Design strategies and merit of system parameters for uniform uncompensated links supporting Nyquist-WDM transmission

Design Strategies and Merit of System Parameters for Uniform Uncompensated Links Supporting Nyquist-WDM Transmission

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***Abstract*—We consider the transmission of Nyquist-wavelength-division-multiplexed (NyWDM) channels based on polarization-multiplexed m-ary QAM multilevel modulation formats with DSP-based coherent detection over point-to-point uncompensated periodically amplified uniform fiber links. Taking into account both the effect of amplified spontaneous emission (ASE) noise accumulation and generation of non-linear interference (NLI) introduced by fiber propagation, we propose three different design strategies: the maximization of both the Q margin and the span-loss margin, for a given span length, and the maximization of the total link length given a target performance. We propose and apply an approximation for the Gaussian-Noise(GN) model in order to evaluate the NLI intensity, deriving for the three design strategies the merit of link and signal parameters. Finally, we validate the proposed methodologies using experimental and simulative results already published in literature.**

***Index Terms*—Multilevel modulation formats, Nyquist-WDM, Coherent detection, Uncompensated links, Non-linear interference.**

I. Introduction

The first three decades of the optical communications age were characterized by the use of receivers (Rx) based on direct-detection. During that time-span, all practically used modulation formats were forcedly flavors of the intensity-modulation direct-detection (IM/DD) format.

Electronic dispersion compensation was largely ineffective for IM/DD. As a result, optical links included optical chromatic dispersion (CD) compensators. However, the complex interplay between CD and fiber non-linearity required optimized ‘dispersion maps’. Generalized engineering rules could not be derived and, consequently, design procedures required a case-by-case optimization of system parameters.

Over the last decade, the development of fast digital-to-analog (DAC) and analog-to digital (ADC) converters together with fast digital signal processing (DSP), has enabled the generation of polarization-multiplexed (PM) multilevel modulation formats using I/Q modulators and the implementation of DSP-based polarization diversity coherent Rx’s. For these transmission systems, linear effects such as CD and polarization-mode dispersion (PMD), can be fully compensated for by the Rx DSP. The use of DSP at the transmitter (Tx) has also permitted the accurate shaping of the channel spectrum, enabling raised-cosine-shaped channels, as required by the Nyquist-WDM (NyWDM) technique. Hence, the channel spacing (Δf) has approached the symbol rate (R_s), further improving the spectral efficiency (SE).

Remarkably, it has been extensively shown that for these systems the best reach performance is achieved in the absence of any in-line chromatic dispersion (CD) compensation, i.e., with uncompensated transmission (UT) [1]–[3]. This fundamental property dramatically simplifies the link structure removing the need for an optimized dispersion map. It also simplifies the task of assessing the impact of non-linear propagation effects.

The field of the analytical modeling of fiber non-linear effects has historically been very active. A partial list of contributions, covering only the last two years, is [4]–[17] . For prior years, the reader may refer to the bibliography of [6] and [12].

One modeling result of substantial practical importance has been the recognition that in typical UT links the effect of fiber non-linearity on any single WDM channel can be approximately modeled as additive Gaussian noise (AGN), called non-linear interference (NLI), whose variance scales as the cube of the power per channel. Such AGN is approximately statistically independent of the channel signal and of the ASE noise. A non-linear phase-noise component may be present too [7], [9], whose correlation time may be several tens of symbol times, but its impact has been found to be small or negligible in conventional long-haul EDFA-amplified systems [14]-[18], as well as in systems with hybrid EDFA-Raman amplification, when the Raman gain is substantially less than the span fiber loss. In this paper, we assume to be operating in such weak phase-noise scenarios, so that the NLI can be considered statistically-independent, short-correlation (i.e., up to one symbol) AGN.

Several models are currently available for estimating the amount of NLI. We took into account the GN [12] and enhanced-GN (EGN) [13] models. They are conceptually similar, but the GN-model makes the simplifying assumption that the highly dispersed signal propagating over UT links can be approximated as a Gaussian random process itself. Thanks to this hypothesis, the GN-model application is relatively simple and it can be approximated by a compact closed-

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form for several practical scenarios, in particular for channel combs exploiting the entire C-bandwidth. The EGN model, instead, takes into full account the statistical properties of the modulated signal and is very accurate for most of practical scenarios but it is more complex than the GN-model.

In this paper, we concentrate on UT of NyWDM channel combs over links made up of all-identical spans - uniform links. For these systems, the use of a simplified version of the GN model - the 'incoherent' GN-model - ensures excellent accuracy in performance prediction and allows to derive closed-form expressions for the design rules. The 'incoherent' GN-model neglects the coherent accumulation of NLI produced by subsequent different spans and phenomenological turns out to improve the accuracy of the GN-model in UT over uniform links and on the entire C-band. For a discussion of this aspect, see [12] and [18]. For a general validation of the incoherent GN model, see [5], [12].

Our investigations had two main purposes. First, we derived link design rules taking into account ASE noise and NLI accumulation and targeted to optimize different performance parameters. In particular, we identified three possible design 'strategies' aimed at maximizing three different system performance indicators: the Q-factor, the span margin or the reach, hence carrying on and refining our previous work [26] which was focused only on reach maximization.

Second, we aimed at identifying scaling laws which effectively captured the merit of each link and signal parameter on target performance parameters. They had to be simple, based on closed-form formulas and may be based on approximations with a maximum tolerable inaccuracy.

As shown in Sec. IV, the proposed scaling laws allow to express the target performance parameters as a linear combination of the system parameters, in convenient dB units. Hence, the designer can get an evaluation "at a glance", with a given and limited maximum inaccuracy, for instance, of how much is the merit on the maximum reach extension of changing the fiber type. Such an evaluation can be done simply adding up the merit of relative variations of loss, dispersion and nonlinear fiber coefficients . If an analytical model for the NLI evaluation was applied, such an operation would be more complex and, most important, the merit of each system parameter on the design target should not be so clear and quickly readable. So far, this problem had been addressed by looking at different fiber 'figures of merit' [22], [23] and only partially analyzed with the objective of obtaining clear and simple design rules [25]–[27].

In this paper, we do not take into account for possible nonlinear mitigation or pre-compensation techniques and we do not consider phase-conjugated twin waves communications.

The paper is organized as in the following.

In Sec. II, we describe the considered system setup and present the possible design strategies aimed at maximizing either the system Q, the span margin, or the reach.

In Sec. III, we recall the main analytical results for the GN-model and propose a suitable linear approximation, relating in convenient dB units the scaling of NLI intensity with system parameters. An acceptable level of accuracy is found over the range of system parameters of practical interest.

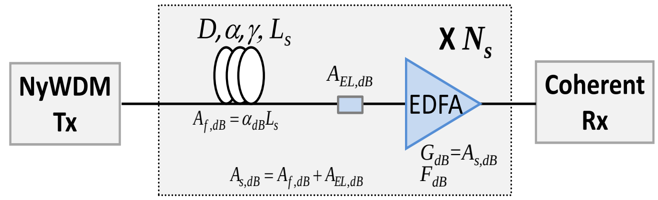

Fig. 1. Uniform and amplified system setup considered for the analysis

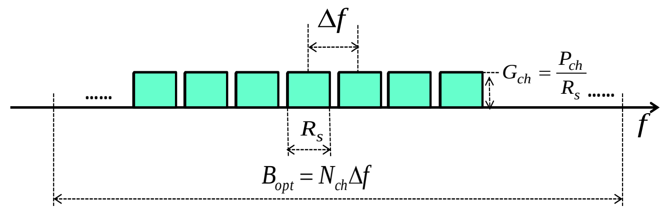

Fig. 2. Power spectral density of the NyWDM comb of channels considered for the analysis

In Sec. IV, we use the GN-model approximation together with the design strategies proposed in Sec. II in order to derive the scaling laws giving the merit on target performances of scaling signal and link parameters, such as amplifier and fiber parameters, symbol rate $\mathrm{R_s}$, channel spacing $\Delta\mathrm{f}$ and operating bit error-rate (BER).

In Sec. V, we validate the scaling laws presented in Sec. IV using already published experimental and simulative results.

Finally, in Sec. VI we comment on the use of merit of system parameters in practical choices typically faced in designing or upgrading optical communication links.

II. Design strategies

We consider the system setup pictorially described in Fig. 1 that is a generic multi-span uncompensated uniform fiber-link with periodical lumped amplification. Each of the N_s spans is made of the transmission fiber followed by a variable optical attenuator (VOA) emulating lumped losses, and by an optical amplifier with noise figure F completely recovering the span loss. Therefore, the overall span loss, in dB units, is $\mathrm{A_{s,dB}} = \alpha_{\mathrm{dB}}\mathrm{L_s} + \mathrm{A_{EL,dB}}$ and the amplifier gain is $\mathrm{G_{dB}} = \mathrm{A_{s,dB}}$. $\mathrm{A_{EL,dB}}$ is the possible lumped excess-loss.[1]

The ASE noise introduced by each optical amplifier can be considered as additive white and Gaussian noise (AWGN) with a power spectral density (PSD) given by:

$$\mathrm{G_{ASE}} \cong \mathrm{E_{ph}} \cdot \mathrm{F} \cdot \mathrm{A_s} \tag{1}$$

where $\mathrm{E_{ph}} = \mathrm{h} \cdot \mathrm{f_0}$ is the single-photon energy at the operating frequency $\mathrm{f_0}$ and h is Planck's constant.

