases} \quad \text{with } \bar{q} = 2 \cdot \frac{1 - \cos(i \cdot \pi)}{i \cdot \pi} \cdot \frac{S_a(T_i)}{\omega_i^2} \quad (7)$$

where $S_a(T_i)$ is the spectral acceleration with a natural period T_i .

NUMERICAL RESULTS

In Table 2 are listed the estimated masses and stiffnesses of the *analytical model* through the centuries and for the different interventions. According to the estimated values, both stiffness and mass after 1958 intervention increased, reducing the reserve capacity of the existing structural members supporting the roof. The normalized data in Table 3 shows the degree by which each subsequent roof intervention changed the mass and stiffness properties of the previous roof. From the table, it is important to note that the major increase in mass occurred between roofs 1 and 2, which can be attributed to the installation of the massive masonry arches. On the other hand, the major increase in stiffness occurred between roofs 2 and 3, which can be attributed to the installation of the reinforced concrete purlins, ring beams, and masonry slab. In addition, it is obvious that each roof intervention increased the stiffness more than the mass of the roof. Each intervention decreased the fundamental period of the Basilica, generating an

increment due to redistribution of the seismic loads on the supporting structural members. Furthermore, the more drastic intervention that has changed the properties of the Basilica is the one in 1958, because while the first roof intervention increased the weight and stiffness by about the same factor, the second roof intervention increased the mass slightly, but the stiffness significantly. This explains the reduction of the first fundamental period from roof 1 to roof 3 of 66.2%.

The 1997 Umbria Marche 2nd shock seismic record related to the R.A.N. (Rete Accelerometrica Nazionale) station “ASS” has been used in the response spectrum analyses. Horizontal response spectra in the North-South and East-West directions and the worst-case combination, which considers the inclination (36°) of the Basilica with respect to the North-South direction, are shown in Figure 14. Response spectrum analyses were performed using the first 19 modes and applying the worst-case response spectrum. The analyses (Figure 15) predict that the maximum absolute displacement of the nave is at the middle point of the beam. This is 0.4 and 0.6 lower, respectively with *roof 3* and *roof 4*, with respect to that predicted with *roof 1* or *roof 2*. The shear envelope of the beam and the horizontal seismic loads applied on the nave section defined in Equation 7 are shown in Figure 16 and Figure 17. The analyses indicate that the maximum nave load is located at the middle point of the beam, while the maximum shear is located at the rigid supports (i.e., transept and façade). In Table 4 are listed the normalized results of the worst-case response spectrum analyses. The increase in the element stiffness and the participant mass explains the massive increase of 35 and 25 times, respectively, in the maximum beam shear and in the ratio between the maximum shear and the maximum nave load from *roof 1* to *roof 3*. The spectral acceleration of the analytical model with roof 4 model is lower with respect to that with

roof 3 model. This justifies the reduction of the maximum shear from roof 3 to roof 4. The response spectrum analysis gives an approximation of the initial load distribution, but it does not provide any information about the dynamic nonlinearities.

CONCLUDING REMARKS

The paper has analyzed the modifications in the global dynamic behavior of the Basilica of San Francesco in Assisi over its history due to the massive retrofit interventions on the central nave. In particular, only the main retrofit interventions regarding the roof and the flying buttresses of the Basilica have been considered, because the roof was the only structural element of the church that was modified in each intervention through the centuries.

During a period of nearly 750 years, the Basilica was hit at least by 75 earthquakes, but it survived without significant damage. However, the historical studies have shown that less than fifty years after the 1958 roof intervention, the vaults in the central nave collapsed during a relatively moderate earthquake (1997 Umbria-Marche earthquake) and the Basilica incurred in more damage than any other earthquake in its history.

The results of the developed analytical models have been cross-referenced with the roof intervention and the seismic history of the Basilica. This cross referencing supports the assumption that the roof intervention in 1958 was one of the causes of the 1997 vault collapses. In fact, the simplified models used in this paper have shown there has been a modification on the global dynamic response of the Basilica after the 1958 roof intervention. In particular, the fundamental period of the Basilica has been significantly reduced with respect to the two

previous interventions, generating an increase in the seismic loads, and a shift of these loads toward the façade and the transept. Therefore, it would have been necessary to verify if the 1958 retrofit intervention would have provided enough capacity to withstand the increased loads and in particular if, the existing structural members underneath the roof had enough residual capacity. In conclusion, the lack of the residual capacity of the structural members (i.e. the rigid supports) underneath the roof, the asymmetries of the nave – i.e. the missing tendons at the transept and at the façade and the missing flying buttress at the bell tower – are some of the reasons which might have led to the 1997 collapse. This is another example of how important is carrying out historical studies on monuments before starting structural interventions.

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Figure 1- Basilica of San Francesco in Assisi.



Figure 2- Dimensions of the Basilica of San Francesco in Assisi (Rocchi, 1982).

