

Practical method for the design of pretensioned fully grouted rockbolts in tunnels

*Original*

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1 **A practical method for the design of pre-tensioned fully grouted**  
2 **rockbolts in tunnels**

3

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5

6 **Abstract**

7 This paper develops an analytical approach to quantitatively model the efficiency of  
8 the pre-tensioning of grouted rockbolts in terms of reduction of tunnel convergence.

9 In this study, the distribution of force along the pre-tensioned fully grouted bolt is  
10 calculated by the assumption of a rigid connection between the bolt and the rock  
11 mass. A compressive force is then applied to the bolt head on tunnel surface to  
12 consider the shear relative displacement between the bolt and the rock mass. The  
13 magnitude of this compressive force is found by modeling of bolt boundaries  
14 stiffness.

15 Finally, the theoretical proposed approach is simplified to be used for the practical purposes.  
16 The results show if the stiff end plate is tightened to the bolt head (complete planner  
17 contact), the grouting effect of the pre-tensioned fully grouted bolts on tunnel stability  
18 can be neglected.

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20 **Keywords:** Analytical techniques; Tunneling; Pre-tensioning; Bolts

## 21 **Introduction**

22 The systematic grouted rockbolting is widely used as an effective technique in the  
23 design and construction of tunnels. The pre-tensioned rockbolts, which transfer initial  
24 compressive pressure to the rock mass in order to increase their performance and  
25 efficiency, are one of the best and the most appropriate supporting systems to be used  
26 in particular circumstances like delay in the bolt installation.

27 During last three decades, a great number of analytical methods have been developed  
28 for the study of passive grouted bolts in tunneling design. In a group of approaches,  
29 the obtaining of the engineering properties of reinforced rock mass has been focused  
30 e.g. with definition of a dimensionless parameter named as "*bolt density*" (which  
31 reflects the relative density of bolts with respect to the opening perimeter) to calculate  
32 the improved geo-mechanical properties of rock mass (Indraratna and Kaiser 1990a,b;  
33 Osgui and Oreste 2010), with introducing a dimensionless coefficient named as  
34 "*ground reinforcement- stiffness*" (the contrast of stiffnesses of ground and rockbolt)  
35 to be used as the multipliers to obtain the confinement stress of composite material  
36 (Carranza-Torres 2009), with presenting of a formulation for mechanical contribution  
37 of the rockbolts based on shear stress on the bolt surface (Bobet and Einstein 2011),  
38 with obtaining the elastic properties of the rock-bolt material using the shear-lag  
39 method (Bobet 2006), and with assuming the influence of bolting on rock mass as a  
40 pressure on the tunnel boundary (Bischoff and Smart 1975), as a fictitious increase of  
41 the rock mass cohesion (Grasso et al. 1989), and as an increase of confinement stress  
42 within rock mass (Fahimifar and Soroush 2005).

43 On the other hand, in another group of approaches, a comprehensive series of studies  
44 have been conducted by assuming that the grouted bolt contributes to rock mass in the

45 form of a radial pressure within the influence zone of the rockbolt (Aydan 1989; Peila  
46 and Oreste 1995; Oreste and Peila 1996; Li and Stillborg 1999; Cai et al. 2004a, b;  
47 Guan et al. 2007; Bobet and Einstein 2011).

48 Unlike the passive grouted bolts, the study of pre-tensioned grouted bolts has not been  
49 of high interest so far, and their performance is still quantitatively unknown. A few  
50 attempts which were carried out were based on the development of the works  
51 originally performed for the passive reinforcements (Carranza-Torres 2009; Fahimifar  
52 and Ranjbarnia 2009; Bobet and Einstein 2011). Hence, they have involved great  
53 limitations which may cause them to give crude predictions.

54 In order to model the pre-tensioned grouted rockbolts as a systematic support of  
55 tunnels (at least for the short-time), the relation between the value of pre-tensioned  
56 pressure on the tunnel surface (produced by the pre-tensioned force) and that of the  
57 fictitious constrained radial pressure (supplied by the proximity of tunnel face) should  
58 be particularly taken into consideration. That is, the progressively advancing tunnel  
59 face in front of bolted section leads to diminishing of the fictitious constrained radial  
60 pressure to zero and ultimately, the pre-tensioned pressure will only remain. Provided  
61 that the value of pre-tensioned pressure on the tunnel surface is greater than the  
62 constrained radial pressure, advancement of the tunnel face will not change the  
63 stresses within the rock mass around tunnel, and the ultimate load will not be  
64 changed. Meanwhile, if the value of pre-tensioned pressure is less than the fictitious  
65 constrained radial pressure, the tunnel convergence will again occur immediately after  
66 the radial pressure becomes less than the initial value prior to bolt installation.  
67 Remarking that above discussion is pertinent to the condition that tunnel convergence  
68 merely occurs due to tunnel face advancement (short-term movement).

69 Thus, the above-mentioned analytical approaches for the pre-tensioned grouted  
70 rockbolts are not appropriate solution due to either neglecting the relation of the pre-  
71 tensioned and the fictitious constrained pressures (Carranza-Torres 2009; Fahimifar  
72 and Ranjbarnia 2009) or considering constant bolt tensioning (Bobet and Einstein  
73 2011).

74 In an effort to bridge this apparent gap in the available methods and tools for analysis  
75 of reinforced tunnel by the pre-tensioned grouted rockbolts, this paper develops an  
76 analytical approach to quantitatively model the efficiency of the pre-tensioning of  
77 grouted rockbolts in terms of reduction of both tunnel convergence.

78 The distribution of force along the bolt is an important issue. In general, the bolt axial  
79 force is originated by the relative shear displacements between the bolt and the rock  
80 mass which itself affected by both the shear stiffness and the bolt boundaries  
81 conditions. Some of the previously mentioned works have obtained the axial force  
82 along the bolt e.g. with modeling of shear stress between the bolt and the rock mass,  
83 and then integrating of the corresponding function (Li and Stillborg 1999), with  
84 considering the constitutive deformation between the bolt and rock mass, and taking  
85 derivation to obtain the differential equation of axial force (Cai et al. 2004a, b). In  
86 these efforts, the boundary conditions were not taken into account i.e. the force on the  
87 tunnel wall was considered zero. Meanwhile; by the in-situ measurements and the  
88 results of numerous numerical calculations, Oreste (2008) presented a simple two-line  
89 graphic for distribution of axial force along the bolt in which the force on tunnel wall  
90 is considered for the stiff end plate.

91 All these works have been carried out for the passive bolts while no attempt has been  
92 performed for the pre-tensioned grouted types. Hence, in this paper, a new  
93 methodology is also presented to compute the distribution of the force along the pre-

94 tensioned grouted bolts. For this purpose, it is calculated by the assumption of a rigid  
95 connection between the bolt and the rock mass. A compressive force is then applied to  
96 the bolt head on tunnel surface (near boundary) to reduce the bolt force. The  
97 magnitude of this compressive force is dependent upon the near boundary condition  
98 (i.e. the stiffness of components of nut, washer, and the plate) and the far boundary  
99 condition of bolt (i.e. the shear stiffness of the initial anchored length). Therefore,  
100 these two boundaries conditions will be also modeled.

101 Finally, as the derived formula of the proposed model is too complicated for practice  
102 and preliminary design, a simple method will be introduced with employing the  
103 support and the rock mass interaction concepts on the basis of the proposed model.

104

## 105 **Modeling of systematic pre-tensioned fully grouted bolts**

### 106 **behavior in tunnels**

#### 107 **General assumptions**

108 A circular tunnel of radius  $r_i$ , under plane strain condition, is driven in a  
109 homogeneous, isotropic, initially elastic rock mass with a strain-softening behavior  
110 subjected to a hydrostatic stress field,  $p_0$ .

111 The problem is modeled with the assumption that tunnel closure is only occurred due  
112 to advancement of the tunnel face (which is equivalent to the reduction of fictitious  
113 radial pressure). Therefore, time-dependent properties of the rock mass are ignored,  
114 and short-term convergence of tunnel is only taken into account.

115 As the rockbolts are installed, a certain convergence of tunnel has already been  
116 occurred, and an initial plastic zone of radius  $\bar{r}_e$  has been developed around the tunnel  
117 (Fig. 1a).

118

119 **Theoretical concept of the pre-tensioned grouted rockbolts behavior in**  
120 **tunnel**

121 The installation process of the pre-tensioned grouted rockbolt, in this paper, consists  
122 of placing a grouted anchor, tensioning the rockbolt and tying end of the bolt by nut  
123 and plate to the tunnel surface, and then grouting the remained of the bolt length.

124 Once the pre-tensioned force is applied by the plate to tunnel surface, a radial pressure  
125 develops within the rock in the influence domain of itself. Therefore

126 
$$p_{pre-ten} = \frac{T_{pre-ten}}{C_0} \quad (1)$$

127 where  $T_{pre-ten}$  and  $p_{pre-ten}$  are the pre-tensioned force and the associated radial  
128 pressure at tunnel surface, respectively.  $C_0$  is the rockbolt effective area at tunnel  
129 surface calculated by

130 
$$C_0 = S_l \cdot S_{c_0} \quad (2)$$

131  $S_l$  and  $S_{c_0}$  are the longitudinal and tangential space of bolts at tunnel surface,  
132 respectively.

133 The advancement of tunnel face is again restarted after full installation of bolts. Then,  
134 the remained fictitious radial pressure will be further reduced and will be ultimately  
135 diminished. Accordingly, two following different circumstances can occur:

136 **Case A:** If the magnitude of the fictitious radial pressure is less than the radial pre-  
137 tensioned pressure, progressive advancement of tunnel face will not result in further  
138 radial displacement (Fig 1a). This is because, due to applying the pre-tensioned  
139 pressure, the remained radial pressure on tunnel surface (after full diminishing of the  
140 fictitious constrained pressure) is greater than the value prior to the bolt installation.

141 Thus, the final bolt force is not greater than the initial applied tension i.e. the bolt  
142 force will remain constant along the bolt and will equal to pre-tensioned force. As  
143 well, grouting the remained bolt length has no influence on its behavior mechanism  
144 but will protect the bolt from corrosion.

145 **Case B:** If the magnitude of the fictitious constrained radial pressure is greater than  
146 the pre-tensioned pressure, somewhat re-advancing of tunnel face will lead to further  
147 inward radial displacement of the rock mass. So the bolt force will increase till to full  
148 diminishing of the fictitious constrained radial pressure, and the plastic radius will  
149 become greater (Fig. 1b).

150 Reminding that the magnitude of the pre-tensioned force is a significant fraction of  
151 the bolt's yielding capacity so that it does not have a final force to yield.

152

### 153 **The analytical simulation of the radial pre-tensioned fully grouted bolts**

#### 154 ***Rigid (Ideal) connection between the bolt and rock mass***

155 In general, the grouted rockbolts reinforce and mobilize the inherent strength of the  
156 rock mass by offering internal and confining pressure (Huang et al. 2002). Assuming  
157 that the bolt contribution is in the form of a radial load spread within its influencing  
158 zone, the differential equation of equilibrium for tunnel with circular cross section,  
159 uniform in-situ stresses, and close spacing of the rockbolts will be

$$160 \quad \frac{d\sigma_r}{dr} = \frac{\sigma_\theta - \sigma_r}{r} + \frac{dT}{dr} \frac{r_i}{C_0} \frac{1}{r} \quad (3)$$

161 where  $\sigma_\theta$  and  $\sigma_r$  are the tangential and radial stresses, respectively.  $r$  is a variable  
162 showing the radial distance from tunnel center,  $T$  is the overall rockbolt tensioned  
163 force.

164 For *Case A*, as discussed in section 2-2, tunnel convergence will not increase, and the  
 165 force along the bolt will be almost constant and will equal to the pre-tensioned value  
 166 i.e.  $T = T_{pre-ten}$ . Thus,  $dT/dr$  will be zero and Eq. (4) will be resulted from Eq. (3)  
 167 (also by replacing of the Hoek- Brown strength criterion (1980) for rock mass)

$$168 \quad \frac{d\sigma_r}{dr} = \frac{[m\sigma_c\sigma_r + s\sigma_c^2]^{1/2}}{r} \quad (4)$$

169 with the following boundary condition

170 (i) At  $r = r_i$ ,  $\sigma_r = p_i$  in which  $p_{inst} \leq p_i \leq p_0$ . (Because,  $p_{pre-ten} > p_{inst}$ )

171 (ii) At  $r = r_e$ ,  $\sigma_r = \sigma_{re}$ .