The signal transmitted on the link is a comb of NyWDM channels whose PSD is qualitatively shown in Fig. 2. Each of the $\mathrm{N_{ch}}$ carriers is modulated by a polarization multiplexed M-ary QAM constellation at the symbol rate $\mathrm{R_s}$. Considering the Tx DSP being able to perfectly shape each channel spectrum,

[1] In this paper, we define losses as parameters ≥ 1, i.e., given the loss $\mathrm{A_s}$ we assume $\mathrm{P_{out}} = \mathrm{P_{in}}/\mathrm{A_s}$. In dBs, $\mathrm{A_{s,dB}} \geq 0$ dB.

we assume such a spectrum to be exactly squared. Consequently, the channel spacing Δf may approach the symbol rate R_s and the overall occupied bandwidth is $B_{opt} \cong N_{ch} \cdot \Delta f$.

It has been extensively shown that non-linear propagation of NyWDM signal combs over uncompensated and uniform multi-span links induces the growth of an additive Gaussian-distributed disturbance called non-linear interference. In general, the NLI PSD depends on the frequency, but its variation with f is weak over the channel bandwidth, so, in order to evaluate the worst-case performance for the center channel, we can approximate the NLI as an AWGN, similar to the ASE noise.

In general, in uniform links, the NLI accumulation with distance is super-linear, but in case of a large number of channels it approaches the linear behavior [12]. Thus, from a practical point of view, we can adopt this simplifying assumption and consider linear accumulation of NLI with distance. It is also well known and demonstrated that the NLI intensity scales with the cube of the power per channel P_{ch} propagating in the fiber, therefore, in general, we can express the NLI PSD introduced by each fiber span as:

$$G_{NLI} \cong \Gamma_{NLI} \cdot \frac{G_{ch}^3}{E_{ph}^2} \tag{2}$$

where $G_{ch} = P_{ch}/R_s$ is the power spectral density of transmitted channels (see fig. 2) which is flat in accordance to the Nyquist shaping, and Γ_{NLI} is the normalized NLI efficiency that depends on the link and signal parameters. Given R_s and Δf, according to the GN-model theory , Γ_{NLI} can be practically assumed to be independent of the modulation format, provided that it is a polarization multiplexed M-ary QAM [18].

As the ASE noise and the NLI disturbance introduced at each span are statistical independent AWGNs accumulating linearly with distance, the center channel at the Rx is affected by an overall AWGN whose PSD is

$$G_{Rx} = N_s(G_{ASE} + G_{NLI}) \, . \tag{3}$$

We assume the use of a DSP-based polarization-diversity coherent receiver whose back-to-back (BER) is given by:

$$BER = \Phi(SNR) \tag{4}$$

where Φ is a monotonically decreasing function that depends on the modulation format [5] and on the Tx/Rx implementations. SNR is the signal to noise ratio measured in the noise bandwidth $B_n = R_s$.

We suppose that the receiver is able to recover signal distortions and impairments due to phase noise, so transmission performances are only limited by the overall amount of Gaussian disturbances G_{Rx} given by Eq. (3). The BER at the receiver is consequently attainable substituting in Eq. (4) the generalized signal to noise ratio SNR_{NL} taking into account both ASE noise and NLI disturbance, whose expression is:

$$SNR_{NL} = \frac{G_{ch}}{N_s E_{ph} \left(F \cdot A_s + \Gamma_{NLI} \frac{G_{ch}^3}{E_{ph}^3} \right)} \, . \tag{5}$$

Eq. (5) is the fundamental law for design processes aimed at optimizing system performances, including nonlinear propagation.

In the following, we describe three different design strategies, all derived applying Eq. (5). For all strategies, we assume that the modulation format is given, as well as R_s, N_{ch}, Δf and the Tx/Rx characteristics, including possible forward error correction (FEC) codes. The fiber type is pre-determined as well, and supposed to be uniform in the link, hence the NLI efficiency Γ_{NLI} is a system constant.

In order to show the behavior of the three design parameters vs. the channel PSD we considered a typical uniform link made of standard single-mode fiber (SMF) (α_{dB} = 0.2 dB/km, D = 16.7 ps/nm/km, γ = 1.3 1/W/km) with span length 100 km - 20 dB of span loss - and EDFA F = 5 dB. The NyWDM channel comb is supposed to operate on the Δf = 50 GHz ITU grid at R_s = 32 Gbaud exploiting the entire C-band (B_{opt} = 4 THz). Performance results for the considered typical link are shown as SNR_{NL} for N_{sys} = 15 spans (Fig. 3a), as span loss for N_{sys} = 15 spans with target signal-to-noise ratio SNR_T = 10 dB (Fig. 3b) and as maximum reach for SNR_T = 10 dB (Fig. 3c). Parameters are plotted against the normalized channel PSD G_{ch}/E_{ch} that assumes the meaning of average number of photon per channel.

A. *Maximization of Q*

Considering a system composed of a given number of spans ($N_s = N_{sys}$) with a defined span loss ($A_s = A_{sys}$), system optimization can address BER minimization, or Q maximization having defined the Q factor, using the convenient dB units, as:

$$Q_{dB} = 20 \log_{10}[\sqrt{2}\, \text{erfc}^{-1}(2\, BER)] \, . \tag{6}$$

A given span loss means that both the span length (L_s) and the extra loss (A_{EL}) per span are defined *a priori* and kept as constants. Under these hypotheses, this optimization strategy corresponds to the maximization of Eq. (5). SNR_{NL} shows a maximum with respect to G_{ch} that is the optimal operating point for the considered system setup. It means that we can achieve a maximum $SNR_{NL} = SNR_{NL,max}$ at the optimal PSD per channel $G_{ch} = G^A_{ch,opt}$, as displayed in Fig. 3a. Such optimal values can be easily evaluated and have the following analytic expressions:

$$G^A_{ch,opt} = E_{ph} \frac{1}{\sqrt[3]{\Gamma_{NLI}}} \left(\frac{F \cdot A_{sys}}{2} \right)^{\frac{1}{3}} \tag{7}$$

and

$$SNR_{NL,max} = \frac{1}{3} \frac{1}{N_{sys}} \frac{1}{\sqrt[3]{\Gamma_{NLI}}} \left(\frac{2}{F \cdot A_{sys}} \right)^{\frac{2}{3}} \, . \tag{8}$$

as shown in [34]. In practical systems, to each target $BER = BER_T$, depending on modulation format, on Tx/Rx implementation and on FEC strength, corresponds a target $SNR = SNR_T$. So, this design strategy can be also interpreted as the maximization of the receiver margin, that using the convenient dB units, is defined as:

$$\mu_{Rx} = SNR_{NL,max,dB} - SNR_{T,dB} \, . \tag{9}$$

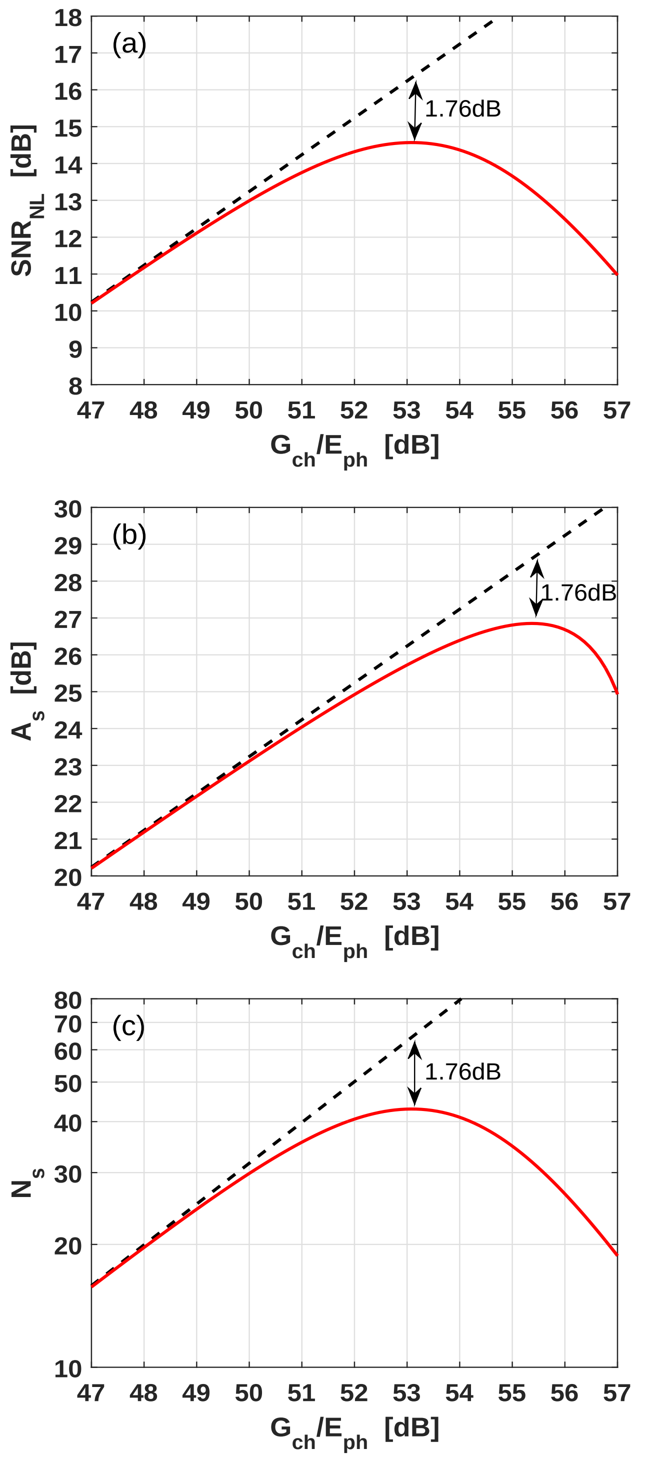

Fig. 3. The three considered optimization parameters for a uniform amplified link on SMF (α_{dB} = 0.2 dB/km, D = 16.7 ps/nm/km, γ = 1.3 1/W/km, L_s = 100 km, EDFA F = 5 dB). N_{ch} = 80 on a Δf = 50 GHz grid corresponding to B_{opt} = 4 THz. Symbol rate is R_s = 32 Gbaud. Solid lines are SNR_{NL} for N_{sys} = 15 spans (a), span loss for N_{sys} = 15 spans and SNR_T = 10 dB(b) and maximum reach for SNR_T = 10 dB(c) vs. the average number of photon per channel (G_{ch}/E_{ch}). Dashed lines refer to the same parameters evaluated in linear regime considering ASE noise only.