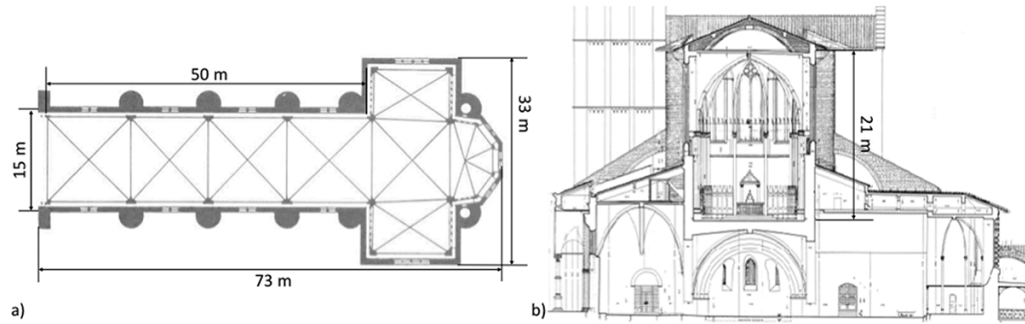


Figure 3- Damage incurred during 1997 earthquake (Croci, 2001).

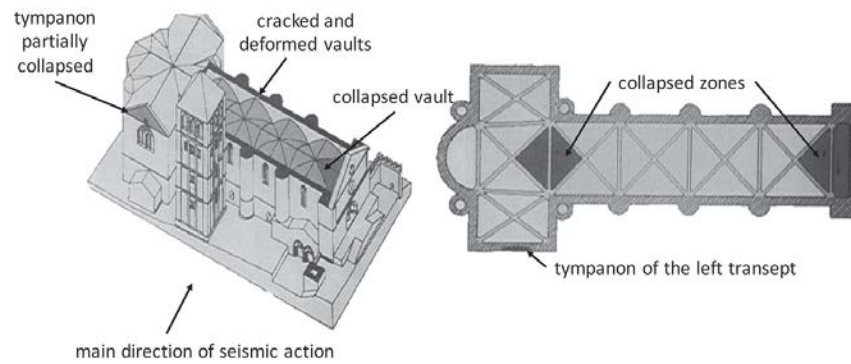


Figure 4- Collapse of the vault of the Doctors of the Church (Bonsanti, 1998).



Figure 5- Restored Frescoes after 1997 Earthquake. a) Cimabue; b) Giotto vaults.



a)

b)

Figure 6- Schemes of the nave roof.

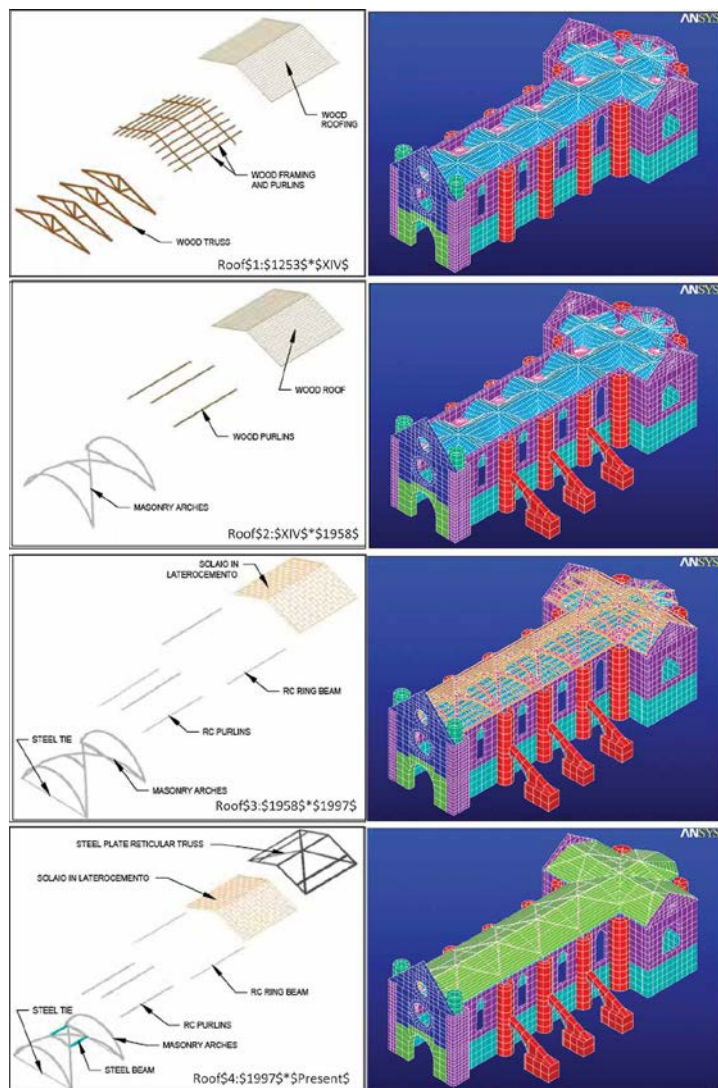


Figure 7- Tie installed during the 1958 (Rocchi, 2002).



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Figure 8- Location of the installed tie.

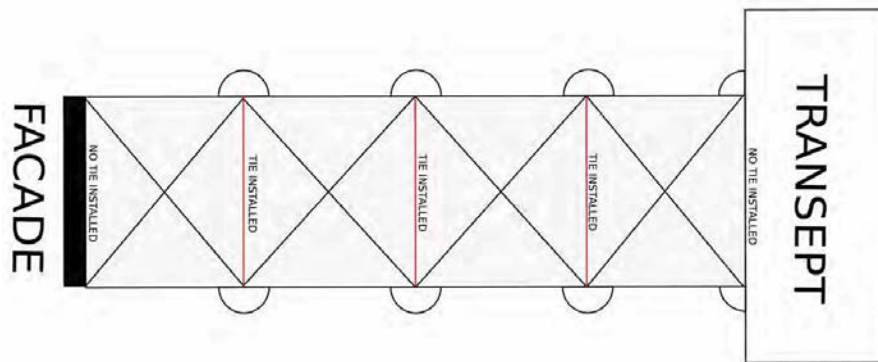


Figure 9- Installation of SMADs at the Tympanum-roof interface (Crocì et al., 2007).