172 where  $\sigma_c$  is uniaxial compressive strength of the intact rock material, and parameters  
 173  $m$  and  $s$  are rock mass constants depending on the nature of the rock mass and its  
 174 geotechnical conditions.  $p_i$  is the magnitude of radial pressure in the tunnel surface,  
 175  $p_{inst}$  is the fictitious radial pressure induced by the working face at bolt installation  
 176 time, and  $\sigma_{re}$  is the radial stress at the outer boundary of plastic zone and is obtained  
 177 by (Hoek and Brown 1980)

$$178 \quad \sigma_{re} = p_0 - M \cdot \sigma_c \quad (5)$$

179 in which

$$180 \quad M = \frac{1}{2} \left[ \left( \frac{m_p}{4} \right)^2 + m_p \frac{p_0}{\sigma_c} + s_p \right]^{1/2} - \frac{m_p}{8} \quad (6)$$

181 where parameters  $m_p$  and  $s_p$  are rock mass constants before failure.

182 For *Case B*, dwindling of the radial pressure on tunnel surface from its remained  
 183 value i.e.  $p_{inst}$  to pre-tensioned pressure i.e.  $p_{pre-ten}$  leads to increase of radial  
 184 deformations of rock mass, and imposes further tension to the bolt. Thus, the

185 differential equation for this condition will be Eq. (3) with the following boundary  
 186 conditions

187 (i) At  $r = r_i$ ,  $\sigma_r = p_i$  in which  $p_{pre-ten} \leq p_i \leq p_{inst}$

188 (ii) At  $r = r_e$ ,  $\sigma_r = \sigma_{re}$

189 The bolt axial force can be obtained by

$$190 \quad T = A_b \cdot E_s \cdot \varepsilon_b \quad (7)$$

191 where  $A_b$  and  $E_s$  are bolt cross section area and the modulus of elasticity of bolt,

192 respectively, and  $\varepsilon_b$  is the bolt axial strain calculated by

$$193 \quad \varepsilon_b = \varepsilon_r' + \varepsilon_{pre-ten} \quad (8)$$

194 where  $\varepsilon_{pre-ten}$  is the pre-tensioned strain of rockbolts, and  $\varepsilon_r'$  is the radial strain within

195 rock mass taking place after the bolts installation computed by

$$196 \quad \varepsilon_r' = \begin{cases} \varepsilon_r - \bar{\varepsilon}_r & r_i < r \leq \bar{r}_e \\ \varepsilon_r - \varepsilon_r^e & \bar{r}_e < r \leq r_e \end{cases} \quad (9)$$

197 where  $\varepsilon_r$  is total radial strain within plastic rock mass,  $\bar{\varepsilon}_r$  and  $\varepsilon_r^e$  are the radial strain

198 within the rock mass before the bolts installation in the initial and developed plastic

199 zone, respectively. Reminding that prior to the bolts installation, a plastic

200 displacement in the initial plastic zone,  $\bar{r}_e$ , and the elastic deformations in the greater

201 plastic zone,  $r_e$ , had been developed (Fig. 1b).

202 To solve differential Eqs. (3) and (4), it is essential to employ a numerical method due

203 to their algebraic complexity. For this purpose, Brown et al. (1983) analytical-

204 numerical method with inclusion of the rockbolt parameters is used to calculate

205 stresses and strains around reinforced circular tunnel. This method is an iterative finite

206 difference solution in which the plastic zone is split into annular rings. The

207 differential equation (3) is rewritten for a ring i.e.

$$\frac{\sigma_{r(j-1)} - \sigma_{r(j)}}{r_{(j-1)} - r_{(j)}} = \frac{\left[ \frac{m_a \sigma_c}{2} (\sigma_{r(j)} + \sigma_{r(j-1)}) + s_a \sigma_c^2 \right]^{1/2}}{\frac{r_{(j)} + r_{(j-1)}}{2}} + \frac{T_{(j-1)} - T_{(j)}}{r_{(j-1)} - r_{(j)}} \cdot \frac{r_i}{C_0} \cdot \frac{2}{r_{(j)} + r_{(j-1)}}$$

208

209

(10)

210 in which

211

$$m_a = \frac{m_{(j-1)} + m_{(j)}}{2} \quad (11)$$

212

$$s_a = \frac{s_{(j-1)} + s_{(j)}}{2}$$

213

(12)

214 Manipulating Eq. (10) results the second order equation giving  $\sigma_{r(j)}$

215

$$a \cdot \sigma_{r(j)}^2 + b \cdot \sigma_{r(j)} + c = 0 \quad (13)$$

216 and solution is

217

$$\sigma_{r(j)} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

218 For  $r_i < r \leq \bar{r}_e$  zone

219

$$a = \frac{1}{4K^2}, \quad b = -\frac{K_1 - \bar{K}_1}{K} - \frac{\sigma_{r(j-1)}}{2K^2} - 2K_2$$

220

$$c = \sigma_{r(j-1)} \left[ \frac{\sigma_{r(j-1)}}{4K^2} + \frac{K_1 - \bar{K}_1}{K} - 2K_2 \right] + (K_1 - \bar{K}_1)^2 - s_a \sigma_c^2$$

221

(14)

222 where

223

$$\gamma = d\varepsilon_{r(j)} = \varepsilon_{r(j)} - \varepsilon_{r(j-1)} \quad (15)$$

224 
$$K_1 = \frac{A_b E_s r_i}{C_0 (r_{(j-1)} - r_{(j)})} \gamma \quad (16)$$

225 
$$\bar{\gamma} = d\bar{\varepsilon}_{r(j)} = \bar{\varepsilon}_{r(j)} - \bar{\varepsilon}_{r(j-1)} \quad (17)$$

226 
$$\bar{K}_1 = \frac{A_b E_s r_i}{C_0 (r_{(j-1)} - r_{(j)})} \bar{\gamma} \quad (18)$$

227 and for  $\bar{r}_e < r \leq r_e$  zone

228 
$$a = \frac{1}{4K^2}, \quad b = -\frac{K_1 - K_1^e}{K} - \frac{\sigma_{r(j-1)}}{2K^2} - 2K_2$$

229 
$$c = \sigma_{r(j-1)} \left[ \frac{\sigma_{r(j-1)}}{4K^2} + \frac{K_1 - K_1^e}{K} - 2K_2 \right] + (K_1 - K_1^e)^2 - s_a \sigma_c^2$$

230 (19)

231 where

232 
$$\gamma^e = d\varepsilon_{r(j)}^e = \varepsilon_{r(j)}^e - \varepsilon_{r(j-1)}^e$$

233 (20)

234 
$$K_1^e = \frac{A_b E_s r_i}{C_0 (r_{(j-1)} - r_{(j)})} \gamma^e \quad (21)$$

235 After finding the distribution of the stress and strain around circular tunnel, the axial  
 236 force along the bolt (in the ideal condition) can be obtained by Eq. (7).

237

238 ***Modeling of shear displacement between the bolt and rock mass***

239 To find the bolt force in the reality (to be used in Eq. (10)), the relative shear  
 240 displacement between the bolt and the rock should be calculated. For this purpose, the  
 241 following new method is proposed. The force applied through the bolt head deforms  
 242 the tunnel surface beneath the plate, and the bolt elongation is reduced (Fig. 2). It can

243 be said that the reduction of the reinforcement elongation ( $\delta_{rein}$ ) is identical to the  
 244 deformation of the tunnel surface ( $\Delta_s$ ).

$$245 \quad |\Delta_s| = |\delta_{rein}| \quad (22)$$

246 In fact, a portion of the force through the bolt head is devoted for the initial bedding in  
 247 the components of nut and washer on the plate, bedding of the plate on the rock mass,  
 248 and compressing of the rock mass. Hence, the force in the bolt head is reduced from  
 249  $T_{max}$  to  $T_s$ . ( $T_{max}$  and  $T_s$  are the forces through the bolt head in the ideal and the real  
 250 conditions, respectively).

251 From Eq. (22), it can be written

$$252 \quad \frac{T_s}{K_s} = \frac{T_{rein}}{K_{rein}} \quad (23)$$

253 where  $K_{rein}$  is the shear stiffness of the reinforcing element, and  $K_s$  describes the  
 254 equivalent stiffness of the components of nut and washer and the plate's basement.  
 255 When a stiff end plate tightened to the bolt head (perfect constraint), it is estimated  
 256 that  $K_s \cong (0.5 - 0.8)K_{rein}$  (For the weak rock mass and high in-situ stress, the lower  
 257 coefficient is used). This is proven in Appendix A (II). However; when a perfect  
 258 constraint is not guaranteed, the magnitude of  $K_s$  is drastically reduced (Oreste 2008)  
 259 and becomes a very small value.  $T_{rein}$  is the magnitude of bolt head force reduction i.e.

$$260 \quad T_{rein} = T_{max} - T_s \quad (24)$$

261 Combining Eq. (23) and Eq. (24), it can be written

$$262 \quad T_s = \frac{K_s}{K_s + K_{rein}} T_{max} \quad (25)$$

263 From above discussion, it can be assumed that  $T_{rein}$  acts as a compression force  
 264 through the bolt head to reduce the bolt elongation and reduces the bolt force from  
 265  $T_{max}$  to  $T_s$ .

266 In the case of the pre-tensioned grouted rockbolts, the computation of the real force  
 267 applied by the bolt to the tunnel surface may be carried out in two steps as follows:

- 268 (1) The computation of the real force due to the pre-tensioned force.  
 269 (2) The computation of the real force due to the subsequent load may probably  
 270 occur after full grouting of the bolt length.

271 For *Case A*, the real force should be only computed due to the pre-tensioned force. As  
 272 fictitious compression force acts from the bolt head towards the rock mass, the  
 273 reduction takes place in two sections of the bolt length i.e. in the free length section  
 274 (un-grouted section) and in the initially anchored length section as observed in Fig.  
 275 (3).

$$276 \quad \delta_{rein} = \delta_{free} + \delta_{anch} \quad (26)$$

277 where  $\delta_{free}$  and  $\delta_{anch}$  are respectively the reduction of the bolt elongation in the free  
 278 length and anchored length of the bolt obtained by

$$279 \quad \delta_{free} = \frac{T_{rein}}{K_{free}} \quad (27)$$

$$280 \quad \delta_{anch} = \frac{T_{rein}}{K_{anch}} \quad (28)$$

281 in which

$$282 \quad K_{free} = \frac{E_b \cdot A_b}{L_{free}} \quad (29)$$

283 
$$K_{anch} = \frac{H}{\lambda} \left( \frac{e^{\lambda L_{anch}} + e^{-\lambda L_{anch}}}{e^{\lambda L_{anch}} - e^{-\lambda L_{anch}}} \right) \quad (30)$$

284 where  $K_{free}$  and  $K_{anch}$  are the axial stiffness of the free and total anchored length of the  
 285 bolt for *Case A*, respectively; (See Appendix A (I) for detailed derivation of  $K_{anch}$ ).

286  $L_{free}$  and  $L_{anch}$  are the free and the anchored length of the bolt, and  $H$  is a material  
 287 parameter associates to the shear stiffness between the bolt and the rock mass, and its  
 288 formulation is available in Appendix A (I).  $\lambda$  is a parameter defined as

289 
$$\lambda = \left( \frac{H}{E_{rein} \cdot A_{rein}} \right)^{0.5} \quad (31)$$

290 where  $A_{rein}$  and  $E_{rein}$  are the area section and the elasticity Modulus of the  
 291 reinforcement, respectively.

292 Substituting Eqs. (27) and (28) into Eq. (26), and then simplifying gives

293 
$$K_{rein}^A = \frac{K_{free} \cdot K_{anch}}{K_{free} + K_{anch}} \quad (32)$$

294 For *Case A*,  $T_{max}$  in Eq. (27) equals to  $T_{pre}$ , and superscript  $A$  in above equations  
 295 refers to *Case A*.