It is the amount of dBs the Rx sensitivity may degrade still keeping the link *in-service*, i.e., BER below the target value $\mathrm{BER_T}$. Such a margin may enable the use of a less powerful FEC code or allows upgrading the link to an higher order modulation format to increase system capacity: in both cases $\mathrm{SNR_{T,dB}}$ can be increased up to $\mathrm{SNR} = \mathrm{SNR_{NL,max,dB}}$, in case the entire margin is "spent". The margin, if kept, is a prevention to receiver aging causing an increase of $\mathrm{SNR_{T,dB}}$ due to performance degradation of components during their lifespan.

An alternative way to express the receiver margin is to refer to the Q factor, defining its margin as:

$$\mu_\mathrm{Q} = \mathrm{Q_{max,dB}} - \mathrm{Q_{T,dB}} \ . \tag{10}$$

where $\mathrm{Q_{max,dB}}$ corresponds to $\mathrm{BER_{min}}$ obtained at $\mathrm{SNR_{NL,max}}$ and $\mathrm{Q_{T,dB}}$ depends on $\mathrm{BER_T}$ through Eq. (6). For multilevel modulation formats as PM-BPSK, PM-QPSK, PM-8QAM and PM-16QAM, BER functions Φ are in the heuristic form $\mathrm{BER} = \mathrm{k_1}\ \mathrm{erfc}\left(\mathrm{k_2}\sqrt{\mathrm{SNR}}\right)$ [5] and so:

$$\mu_\mathrm{Q} \cong \mathrm{SNR_{NL,max,dB}} - \mathrm{SNR_{T,dB}} \ . \tag{11}$$

Therefore, for all practical scenarios the Q-margin μ_Q corresponds to the Rx-margin μ_Rx.

B. *Maximization of span margin*

Considering a system composed of a given number of spans ($\mathrm{N_s} = \mathrm{N_{sys}}$) and operating at a target performance $\mathrm{BER} = \mathrm{BER_T}$, corresponding to $\mathrm{SNR} = \mathrm{SNR_T}$, system optimization may target the span margin maximization. Using convenient dB units, we define such a margin as:

$$\mu_\mathrm{s,max} = \mathrm{A_{s,max,dB}} - \mathrm{A_{sys,dB}} \tag{12}$$

where $\mathrm{A_{s,max,dB}}$ is the maximum tolerable span loss at $\mathrm{BER} = \mathrm{BER_T}$, and $\mathrm{A_{sys,dB}}$ is the actual span loss. In case the span margin is positive, the link may deliver a BER lower than $\mathrm{BER_T}$ if the optimal channel PSD is used. Hence, the span loss can be possibly increased up to $\mathrm{A_{s,max,dB}}$ still keeping the link *in-service*.

In order to derive $\mathrm{A_{s,max,dB}}$ we consider Eq. (5) and evaluate $\mathrm{A_s}$ as a function of the other system parameters, obtaining:

$$\mathrm{A_s} = \frac{1}{\mathrm{E_{ph}} \cdot \mathrm{F}}\left(\frac{\mathrm{G_{ch}}}{\mathrm{N_{sys}} \cdot \mathrm{SNR_T}} - \frac{\Gamma_\mathrm{NLI}}{\mathrm{E_{ph}^2}}\mathrm{G_{ch}^3}\right) \tag{13}$$

that is a function of the channel PSD $\mathrm{G_{ch}}$ as shown in Fig. 3b. The optimal operating point corresponds to the maximum $\mathrm{A_s} = \mathrm{A_{s,max}}$ at the optimal channel PSD $\mathrm{G^B_{ch,opt}}$. From Eq. (13) it is straightforward to derive the optimal values for the channel PSD and for the span loss that are:

$$\mathrm{G^B_{ch,opt}} = \frac{\mathrm{E_{ph}}}{\sqrt{\Gamma_\mathrm{NLI}}}\sqrt{\frac{1}{3 \cdot \mathrm{N_{sys}} \cdot \mathrm{SNR_T}}} \tag{14}$$

and

$$\mathrm{A_{s,max}} = \frac{2}{3}\frac{1}{\mathrm{F}\sqrt{\Gamma_\mathrm{NLI}}}\sqrt{\frac{1}{3(\mathrm{N_{sys}} \cdot \mathrm{SNR_T})^3}} \ . \tag{15}$$

Substituting Eq. (15) in Eq. (12), we obtain a closed form for the span margin μ_s.

The span margin can be seen as a *safeness* against the aging of the link, since degradation of components during their lifespan may induce increase of span loss. It can also be used in the design phase to enlarge the span length or to allow higher lumped losses.

Note that the span margin corresponds to the SNR margin typically evaluated in lab experiments [25]. It is measured through a controlled Rx ASE noise-loading increasing the BER up to $\mathrm{BER_T}$. The difference between the line SNR, measured before the Rx noise-loading, and the SNR after the Rx noise-loading is defined as SNR margin: it can be easily shown that it corresponds to the span margin as defined in this section.

Note that the optimal channel PSD for span margin maximization (Eq. (14)) is different than the one referred to the optimization of Q-margin (Eq. (7)). It means that depending on the design target the choice of the optimal launched power may vary.

C. Maximization of reach

The last design strategy we consider is aimed at maximizing the system reach, given the target $\mathrm{BER} = \mathrm{BER_T}$ ($\mathrm{SNR_{NL}} = \mathrm{SNR_T}$) and the actual span loss $\mathrm{A_s} = \mathrm{A_{sys}}$. To this purpose, from Eq. (5) we derive the number of spans $\mathrm{N_s}$ as a function of the other system parameters. Also for this case, the target parameter is a function of the channel PSD:

$$\mathrm{N_s} = \frac{\mathrm{G_{ch}}}{\mathrm{SNR_T} \cdot \mathrm{E_{ph}} \left(\mathrm{F} \cdot \mathrm{A_{sys}} + \Gamma_{\mathrm{NLI}} \frac{\mathrm{G_{ch}^3}}{\mathrm{E_{ph}^3}} \right)} . \tag{16}$$

Eq. (16) clearly presents a maximum $\mathrm{N_{s,max}}$ at the optimal channel PSD $\mathrm{G_{ch}} = \mathrm{G^C_{ch,opt}}$ as pictorially shown in Fig. 3c. For this design strategy, the optimal channel PSD corresponds to the one obtained for the Q-margin optimization, so its expression is:

$$\mathrm{G^C_{ch,opt}} = \mathrm{G^A_{ch,opt}} = \mathrm{E_{ph}} \frac{1}{\sqrt[3]{\Gamma_{\mathrm{NLI}}}} \left(\frac{\mathrm{F} \cdot \mathrm{A_{sys}}}{2} \right)^{\frac{1}{3}} \tag{17}$$

while the maximum reach has the following close-form [23], [34]

$$\mathrm{N_{s,max}} = \frac{1}{3} \frac{1}{\mathrm{SNR_T}} \frac{1}{\sqrt[3]{\Gamma_{\mathrm{NLI}}}} \left(\frac{2}{\mathrm{F} \cdot \mathrm{A_{sys}}} \right)^{\frac{2}{3}} . \tag{18}$$

A fundamental property, common to the three design strategies, can be derived analyzing in detail the amount of ASE noise and NLI disturbance when operating the link at optimal channel PSD. Under this condition, the NLI intensity always corresponds to half the ASE noise power, both measured in R_s. Consequently, independently of the design target, the NLI penalty is always $2/3$, i.e., 1.76 dB, at the optimal transmitted power, confirming results shown in [33],[34].

III. A SUITABLE APPROXIMATION OF THE GN-MODEL

In Sec. II we assumed that the NLI efficiency Γ_{NLI} is known, and on such a basis we found the three proposed design strategies. Therefore, so far, besides assuming the NLI disturbance to be an AWGN on the center channel, no specific model for NLI calculation has been used. Nevertheless, the proposed strategies clearly show trade-offs among link length, span loss, EDFA noise figure and the Rx sensitivity. A further step in assisting the design process of optical links is the derivation of simple scaling laws of design targets with all link and signal parameters in order to clearly point out the merit of each system characteristic. To pursue such an objective a mathematical model for the NLI is needed.

In the technical literature, several models for NLI generation have been proposed allowing to calculate its intensity - and so its efficiency Γ_{NLI} - from the knowledge of signal spectrum and fiber characteristics. In this paper, we consider the well-known Gaussian-Noise (GN) model for the NLI [20], [21], [5], [19], [12]. This model has been extensively validated experimentally and by simulation, and allows to evaluate analytically the NLI PSD $\mathrm{G_{NLI}}$.