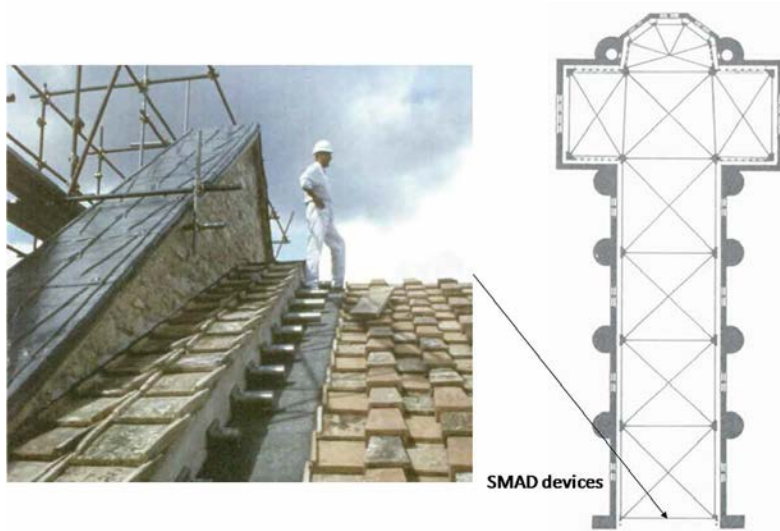


Figure 10- Steel beam and spring/damper system installed throughout the entire roof.

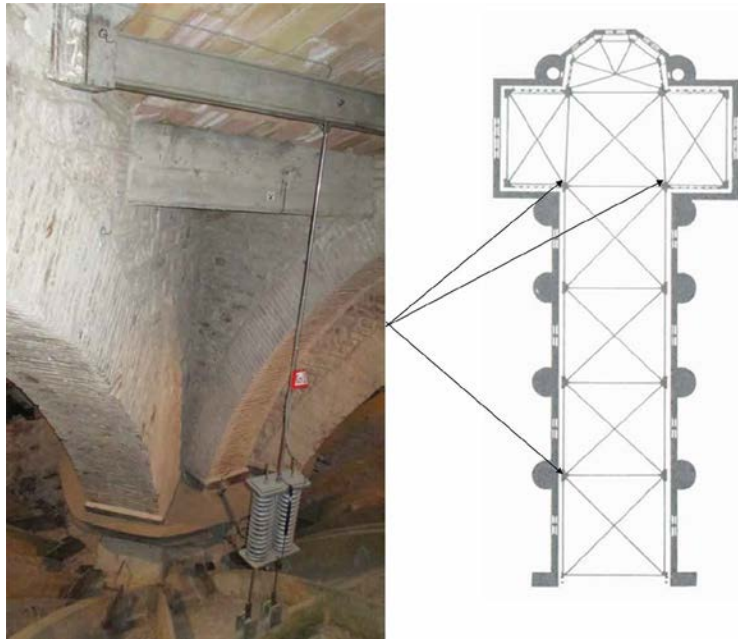


Figure 11- Ring beam installed along the cornice.

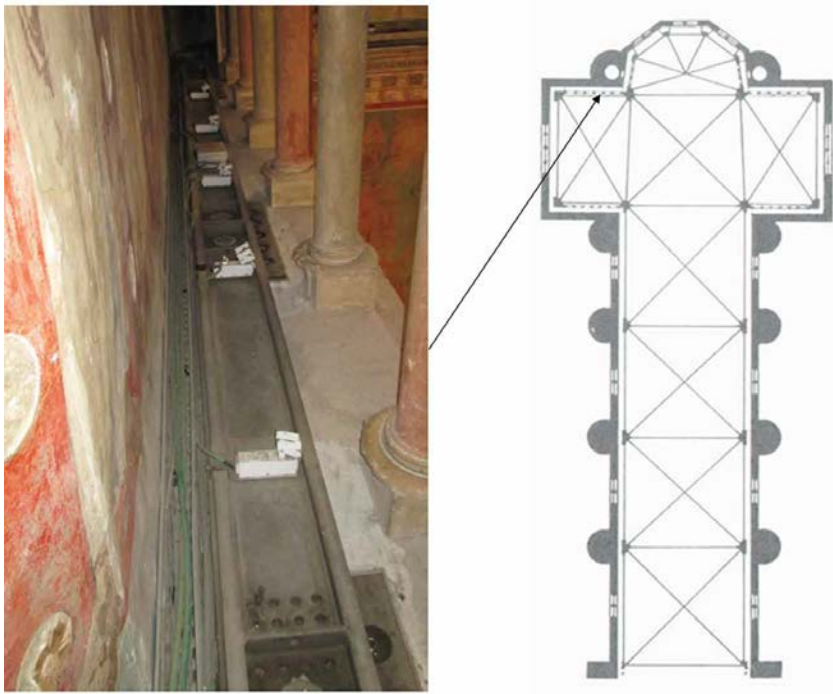


Figure 12- Schematic view of: a) elements involved in the analytical model, and b) the analytical model.

