

Optimization of breastshot water wheels performance using different inflow configurations

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Abstract

1 Breastshot water wheels are gravity hydraulic machines employed in low head
2 sites. The scope of this work is to test the performance of a breastshot water
3 wheel with two geometric inflow configurations: a sluice gate at different openings
4 and two vertical overflow weirs. With the sluice gate, the maximum efficiency of
5 the plant is 75%, constant over a wide range of flow rates, while the efficiency
6 with the weir is increasing in the same flow rate range. Therefore, the wheel
7 with the weir can exploit higher water volumes, and also it performs better
8 at high power input. In practical applications, the inflow configuration can be
9 effectively controlled to optimize the operative working conditions of breastshot
10 water wheels, depending on the external hydraulic ones. The experimental results
11 are also discussed in dimensionless terms, in order to support engineers in the
12 design of similar breastshot water wheels.

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1. Introduction

13 The wheel has been one of the most ancient technology used by mankind to
14 produce energy. The first vertical water wheel was the *stream* water wheel, still
15 used nowadays in flowing water [1]. The water interacts with the blades below
16 the wheel and the kinetic energy of streams drives the wheel. In *gravity* wheels
17 (*overshot*, *breastshot* and *undershot* water wheels) the weight of water is mainly
18 employed for the generation of energy, in sites where a geometric head difference
19 exists (the difference of the channel's bed elevation upstream and downstream of
20 the wheel). In overshot water wheels the water enters into the cells from the top
21 of the wheel. They are generally used for head differences between 2.5 and 10 m
22 and at low flow rates (approximately from 0.2 to 1.0 m³/s per unit width). In
23 breastshot wheels the water enters into the buckets near the rotation axle. These
24 wheels are usually employed for head differences lower than 4 m and at flow rates
25 from 0.5 to 2.0 m³/s per unit width. When the geometric head difference is very
26 low (e.g. $1/8 \div 1/10$ of the diameter, although there not exists a precise limit),
27 breastshot water wheels can be called *low* breastshot wheels, or undershot wheels:
28 the water fills the buckets in the lowest part of the wheel and these wheels are
29 generally used at flow rates from 1 to 3 m³/s per unit width.

30 During the Eighteenth and Nineteenth century, some experimental tests and
31 theoretical estimations for the determination of the efficiency of water wheels
32 were developed [2] [3] [4] [5] [6] [7] [8]. However, the previous studies generally
33 were not totally satisfactory, since theoretical analyses were not supported by
34 experimental tests, and comparisons among different geometric configurations
35 under the same hydraulic conditions were generally not presented. Therefore,

36 the most of the available engineering and scientific information is ancient, with
37 uncertainty and often published in not well known text-books.

38 At the beginning of the Twentieth century, the rising demand of energy, the
39 economic development and the rapid improvement in the engineering knowledge
40 (especially the design of big hydroelectric plants and the transmission of electric-
41 ity), led to the introduction and diffusion of modern turbines, employed in big
42 hydroelectric plants with heads of tens/hundreds meters. Therefore, the classical
43 water wheels, used in low head sites especially for self sustainment, were replaced
44 and by then considered ancient and bygone machines.

45 In the last years, due to their numerous purposes, quite high efficiency, low
46 payback periods, low environmental impact and simplicity of construction [9],
47 water wheels are regarded again as interesting hydraulic machines for the pro-
48 duction of decentralized energy, especially when combined with a mill for grind-
49 ing wheat. Indeed, it is a general view that bread made by water mill's flour is
50 tastier than that produced by electric engines and it has also a finer quality and
51 higher nutritive value [10]. When installed in old water mills, water wheels may
52 also contribute to the preservation of the cultural heritage, the development of
53 tourism, the promotion of local manufacture and the creation of employment.
54 Hence water wheels may become a profitable industry, especially due to the wide
55 diffusion on the territory of sites suitable for water wheels [11]. These machines
56 may be also an interesting investment in rural areas, since their payback periods
57 are low ($7 \div 14$ years with respect to 30 years for a Kaplan installation) [9].

58 Therefore, thanks to the previous motivations, the interest of the scientific
59 community in water wheels is starting to increase. For example, recent scientific
60 studies on undershot and stream wheels can be found in [1] [12] [13] [14] [15].

61 In [16] a study of an overshot water wheel is presented. Concerning breastshot
62 water wheels, in [17] [18] theoretical and dimensional analysis, respectively, have
63 been performed for a breastshot wheel equipped with a sluice gate.

64 **2. Breastshot water wheels**

65 Since this work will investigate different inflow configurations of a breastshot
66 water wheel, it is worthwhile to cite the book of *Garuffa* [7], where breastshot
67 water wheels are classified as *fast* and *slow*. Figure 1 shows a *fast* breastshot
68 wheel, where the inflow configuration is constituted of a sluice gate. Figure 2 de-
69 picts a *slow* breastshot water wheel, where the inflow configuration is constituted
70 of an overflow weir. In this book, the previous terminology is inspired by the fact
71 that in fast breastshot wheels the flow accelerates passing under the sluice gate.
72 The flow velocity to the wheel is hence faster with respect to the flow velocity
73 in *slow* wheels, where the water passes over an overflow weir just upstream of
74 the wheel, entering into the buckets from higher elevations. This means that,
75 considering the same flow rate, head difference and wheel rotational speed, the
76 torque contribution of the water weight in *slow* wheels is higher with respect to
77 the torque contribution of the water weight in *fast* breastshot wheels. In *slow*
78 breastshot wheels the torque due to the kinetic energy of water is lower with
79 respect to *fast* wheels.