According to the GN-model, in general, the expression of $\mathrm{G_{NLI}}$ introduced by a single fiber span assumes the following form:

$$\begin{aligned} &G_{NLI}(f) = \frac{16}{27}\gamma^2 \mathrm{L_{eff}^2} \\ &\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathrm{G_{WDM}}(\mathrm{f_1})\, \mathrm{G_{WDM}}(\mathrm{f_2}) \\ &\mathrm{G_{WDM}}(\mathrm{f_1}+\mathrm{f_2}-\mathrm{f})\, \rho(\mathrm{f_1},\mathrm{f_2},\mathrm{f})\, \mathrm{df_2 df_1} \end{aligned} \tag{19}$$

where γ is the fiber nonlinear coefficient, $\mathrm{L_{eff}}$ is the effective length of the fiber span, $\rho(\mathrm{f})$ is the four wave mixing efficiency and $\mathrm{G_{WDM}}(\mathrm{f})$ is the PSD of the channel comb at the fiber input. In Eq. (19) it can be noted that the NLI PSD depends on frequency, but in the channel bandwidth $\mathrm{B_{ch}} = \mathrm{R_s}$ such a dependence is practically negligible. In particular, if we focus on the center channel of the NyWDM comb, - the worst case - the NLI can be considered as AWGN and we need to evaluate only $\mathrm{G_{NLI}}$ at $\mathrm{f} = 0$.

Moreover, it has been shown that for NyWDM transmission over a large optical bandwidth, Eq. (19) has a very accurate closed-form approximation: on the center NyWDM channel, the NLI PSD generated by a single fiber span is approximated as an AWGN whose PSD is [21]:

$$\mathrm{G_{NLI}} \cong \frac{8}{54} \mathrm{C} \cdot \mathrm{asinh} \left\{ \frac{\pi}{\mathrm{C}} \frac{|\mathrm{D}|\, \mathrm{R_S^2}}{\alpha_{\mathrm{dB}}} \left[\frac{\mathrm{B_{opt}}}{\mathrm{K_S R_S}} \right]^{\frac{2}{\mathrm{K_S}}} \right\} \frac{\alpha_{\mathrm{dB}}}{|\mathrm{D}|} \gamma^2 \mathrm{L_{eff}^2} \mathrm{G_{ch}^3} \tag{20}$$

where $\mathrm{K_s} = \Delta f / \mathrm{R_s}$, α_{dB} is the fiber loss (dB/km), D is the fiber dispersion (ps/nm/km), γ is the fiber nonlinear coefficient (1/W/km), $\mathrm{L_{eff}}$ is the fiber-span effective length (km), $\mathrm{C} = \frac{2}{5\log_{10}(e)} \frac{\mathrm{f_0^2}}{\mathrm{c}}$, and c is the speed of the light. Using the NLI approximation of Eq. (20), the normalized NLI efficiency assumes the following closed-form:

$$\Gamma_{\mathrm{NLI}} \cong \mathrm{E_{ph}^2} \frac{8 \cdot \mathrm{C} \cdot \mathrm{asinh} \left\{ \frac{\pi}{\mathrm{C}} \frac{|\mathrm{D}| \mathrm{R_s^2}}{\alpha_{\mathrm{dB}}} \left[\frac{\mathrm{B_{opt}}}{\mathrm{K_S R_S}} \right]^{\frac{2}{\mathrm{K_S}}} \right\}}{54} \frac{\alpha_{\mathrm{dB}} \cdot \gamma^2 \cdot \mathrm{L_{eff}^2}}{|\mathrm{D}|} . \tag{21}$$

Substituting Eq. (21) in results obtained in Sec. II, we get analytic expressions for the optimal channel PSD and for the optimal Q-margin, SNR-margin and maximum reach, resulting from the three design rules. Thus, optimal design targets assume close analytic forms with respect to all system parameters, but merits of each parameters are not explicit because in Eq. (21) the dependence on system parameters in part occurs through the function $\mathrm{asinh(x)}$. To the purpose of getting explicit scaling laws with all system parameters pointing out their merit, we investigated the existence of a further approximation of Eq. (21) with a simplified mathematical dependence on system parameters and a controlled accuracy.

Observing Eq. (21), we note that dependence on system parameters can be subdivided into two categories: direct dependence through linear or square proportionality and a dependence through the function $\mathrm{asinh(x)}$, that is weak for practical scenarios.

In order to point out the merit of each system parameters in convenient dB units the approximation of $\Gamma_{\rm NLI}$ must be in the form:

$$\Gamma_{\rm NLI} \cong \frac{\rm k_1}{\rm E_{ph}^2}\frac{\rm B_{opt}^{k_2}\cdot\gamma^2}{\rm K_S^{k_3}\cdot\alpha_{dB}^{k_4}\cdot D^{k_5}}\ . \tag{22}$$

To this purpose we investigated the accuracy of such an expression extensively over a wide range of system parameters, as listed in the following. Fiber span length $\rm L_s \in [50;125]$ km, fiber attenuation $\alpha_{\rm dB} \in [0.15;0.25]$ dB/km, fiber dispersion D [4;24] ps/nm/km, symbol rate $\rm R_s \in [20;40]$ Gbaud, normalized channel spacing $\rm K_s \in [1;1.5]$ and overall optical bandwidth $\rm B_{opt} \in [1;4]$ THz. In this way we are able to obtain a linear expression for $\Gamma_{\rm NLI}$, if dB units are considered. The approximation minimizing the inaccuracy, with respect the exact GN-model, to ± 0.45 dB in the previously listed space of explored system parameters is the following:

$$\Gamma_{\rm NLI} \cong \frac{1.5\cdot 10^{24}}{\rm E_{ph}^2}\frac{\rm B_{opt}^{1/4}\cdot\gamma^2}{\rm K_S\cdot\alpha_{dB}\cdot D^{5/6}}\ . \tag{23}$$

Such approximation, showing the maximum inaccuracy on lower border of rabges of loss and dispersion coefficients, is slightly more precise than the one obtained in [26], whose error with respect to the exact GN-model was found to be in the range ± 0.5 dB.

To further understand the reliability of this approximation, we calculated optimal design parameters of Eq. (8), Eq. (15) and Eq. (18) using for $\Gamma_{\rm NLI}$ both the approximation of Eq. (23) and the exact expression of Eq. (19), varying system parameters within the ranges previously listed. Comparing the results, the error induced by the approximated expression of $\Gamma_{\rm NLI}$ is always within ± 0.25 dB, confirming its good level of accuracy with respect to the exact model.

Substituting Eq. (23) in the equations derived in Sec. II, we get simplified expressions for the optimal channel PSD and for the three optimal design targets. Such approximated expressions will be used in the following section to point out the merit of each system parameter, and so allowing a quick evaluation of the scaling laws of design targets vs. the variation of each parameter, in convenient dB units.

IV. Scaling Laws

In order to better understand how varying system parameters impacts on the design targets, it is useful to consider a reference system and estimate differential variations. Using the design strategies introduced in Sec.II, we derived the differential variation of the optimal power per channel, defined as $\rm P_{ch,opt} = G_{ch,opt}\cdot R_s$, and of the design targets as ratios with respect to reference values. In the following, results for each design strategy are reported, identifying the reference scenario with the "ref" subscript.

A: Maximization of Q-factor

$$\frac{\rm P^A_{ch,opt}}{\rm P^A_{ch,opt,ref}} = \left(\frac{\Gamma_{\rm NLI,ref}}{\Gamma_{\rm NLI}}\frac{\rm F}{\rm F_{ref}}\frac{\rm A_{sys}}{\rm A_{sys,ref}}\right)^{\frac{1}{3}}\left(\frac{\rm R_s}{\rm R_{s,ref}}\right) \tag{24}$$

$$\frac{\rm Q_{max}}{\rm Q_{max,ref}} = \left(\frac{\Gamma_{\rm NLI,ref}}{\Gamma_{\rm NLI}}\right)^{\frac{1}{3}}\left(\frac{\rm F_{ref}}{\rm F}\frac{\rm A_{sys,ref}}{\rm A_{sys}}\right)^{\frac{2}{3}}\frac{\rm N_{sys,ref}}{\rm N_{sys}} \tag{25}$$

B: Maximization of span margin

$$\frac{\rm P^B_{ch,opt}}{\rm P^B_{ch,opt,ref}} = \left(\frac{\Gamma_{\rm NLI,ref}}{\Gamma_{\rm NLI}}\frac{\rm N_{sys,ref}}{\rm N_{sys}}\frac{\rm SNR_{T,ref}}{\rm SNR_T}\right)^{\frac{1}{2}}\left(\frac{\rm R_s}{\rm R_{s,ref}}\right) \tag{26}$$

$$\begin{aligned}\frac{\mu_{\rm s,max}}{\mu_{\rm s,max,ref}} &= \left(\frac{\Gamma_{\rm NLI,ref}}{\Gamma_{\rm NLI}}\right)^{\frac{1}{2}}\cdot\\ &\cdot\left(\frac{\rm N_{sys,ref}}{\rm N_{sys}}\frac{\rm SNR_{T,ref}}{\rm SNR_T}\right)^{\frac{3}{2}}\frac{\rm F_{ref}}{\rm F}\frac{\rm A_{sys,ref}}{\rm A_{sys}}\end{aligned} \tag{27}$$