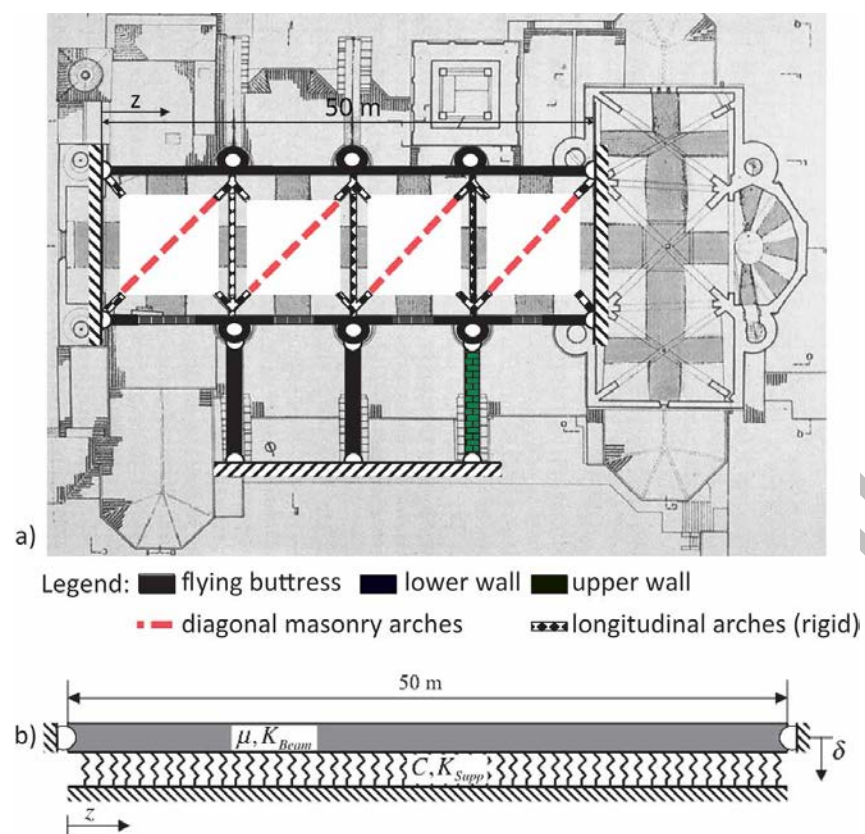


Figure 1 consists of three parts: (a) Deformed shape of the wall and rigid link, showing a rigid link of length d and a deformed shape with a horizontal displacement $\delta=1$. (b) Cross-section of the wall showing various layers: flying buttress, lower wall, upper wall, diagonal masonry arches, and longitudinal arches (rigid). Dimensions include $H=10.5$ m, $H_1=10.5$ m, $H_2=10.5$ m, $H_3=10.5$ m, $H_4=10.5$ m, $H_5=10.5$ m, $H_6=10.5$ m, $H_7=10.5$ m, $H_8=10.5$ m, $H_9=10.5$ m, $H_{10}=10.5$ m, $H_{11}=10.5$ m, $H_{12}=10.5$ m, $H_{13}=10.5$ m, $H_{14}=10.5$ m, $H_{15}=10.5$ m, $H_{16}=10.5$ m, $H_{17}=10.5$ m, $H_{18}=10.5$ m, $H_{19}=10.5$ m, $H_{20}=10.5$ m, $H_{21}=10.5$ m, $H_{22}=10.5$ m, $H_{23}=10.5$ m, $H_{24}=10.5$ m, $H_{25}=10.5$ m, $H_{26}=10.5$ m, $H_{27}=10.5$ m, $H_{28}=10.5$ m, $H_{29}=10.5$ m, $H_{30}=10.5$ m, $H_{31}=10.5$ m, $H_{32}=10.5$ m, $H_{33}=10.5$ m, $H_{34}=10.5$ m, $H_{35}=10.5$ m, $H_{36}=10.5$ m, $H_{37}=10.5$ m, $H_{38}=10.5$ m, $H_{39}=10.5$ m, $H_{40}=10.5$ m, $H_{41}=10.5$ m, $H_{42}=10.5$ m, $H_{43}=10.5$ m, $H_{44}=10.5$ m, $H_{45}=10.5$ m, $H_{46}=10.5$ m, $H_{47}=10.5$ m, $H_{48}=10.5$ m, $H_{49}=10.5$ m, $H_{50}=10.5$ m, $H_{51}=10.5$ m, $H_{52}=10.5$ m, $H_{53}=10.5$ m, $H_{54}=10.5$ m, $H_{55}=10.5$ m, $H_{56}=10.5$ m, $H_{57}=10.5$ m, $H_{58}=10.5$ m, $H_{59}=10.5$ m, $H_{60}=10.5$ m, $H_{61}=10.5$ m, $H_{62}=10.5$ m, $H_{63}=10.5$ m, $H_{64}=10.5$ m, $H_{65}=10.5$ m, $H_{66}=10.5$ m, $H_{67}=10.5$ m, $H_{68}=10.5$ m, $H_{69}=10.5$ m, $H_{70}=10.5$ m, $H_{71}=10.5$ m, $H_{72}=10.5$ m, $H_{73}=10.5$ m, $H_{74}=10.5$ m, $H_{75}=10.5$ m, $H_{76}=10.5$ m, $H_{77}=10.5$ m, $H_{78}=10.5$ m, $H_{79}=10.5$ m, $H_{80}=10.5$ m, $H_{81}=10.5$ m, $H_{82}=10.5$ m, $H_{83}=10.5$ m, $H_{84}=10.5$ m, $H_{85}=10.5$ m, $H_{86}=10.5$ m, $H_{87}=10.5$ m, $H_{88}=10.5$ m, $H_{89}=10.5$ m, $H_{90}=10.5$ m, $H_{91}=10.5$ m, $H_{92}=10.5$ m, $H_{93}=10.5$ m, $H_{94}=10.5$ m, $H_{95}=10.5$ m, $H_{96}=10.5$ m, $H_{97}=10.5$ m, $H_{98}=10.5$ m, $H_{99}=10.5$ m, $H_{100}=10.5$ m, $H_{101}=10.5$ m, $H_{102}=10.5$ m, $H_{103}=10.5$ m, $H_{104}=10.5$ m, $H_{105}=10.5$ m, $H_{106}=10.5$ m, $H_{107}=10.5$ m, $H_{108}=10.5$ m, $H_{109}=10.5$ m, $H_{110}=10.5$ m, $H_{111}=10.5$ m, $H_{112}=10.5$ m, $H_{113}=10.5$ m, $H_{114}=10.5$ m, $H_{115}=10.5$ m, $H_{116}=10.5$ m, $H_{117}=10.5$ m, $H_{118}=10.5$ m, $H_{119}=10.5$ m, $H_{120}=10.5$ m, $H_{121}=10.5$ m, $H_{122}=10.5$ m, $H_{123}=10.5$ m, $H_{124}=10.5$ m, $H_{125}=10.5$ m, $H_{126}=10.5$ m, $H_{127}=10.5$ m, $H_{128}=10.5$ m, $H_{129}=10.5$ m, $H_{130}=10.5$ m, $H_{131}=10.5$ m, $H_{132}=10.5$ m, $H_{133}=10.5$ m, $H_{134}=10.5$ m, $H_{135}=10.5$ m, $H_{136}=10.5$ m, $H_{137}=10.5$ m, $H_{138}=10.5$ m, $H_{139}=10.5$ m, $H_{140}=10.5$ m, $H_{141}=10.5$ m, $H_{142}=10.5$ m, $H_{143}=10.5$ m, $H_{144}=10.5$ m, $H_{145}=10.5$ m, $H_{146}=10.5$ m, $H_{147}=10.5$ m, $H_{148}=10.5$ m, $H_{149}=10.5$ m, $H_{150}=10.5$ m, $H_{151}=10.5$ m, $H_{152}=10.5$ m, $H_{153}=10.5$ m, $H_{154}=10.5$ m, $H_{155}=10.5$ m, $H_{156}=10.5$ m, $H_{157}=10.5$ m, $H_{158}=10.5$ m, $H_{159}=10.5$ m, $H_{160}=10.5$ m, $H_{161}=10.5$ m, $H_{162}=10.5$ m, $H_{163}=10.5$ m, $H_{164}=10.5$ m, $H_{165}=10.5$ m, $H_{166}=10.5$ m, $H_{167}=10.5$ m, $H_{168}=10.5$ m, $H_{169}=10.5$ m, $H_{170}=10.5$ m, $H_{171}=10.5$ m, $H_{172}=10.5$ m, $H_{173}=10.5$ m, $H_{174}=10.5$ m, $H_{175}=10.5$ m, $H_{176}=10.5$ m, $H_{177}=10.5$ m, $H_{178}=10.5$ m, $H_{179}=10.5$ m, $H_{180}=10.5$ m, $H_{181}=10.5$ m, $H_{182}=10.5$ m, $H_{183}=10.5$ m, $H_{184}=10.5$ m, $H_{185}=10.5$ m, $H_{186}=10.5$ m, $H_{187}=10.5$ m, $H_{188}=10.5$ m, $H_{189}=10.5$ m, $H_{190}=10.5$ m, $H_{191}=10.5$ m, $H_{192}=10.5$ m, $H_{193}=10.5$ m, $H_{194}=10.5$ m, $H_{195}=10.5$ m, $H_{196}=10.5$ m, $H_{197}=10.5$ m, $H_{198}=10.5$ m, $H_{199}=10.5$ m, $H_{200}=10.5$ m, $H_{201}=10.5$ m, $H_{202}=10.5$ m, $H_{203}=10.5$ m, $H_{204}=10.5$ m, $H_{205}=10.5$ m, $H_{206}=10.5$ m, $H_{207}=10.5$ m, $H_{208}=10.5$ m, $H_{209}=10.5$ m, $H_{210}=10.5$ m, $H_{211}=10.5$ m, $H_{212}=10.5$ m, $H_{213}=10.5$ m, $H_{214}=10.5$ m, $H_{215}=10.5$ m, $H_{216}=10.5$ m, $H_{217}=10.5$ m, $H_{218}=10.5$ m, $H_{219}=10.5$ m, $H_{220}=10.5$ m, $H_{221}=10.5$ m, $H_{222}=10.5$ m, $H_{223}=10.5$ m, $H_{224}=10.5$ m, $H_{225}=10.5$ m, $H_{226}=10.5$ m, $H_{227}=10.5$ m, $H_{228}=10.5$ m, $H_{229}=10.5$ m, $H_{230}=10.5$ m, $H_{231}=10.5$ m, $H_{232}=10.5$ m, $H_{233}=10.5$ m, $H_{234}=10.5$ m, $H_{235}=10.5$ m, $H_{236}=10.5$ m, $H_{237}=10.5$ m, $H_{238}=1$

Figure 14-Response spectra of the 1997 Umbria-Marche earthquake.