296 Combining Eq. (25) and Eq. (32) gives the real force on the tunnel surface for *Case A*  
 297 (or for the pre-tensioned force)

298 
$$T_s^A = \frac{K_s (K_{free} + K_{anch})}{K_s K_{free} + K_s K_{anch} + K_{free} K_{anch}} T_{pre} \quad (33)$$

299 and from Eq. (24)

300 
$$T_{rein}^A = \eta \cdot T_{pre} \quad (34)$$

301 
$$\eta = \frac{K_{free} K_{anch}}{K_s K_{free} + K_s K_{anch} + K_{free} K_{anch}} \quad (35)$$

302 Now, from Eq. (24), distribution of force can be obtained. As the initial anchored  
 303 length is assumed to be located beyond the plastic zone, the distribution of force in  
 304 that length of bolt is not here studied.

305 For *Case B*, the bolt is tensioned by both the pre-tensioned force (Fig. 4a) and the  
 306 movement of rock mass towards tunnel (Fig. 4b). The real force applied on the tunnel  
 307 surface by the pre-tensioned force can be calculated by a similar formulation of *Case*  
 308 *A*. The real force applied on the tunnel surface by the movement of rock mass can be  
 309 obtained by

$$310 \quad T_s^{(2)} = \frac{K_s}{K_s + K_{rein}^{(2)}} T_{max}^{(2)} \quad (36)$$

311 where  $K_{rein}^{(2)}$  is the axial stiffness of free length of the bolt where it is grouted after  
 312 pre-tensioning calculated by

$$313 \quad K_{rein}^{(2)} = \frac{H}{\lambda} \left( \frac{e^{\lambda L} + e^{-\lambda L}}{e^{\lambda L} - e^{-\lambda L}} \right) \quad (37)$$

314 where  $L$  is the length of the bolt located in the plastic zone, and  $T_{max}^{(2)}$  is the  
 315 maximum force of the bolt in the second step. It can be obtained by subtracting the  
 316 pre-tensioned force from the total maximum force in the ideal condition. (Superscripts  
 317 (1) and (2) refer to the first and second steps of the bolt tensioning, respectively).

318 The total real force on tunnel surface may be computed by

$$319 \quad T_s^{(1)} + T_s^{(2)} = \frac{K_s (K_{free} + K_{anch})}{K_s K_{free} + K_s K_{anch} + K_{free} K_{anch}} T_{pre} + \frac{K_s}{K_s + K_{rein}^{(2)}} T_{max}^{(2)} \quad (38)$$

320 Considering that Eq. (33) is the extension of Eq. (25), Eq. (38) will be

$$321 \quad T_s^B = \frac{K_s}{K_s + K_{rein}^{(1)}} T_{pre} + \frac{K_s}{K_s + K_{rein}^{(2)}} (T_{max} - T_{pre}) \quad (39)$$

322 where  $T_s^B$  is the real force on the tunnel surface for *Case B*.

323 The distribution of real force along the bolt for *Case B* will be obtained by

324 • For the first step (similar to *Case A*)

$$325 \quad T^{(1)} = T_{pre} - T_{rein}^{(1)} \quad 0 \leq x < L_{free}$$

326 (40a)

327 • For the second step (coupling behavior of the bolt and the rock mass)

$$328 \quad T^{(2)} = T_{ideal}^{(2)} - \left[ - \left( T_{max}^{(2)} - T_s^{(2)} \right) \frac{e^{-\lambda L}}{e^{-\lambda L} + e^{\lambda L}} e^{\lambda \cdot x} - \left( T_{max}^{(2)} - T_s^{(2)} \right) \frac{e^{\lambda L}}{e^{-\lambda L} + e^{\lambda L}} e^{-\lambda \cdot x} \right]$$

329  $0 \leq x < L_{free}$

330 (40b)

331 where  $x$  denotes the arbitrary section of bolt length i.e.  $x = 0$  at  $r = r_i$  and  $x = L$  at

332  $r = r_e$ .

333 Therefore, summing of Equations (40a) and (40b), and after some manipulations, the

334 distribution of axial force along the bolt for *Case B* can be calculated as

$$335 \quad T = T_{ideal} - \eta \cdot T_{pre} - \left[ - \left( T_{max} - T_{pre} \right) (1 - \beta) \left( \frac{e^{-\lambda L}}{e^{-\lambda L} + e^{\lambda L}} \right) \left( e^{\lambda \cdot x} + e^{2\lambda L} e^{-\lambda x} \right) \right]$$

336 (41)

337 in which

$$338 \quad \beta = \frac{K_s}{K_s + K_{rein}^{(2)}} \quad (42)$$

339

### 340 **Calculation of tunnel convergence considering the real force**

341 As the obtained equilibrium equation is solved by the finite difference method (i.e.

342 Eq. (10)), Eq. (41) should be written as the iterative way. For ring  $r_{(j)}$

$$T_{(j)} = T_{ideal(j)} - \eta \cdot T_{pre} - \left[ - (T_{max} - T_{pre}) (1 - \beta) \left( \frac{e^{-\lambda L}}{e^{-\lambda L} + e^{\lambda L}} \right) \left( e^{\lambda \cdot (r_{(j)} - r_i)} + e^{2\lambda L} e^{-\lambda (r_{(j)} - r_i)} \right) \right]$$

344 (43)

345 Substituting Eq. (43) into Eq. (10) gives

$$\frac{\sigma_{r(j-1)} - \sigma_{r(j)}}{r_{(j-1)} - r_{(j)}} = \frac{\left[ \frac{m_a \sigma_c (\sigma_{r(j)} + \sigma_{r(j-1)}) + s_a \sigma_c^2 \right]^{1/2}}{\frac{r_{(j)} + r_{(j-1)}}{2}} + \left| \frac{T_{(j-1)} - T_{(j)}}{r_{(j-1)} - r_{(j)}} \cdot \frac{r_i}{C_0} \cdot \frac{2}{r_{(j)} + r_{(j-1)}} \right|$$

347 (44)

348 The value of  $T_{(j-1)} - T_{(j)}$  in the either side of neutral point (the location of maximum  
349 force along the bolt) is the opposite to each other; the absolute value is used in Eq.

350 (44)

351 The similar performing process and defined parameters which were used to solve Eq.  
352 (10) are applied to Eq. (44) except that the parameter  $K_1 - \bar{K}_1$  (or  $K_1 - K_1^e$ ) is  
353 replaced by Eq. (45).

$$K_1 = \frac{T_{(j-1)} - T_{(j)}}{r_{(j-1)} - r_{(j)}} \cdot \frac{r_i}{C_0}$$

354 (45)

355 Hence, multipliers of the second order Eq. (13) will change to

$$b = \frac{K_1}{K} - \frac{\sigma_{r(j-1)}}{2K^2} - 2K_2 \quad a = \frac{1}{4K^2}$$

356

$$c = \sigma_{r(j-1)} \left[ \frac{\sigma_{r(j-1)}}{4K^2} - \frac{K_1}{K} - 2K_2 \right] + K_1^2 - s_a \sigma_c^2$$

357 (46)

359 Appendix B sets out the stepwise sequence of calculations provided in the section 2.

360 The consideration of a relative shear displacement results in a rotation of principal  
361 stresses. That is, the radial and tangential stresses will not be longer principal stresses

362 as assumed in the ideal derivation of bolt force. However, it is assumed in this paper,  
363 the produced shear stress is not so great that the principal stresses direction is greatly  
364 changed to eclipse the results. It is a venial assumption at least in some conditions e.g.  
365 where the pre-tensioned force value, the bolt's density, or rock mass Young's Modulus  
366 is great.

367

## 368 **Examples**

369 A computer program was prepared to solve the differential equations developed by  
370 the finite difference method.

371 **Example 1.** The proposed theoretical solution is applied to the Kielder experimental  
372 tunnel to compare the accuracy of its results with the actual performance of bolts. The  
373 Kielder Experimental Tunnel was driven through four rock mass types. The tunnel in  
374 the mudstone was highly unstable, and required most support. The engineering  
375 properties of mudstone are available in Table (1). Eight sections with different  
376 support systems were constructed in which extensometers were also installed to  
377 monitor movement of the rock mass. One of the sections was left unsupported while  
378 two sections included combination of the passive grouted rockbolts and shotcrete.  
379 One of the sections is also supported by passive grouted rockbolts only. The  
380 geometrical parameters of two systems are available in Table (2).

381 According to Ward et al. (1976), total short-term movement of tunnel surface in the  
382 unsupported section of mudstone was about 8 *mm* in which less than 1 *mm* had  
383 occurred before the face reached, and about 6 *mm* when the face had advanced 2 *m*  
384 beyond this position. If the reinforcement system was installed just in front of the

385 face, it can be expected that tunnel closure was about 1-2 *mm* prior to bolt installation  
386 (assumed value is 1.5 *mm* in this paper).

387 Fig. (5) shows the corresponding ground response curves, and Table (3) gives the  
388 calculated and the measured deformations data at tunnel surface for the supported  
389 and unsupported rock mass (sequence of calculations was performed by the algorithm  
390 presented in Appendix B). As observed, the proposed method can almost predict the  
391 identical results and agree with the in-situ measurements in a satisfactory way.

392 A perfect constraint from the end- plate is predicted in the case of using rockbolt  
393 together with shotcrete. This is because, a complete planner contact between the bolt  
394 head and the tunnel surface is obtained, and the bolts will take higher loads at the  
395 tunnel surface in comparison with the condition that not perfect constraint is foreseen.

396 **Example 2.** A highway tunnel with 10.7 *m* in diameter is driven in a fair to good  
397 quality limestone at a depth of 122 *m* below the surface (Brown et al. 1983). The  
398 material property data for the rock mass and in-situ stress are available in Brown et al.  
399 (1983).

400 The pre-tensioned grouted rockbolts are installed by  $T_{pre-ten} = 17 \text{ ton}$  with  $C_0 = 1 \text{ m}^2$ ,  
401  $L = 3.15 \text{ m}$  when the fictitious constrained pressure is  $p_{inst} = 16.5 \text{ ton/m}^2$  (the  
402 other parameters is assumed to be similar to Example 1). If it is assumed a complete  
403 constrained is provided by the end plate, the pre-tensioned pressure is greater than the  
404 fictitious constrained pressure of tunnel face. Consequently, the circumstance of *Case*  
405 *A* will take place. The output results are shown in Fig. (6) and Table (4). The  
406 efficiency of pre-tensioning can now be best assessed and observed. Therefore, the  
407 convergence of tunnel by the pre-tensioning of bolts is reduced considerably.

408 If the pre-tensioned grouted rockbolts are installed by  $T_{pre-ten} = 10 \text{ ton}$ , the  
409 circumstance of *Case B* will take place. Fig. (6) shows the ground response curve for  
410 this case.

411

## 412 **A new practical method for the design of the pre-tensioned** 413 **grouted bolts**

414 Although the proposed model almost predicts the accurate performance of the passive  
415 and the pre-tensioned grouted bolts, its formulas are too complicated to be used as a  
416 preliminary design tool, and always need a computer program to carry out the  
417 computation procedures. Hence, it will be worth introducing a simple method on the  
418 basis of new presented approach parameters to be used as a rule of thumb method in  
419 practice.

420 For *Case A*, the bolt and the rock mass interaction behavior is similar to that of the  
421 support systems (such as shotcrete or the pre-tensioned un-grouted bolts) rather than  
422 to the reinforcement systems. Therefore, Ground Response Curve (GRC) of the un-  
423 supported rock mass and Support Characteristic Curve of a pre-tensioned bolt are  
424 plotted (solid line for this Case) to obtain the ultimate tunnel convergence. As seen in  
425 Fig. (7), the ultimate convergence is equal to that in the installation time.

426 The pre-tensioned un-grouted bolt characteristic curve can be obtained by the Eq. (47)  
427 (Stille et al. 1989)

$$428 \quad p_i = k_{sys} \cdot \Delta u_i + p_{pre-ten} \quad (47)$$

429 where  $k_{sys}$  is the support system stiffness calculated by (Stille et al. 1989)

$$430 \quad k_{sys} = \frac{A_b \cdot E_b}{C_0} \frac{1}{L_{free}} \frac{1}{\xi} \quad (48)$$

431 Eq. (48) is the stiffness of support system which the reinforcement effect is smeared  
 432 within the zone of its influence. Thus, the stiffness of a single element is calculated by

$$433 \quad k = \frac{A_b \cdot E_b}{L_{free}} \frac{1}{\xi} = K_{free} \frac{1}{\xi}$$

$$434 \quad (49)$$

435  $\xi$  is a factor describing the local deformations occurring in the anchoring zone (the  
 436 far boundary), under the end plate and the bolt head (the near boundary). Stille et al.  
 437 (1989) pointed out that  $\xi$  is an empirical factor which can be determined from Hoek  
 438 and Brown's (1980) published pull-out tests data of a variety of mechanical and  
 439 chemically anchored rockbolts. However, those data were not guaranteed to give the  
 440 accurate results, and were strongly recommended to be determined from field tests on  
 441 the bolts for critical applications.