80 Although it is not mandatory to install one of the previous hydraulic struc-
81 tures upstream of a water wheel, they are useful. These inflow structures allow to
82 regulate and to optimize the operative working conditions. Sluice gates and weirs
83 are usually present in irrigation canals, where suitable conditions for breastshot
84 water wheels exist. Due to the higher flow velocity, in fast breastshot wheels the

kinetic energy of the flow can contribute significantly to the driving torque of the wheel. In order to exploit efficiently the kinetic energy of the flow, the inclination of the blades surface has to be parallel to the flow relative velocity (\vec{w}) at the entry point, as shown in Fig. 1. The relative velocity is defined as the vector difference between the absolute entry velocity of water (\vec{v}) and the tangential velocity of the wheel (\vec{u}). The opening (a) of the sluice gate can be regulated to control the absolute velocity of the flow to the wheel, hence the relative velocity.

No complete and detailed experimental comparisons on the performance of slow and fast breastshot wheels have been found in modern literature, under the same hydraulic conditions. Therefore, in order to shed light on this issue, the aim of the present paper is to perform experimental tests on a breastshot water wheel, investigating its performance with an inflow weir and a sluice gate. In practical operative conditions, the inflow configuration can be managed depending on the external hydraulic conditions, optimizing the efficiency of the hydro plant. Scope of the present paper is thus to determine in which conditions it is more advisable to use the weir, and when it is better to regulate the flow to the wheel acting on the opening of the sluice gate.

3. Method

3.1. Experimental equipment and procedure

An experimental channel has been installed in the Laboratory of Hydraulics at Politecnico di Torino with the aim of testing different kinds of water wheels; in this work the results of a breastshot water wheel are presented. The diameter of the wheel was $D=2R=2.12$ m, the width was $b=0.65$ m and the number of the blades was 32 (Fig. 3).

109 The flow rate Q to the wheel was set acting on a pump and a gate valve
 110 installed in the supply pipe of the channel (flow rates $Q = 0.02 \div 0.1 \text{ m}^3/\text{s}$ were
 111 investigated). The flow rate was detected by an electromagnetic flow meter,
 112 whose accuracy was $\delta Q = \pm 0.5 \cdot 10^{-3} \text{ m}^3/\text{s}$. A brake system, constituted of a
 113 generator and a resistor, was connected at the wheel's shaft. An electrical energy
 114 analyzer and a control of the electrical resistance were installed to manage the
 115 electrical power output of the generator and the load on the wheel, regulating the
 116 rotational speed N of the wheel ($N = 0.2 \div 2.1 \text{ rad/s}$). The minimum rotational
 117 speed depended on the maximum braking torque that the brake could apply.
 118 The maximum rotational speed was close to the runaway velocity. Between the
 119 wheel and the brake, a gearbox was installed to provide an optimum speed and
 120 torque range on the generator shaft. Along the shaft an inductive proximity
 121 sensor was installed in order to acquire the rotational speed of the wheel. A
 122 torque transducer was also installed along the transmission shaft to measure the
 123 shaft torque (C_{exp}) with a precision of $\delta C_{exp} = \pm 6 \text{ Nm}$. The experimental power
 124 output $P_{exp} = C_{exp} \cdot N$ was determined by multiplying the applied torque C_{exp}
 125 by the rotational speed N . The rotational speed was measured by the internal
 126 clock of the acquisition board, which could discretize the output signal frequency
 127 of the proximity sensor till 100 MHz, with very high accuracy.

128 In order to evaluate the power input to the wheel, the upstream and down-
 129 stream water depths, h_u and h_d , respectively, were measured, obtaining the head
 130 difference, or difference of energy head:

$$H_{gr} = (H_U - H_D) = \left[\left(z_u + h_u + \frac{v_u^2}{2g} \right) - \left(z_d + h_d + \frac{v_d^2}{2g} \right) \right] \quad (1)$$

131 where H_U is the energy head upstream of the wheel (measured 2.5 m from the
 132 axle of the wheel), H_D the downstream one (energy head at the tailrace, 0.89
 133 m from the axle of the wheel) and $H_{gr} = H_U - H_D$ is the head difference. The
 134 energy head H_x is the sum of the channel's bed elevation z_x , the water depth h_x
 135 and the kinetic term $v_x^2/2g$, where $g = 9.81 \text{ m/s}^2$ is the acceleration of gravity
 136 and v_x is the mean flow velocity (Fig.4). The mean flow velocity is calculated
 137 as $v_u = Q/l_u h_u$ and $v_d = Q/l_d h_d$, where $l_u = 1.5 \text{ m}$ and $l_d = 0.67 \text{ m}$ are the
 138 widths of the channel in the measurement points. In our case, the geometric head
 139 difference is $H_g = z_u - z_d = 0.35 \text{ m}$, thus the ratio $r = H_g/D = 0.165$. The water
 140 depth h_u was monitored by an ultrasonic sensor with a precision of $\delta h_u = \pm 0.004$
 141 m and the downstream depth h_d by a classical ruler, with the operator precision
 142 of $\delta h_d \simeq 0.002 \text{ m}$. While the upstream water depth depended on the opening of
 143 the sluice gate or the height of the inflow weir, the downstream water depth was
 144 not regulated by any hydraulic structure.