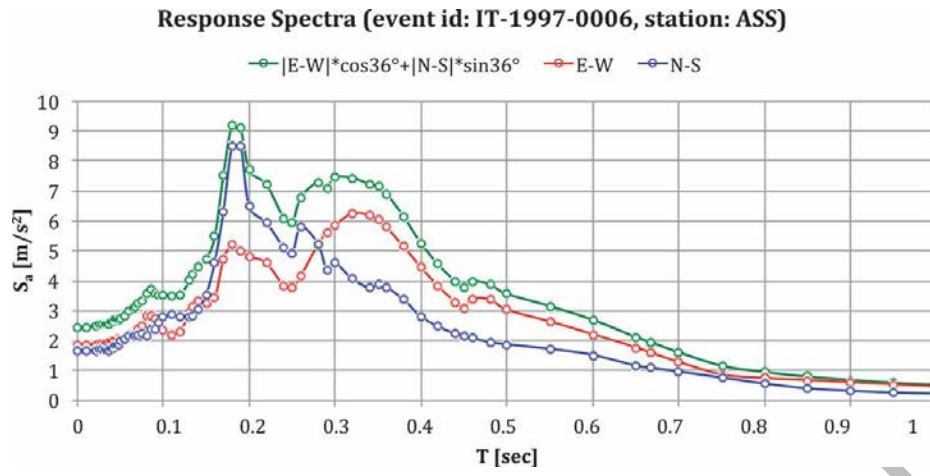


Figure 15- Results of the response spectrum analyses: horizontal displacement of the nave's axis.

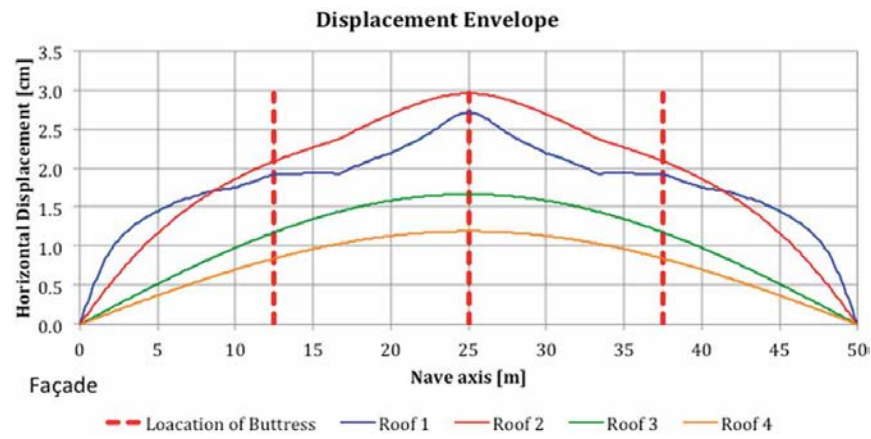


Figure 16- Results of the response spectrum analyses: load acting on the supports.

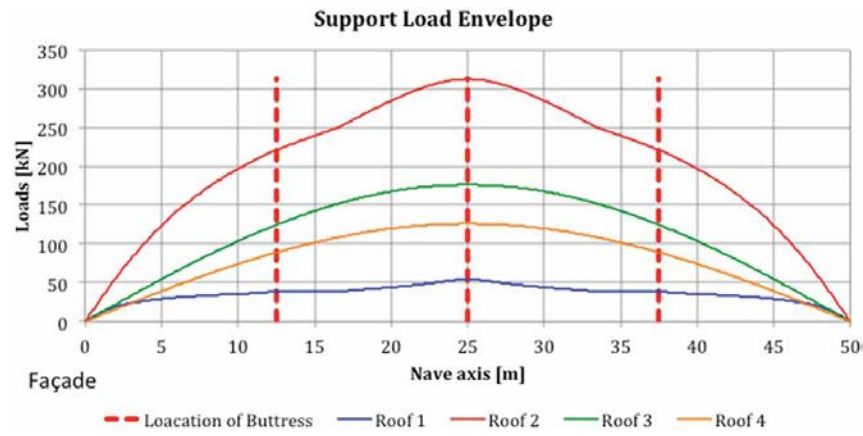


Figure 17- Results of the response spectrum analyses: shear acting on the beam.