442 It seems that it will be worth developing an analytical approach to obtain  $\xi$ .

443 Flexibility of the complex of bolt head component and the initial anchored length lead  
 444 to decreasing the axial stiffness of single reinforcement. This is because their  
 445 deformations under the applied force reduce bolt elongation.

446 On the other hand, according to the proposed method, the axial stiffness of pre-  
 447 tensioned un-grouted rockbolt can be calculated by

$$448 \quad \frac{1}{K_b} = \frac{1}{K_{anch}} + \frac{1}{K_{free}} + \frac{1}{K_s} \quad (50)$$

449 where  $K_b$  is total axial stiffness of reinforcement element.

450 Equating right hand of Eq. (50) with that of Eq. (49), and then simplifying gives

$$451 \quad \xi = 1 + \frac{K_{free}}{K_{anch}} + \frac{K_{free}}{K_s} \quad (51)$$

452 For *Case B*, the ultimate tunnel convergence will be at the intersection point of the  
453 diagonal line of Support characteristic curve and the Ground Response Curve of the  
454 un-supported rock mass (Dashed line in Fig. 7).  
455 However, this solution is not very exact in case the entire length of the bolt is grouted,  
456 and the bolt interacts with its surrounding grout and rock mass. In other words, the  
457 bolts confine tunnel convergence not only by applying radial pressure to tunnel  
458 surface (like the support systems e.g. un-grouted pre-tensioned bolts), but also by  
459 improving rock mass strength quality (like the reinforcement systems e.g. the passive  
460 grouted rockbolts).  
461 Therefore, to extend this new approach for *Case B*, the pre-tensioned grouted  
462 rockbolts behavior is simulated as a combination of both the support and the  
463 reinforcement systems. That is, the improved rock mass and the pre-tensioned un-  
464 grouted bolts act independently. The ground response curve of reinforced rock mass  
465 by the passive grouted rockbolts is calculated and plotted, and then the support  
466 characteristic curve of the pre-tensioned un-grouted bolts plotted separately. The  
467 intersection point of two curves gives tunnel convergence which is reinforced by pre-  
468 tensioned grouted rockbolts.  
469 No end-plate should be considered for the passive grouted rockbolts. This is because  
470 the end-plate effect is taken into account in the behavior of un-grouted pre-tensioned  
471 bolt. If the end-plate does not exist, then  $K_s$  will be zero, and  $T_s = 0$ . Consequently  
472 the distribution of axial stress along the passive grouted bolts without the end-plate is

$$T^{pass} = T_{ideal}^{pass} - \left[ -T_{max}^{pass} \frac{e^{-\lambda L}}{e^{-\lambda L} + e^{\lambda L}} e^{\lambda \cdot x} - T_{max}^{pass} \frac{e^{\lambda L}}{e^{-\lambda L} + e^{\lambda L}} e^{-\lambda \cdot x} \right]$$

473

474 (52)

475 Superscript *pass* refers to the passive grouted bolts.

476 This new approach is employed to solve Example 2 for *Case B*. The output results are  
477 available in Table (5) and Fig. (8). The intersection point of the support characteristic  
478 curve and the ground response curve of the reinforced tunnel (by the passive grouted  
479 bolts without face-plate) gives the ultimate convergence of tunnel. As observed, this  
480 approach predicts almost the identical convergence obtained in Example 2.

481 The convergence of tunnel supported by un-grouted pre-tensioned rockbolts is almost  
482 identical to that of employing the grouted types. In other words, the grouting effect of  
483 bolt is not very effective, and can be neglected. However, on the basis of the proposed  
484 model concepts and as it can be seen from Fig. (8), when either the pre-tensioned  
485 force is not great enough or the stiffness of the pre-tensioned bolt system is small (e.g.  
486 the value of  $\xi$  is great), the grouting effect can be considerable.

487 As a practical design tool, if complete constraint is provided for the near end of bolt  
488 head, the pre-tensioned fully grouted bolts can be treated as un-grouted types and its  
489 grouting effect is only considered as a factor improving safety.

490

## 491 **Conclusions**

492 New analytical approach was proposed for the design the pre-tensioned grouted  
493 rockbolts in tunnels based on convergence confinement method. The relationship  
494 between the value of constrained radial stress at bolt installation time and the value of  
495 applied pre-tensioned pressure was focused on in process of modeling. The near and  
496 far boundaries conditions of bolt were also analytically modeled because they can  
497 affect the performance of pre-tensioned bolts.

498 Due to the complexity of theoretical approach for design purposes, a simple method  
 499 on the basis of new given approach was finally introduced.  
 500 The practical outcome of this paper is that if the complete constraint is provided for  
 501 the near end of bolt head, the grouting effect of the pre-tensioned fully grouted bolts  
 502 on tunnel stability can be neglected. Therefore, they can be designed by the similar  
 503 approach of un-grouted pre-tensioned bolts. However, if it is not possible to apply  
 504 sufficient pre-tensioned force to the bolts (the pre-tensioned force is not great  
 505 enough), if the anchoring system of bolt is not proper e.g. using the expansion shell or  
 506 weak grout, or if the complete planner contact between the bolt head and the tunnel  
 507 surface is not predicted, the grouting effect will be considerable and the attention  
 508 should be taken to grout quality.

509

510 **Appendix A.**

511 **(I) Calculation of the axial stiffness of anchored length and full length of grouted**  
 512 **rockbolt**

513 As the pre-tensioned force is applied, the free and the anchored length of the bolt are  
 514 tensioned. The equilibrium of the axial force in the anchored length is

515 
$$T + dT = T + \tau \cdot \pi \cdot d_b \cdot dx \quad (53)$$

516 where  $T$  is the force in the anchored length,  $d_b$  is diameter of bolt, and  $\tau$  is the shear  
 517 stress on reinforcement perimeter which can be obtained by

518 
$$\tau = K_{ini} \cdot v \quad (54)$$

519 where  $v$  is the relative displacement between the rock mass and the bolt,  $K_{ini}$  is the  
 520 initial shear stiffness between the bolt and the rock mass expressed as (Cai et al.  
 521 2004a,b)

522 
$$K_{ini} = \frac{H}{\pi d_{rein}} \quad (55)$$

523 where  $H$  is a material parameter associated to the shear stiffness between the bolt and  
 524 the rock mass and can be computed by Eq. (56)

525 
$$H = \frac{2\pi G_g G_m}{[\ln(R/r_b) - 1/2]G_g + \ln(r_g/r_b)G_m} \quad (56)$$

526 where  $r_b$  and  $r_g$  are radius of the bolt and radius of the grout borehole;  $G_g$  and  $G_m$  is  
 527 shear modulus of the grout mortar and the rock mass, respectively; and  $R$  is the  
 528 influence radius of a single rock bolt.

529 Substituting Eq. (54) into Eq. (53) and then taking derivation gives

530 
$$\frac{d^2T}{dx^2} - \lambda^2 T = 0 \quad (57)$$

531 in which

532 
$$\frac{dv}{dx} = \frac{T}{E_b \cdot A_b} \quad (58)$$

533 
$$\lambda = \left( \frac{K_{ini} \cdot \pi d_b}{E_b A_b} \right)^{0.5} = \left( \frac{H}{E_b A_b} \right)^{0.5} \quad (59)$$

534  $A_b$  and  $E_b$  are the area section and the elastic modulus of the bolt, respectively.

535 The solution of above differential equation is

536 
$$T = C_1 e^{\lambda \cdot x} + C_2 e^{-\lambda \cdot x} \quad (60)$$

537  $C_1$  and  $C_2$  are constants obtained by the following boundary conditions

538 At  $x = L_{free}$   $T = -T_{rein} = -(T_{max} - T_s)$

539 and at  $x = L_{free} + L_{anch}$   $v = 0$

540 where  $L_{free}$  is the free length of the bolt which is not grouted in pre-loading process,

541  $L_{anch}$  is the initially anchored length of the bolt securing the anchoring capacity

542 against pre-loading.

543 Note that  $T_{max}$  is equal to  $T_{pre}$  for *Case A*, (and also for the first step of *Case B*).

544 Because, the maximum force is  $T_{pre}$  for this case.

545 Substituting  $C_1$  and  $C_2$  into Eq. (60) and then calculating  $v$  at  $x = L_{free}$  gives the

546 magnitude of displacement of the bolt in anchored section i.e.

$$547 \quad v_{anch} = \frac{\lambda T_{rein}}{H} \left( \frac{e^{\lambda L_{anch}} - e^{-\lambda L_{anch}}}{e^{\lambda L_{anch}} + e^{-\lambda L_{anch}}} \right) \quad (61)$$

548 therefore

$$549 \quad K_{anch} = \frac{H}{\lambda} \left( \frac{e^{\lambda L_{anch}} + e^{-\lambda L_{anch}}}{e^{\lambda L_{anch}} - e^{-\lambda L_{anch}}} \right) \quad (62)$$

550  $K_{anch}$  is the axial stiffness of bolt in the anchored section.

551 Performing the same process for the second step of *Case B* with the following

552 boundary condition

$$553 \quad \text{At} \quad x = 0 \quad T = -T_{rein}^{(2)} = -(T_{max} - T_{pre}) - T_s^{(2)}$$

$$554 \quad \text{and at} \quad x = L \quad v = 0$$

555 gives

$$556 \quad K_{rein}^{(2)} = \frac{H}{\lambda} \left( \frac{e^{\lambda L} + e^{-\lambda L}}{e^{\lambda L} - e^{-\lambda L}} \right) \quad (63)$$

557 where  $T_{max}$  is the force on the bolt head in the ideal connection between the bolt and

558 the rock mass, and  $T_s^{(2)}$  is the force on the bolt head applied on the tunnel surface in

559 real condition. Superscript (2) refers to the second step of the bolt tensioning which  
560 is due to rock mass movement towards tunnel.

561

## 562 **(II) Estimation of $K_s$**

563 From the analysis of the in situ measurements and the results of numerous bi-  
564 dimensional numerical calculation in the performed parametric study, Oreste (2008)  
565 presented the axial force along the passive grouted bolts, with a certain  
566 approximation, by a simple two-line graphic (Fig. 9). When a perfect constraint on the  
567 bolt head is predicted, the maximum value of bolt force is at the distance of about  
568  $L/6$  from the tunnel wall while the value in the bolt head ( $T_s$ ) is  $2/3$  of the  
569 maximum force along the bolt i.e.

$$570 \quad T_s = \frac{2}{3} T'_{\max} \quad (64)$$

571 As the distribution of axial force in the “pick up length” is exponential, it can be  
572 written  $T_s = (0.35 - 0.5)T_{\max}$  (Ranjbarnia 2014). Therefore

$$573 \quad K_s = (0.5 - 0.8)K_{rein} \quad (65)$$

574 Evaluation of the results of theoretical approaches carried out for the modelling of  
575 passive grouted bolts (Stille et al. 1989; Oreste and Peila 1996; Li and Stillborg 1999,  
576 Cai et al. 2004a) and in-situ measurements (Ward et al. 1976) shows the suitability of  
577 Eq. (65).

578  $K_{rein}$  in Eq. (65) is obtained by Eq. (42). This is because; it gives the reinforcement  
579 stiffness for the second step of loading which is identical to the loading process of the  
580 passive grouted bolts.

581

582 **Appendix B. Ground response curve calculation for reinforced**

583 **tunnel**

584 *Input data.*

585  $\sigma_c$  : un- axial compressive strength of intact rock pieces.

586  $m, s$  : material constants for original rock mass.

587  $E_m, \nu$  : Young's modulus and Poisson's ratio of original rock mass.

588  $G_g$  : shear modulus grout.

589  $m_r, s_r$  : material constants for broken rock mass.