C: Maximization of reach

$$\frac{\rm P^C_{ch,opt}}{\rm P^C_{ch,opt,ref}} = \left(\frac{\Gamma_{\rm NLI,ref}}{\Gamma_{\rm NLI}}\frac{\rm F}{\rm F_{ref}}\frac{\rm A_{sys}}{\rm A_{sys,ref}}\right)^{\frac{1}{3}}\left(\frac{\rm R_s}{\rm R_{s,ref}}\right) \tag{28}$$

$$\frac{\rm N_{s,max}}{\rm N_{s,max,ref}} = \left(\frac{\Gamma_{\rm NLI,ref}}{\Gamma_{\rm NLI}}\right)^{\frac{1}{3}}\left(\frac{\rm F_{ref}}{\rm F}\frac{\rm A_{sys,ref}}{\rm A_{sys}}\right)^{\frac{2}{3}}\frac{\rm SNR_{T,ref}}{\rm SNR_T} \tag{29}$$

These scaling laws are not approximated, since they assume only the knowledge of $\Gamma_{\rm NLI}$. In order to get the merit of system parameters, we used the GN-model approximation of Eq. (23), obtaining the following expression for the relative $\Gamma_{\rm NLI}$ variations:

$$\frac{\Gamma_{\rm NLI,ref}}{\Gamma_{\rm NLI}} \cong \frac{\rm K_S}{\rm K_{S,ref}}\frac{\alpha_{\rm dB}}{\alpha_{\rm dB,ref}}\left(\frac{\rm D}{\rm D_{ref}}\right)^{\frac{5}{6}}\left(\frac{\gamma_{\rm ref}}{\gamma}\right)^2\left(\frac{\rm B_{opt,ref}}{\rm B_{opt}}\right)^{\frac{1}{4}}. \tag{30}$$

Substituting Eq. (30) in Eqs. (24-29), and using convenient dB units, we obtain the relative variations of optimal power per channel with differential variations of all system parameter:

$$\Delta \rm P^i_{ch,opt} = 10\ \log_{10}\left(\frac{\rm P^i_{ch,opt}}{\rm P^i_{ch,opt,ref}}\right) \cong \sum_{j=1}^{10} \rm w_{p,j}\cdot\Delta_j\ \ [dB], \tag{31}$$

where $\rm i = A, B, C$ is a label for the considered design strategy. For relative variations of the optimal design targets the expression is the following:

$$\Delta \rm par_{max} = 10\ \log_{10}\left(\frac{\rm par_{max}}{\rm par_{max,ref}}\right) \cong \sum_{j=1}^{10} \rm w_{par,j}\cdot\Delta_j\ \ [dB] \tag{32}$$

where $\rm par_{max} = Q_{max}$ or $\mu_{\rm s,max}$ or $\rm N_{s,max}$. $\Delta_{\rm j}$ are the ten differential variations of system parameters detailed described below. The weights $\rm w_{p,j}$ and $\rm w_{par,j}$ are listed in in

Tab. I and II for the optimal power and the optimal design parameters, respectively. From a system design point of view, these weights represent the merit of each system parameter allowing, for instance, to evaluate "at a glance" the benefit, or penalty, induced by 1 dB variation of each system parameter. Note that Eqs. (31-32) are affected by a limited inaccuracy, as opposite to Eqs. (24-29), since they are obtained by mean of the approximation of Eq. 30.

Incremental Parameter [dB]	**Max Q-factor**	**Max span margin**	**Max reach**
	$\Delta P_{\mathrm{ch,opt}}$ **[dB]**		
$\Delta\alpha$	$+\frac{1}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$
ΔA_{sys}	$+\frac{1}{3}$	0	$+\frac{1}{3}$
ΔD	$+\frac{5}{18}$	$+\frac{5}{12}$	$+\frac{5}{18}$
$\Delta\gamma$	$-\frac{2}{3}$	-1	$-\frac{2}{3}$
ΔK_{s}	$+\frac{1}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$
ΔR_{s}	+1	+1	+1
ΔB_{opt}	$-\frac{1}{12}$	$-\frac{1}{8}$	$-\frac{1}{12}$
$\Delta \mathrm{SNR_T}$	*N.A.*	$-\frac{1}{2}$	0
ΔN_{sys}	0	$-\frac{1}{2}$	*N.A.*
ΔF	$+\frac{1}{3}$	0	$+\frac{1}{3}$

TABLE I
MERIT COEFFICIENTS OF SYSTEM PARAMETERS IN THE EVALUATION OF OPTIMAL POWER LEVELS FOR THE THREE CONSIDERED DESIGN STRATEGIES.

Incremental Parameter [dB]	**Max Q-factor** $\Delta Q_{\max}$ **[dB]**	**Max Span-margin** $\Delta\mu_{\mathrm{s,max}}$ **[dB]**	**Max Reach** $\Delta N_{\mathrm{s,max}}$ **[dB]**
$\Delta\alpha$	$+\frac{1}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$
ΔA_{sys}	$-\frac{2}{3}$	-1	$-\frac{2}{3}$
ΔD	$+\frac{5}{18}$	$+\frac{5}{12}$	$+\frac{5}{18}$
$\Delta\gamma$	$-\frac{2}{3}$	-1	$-\frac{2}{3}$
ΔK_{s}	$+\frac{1}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$
ΔR_{s}	0	0	0
ΔB_{opt}	$-\frac{1}{12}$	$-\frac{1}{8}$	$-\frac{1}{12}$
$\Delta \mathrm{SNR_T}$	*N.A.*	$-\frac{3}{2}$	-1
ΔN_{sys}	-1	$-\frac{3}{2}$	*N.A.*
ΔF	$-\frac{2}{3}$	-1	$-\frac{2}{3}$

TABLE II
INDIVIDUAL MERIT COEFFICIENTS OF SYSTEM PARAMETERS FOR THE THREE CONSIDERED DESIGN STRATEGIES.

The meaning of each incremental system parameter with respect to the reference scenario (subscript "ref") is defined in the following.

- $\Delta\alpha = 10 \cdot \log_{10}\left(\frac{\alpha_{\mathrm{dB}}}{\alpha_{\mathrm{dB,ref}}}\right)$: it takes into account the change of the fiber attenuation. The smaller is the fiber attenuation, the larger is the fiber effective length, and nonlinear effects are consequently enhanced. An attenuation reduction implies $\Delta\alpha < 0$ that means poorer system performances for all strategies. $\Delta\alpha$ is weighted $1/3$ for Q-margin and reach, while it is weighted $1/2$ for span margin.
- $\Delta A_{\mathrm{sys}} = A_{\mathrm{sys,dB}} - A_{\mathrm{sys,dB,ref}}$: this is the variation of the overall span loss with respect to the reference set-up. It can be caused by a different lumped loss or by a different fiber loss $A_{\mathrm{dB}} = \alpha_{\mathrm{dB}} \cdot L_{\mathrm{s}}$ depending on both the fiber attenuation α_{dB} and span length L_{s}. A reduction in the overall span loss, i.e., $\Delta A_{\mathrm{sys}} < 0$, improves system performance in all three strategies because the term ΔA_{sys} appears with negative sign in the expressions. Reducing the span loss of 1 dB induces 0.67 dB of Q-margin increasing and allows 16.6 % (0.67 dB) reach extension while the span margin grows of 1 dB. Note that reducing fiber attenuation α has two opposite impacts on system performances: negative through $\Delta\alpha$ and positive through ΔA_{sys}.
- $\Delta D = 10 \cdot \log_{10}\left(\frac{D}{D_{\mathrm{ref}}}\right)$: it is the alteration of the fiber chromatic dispersion coefficient. Increasing D of 25% (1 dB) reduces nonlinear effects, implying 0.28 dB enhancement for Q-margin and reach and 0.42 dB extra span margin span margin.
- $\Delta\gamma = 10 \cdot \log_{10}\left(\frac{\gamma}{\gamma_{\mathrm{ref}}}\right)$: it is the variation of the fiber nonlinear coefficient. A 20% reduction (1 dB)of γ enhances system performances of 0.67 dB for Q-margin and reach and 1 dB for span margin.
- $\Delta N_{\mathrm{sys}} = 10 \cdot \log_{10}\left(\frac{N_{\mathrm{sys}}}{N_{\mathrm{sys,ref}}}\right)$: this differential parameter considers the change in the number of span the link is made of. Hence, a 25% link extension (1 dB) reduces the Q-margin of 1 dB and the span margin of 1.56 dB.
- $\Delta \mathrm{SNR_T} = \mathrm{SNR_{T,dB}} - \mathrm{SNR_{T,dB,ref}}$ considers the variation in the signal to noise ratio required to keep the link in-service. Asking for 1 dB better performance, $\Delta \mathrm{SNR_T}$ = 1 dB, reduces of 1.5 dB the span margin and of 20% (1 dB) the max reach. $\mathrm{SNR_T}$ depends on the modulation format cardinality, on the Tx/Rx implementation and on the chosen FEC. In general, a variation $\Delta \mathrm{SNR_T}$ is given by changes in TX/Rx setup.
- $\Delta F = F_{\mathrm{dB}} - F_{\mathrm{dB,ref}}$ accounts for the change in the quality of the amplifiers. Poorer quality amplifier means positive ΔF. ΔF = 1 dB induces 1 lower span margin, 0.67 dB smaller Q-margin and 15% (0.67 dB) reduction in max reach reach.
- $\Delta R_{\mathrm{s}} = 10 \cdot \log_{10}\left(\frac{R_{\mathrm{s}}}{R_{\mathrm{s,ref}}}\right)$: it is the variation of the symbol rate operating the link and does not modify design targets. It only affects the optimal power per channel.
- $\Delta K_{\mathrm{s}} = 10 \cdot \log_{10}\left(\frac{K_{\mathrm{s}}}{K_{\mathrm{s,ref}}}\right)$: it considers changes in the normalized channel spacing that gives the system spectral efficiency (SE): $\Delta K_{\mathrm{s}} = -\Delta \mathrm{SE}$ Reducing SE of 20% (1 dB) improves span margin of 0.5 dB, Q-margin of 0.33 dB and max reach of 8% (0.33 dB).
- $\Delta B_{\mathrm{opt}} = 10 \cdot \log_{10}\left(\frac{B_{\mathrm{opt}}}{B_{\mathrm{opt,ref}}}\right)$: it is the change in the overall occupied optical bandwidth given the number of transmitted channels N_{ch}. Increasing the total number of channels induces a weak enhancement of nonlinear effects as 25% more channels ($\Delta B_{\mathrm{opt}} = 1$ dB) reduces the span margin of 0.125 dB, the Q-margin of 0.08 dB and the maximum reach of 2% (0.08 dB).