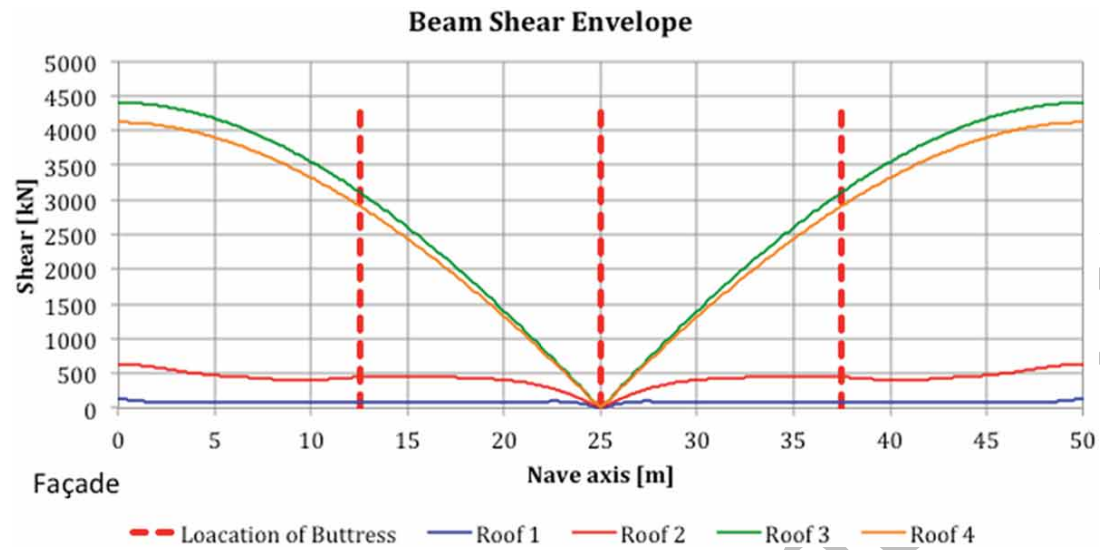


Table 1-List of earthquakes historically recorded for Assisi (Source: Locati et al.,2011)

I [MCS]	Date	Location	Np	Io	Mw
7	1747 04 17	NOCERA UMBRA	64	9	5.94 \pm 0.26
D	1747 09 22	CAMPODONICO	11	7-8	5.30 \pm 0.58
7	1751 07 27 01:00	Appennino umbro-marchigiano	68	10	6.25 \pm 0.22
NC	1781 06 03	CAGLIESE	157	10	6.42 \pm 0.13
F	1785 05 03 02:30	Alta valle del Chienti	11	7	5.14 \pm 0.34
4-5	1791 10 11 13:05	Appennino umbro	54	8	5.49 \pm 0.27
8	1832 01 13 13:00	Valle del Topino	102	10	6.33 \pm 0.14
7	1854 02 12 05:00	Valle del Topino	21	8	5.55 \pm 0.44
F	1873 03 12 20:04	Marche meridionali	196	8	5.95 \pm 0.10

F	1875 03 17 23:51	Romagna sud-orientale	144		5.93 \pm 0.16
4	1878 09 15 07:20	Valle del Clitunno	34	8	5.42 \pm 0.28
4	1881 03 11 22:50	SPOLETO	15	5	4.66 \pm 0.37
NF	1895 04 14 22:17	Slovenia	296	8	6.23 \pm 0.08
3-4	1897 12 18 07:24:20	Appennino umbro- marchigiano	132	7	5.13 \pm 0.14
4	1898 06 27 23:38	RIETI	186	8	5.49 \pm 0.12
3	1902 10 23 08:51	REATINO	77	6	4.80 \pm 0.26
4	1903 11 02 21:52	Valnerina	33	6	4.89 \pm 0.26
3	1909 01 13 00:45	BASSA PADANA	799	6-7	5.53 \pm 0.09
F	1909 08 25 00:22	MURLO	283	7-8	5.37 \pm 0.10
F	1910 06 29 13:52	MUCCIAFORA	58	7	4.86 \pm 0.33

3	1914 10 27 09:22:36	Garfagnana	618	7	5.76 \pm 0.09
5-6	1915 01 13 06:52	Avezzano	1041	11	7.00 \pm 0.09
6	1915 03 26 23:37	Assisi	40	6	4.60 \pm 0.24
2-3	1916 05 17 12:49:50	Alto Adriatico	132		5.95 \pm 0.14
2-3	1916 08 16 07:06	Alto Adriatico	257		6.14 \pm 0.14
4	1917 04 26 09:35:59	Valtiberina	134	9-10	5.89 \pm 0.11
4	1918 04 14 01:56	GIANO DELL'UMBRIA	23	6	4.55 \pm 0.32
NF	1919 09 10 16:57:20	PIANCASTAGNAIO	67	7-8	5.32 \pm 0.18
NF	1919 10 22 06:05:54	Anzio	142		5.48 \pm 0.15

2	1919 10 25 13:51:30	MONTERCHI	30	6	5.02 \pm 0.24
3	1920 09 07 05:55:40	Garfagnana	756	10	6.48 \pm 0.09
2	1922 06 08 07:47	CALDAROLA	52	6	4.89 \pm 0.19
2-3	1924 01 02 08:55:08	Medio Adriatico	76	7-8	5.36 \pm 0.16
3	1927 08 16 00:53	CASTEL SANT'ANGELO	17	6	4.56 \pm 0.27
2-3	1929 07 18 21:01:58	Mugello	56	6-7	5.02 \pm 0.17
3	1930 10 30 07:13:13	SENIGALLIA	263	8	5.81 \pm 0.09
F	1935 06 06 11:05	FOLIGNO	4	5	4.30 \pm 0.34
5	1936 04 05 18:10	FOLIGNO	3	5	4.36 \pm 0.25

5	1936 12 09 07:34	CALDAROLA	32	6-7	4.79 \pm 0.22
4-5	1950 09 05 04:08	GRAN SASSO	386	8	5.68 \pm 0.07
3	1951 08 08 19:56	Gran Sasso	94	7	5.30 \pm 0.14
5	1951 09 01 06:56:04	SARNANO	81	7	5.34 \pm 0.20
5	1979 09 19 21:35:37	Valnerina	694	8-9	5.86 \pm 0.09
NF	1980 11 23 18:34:52	Irpinia-Basilicata	1394	10	6.89 \pm 0.09
4-5	1982 10 17 04:54:35	PERUGINO	16	6	4.61 \pm 0.20
5	1982 10 17 06:45:37	Valfabbrica	32	6	4.67 \pm 0.09
5-6	1984 04 29	GUBBIO/VALFABBRICA	709	7	5.65 \pm 0.09