590  $f, h$  : gradients of  $-\varepsilon_3^p$  vs.  $\varepsilon_1^p$  lines in the residual and the strain softening stages,

591 respectively.

592  $\mu$  : constant defining strain at which residual strength is reached.

593  $p_0$  : in situ hydrostatic stress.

594  $r_i$  : tunnel radius.

595  $d_b$  : the bolt diameter.

596  $d_g$  : the hole diameter.

597  $A_b$  : cross section area of each bolt.

598  $E_s$  : Young's modulus of bolt.

599  $C_0$  : bolt's spacing.

600  $T_{pre-ten}$  : pre-tensioning force.

601  $L_{anch}$  : initial anchored length.

602  $L$  : total bolt length.

603  $R$  : the influence limit of each bolt usually is  $10d_g$

604

605 **Preliminary Calculations**

606 1)  $M = \frac{1}{2} \left[ \left( \frac{m}{4} \right)^2 + m \frac{p_0}{\sigma_c} + s \right]^{1/2} - \frac{m}{8}$

607 2)  $G_m = \frac{E_m}{2(1+\nu)}$

608 3)  $H = \frac{2\pi G_g G_m}{[\ln(R/r_b) - 1/2]G_g + \ln(r_g/r_b)G_m}$

609 4)  $\lambda = \left( \frac{H}{E_s \cdot A_b} \right)^{0.5}$

610 5)  $p_{pre-ten} = T_{pre-ten} / (C_0)$

611 6)  $\varepsilon_{pre-ten} = T_{pre-ten} / (A_b \cdot E_s)$

612 7)  $L_{free} = L - L_{anch}$

613 8)  $K_{rein} = \frac{H}{\lambda} \left( \frac{e^{\lambda L} + e^{-\lambda L}}{e^{\lambda L} - e^{-\lambda L}} \right)$

614 9)  $K_s = \chi \cdot K_{rein} \quad 0.5 < \chi < 0.8$

615 10)  $K_{anch} = \frac{H}{\lambda} \left( \frac{e^{\lambda L_{anch}} + e^{-\lambda L_{anch}}}{e^{\lambda L_{anch}} - e^{-\lambda L_{anch}}} \right)$

616 11)  $K_{free} = \frac{E_b \cdot A_b}{L_{free}}$

617 12)  $K_{rein}^{(1)} = \frac{K_{free} \cdot K_{anch}}{K_{free} + K_{anch}}$

618 13)  $K_{rein}^{(2)} = \frac{H}{\lambda} \left( \frac{e^{\lambda L} + e^{-\lambda L}}{e^{\lambda L} - e^{-\lambda L}} \right)$

619 14)  $\alpha = \frac{K_s}{K_s + K_{rein}}^{(1)}$

620 15)  $\beta = \frac{K_s}{K_s + K_{rein}}^{(2)}$

621 16)  $\eta = \frac{K_{free} K_{anch}}{K_s K_{free} + K_s K_{anch} + K_{free} K_{anch}}$

622

623 **Calculations for the first ring**

624 1)  $r_{(1)} = r_e$

625 2)  $\varepsilon_{\theta(1)} = \varepsilon_{\theta(e)} = M\sigma_c / 2G$

626 3)  $\varepsilon_{r(1)} = \varepsilon_{r(e)} = -M\sigma_c / 2G$

627 4)  $\sigma_{r(1)} = \sigma_{re} = p_0 - M.\sigma_c$

628 5)  $\sigma_{\theta(1)} = \sigma_{\theta e} = p_0 + M.\sigma_c$

629 6)  $m_{(1)} = m$

630 7)  $s_{(1)} = s$

631 8)  $\omega_{(1)} = 0$

632 9)  $\zeta_1 = r_{(1)} / r_e = 1$

633

634 **Sequence of calculations for each ring**

635 1)  $d\varepsilon_{\theta} = 0.005\varepsilon_{\theta(1)}$

636 2)  $\varepsilon_{\theta(j)} = \varepsilon_{\theta(j-1)} + d\varepsilon_{\theta}$

637 3) If  $\varepsilon_{\theta(j)} \leq \mu.\varepsilon_{\theta(1)}$  then  $\varepsilon_{r(j)} = \varepsilon_{r(j-1)} - hd\varepsilon_{\theta}$  otherwise  $\varepsilon_{r(j)} = \varepsilon_{r(j-1)} - fd\varepsilon_{\theta}$

638 4)  $\kappa = \frac{2\varepsilon_{\theta(j-1)} - \varepsilon_{r(j-1)} - \varepsilon_{r(j)}}{2\varepsilon_{\theta(j)} - \varepsilon_{r(j-1)} - \varepsilon_{r(j)}}$

639 5)  $\zeta_{(j)} = \kappa \cdot \zeta_{(j-1)}$

640 6)  $\lambda_{(j)} = r_{(j)} / r_e$   $\lambda_{(j-1)} = r_{(j-1)} / r_e$

641 7) If  $\varepsilon_{\theta(j)} \leq \mu\varepsilon_{\theta(1)}$  then  $m_{(j)} = m + (m_r - m) \frac{\varepsilon_{\theta(j)} - \varepsilon_{\theta(e)}}{(\mu - 1)\varepsilon_{\theta(e)}}$  otherwise  $m_{(j)} = m_r$

642 8) If  $\varepsilon_{\theta(j)} \leq \mu\varepsilon_{\theta(1)}$  then  $s_{(j)} = s + (s_r - s) \frac{\varepsilon_{\theta(j)} - \varepsilon_{\theta(e)}}{(\mu - 1)\varepsilon_{\theta(e)}}$  otherwise  $s_{(j)} = s_r$

643 9)  $m_a = \frac{1}{2}(m_{(j-1)} + m_{(j)})$

644 10)  $s_a = \frac{1}{2}(s_{(j-1)} + s_{(j)})$

645 11)  $K_2 = \frac{m_a \sigma_c}{4}$

646 12)  $K = \frac{\lambda_{(j-1)} - \lambda_{(j)}}{\lambda_{(j)} + \lambda_{(j-1)}}$

647 13) If  $p_{pre-ten} \geq p_{inst}$ , then go to step 14, otherwise go to step 18

648 14)  $a = \frac{1}{4K^2}$

649 15)  $b = -\frac{\sigma_{r(j-1)}}{2K^2} - 2K_2$

650 16)  $c = \sigma_{r(j-1)} \left[ \frac{\sigma_{r(j-1)}}{4K^2} - 2K_2 \right] - s_a \sigma_c^2$

651 17) Now, go to step 26

652 18) If  $\sigma_{r(j-1)} > p_{inst}$  and then  $\omega_{(j)} = 0$ , other wise  $\omega_{(j)} = \omega_{(j-1)} + 1$

653 19)  $\gamma = d\varepsilon_{r(j)} = \varepsilon_{r(j)} - \varepsilon_{r(j-1)}$

$$654 \quad 20) K_1 = \frac{A_b E_s r_i}{C_0 (r_{(j-1)} - r_{(j)})} \gamma$$

$$655 \quad 21) \bar{\gamma} = d\bar{\varepsilon}_{r(j)} = \bar{\varepsilon}_{r(j-\omega_j)} - \bar{\varepsilon}_{r(j-1-\omega_j)} \quad r_i \leq r < \bar{r}_e$$

$$656 \quad \gamma^e = d\varepsilon_{r(j)}^e = \varepsilon_{r(j)}^e - \varepsilon_{r(j-1)}^e \quad \bar{r}_e \leq r \leq r_e$$

$$657 \quad 22) \bar{K}_1 = \frac{A_b E_s r_i}{C_0 (r_{(j-1)} - r_{(j)})} \bar{\gamma} \quad r_i \leq r < \bar{r}_e$$

$$658 \quad \bar{r}_e \leq r \leq r_e \quad K_1^e = \frac{A_b E_s r_i}{C_0 (r_{(j-1)} - r_{(j)})} \gamma^e$$

$$659 \quad 23) a = \frac{1}{4K^2}$$

$$660 \quad 24) b = -\frac{K_1 - \bar{K}_1}{K} - \frac{\sigma_{r(j-1)}}{2K^2} - 2K_2 \quad r_i \leq r < \bar{r}_e$$

$$661 \quad \bar{r}_e \leq r \leq r_e \quad b = -\frac{K_1 - K_1^e}{K} - \frac{\sigma_{r(j-1)}}{2K^2} - 2K_2$$

$$662 \quad 25) \quad c = \sigma_{r(j-1)} \left[ \frac{\sigma_{r(j-1)}}{4K^2} + \frac{K_1 - \bar{K}_1}{K} - 2K_2 \right] + (K_1 - \bar{K}_1)^2 - s_a \sigma_c^2$$

$$663 \quad r_i \leq r < \bar{r}_e$$

$$664 \quad c = \sigma_{r(j-1)} \left[ \frac{\sigma_{r(j-1)}}{4K^2} + \frac{K_1 - K_1^e}{K} - 2K_2 \right] + (K_1 - K_1^e)^2 - s_a \sigma_c^2$$

$$665 \quad \bar{r}_e \leq r \leq r_e$$

$$666 \quad 26) \Delta = b^2 - 4a.c$$

$$667 \quad 27) \sigma_{r(j)} = \frac{-b - \sqrt{\Delta}}{2a}$$

668 28) If  $\sigma_{r(j)} > p_i$ , then increment  $j$  by 1 and repeat the calculation sequence for next

669 ring.

670 29) If  $\sigma_{r(j)} \approx p_i$ , then  $r_{(j)} = r_i$ ;  $r_e = r_{(j)} / \lambda_{(j)}$ .

671 Note:  $p_i$  should not be decreased from  $p_{pre-ten}$ .

672 30) If  $\sigma_{r(j)} \approx p_{inst}$ , then  $\bar{r}_e = r_e$

673 31) The radii of all the rings may now be calculated, using  $r_{(j)} = \lambda_{(j)} \cdot r_e$

674 32) The displacement values of rings may be determined from the previously

675 computed values of  $u_j = -\varepsilon_{\theta(j)} \cdot r_{(j)}$

676 33)  $T_{ideal(j)} = A_b \cdot E_s \cdot (\varepsilon_{r(j)} - \varepsilon_{r(j-\omega_j)} + \varepsilon_{pre-ten})$

677 34) If  $\sigma_{r(j)} \approx \min\{p_{inst}, p_{pre-ten}\}$ , then  $T_{ideal(j)} = T_{max}$

678 35) After calculating  $T_{max}$ , the steps 1 to 12 are repeated.

679 36)  $T_s = (1 - \eta) T_{pre}$

680 37)  $p'_{pre-ten} = T_s / C_0$

681 38) If  $p'_{pre-ten} \geq p_{inst}$ , then  $T_{(j)} = (1 - \eta) T_{pre}$  and go to step 39, otherwise go to step 40

682 39) the steps 14 to 16 and steps 26 to 29 are repeated except that  $p_{inst} < p_i < p_0$ . Go to

683 step 46.

684 40)  $T_{(j)} = T_{ideal(j)} - \eta T_{pre} - \left[ - (T_{max} - T_{pre}) (1 - \beta) \left( \frac{e^{-\lambda L}}{e^{-\lambda L} + e^{\lambda L}} \right) \left( e^{\lambda(r_{(j)} - r_i)} + e^{2\lambda L} e^{-\lambda(r_{(j)} - r_i)} \right) \right]$

685 41)  $K_1 = \frac{T_{(j-1)} - T_{(j)}}{r_{(j-1)} - r_{(j)}} \frac{r_i}{C_0}$

686 42)  $a = \frac{1}{4K^2}$

687 43)  $b = \frac{K_1}{K} - \frac{\sigma_{r(j-1)}}{2K^2} - 2K_2$

688 44).  $c = \sigma_{r(j-1)} \left[ \frac{\sigma_{r(j-1)}}{4K^2} - \frac{K_1}{K} - 2K_2 \right] + K_1^2 - s_a \sigma_c^2$

689 45) Steps 26 to 29 are repeated except that  $p'_{pre-ten} < p_i < p_0$ .

690 46) The radii of all the rings may now be calculated, using  $r_{(j)} = \lambda_{(j)} \cdot r_e$

691 47) The displacement values of rings may be determined from the previously

692 computed values of  $u_j = -\varepsilon_{\theta(j)} \cdot r_{(j)}$ .