V. VALIDATION

In order to validate results of Sec. IV we considered already published experimental and simulative results, comparing published results to the quick derivations obtained using the proposed scaling laws. We apply the validation process to the Q-margin and reach maximization only, as the suitable published experimental results refer to these two design strategies and we could not find relevant data targeting optimization of span margin.

The proposed scaling laws give variations with respect to a reference scenario, therefore, a reference setup must be considered.

A. Scaling laws for maximum Q

Experiment	Channel spacing Bandwidth Distance	max Q factor [dB]		
		Exp	Scaling Law	Eq. (25)
Reference 16.64 Gbaud PM-16QAM 8 channels	Δf = 17 GHz B_{opt} = 136 GHz N_{sys} = 120	6.1 (experiment)		
(a) 16.64 Gbaud PM-16QAM 8 channels	Δf = 23.5 GHz B_{opt} = 188 GHz N_{sys} = 120	6.5	6.45	6.25
(b) 16.64 Gbaud PM-16QAM 294 channels	Δf = 17 GHz $B_{opt} \cong$ 5 THz N_{sys} = 131	5.2	4.95	5
(c) 28 Gbaud PM-16QAM 16 channels	Δf = 33 GHz B_{opt} = 528 GHz N_{sys} = 60	8	8.35	8.80

TABLE III
MAXIMIZATION OF Q FACTOR: COMPARISON BETWEEN EXPERIMENTAL RESULTS AND SCALING LAW PREDICTIONS DEFINED IN EQ. (32) WITH $\mathrm{par} = \mathrm{Q}$ AND MERIT COEFFICIENTS OF TAB II. PREDICTIONS BASED ON EQ. (25) USING EQ.(21) ARE DISPLAYED AS A REFERENCE.

The reference setup considered for the Q-margin maximization strategy was the one described in [28], i.e., a system carrying 8 NyWDM channels, modulated with a PM-16QAM, with channel spacing Δf = 17 GHz and symbol rate R_s = 16.64 Gbaud, propagating on a periodical EDFA amplified link composed of 120 identical spans of Large Effective Area Fiber ($\sim$ 152 μm^2) with length $L_s \cong$ 55 km, for a total length of 6600 km. The LEAF has an attenuation coefficient α_{dB} = 0.18 dB/km and dispersion coefficient D = 21.6 ps/nm/km. In order to validate the scaling laws for the maximum Q-margin, different experimental results are compared to the analytical derivations using the proposed scaling laws with merit coefficients of Tab. II. The first considered experiment [28], labeled (a), was based on a setup identical to the reference one except for the channel spacing, which is increased to the value of 23.5 GHz. The second experiment we took into account [29], labeled (b), was based on 294 NyWDM channels, modulated with an half 4D PM-16QAM, with channel spacing Δf = 17 GHz and symbol rate R_s = 16.64 Gbaud, propagating on a uniform link made of 131 spans of Large Effective Area Fiber ($\sim$ 152 μm^2), with length L_s = 55 km, resulting in a total transmission distance of 7230 km. The link amplification was based on EDFAs. The third considered experiment [15] (label (c)), was a 16 NyWDM channels setup, modulated with a PM-16QAM, with channel spacing Δf = 33 GHz and symbol rate R_s = 28 Gbaud, propagating on a uniform link consisting in 60 spans, amplified by EDFAs, composed of 25 km of Corning Vascade EX3000 (150 μm^2, 0.16 dB/km) and 25 km of Corning Vascade EX2000 (112 μm^2, 0.16 dB/km), resulting in a total transmission distance of 3000 km. Since scaling laws does not consider non-uniform links made of different fibers, we approximated the actual span with an equivalent uniform span of 50 km Corning Vascade EX3000 as in lumped amplification only the first portion of the fiber span is responsible for NLI generation.

The reference Q factor was 6.1 dB, as shown in [28]. The variations of the maximum Q (ΔQ_{max}) with respect to the reference setup are evaluated using merit coefficients displayed in Tab II and Eq. 32 with $\mathrm{par} = \mathrm{Q}$. The analytical results obtained using the individual merit of the system parameters, together with the experimental results listed in [28], [29] and [15] are reported in Tab. III. The scaling law predictions demonstrate to be really accurate showing to be able to keep the inaccuracy within 0.3 dB. In Tab. III, we reported as a reference also the predictions based on the GN-model, i.e., on Eq. (25) using Eq.(21) for $Gamma_{NLI}$ evaluation. They confirm the accuracy of ± 0.45 dB previously mentioned. These data confirm the accuracy of the proposed method also when applied to different symbol rates, channel spacing, modulation formats, system bandwidths and target distances.

B. Scaling laws for maximum reach

Experiments	Symbol Rate Channel spacing Bandwidth	Fiber Type	N_{max}		
			Exp	Scaling Laws	Eq. (29)
Reference 200G PM-16QAM 9 channels	R_s = 32 Gbaud K_s = 1.05 B_{opt} = 302 GHz	SSMF	15 (simulation)		
(a) 100G PM-QPSK 10 channels	R_s = 30 Gbaud K_s = 1.1 B_{opt} = 330 GHz	NZDSF	8	8	7
		SSMF	20	20	20
		PSCF	32	31	31
(b) 100G PM-16QAM 22 channels	R_s = 15.625 GBaud K_s = 1.024 B_{opt} = 352 GHz	NZDSF	12	12	12
		SSMF	38	38	41
		PSCF80	44	44	48
		PSCF110	58	56	61
		PSCF130	62	61	67
		PSCF150	70	68	75
(c) 100G PM-64QAM 20 channels	R_s = 10.4 Gbaud K_s = 1.15 B_{opt} = 240 GHz	PSCF150	28	30	32

TABLE IV
MAXIMIZATION OF REACH: COMPARISON BETWEEN EXPERIMENTAL RESULTS AND SCALING LAW PREDICTIONS DEFINED IN EQ. 32 WITH $\mathrm{par} = \mathrm{N_S}$ AND MERIT COEFFICIENTS OF TAB II. PREDICTIONS BASED ON EQ. (29) USING EQ.(21) ARE DISPLAYED AS A REFERENCE.

In order to validate the scaling laws for the maximum reach we used simulative results as a reference. We simulated a 9 PM-16QAM NyWDM channel comb at 200G ($R_{s,ref}$ = 32 Gbaud) channel spacing $K_{s,ref}$ = 1.05 ($B_{opt,ref}$ = 302 GHz) propagating on a SSMF uniform and uncompensated link with span length $L_{s,ref}$ = 80 km and optical amplifiers with noise figure $F_{dB,ref}$ = 5 dB. Fiber parameters were: $\alpha_{dB,ref}$ = 0.22 dB/km, $A_{s,dB,ref}$ = 17.6 dB, D_{ref} = 16.7 ps/nm/km, γ_{ref} = 1.3 1/W/km. The reference BER was 10^{-3} corresponding to $\mathrm{SNR}_{T,dB,ref}$ = 16.85 dB for the considered modulation format and Tx/Rx implementation. The reference value of max reach obtained by simulation was $N_{s,max,ref}$ = 15. Then, we considered data coming from different experimental results and estimated the maximum reach as

$$\mathrm{N_{s,max}} = \mathrm{N_{s,max,ref}} \cdot 10^{\frac{\Delta \mathrm{N_{s,max}}}{10}} \tag{33}$$

where $\Delta\mathrm{N_{s,max}}$ was obtained applying Eq. 32 with merit coefficients listed in Tab II. Results are displayed in Tab. IV. Experiment (a) [30] was a 10-channel NyWDM 100G PM-QPSK setup ($\mathrm{R_s}$ = 30 GBaud, $\mathrm{K_s}$ = 1.1, $\mathrm{B_{opt}}$ $\cong$ 330 GHz) determining the maximum reach with 3 different fibers at $\mathrm{BER} = 10^{-3}$ ($\mathrm{SNR_{T,dB}}$ = 12.7 dB). The second one - labeled (b)- [31] was a 22-channel NyWDM 100G PM-16QAM setup, with symbol rate $\mathrm{R_s}$ = 15.625 GBaud, $\mathrm{K_s}$ = 1.024, $\mathrm{B_{opt}}$ $\cong$ 352 GHz, that investigated the maximum reach over 7 different fiber types at $\mathrm{BER} = 10^{-2}$ ($\mathrm{SNR_{T,dB}}$ = 17.3 dB). Regarding the second experiment, we did not consider DCF results, as parameters for this fiber type are out of the ranges considered in this work, being the DCF not a standard transmission fiber. For the other fibers, besides parameters listed in [31], we included the following measured insertion extra losses: 2.22 dB (NZDSF), 0.28 dB (SSMF), 0.57 dB (PSCF80), 0.93 dB (PSCF110), 0.84 dB (PSCF130) and 0.79 dB (PSCF150). The last experiment we took into account - labeled (c) - [32] was a 20-channel NyWDM 100G PM-64QAM setup, with $\mathrm{R_s}$ = 10.4 GBaud, $\mathrm{K_s}$ = 1.15, $\mathrm{B_{opt}}$ $\cong$ 240 GHz operated on PSCF150 fiber.