	05:02:60				
NF	1984 05 07 17:49:43	Appennino abruzzese	912	8	5.89 \pm 0.09
NF	1984 05 11 10:41:50	Appennino abruzzese	342		5.50 \pm 0.09
2-3	1986 10 13 05:10:01	Appennino umbro- marchigiano	322	5-6	4.65 \pm 0.09
4-5	1993 06 04 21:36:51	Nocera Umbra	90	5-6	4.50 \pm 0.13
5	1993 06 05 19:16:17	GUALDO TADINO	326	6	4.74 \pm 0.09
4-5	1997 05 12 13:50:15	MASSA MARTANA	57	6	4.79 \pm 0.17
NF	1997 07 15 08:51	Appennino umbro- marchigiano	22	4-5	3.69 \pm 0.21

4-5	1997 09 03 22:07:30	Appennino umbro- marchigiano	171	5-6	4.56 \pm 0.09
3	1997 09 07 23:28:06	Appennino umbro- marchigiano	57	5-6	4.38 \pm 0.15
NF	1997 09 09 16:54	Appennino umbro- marchigiano	39	5-6	4.07 \pm 0.18
NF	1997 09 10 06:46:51	Appennino umbro- marchigiano	47	5	4.16 \pm 0.18
5-6	1997 09 26 00:33:13	Appennino umbro- marchigiano	760		5.70 \pm 0.09
6-7	1997 09 26 09:40:27	Appennino umbro- marchigiano	869	8-9	6.01 \pm 0.09
6	1997 10 03 08:55:22	Appennino umbro- marchigiano	490		5.25 \pm 0.09
6	1997 10 06	Appennino umbro-	437		5.46 \pm 0.09

	23:24:53	marchigiano			
5	1997 10 14 15:23:11	Appennino umbro- marchigiano	786	7-8	5.65 \pm 0.09
3	1997 10 23 08:58:44	Appennino umbro- marchigiano	56		4.31 \pm 0.25
3-4	1997 11 09 19:07:33	Appennino umbro- marchigiano	180	5-6	4.90 \pm 0.09
5	1998 02 07 00:59:45	Appennino umbro- marchigiano	62	5-6	4.43 \pm 0.09
5	1998 03 21 16:45:09	Appennino umbro- marchigiano	141	6	5.03 \pm 0.09
5-6	1998 03 26 16:26:17	Appennino umbro- marchigiano	408	6	5.29 \pm 0.09
5	1998 04 05 15:52:21	Appennino umbro- marchigiano	395	6	4.81 \pm 0.09

4	1998 06 01 13:57:10	Appennino umbro- marchigiano	23	5	4.29 \pm 0.18
3	1998 06 02 23:11:23	Appennino umbro- marchigiano	83	5-6	4.28 \pm 0.09
3	1998 08 11 05:22:59	Appennino umbro- marchigiano	24	5-6	4.53 \pm 0.41
NF	2001 11 26 00:56:55	Casentino	213	5-6	4.72 \pm 0.09
4	2005 12 15 13:28:39	Valle del Topino	361	5-6	4.66 \pm 0.09
NF	2006 04 10 19:03:36	Maceratese	211	5	4.51 \pm 0.10

Table 2-Dynamic properties of the roof through the centuries.

Roof n°	Years	μ_{Beam} [kg/m]	K_{Beam} [N/rad]	μ_{Supp} [kg/m]	K_{Supp} [N/m ²]	α [°]	d [m]	ζ [-]
Roof 1	1253 - XIV	1016	2.19E+07	32857	2.01E+06	90	0.00	1
Roof 2	1475 - 1958	9834	2.34E+08	36735	1.06E+07	27	4.25	1
Roof 3	1958 - 1997	13499	4.19E+09	36735	1.06E+07	27	4.25	1
Roof 4	1997 - Present	13898	5.47E+09	36735	1.06E+07	27	4.25	1

Table 3- Results of modal analyses.

Roof n°	Years	μ [kg/m]	k_1 [N/m ²]	T_1 [sec]	Normalized Mass	Normalized Stiffness	Normalized Period
Roof 1	1253 - XIV	33873	2.09E+06	0.799	1.00	1.0	1.00
Roof 2	1475 - 1958	46569	1.15E+07	0.400	1.37	5.5	0.50
Roof 3	1958 - 1997	50234	2.71E+07	0.270	1.48	12.9	0.34
Roof 4	1997 - Present	50633	3.22E+07	0.249	1.49	15.4	0.31

Table 4- Response Spectrum results after the different interventions.

Roof n°	Years	$\max[\delta]$ [cm]	$\max[F_{Supp}]$ [kN]	$\max[S_{Beam}]$ [kN]	Normalized Displacement	Normalized Beam Shear	$\frac{\max[F_{Supp}]}{\max[S_{Beam}]}$
Roof 1	1253 - XIV	2.71	54	127	1.0	1.0	2.3
Roof 2	1475 - 1958	2.95	312	634	1.1	5.0	2.0
Roof 3	1958 - 1997	1.66	176	4400	0.6	34.7	25.0
Roof 4	1997 - Present	1.19	126	4117	0.4	32.4	32.7