693

## 694 **References**

695 Aydan, Ö. (1989). "The Stabilisation of Rock Engineering Structures by Rockbolts."

696 Ph. D. thesis, Queens's Univ., Kingston, Ontario.

697

698 Bischoff, J.A., Smart, J.D. (1975). "A method of computing a rock reinforcement

699 system which is structurally equivalent to an internal support system." *Proc. of 16th*

700 *Symp. on Rock Mechanics*, Univ. of Minnesota, 179-184.

701

702 Bobet, A. (2006). "A simple method for analysis of point anchored rockbolts in

703 circular tunnels in elastic ground." *Rock Mech. Rock Eng.*, 39, 315- 338.

704

705 Bobet, A., Einstein, E. (2011). "Tunnel reinforcement with rockbolts." *Tunnel.*

706 *Underground Space Tech.*, 26, 100-123.

707

708 Brown, E.T., Bray, J.W., Ladanyi, B., and Hoek, E. (1983). "Ground response curves

709 for rock tunnels." *J. of Geotech. Eng.*, 109, 15- 39.

710

711 Cai, Y., Esaki, T., and Jiang, Y. (2004a). “An analytical model to predict axial load in  
712 grouted rock bolt for soft rock tunneling.” *Tunnel. Underground Space Tech.*, 19,  
713 607- 618.  
714

715 Cai, Y., Jiang, Y.J., and Esaki, T. (2004b). “A rock bolt and rock mass interaction  
716 model.” *Int. J. of Rock Mech. Mining Sci.*, 41, 1055-1067.  
717

718 Carranza-Torres, C. (2009). “Analytical and Numerical Study of the Mechanics of  
719 Rockbolt Reinforcement around Tunnels in Rock Masses.” *Rock Mech. Rock Eng.*,  
720 42, 175- 228.  
721

722 Fahimifar, A., Soroush, H. (2005). “A theoretical approach for analysis of the  
723 interaction between grouted rockbolts and rock masses.” *Tunnel. Underground Space*  
724 *Tech.*, 20, 333-343.  
725

726 Fahimifar, A., Ranjbarnia, M. (2009). “Analytical approach for the design of active  
727 grouted rockbolts in tunnel stability based on convergence-confinement method.”  
728 *Tunnel. Underground Space Tech.*, 24, 363-375.  
729

730 Freeman, T.J. (1978). “The behavior of fully-bonded rock bolts in the Kielder  
731 experimental tunnel.” *Tunnels Tunn*, 37–40.  
732

733 Grasso, P.G., Mahtab, A., and Pelizza, S. (1989). “Riqualficazione della massa  
734 rocciosa: un criterio per la atabilizzaazione della gallerie.” *Gallerie e Grandi Opere*  
735 *Sotterranee*, 29, 35- 41.

736

737 Guan, Zh., Jiang, Y., Tanabasi, Y., and Huang, H. (2007). "Reinforcement mechanics  
738 of passive bolts in conventional tunneling." *Int. J. of Rock Mech. Mining Sci.*, 44,  
739 625-636.

740

741 Hoek, E., Brown, E.T. (1980). "Underground Excavations in Rock." The Institution  
742 of Mining and Metallurgy, London.

743

744 Huang, Z., Broch, E., and Lu, M. (2002). "Cavern roof stability- mechanism of  
745 arching and stabilization by rockbolting." *Tunnel. Underground Space Tech.*, 17, 249-  
746 261.

747

748 Indraratna, B., Kaiser, P.K. (1990a). "Design of grouted rock bolts based on the  
749 convergence control method." *Int. J. Rock Mech. Mining Sci and GeoMech.*, 27, 269-  
750 281.

751

752 Indraratna, B., Kaiser, P.K. (1990b). "Analytical model for the design of grouted rock  
753 bolts." *Int. J. Numer. Anal. Meth. Geomech.*, 14, 227- 251.

754

755 Li, C., Stillborg, B. (1999). "Analytical models for rock bolts." *Int. J. of Rock Mech.  
756 Mining Sci.*, 36, 1013- 1029.

757

758 Oreste P.P., Peila, D. (1996). "Radial passive rockbolting in tunneling design with a  
759 new convergence-confinement model." *Int. J. Rock Mech. Mining Sci and GeoMech.*,  
760 33, 443- 454.

761

762 Osgoui, R.R., Oreste, P. (2010). “Elasto-plastic analytical model for the design of  
763 grouted bolts in a Hoek–Brown medium.” *Int. J. Numer. Anal. Meth. Geomech.*, 34,  
764 1651–1686.

765

766 Oreste, P.P. (2008). “Distinct analysis of fully grouted bolts around a circular tunnel  
767 considering the congruence of displacements between the bar and the rock.” *Int. J. of*  
768 *Rock Mech. Mining Sci.*, 45, 1052–1067.

769

770 Peila, D., Oreste P.P. (1995). “Axisymmetric analysis of ground reinforcing in  
771 tunneling design.” *Comput. Geomach.*, 17, 253- 274.

772

773 Ranjbarnia, M., (2014). “Theoretical analysis of pre-tensioned grouted rockbolts  
774 behavior in tunnels considering the interaction of bolt, grout, and rock mass”, *PhD*  
775 *thesis*, Department Civil and Environmental Engineering, Amirkabir University of  
776 Technology, Tehran. Iran

777

778 Stille, H., Holmberg, M., and Nord G. (1989). “Support of Weak Rock with Grouted  
779 Bolts and Shotcrete.” *Int. J. of Rock Mech. Mining Sci. and Geomech.* 26, 99-111

780

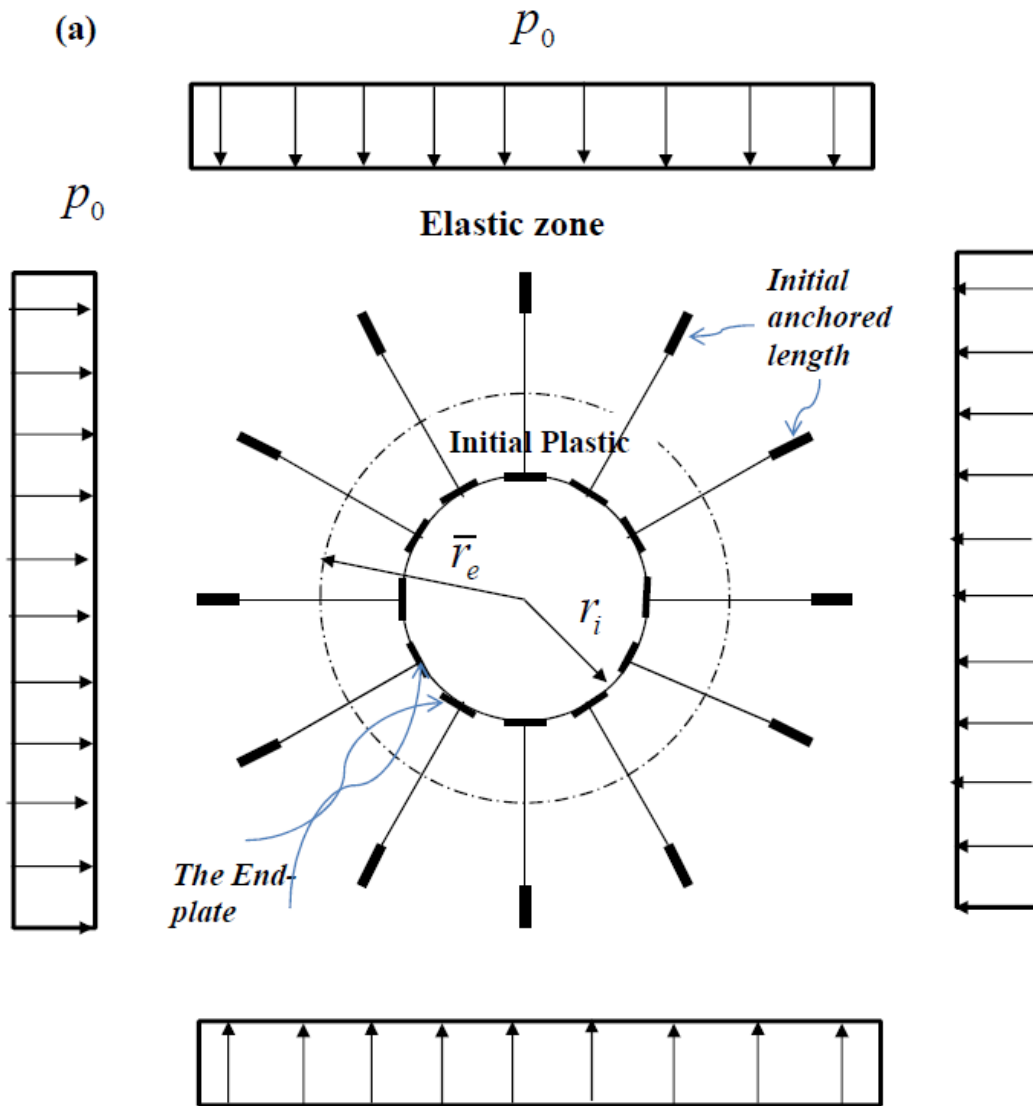
781 Ward, W.H., Coats, D.J., and Tedd, P. (1976). “Performance of support systems in the  
782 Four Fathom Mudstone.” *Proc. of Tunnelling 76*, London, 329-340.

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788 Figure 1. Circular tunnel reinforced by systematic pre-tensioned grouted bolts (a) the

789 plastic radius at the bolt installation time (b) Increasing tunnel plastic radius