As it can be observed in Tab. IV, using merit coefficients of Tab II in Eq. 32 we get an excellent accuracy whose maximum error with respect to the experimental results is 2 spans in a 30 span reach, corresponding to 6.7 % (0.3 dB).

In Tab. IV, we also display as a reference the predictions based on the GN-model, i.e., on Eq. (29 using Eq.(21) for $Gamma_{NLI}$ evaluation. The previously mentioned maximum inaccuracy of ± 0.45 dB.

These findings confirm the reliability of the proposed approach: even when applied to different modulation formats and rates, and also over a wide range of fiber types it is able to predict with very good accuracy the maximum reach.

VI. Example of scaling laws

In this section, we discuss and comment on practical application of the proposed scaling laws. Among the three proposed design strategies, we comment only the maximization of span margin and reach, as Q maximization behaves exactly as the reach maximization.

A. Merit of fiber type

In the following, we comment on how fiber characteristics impact on design targets, considering as an example replacement of standard SMF with a typical PSCF.

1) Effective Area: $\mathrm{A_{eff}}$ [$\mu\mathrm{m}^2$].
$\mathrm{A_{eff}}$ determines the fiber nonlinear coefficient $\gamma = \frac{2\pi}{\lambda}\frac{\mathrm{n_2}}{\mathrm{A_{eff}}}$. Neglecting weak $\mathrm{n_2}$ variations, we can assume γ to be inversely proportional to $\mathrm{A_{eff}}$. Hence, we can express differential variation of nonlinear coefficient as $\Delta\gamma = -10 \cdot \log_{10}\left(\frac{\mathrm{A_{eff}}}{\mathrm{A_{eff,ref}}}\right)$. For instance, using a PSCF ($\mathrm{A_{eff}}$ $\cong$ 130 $\mu\mathrm{m}^2$) with respect to a standard SMF ($\mathrm{A_{eff}}$ $\cong$ 80 $\mu\mathrm{m}^2$), $\Delta\gamma \cong -2$ dB. In maximization of the span margin the merit coefficient of nonlinearity is unitary for $\mathrm{A_{max}}$, hence, migrating to PSCF from SSMF gives a 2 dB span margin increasing. In designing processes aimed at reach maximization, merit coefficient of fiber nonlinearity reduction is $2/3$, consequently replacing SSMF with PSCF we can extend the reach, in number of spans, of 36 % (1.33 dB) and optimal power per channel grows of 1.33 dB as well.
2) Chromatic dispersion: D [ps/nm/km].
It is well-known that the increasing of fiber chromatic dispersion counteracts negative effects of nonlinearity. Using the proposed scaling laws, we can give a quick estimate to its benefit, as $\Delta\mathrm{D}$ has merit coefficient $5/12$ in targeting the max span and merit $5/18$ in reach maximization. Using a PSCF with $\mathrm{D} \cong$ 20.6 ps/nm/km with respect to a SSMF with D $\cong$ 16.7 ps/nm/km, $\Delta\mathrm{D}$ $\cong$ 0.9 dB. The max span margin increases of 0.4 dB while the number of spans extends of about 6 % (0.25 dB).
3) Attenuation coefficient and span length: α_{dB} [dB/km] and $\mathrm{L_s}$ [km].
The effects of the fiber attenuation coefficient and of span length must be considered together, because these parameters jointly determine the span loss.
On one side, a smaller α_{dB} is beneficial for system performance as reduces the overall span loss $\mathrm{A_{sys}}$, but on the other side it increases the impact of non-linear effects giving a larger effective length $\mathrm{L_{eff}} \propto \frac{1}{\alpha_{\mathrm{dB}}}$.
In all practical scenario the overall merit is positive and reducing fiber attenuation is beneficial for system performance because $\Delta\mathrm{A_{sys}}$ always overwhelms $\Delta\alpha$ and has merit coefficients two times the ones of $\Delta\alpha$. Considering to replace a SSMF (α_{dB} = 0.22 dB/km) with a PSCF (α_{dB} = 0.16 dB/km) over a span of 100 km, we have $\Delta\alpha \cong$ -1.4 dB and $\Delta\mathrm{A_{sys}}$ = -6 dB, giving $\Delta\mu_{\mathrm{s,max}}$=5.36 dB and $\mathrm{N_{s,max}}$=3.5 dB (125% reach extension).
Considering only the benefit in shortening the span length, its merit corresponds to the one of $\Delta\mathrm{A_{sys}}$.

B. Modulation format and FEC

Keeping the symbol rate as a constant, changing modulation format and/or FEC coding causes a variation $\Delta\mathrm{SNR_T}$ in the required signal to noise ratio $\mathrm{SNR_T}$ at the target BER. As a practical example, we consider an upgrade from PM-QPSK to PM-16QAM in order to double the link capacity. Considering ideal Tx/Rx setups, $\mathrm{SNR_T}$ at $\mathrm{BER_T} = 10^{-3}$ goes from 9.8 dB to 16.5 dB [5]. We have to face a positive $\Delta\mathrm{SNR_T} = 6.7$ dB impairing system performances. In particular, span margin is decreased of 10 dB, or the reach is reduced to about one fifth (6.7 dB).

C. Bandwidth, number of channels and channel spacing

These parameters must be considered simultaneously, as one influences the others through the expression:

$$\mathrm{B_{opt}} = \mathrm{K_s} \cdot \mathrm{N_{ch}} \cdot \mathrm{R_s} \tag{34}$$

where N_{ch} is the number of channels. Keeping the channel spacing $\Delta f = K_s \cdot R_s$ constant, the scaling of the number of channels corresponds to the scaling of the overall bandwidth, i.e., $\Delta B_{opt} = \Delta N_{ch} = 10 \cdot \log_{10}\left(\frac{N_{ch}}{N_{ch,ref}}\right)$. It means, for instance, that doubling the number of channels implies $\Delta B_{opt} = 3$ dB. In the optimization of the span margin the merit coefficient of ΔB_{opt} is $1/8$, hence doubing the number of channels impair of only 0.4 dB. For the reach maximization, the effect of ΔB_{opt} is even weaker, with a merit coefficient $1/12$, therefore doubling the number of channels induces only a 6 % (0.25 dB) in reach reduction. Supposing to modify the number of channels keeping the symbol rate and the overall occupied bandwidth, correspond to change the normalized channel spacing, i.e., $\Delta K_s = -\Delta N_{ch}$. In this case, halving the number of channels implies $\Delta K_s = 3$ dB induces 1.5 dB increasing in span margin and 25 % (1 dB) in reach extension.

D. Optical amplifiers

A change in the quality of amplifiers is taken into account by the variation ΔF of noise figure. In span margin maximization, ΔF gives a direct margin advantage, i.e., a 1 dB improvement in the amplifier noise figure corresponds to 1 dB margin increase. In maximizing the reach, ΔF is weighted by a factor $2/3$, that means that 1 dB smaller F enables a 6 % (2/3 dB) reach extension.

Note that the presented analysis refers to lumped amplification only and so does not formally include the use of Raman distributed amplification. However, if we consider hybrid fiber amplifiers (HFA) based of counter-propagating Raman amplification used as minor supplement to EDFA amplification, it has been shown [19], [35] that the NLI enhancement due to distributed amplification is negligible. Under these conditions, the presented analysis can be applied also to HFA based systems simply considering the Raman noise figure improvement.

VII. Conclusions

Results of this paper are focused to the analysis of transmission of NyWDM comb of channels based on multilevel modulation formats on uniform uncompensated links with DSP-based polarization-diversity coherent receivers. For such system scenarios, performances are limited by ASE noise accumulation and generation of the disturbance called NLI, that scales with the cube of the power per channel weighted by the NLI efficiency Γ_{NLI}.

With these assumptions, we propose three different link design strategies aimed at maximizing the Q margin, the span margin and the reach. We showed that the penalty induced by nonlinearities is always 1.76 dB at the optimal power per channel, independently of the chosen design target. Then, we used the mathematical model called GN-model for the NLI evaluation in order to give close analytical forms for the proposed design targets.

In order to include in a single merit coefficient the effect of each system parameter we proposed a suitable approximation of the GN-model and used it to introduce scaling laws of the design targets with all the system parameters.

We validated such a derivation using already published experimental results showing how the use of merit coefficients give results in very good agreement with the experiments, even when applied to different modulation formats and rates, and also over a wide range of fiber types.

Finally, we commented on the application of the proposed scaling laws to practical choices typically faced in designing or upgrading optical communication links, showing how results presented in this paper can be used for a reliable first assessment of system design.

References

[1] V. Curri, P. Poggiolini, A. Carena, F. Forghieri, “Dispersion compensation and mitigation of non-linear effects in 111 Gb/s WDM coherent PM-QPSK systems,” IEEE Phot. Tech. Letters, vol. 20, Sept. 2008, pp. 1473-1475.

[2] G. Gavioli et al., “100Gb/s WDM NRZ-PM-QPSK Long-Haul Transmission Experiment over Installed Fiber Probing Non-Linear Reach With and Without DCUs,” *P*roc. of ECOC 2009, Vienna, Sep. 20-24, 2009, paper 3.4.2.

[3] D. van den Borne, V. A. J. M. Sleiffer, M. S. Alfiad, S. L. Jansen and T. Wuth, “POLMUX-QPSK modulation and coherent detection: the challenge of long-haul 100G transmission,” *P*roc. of ECOC 2009, Vienna, Sep. 20-24, 2009, paper 3.4.1.

[4] A. Bononi, P. Serena, N. Rossi, E. Grellier, and F. Vacondio, ‘Modeling nonlinearity in coherent transmissions with dominant intrachannel-four-wave-mixing,’ *Optics Express*, vol. 20, pp. 7777-7791, 26 March 2012.

[5] A. Carena, V. Curri, G. Bosco, P. Poggiolini, F. Forghieri, “Modeling of the Impact of Nonlinear Propagation Effects in Uncompensated Optical Coherent Transmission Links,” IEEE/OSA J. of Lightw. Tech., Vol. 30, Issue 10, pp. 1524-1539, May 2012

[6] A. Mecozzi and R.-J. Essiambre, ‘Nonlinear Shannon limit in pseudo-linear coherent systems,’ *J. Lightwave Technol.,* vol. 30, no. 12, pp. 2011-2024, June $15^{\rm th}$ 2012.

[7] M. Secondini and E. Forestieri, ‘Analytical fiber-optic channel model in the presence of cross-phase modulations,’ *IEEE Photon. Technol. Lett.,* vol. 24, no. 22, pp. 2016-2019, Nov. $15^{\rm th}$ 2012.

[8] P. Johannisson and M. Karlsson, ‘Perturbation analysis of nonlinear propagation in a strongly dispersive optical communication system,’ *J. Lightwave Technol.,* vol. 31, no. 8, pp. 1273-1282, Apr. $15^{\rm th}$ 2013.

[9] R. Dar, M. Feder, A. Mecozzi, and M. Shtaif, ‘Properties of nonlinear noise in long, dispersion-uncompensated fiber links,’ *Optics Express*, vol. 21, no. 22, pp. 25685-25699, Nov. 2013.

[10] P. Serena and A. Bononi, ‘An alternative approach to the Gaussian noise model and its system implications,’ *J. Lightwave Technol.,* vol. 31, no. 22, pp. 3489-3499, Nov. $15^{\rm th}$ 2013.

[11] M. Secondini, E. Forestieri and G. Prati, ‘Achievable information rate in nonlinear WDM fiber-optic systems with arbitrary modulation formats and dispersion maps,’ *J. Lightwawe Technol.,* vol. 31, no. 23, pp. 3839-3852, Dec. $1^{\rm st}$, 2013.

[12] P. Poggiolini, G. Bosco, A. Carena, V. Curri, Y. Jiang, F. Forghieri, ‘The GN model of fiber non-linear propagation and its applications,’ *J. of Lightw. Technol.,* vol. 32, no. 4, pp. 694-721, Feb. 2014.

[13] A. Carena, G. Bosco, V. Curri, Y. Jiang, P. Poggiolini and F. Forghieri, ‘EGN model of non-linear fiber propagation,’ *Optics Express,* vol. 22, no. 13, pp. 16335-16362, June 2014. Extended appendices with full formulas derivations can be found in the version available on *www.arXiv.org*.

[14] R. Dar, Omri Geller, M. Feder, A. Mecozzi, and M. Shtaif, ‘Mitigation of inter-channel nonlinear interference in WDM systems,’ *P*roc. of ECOC 2014, paper P.5.6, Cannes (FR), Sept. 2014.

[15] J. Renaudier, O. Bertran-Pardo, A. Ghazisaeidi, P. Tran, H. Mardoyan, P. Brindel, A. Voicila, G. Charlet, and S. Bigo ”Experimental Transmission of Nyquist Pulse Shaped 4-D Coded Modulation using Dual Polarization 16QAM Set-Partitioning Schemes at 28 Gbaud,” *P*roc. of OFC 2013, Anheim, Mar. 17-21, 2013, paper OTu3B.1.

[16] Y. Jiang, A. Carena, P. Poggiolini, and F. Forghieri, ‘On the impact of non-linear phase-noise on the assessment of long-haul uncompensated coherent systems performance,’ in *Proc. of ECOC 2014*, Cannes (FR), Sept. 2014.

[17] M. Secondini and E. Forestieri ‘On XPM mitigation in WDM fiber-optic systems,’ *IEEE Photon. Technol. Lett.*, vol. 26, no. 22, pp. 2252-2255, Nov. 2014.

[18] R. Dar, M. Feder, A. Mecozzi and M. Shtaif 'Accumulation of nonlinear interference noise in fiber-optic systems,' *Optics Express*, vol. 22, Issue 12, pp. 14199-14211, 16 June 2014.

[19] V. Curri, A. Carena, P. Poggiolini, G. Bosco, F. Forghieri, "Extension and Validation of the GN Model for Non-Linear Interference to Uncompensated Links using Raman Amplification," Optics Express, Vol. 21 Issue 3, pp.3308-3317, 2013.

[20] P. Poggiolini, A. Carena, V. Curri, G. Bosco, F. Forghieri, "Analytical modeling of non-linear propagation in uncompensated optical transmission links," IEEE Photon. Technol. Lett., Vol. 23, Issue 11, 74217744, 2011.

[21] P. Poggiolini, "The GN Model of Non-Linear Propagation in Uncompensated Coherent Optical Systems", accepted for publication on *IEEE/OSA J. of Lightw. Tech.*, special issue on *100G-Terabit Optical Networks*.

[22] A. Carena, V. Curri, G. Bosco, P. Poggiolini, F. Forghieri, "A Novel figure of merit to compare fibers in coherent detection systems with uncompensated links," Optics Express, Vol. 20, Issue 1, pp. 339-346, 2012.

[23] V. Curri, A. Carena, G. Bosco, P. Poggiolini, M. Hirano,Y. Yamamoto, F. Forghieri, "Fiber Figure of Merit Based on Maximum Reach," *P*roc. of OFC 2013, Anaheim, Mar. 17-21, 2013, paper OTh3G2.

[24] A. Carena, P. Poggiolini, V. Curri, G. Bosco, F. Forghieri, "Evaluation of the Dependence on System Parameters of Non-Linear Interference Accumulation in Multi-Span Links," *P*roc. of ECOC 2012, Amsterdam, Sep. 16-20, 2012, paper We.2.C.6.

[25] A. J. Stark et. al., "System Performance Prediction with the Gaussian Noise Model in 100G PDM-QPSK Coherent Optical Networks," IEEE/OSA J. of Lightw. Tech., Vol. 31, Issue 21, pp. 3352-3360, 2013.

[26] V. Curri, A. Carena, G. Bosco, P. Poggiolini, A. Nespola, F. Forghieri, "Design Rules for Reach Maximization in Uncompensated Nyquist-WDM Links," *P*roc. of ECOC 2013, London, Sep. 22-26, 2013, paper Th.1.D.1.

[27] B. DeMuth, M. Y. Frankel, V. Pelekhaty, "Semi-empirical system scaling rules for DWDM system design," Optics Express, Vol. 20, Issue 3, pp. 2992-3003, 2012.

[28] M. Mazurczyk, "Spectral Shaping in Long Haul Optical Coherent Systems With High Spectral Efficiency," IEEE/OSA J. of Lightw. Tech., Vol. 32, Issue 16, pp. 2915 - 2924, 2014.

[29] H. Zhang, H. G. Batshon, D. G. Foursa, M. Mazurczyk, J.-X. Cai, C. R. Davidson, A. Pilipetskii, G. Mohs and Neal S. Bergano, "30.58 Tb/s Transmission over 7,230 km using PDM Half 4D-16QAM Coded Modulation with 6.1 b/s/Hz Spectral Efficiency," *P*roc. of OFC 2013, Anheim, Mar. 17-21, 2013, paper OTu2B.3.

[30] E. Torrengo, R. Cigliutti, G. Bosco, A. Carena, V. Curri, P. Poggiolini, A. Nespola, D. Zeolla, F. Forghieri, "Experimental validation of an analytical model for nonlinear propagation in uncompensated optical links," Optics Express, Vol. 19, Issue 26, 2011.

[31] A. Nespola, S. Straullu, A. Carena, G. Bosco, R. Cigliutti, V. Curri, P. Poggiolini, M. Hirano, Y. Yamamoto, T. Sasaki, J. Bauwelinck, K. Verheyen, F. Forghieri, "Extensive Fiber Comparison and GN-model Validation in Uncompensated Links using DAC-generated Nyquist-WDM PM-16QAM Channels," *P*roc. of OFC 2013, Anaheim, Mar. 17-21, 2013, paper OTh3G5.

[32] A. Nespola, S. Straullu, G. Bosco, A. Carena, Y. Jiang, P. Poggiolini, F. Forghieri, Y. Yamamoto, M. Hirano, T. Sasaki, J. Bauwelinck, K. Verheyen, "1306-km 20x124.8-Gb/s PM-64QAM Transmission over PSCF with Net SEDP 11,300 (b?km)/s/Hz using 1.15 samp/symb DAC," *P*roc. of ECOC 2013, London, Sep. 22-26, 2013.

[33] G. Bosco, A. Carena, R. Cigliutti, V. Curri, P. Poggiolini, and F. Forghieri, "Performance prediction for WDM PM-QPSK transmission over uncompensated links," *P*roc. of OFC 2011, Los Angeles, Mar. 8-10, 2011, paper OThO7.

[34] A. Bononi, N. Rossi, and P. Serena, "On the nonlinear threshold versus distance in long-haul highly-dispersive coherent systems," Opt. Express, Vol. 20, Issue. 26, pp. B204B216, Nov. 2012.

[35] V. Curri, A. Carena "HFA Optimization for Nyquist WDM Transmission," *P*roc. of OFC 2015, Los Angeles, Mar. 22-26, 2015, paper W4E.4.