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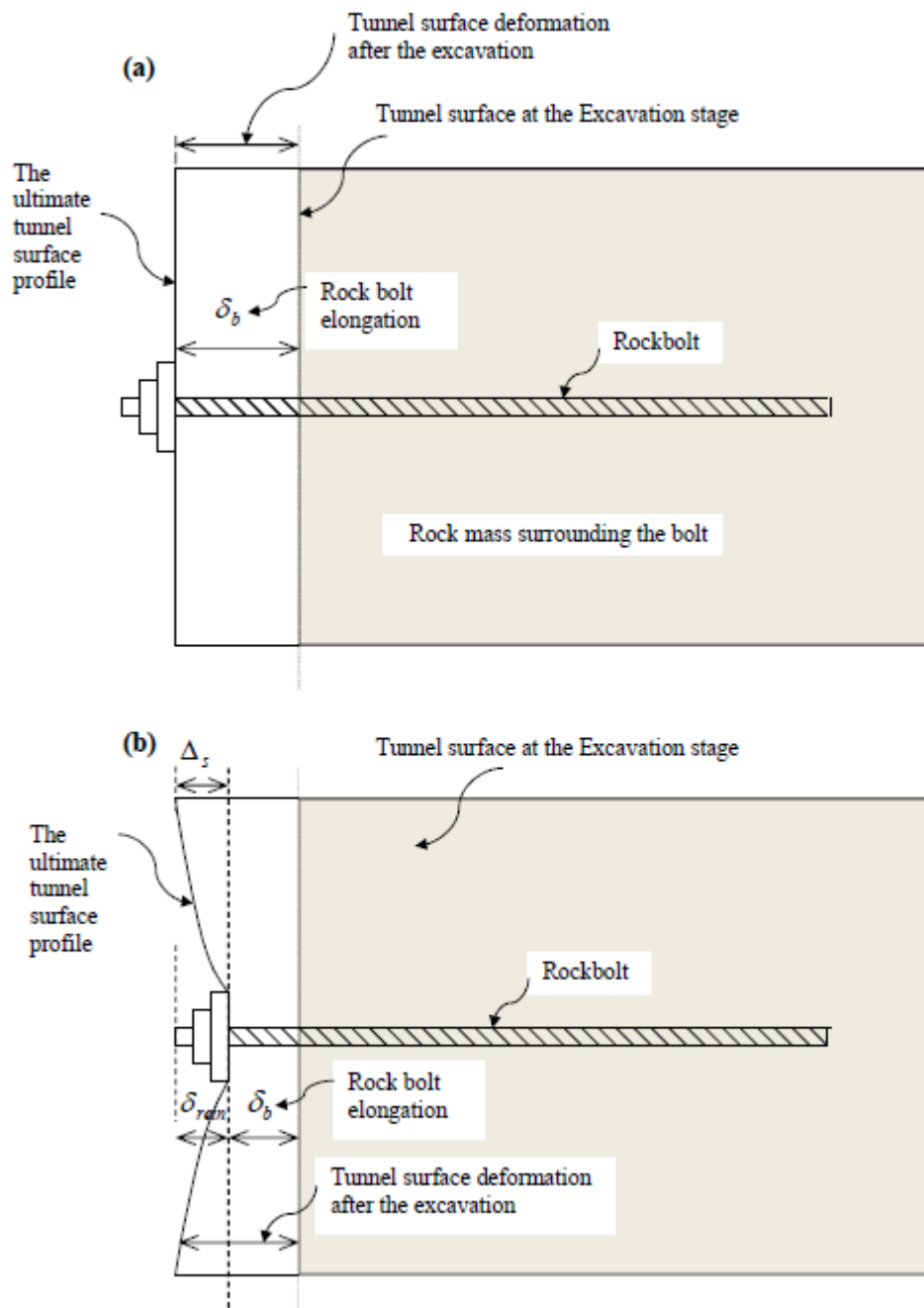
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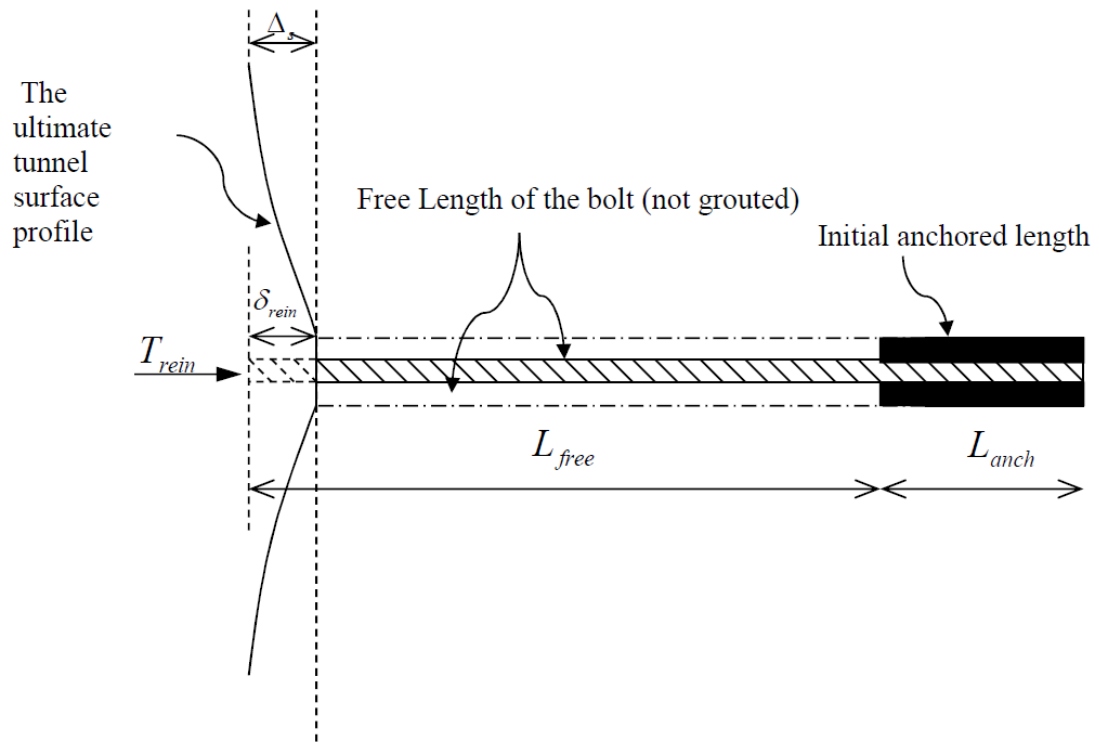
798 Figure 2. Rock mass and bolt interaction at tunnel surface in (a) ideal connection and

799 (b) real connection between the bolt and rock mass

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804 Figure 3. Total reduction of bolt elongation

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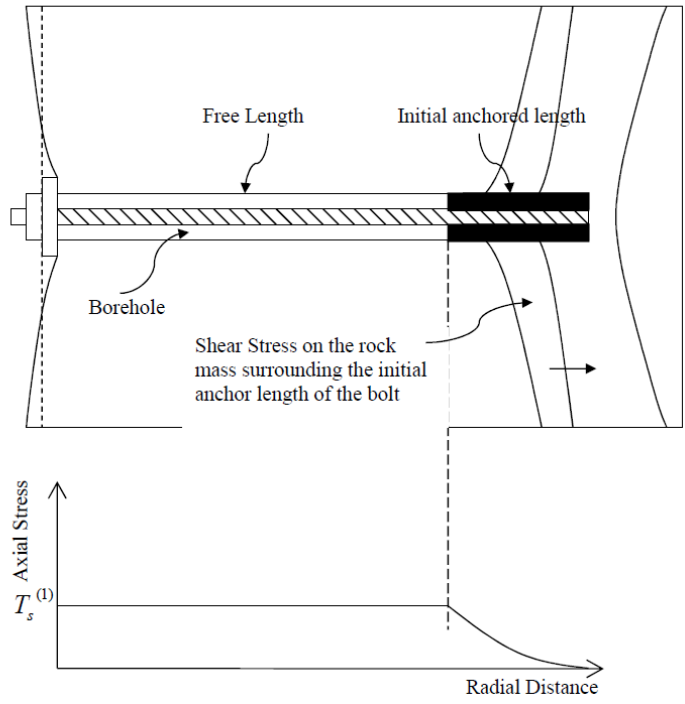
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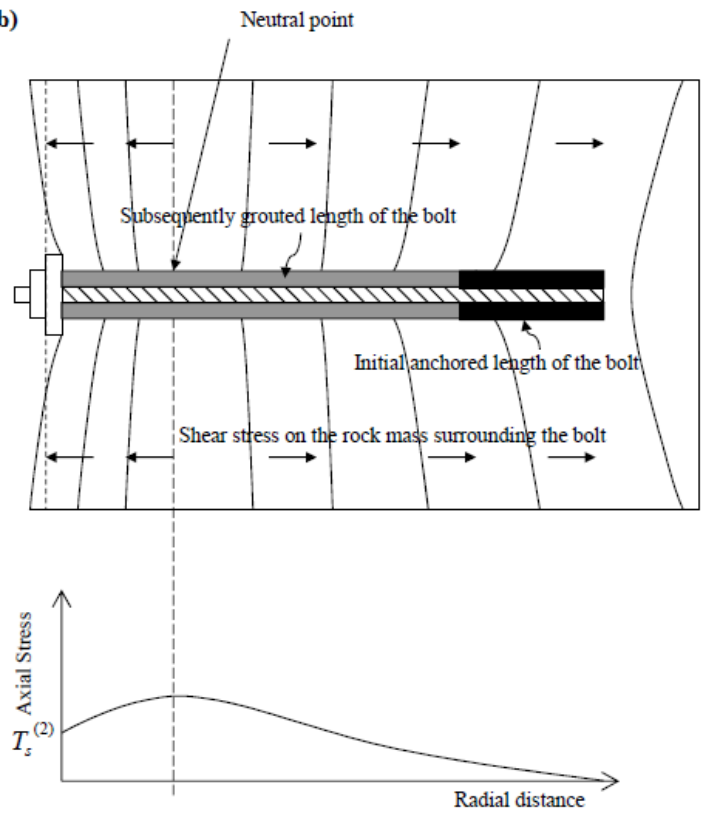
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(a)



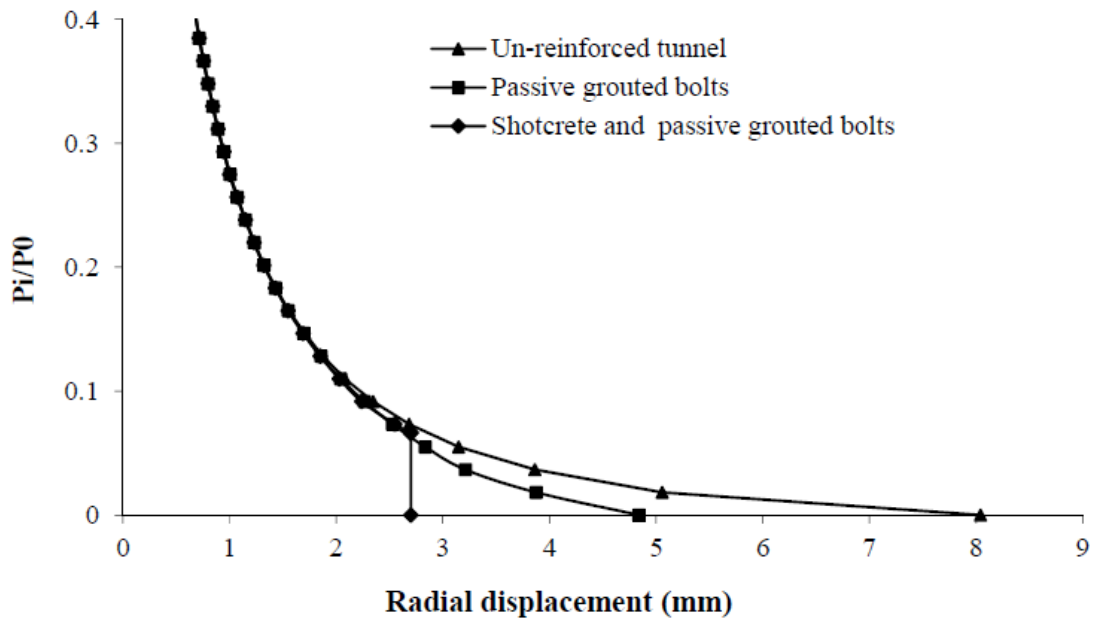
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(b)



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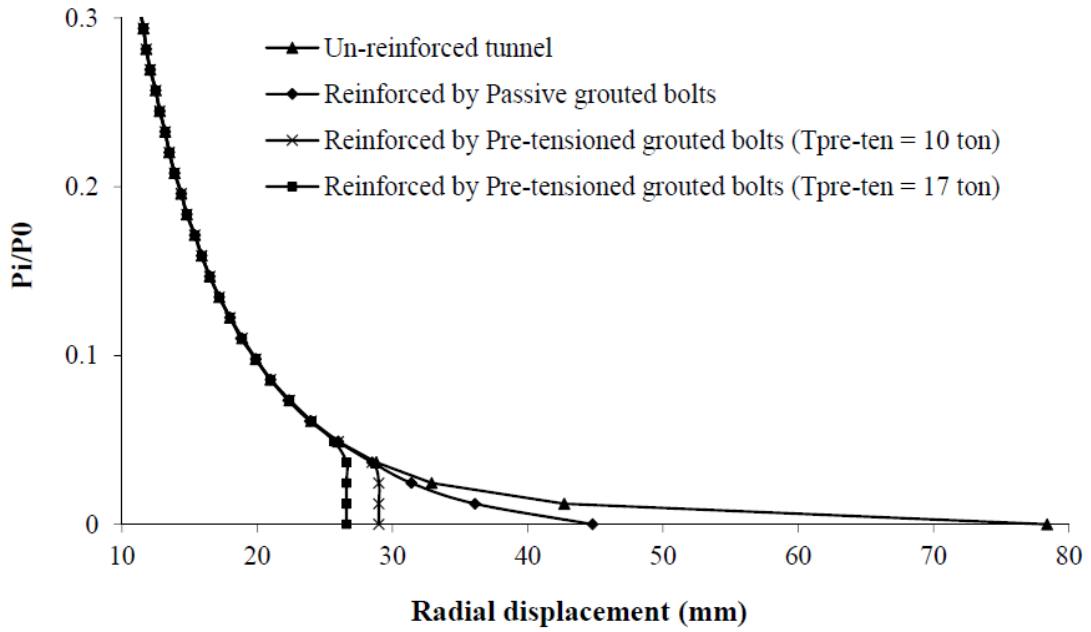
819 Figure 4. Interaction of bolt with its surrounding rock mass for *Case B*. Loading  
820 mechanism of the bolt (a) for the first step (b) for the second step



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822 Figure 5. Ground response curves for the rock mass around Kielder Experimental  
823 Tunnel (Example 1) in the rockbolted and rockbolt with shotcreted section

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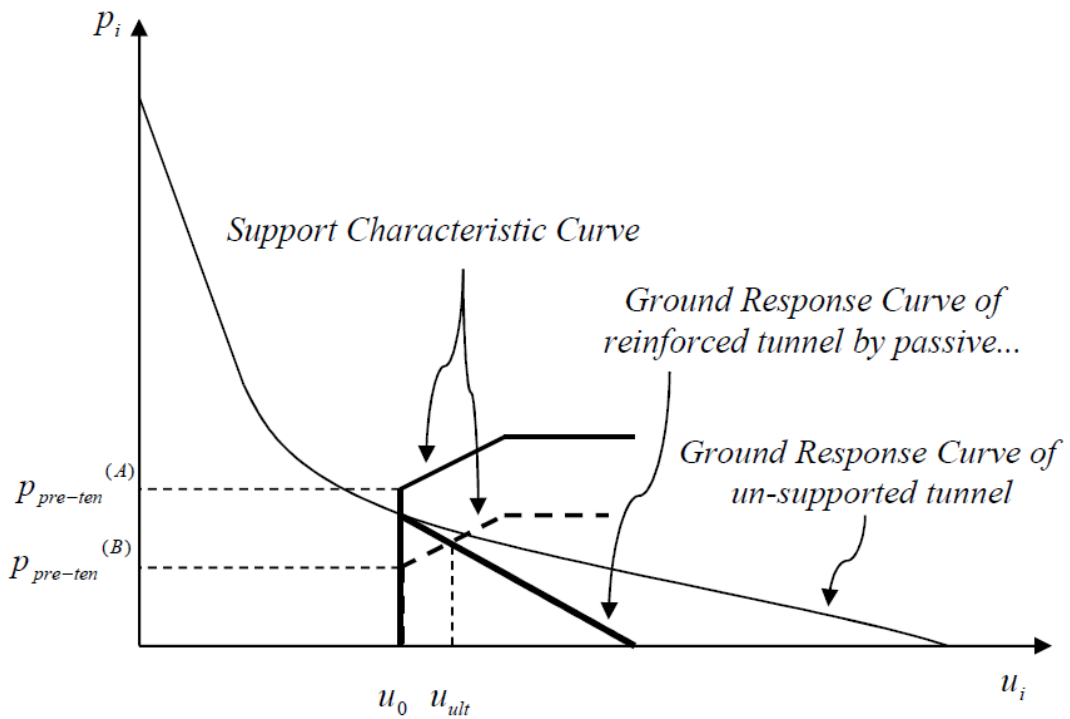


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826 Figure 6. Ground response curve for the rock mass around tunnel in Examples 2 for

827 Case A and Case B

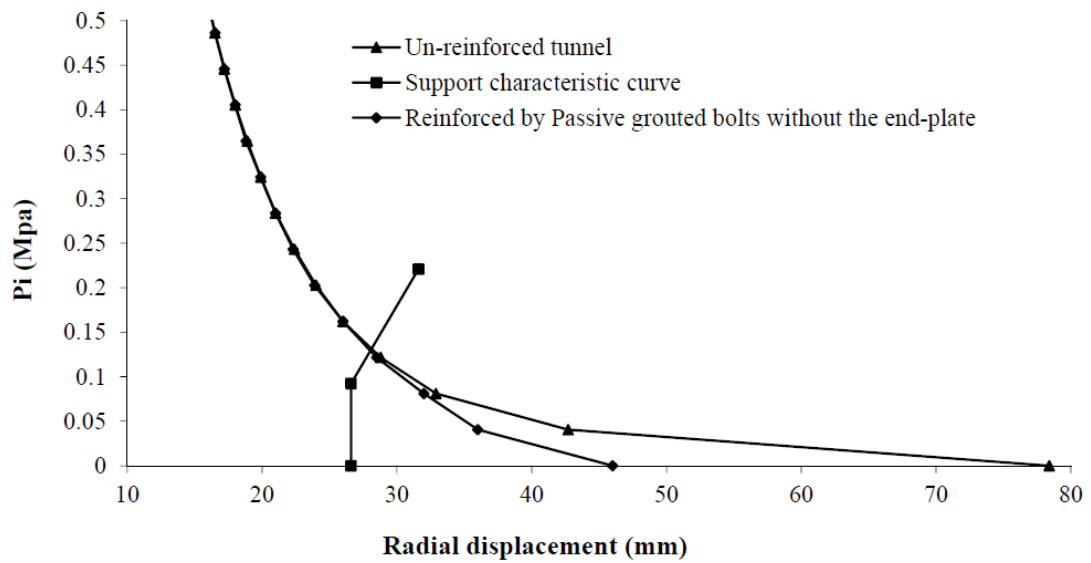
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830 Figure7. Un-grouted pre-tensioned characteristic curve (solid line for Case A

831 condition and dashed line for *Case B* condition) and ground response curves of un-  
832 supported tunnel and reinforced by passive grouted bolts



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834 Figure 8. Un- grouted pre-tensioned characteristic curve and the ground response  
835 curves of tunnel for un-reinforced and reinforced by passive grouted bolts (Example 2  
836 by simple practical method)

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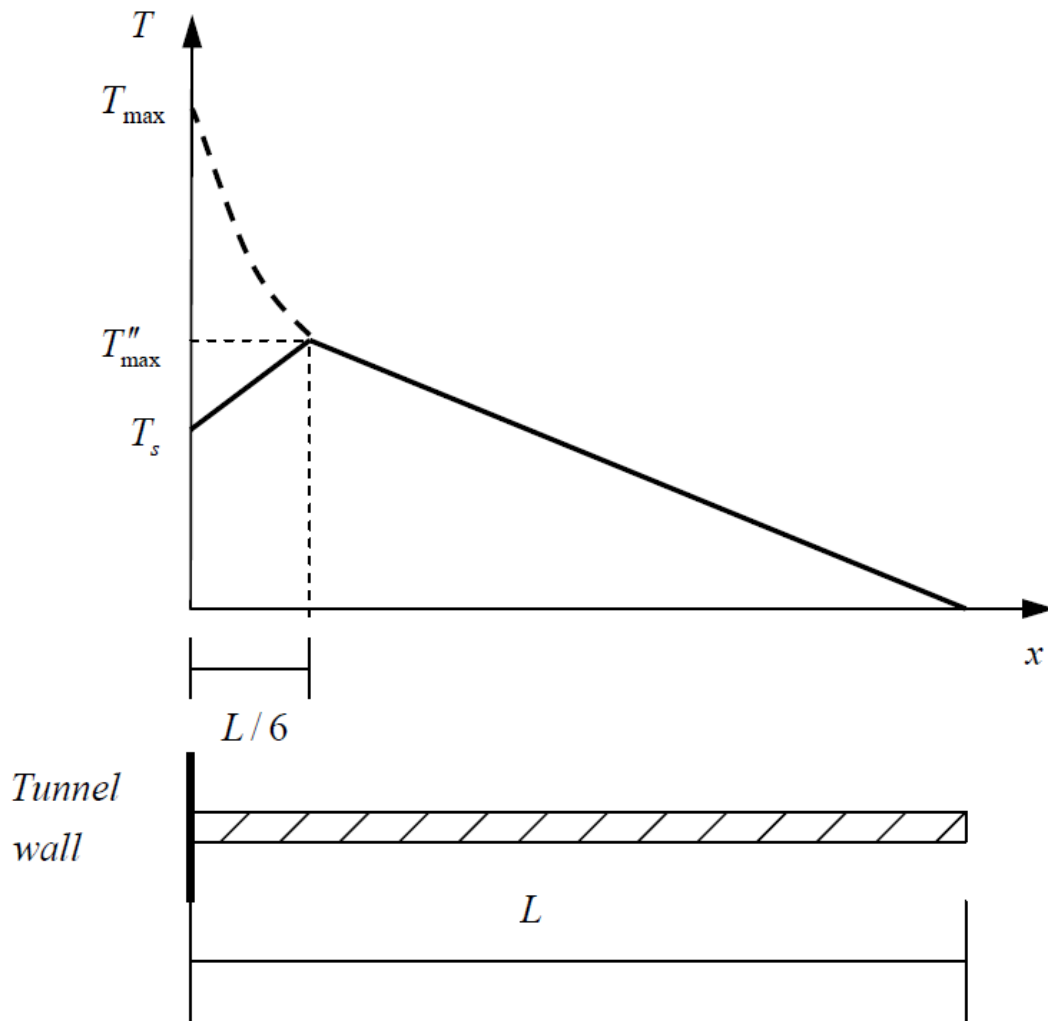
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850 Figure 9. Simplified graphic of force along the bolt (two solid line by Oreste (2008)

851 and dashed curve by Ranjbarnia (2014))

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860 **Table 1.** Mechanical properties of mudstone in the Kielder Experimental Tunnel (data

861 from Freeman 1978; Hoek and Brown 1980)

Parameter	Value
Axial compressive strength $\sigma_c$ (MPa)	37
Tunnel radius, $r_i$ (m)	1.65
In-situ stress, $p_0$ (MPa)	2.56
Deformation modulus, $E_m$ (MPa)	5000
Poisson's ratio, $\nu$	0.25
Peak Strength parameter, $m_p$	0.1
Peak Strength parameter, $s_p$	0.00008
Residual Strength parameter, $m_r$	0.05
Residual Strength parameter, $s_r$	0.00001
Dilation angle (degree), $\psi$	10
Strain softening parameters*, gradients of $-\varepsilon_3^p$ vs. $\varepsilon_1^p$ lines in the residual stage, $f$	1.1
Strain softening parameters*, gradients of $-\varepsilon_3^p$ vs. $\varepsilon_1^p$ lines in the softening stage, $h$	1.2
Strain softening parameters*, constant defining strain at which residual strength is reached, $\mu$	7.5

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\*these parameters were computed by the authors from the value of dilation angle

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872 **Table 2.** Geometrical parameters of passive grouted rockbolts and shotcrete in the  
 873 Kielder Experimental Tunnel (data from Ward et al. 1976; Freeman 1978; Hoek and  
 874 Brown 1980)

Parameter	Value
Rockbolt Length, $L$ (m) *	1.8
Initial anchored length, $L_{anch}$ (m)	0.5
Young's modulus of rockbolt, $E_s$ (GPa)	210
Bolt diameter, $d_b$ (mm)	20
Borehole diameter, $d_g$ (mm) **	60
Distance between rockbolt, $S_l \times S_c$ (m $\times$ m)	0.9*0.9
Early age Young's modulus of shotcrete, $E_{shot}$ (GPa)	2
Shotcrete thickness, (mm)	140
Bolt head stiffness, $K_s$ (MN) ***	320
Shotcrete pressure on the tunnel surface (MPa) ****	0.17

875 \* According to Hoek and Brown (1980) study, this value was smaller than what was required. Authors  
 876 of this paper used the required value.

877 \*\* assumed typical value

878 \*\*\*calculated by the authors for the shotcreted section of tunnel where perfect constrained was  
 879 predicted for the end plate of bolt.

880 \*\*\*\* calculated by the authors from the classic formula presented by Hoek and Brown (1980).

881 Shotcrete layer radial deformation was 1.5 mm.

882

883 Table 3. Calculated and measured deformations (data from Stille et al. 1989) at the  
 884 rock surface for reinforced and un-reinforced rock mass

Parameter	Measured (mm)	Calculated* (mm)
Un-reinforced tunnel	8	8.05
Passive grouted bolt section	4-5	4.84
Passive grouted bolt and Shotcrete section	2-3	2.7

885 \*By authors

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888 Table 4. The output results of Example 2

Example		Ultimate convergence (mm)	Plastic radius (mm)
Un-reinforced tunnel		78.4	12.27
Reinforced tunnel	by the passive bolts	44.8	9.37
	by the pre-tensioned bolts (Case A)	26.6	8.1
	by the pre-tensioned bolts (Case B)	29.8	8.42

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892 Table 5. The input and output data of Example 2 calculated by simple practical

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method

Parameter	Value
$\sigma_{r_0-pre}$ (Mpa) *	0.0924
$\xi$	1.2
$k_{sys}$ (MN / m <sup>3</sup> )	25.7
$u_1$ (mm)	26.6
$u_{ult}$ (mm)	28.5

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\*Calculated by  $p_{pre-ten} = T_s / C_0$  where  $T_s$  is obtained by Eq. (33)

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