145 The power input of the hydroelectric plant was:

$$P_{gr} = \rho g Q H_{gr} \quad (2)$$

146 where Q is the total flow rate and $\rho = 1000 \text{ Kg/m}^3$ is the density of water.

147 The global efficiency of the installed hydroelectric plant is defined as η :

$$\eta = \frac{P_{exp}}{P_{gr}} \quad (3)$$

148 and it is a function of the flow rate, rotational speed and inflow configuration.

149 3.1.1. *Error analysis*

150 Scope of this section is to apply the error propagation laws to estimate the
151 error δ on each quantity derived from the experimental measurements.

152 The width of the channel b , the wheel radius R and the geometric head H_g
153 are considered known, thus without error.

154 As written in the previous section, the error of the measurements is $\delta h_u =$
155 ± 0.004 m for the upstream water depth, $\delta h_d = \pm 0.002$ m for the downstream
156 one, $\delta Q = \pm 0.5 \cdot 10^{-3}$ m³/s for the flow rate and $\delta C_{exp} = \pm 6$ Nm for the torque.

157 The estimated error of the power output can be calculated as:

$$\delta P_{exp} = \pm P_{exp} \cdot \frac{\delta C_{exp}}{C_{exp}} = \pm 6 \cdot N \quad (4)$$

158 since the rotational speed was calculated with very high accuracy.

159 Considering that the generic velocity is $v_x = Q/(bh_x)$, the error of the velocity
160 measurement can be calculated as:

$$\frac{\delta v_x}{v_x} = \frac{\delta Q}{Q} + \frac{\delta h_x}{h_x} \quad (5)$$

161 where the subscript x can refer both to the upstream quantities (flow velocity v_u
162 and water depth h_u), and to the downstream ones (v_d and h_d). The error of the
163 velocity to the second power (the kinetic term) is:

$$\frac{\delta v_x^2}{v_x^2} = 2 \frac{\delta v_x}{v_x} \rightarrow \delta v_x^2 = 2 v_x \delta v_x \quad (6)$$

164 where δv_x can be calculated by eq.5.

165 The error of the head difference (eq.1) estimation can be calculated as:

$$\delta H_{gr} = \delta h_u + \frac{1}{2g} \delta v_u^2 + \delta h_d + \frac{1}{2g} \delta v_d^2 \quad (7)$$

166 where $\delta v_x^2 = 2v_x^2 \left(\frac{\delta Q}{Q} + \frac{\delta h_x}{h_x} \right)$.

167 The error of the measurement of the power input $P_{gr} = \rho g Q H_{gr}$ can be
168 quantified in:

$$\delta P_{gr} = \rho g (\delta [Q H_{gr}]) = \rho g Q H_{gr} \left(\frac{\delta Q}{Q} + \frac{\delta H_{gr}}{H_{gr}} \right) \quad (8)$$

169 Finally, the error of the efficiency estimation ($\eta = P_{exp}/P_{gr}$) can be expressed
170 as:

$$\delta \eta = \eta \left(\frac{\delta P_{exp}}{P_{exp}} + \frac{\delta P_{gr}}{P_{gr}} \right) \quad (9)$$

171 3.2. Inflow geometric configurations

172 The first experiments dealt with the breastshot wheel equipped with a sluice
173 gate, as illustrated in Fig. 4. In the figure, by the point E we identify the water
174 entry point to the wheel. The sluice gate was installed 0.7 m upstream of point E
175 and its opening was varied between $0.050 < a < 0.150$ m. The opening of the sluice
176 gate allowed to regulate the upstream water depth h_u , thus the flow velocity to
177 the wheel. Therefore, while the flow velocity was often negligible upstream of the
178 sluice gate (especially at small sluice gate openings), it was not negligible when
179 the flow entered into the wheel, because of the flow acceleration passing under
180 the sluice gate. The total number of experiments was: 39 for the sluice gate
181 opening $a = 0.05$ m, 53 for $a = 0.075$ m, 59 for $a = 0.100$ m, 55 for $a = 0.125$ m,
182 48 for $a = 0.150$ m. During experiments, first of all the flow rate was set by the

183 pump, then the sluice gate adjusted to the requested opening and, at the end,
184 the rotational speed was regulated by the brake.

185 After the first set of experiments, the sluice gate was removed, and the channel
186 was equipped with a vertical weir (as illustrated in Fig. 5), changing the water
187 entry point to the wheel. Weirs of 0.18 m and 0.28 m high were investigated.
188 The advantage of a vertical weir is its simplicity and facility of regulation. Each
189 weir was located just before the wheel, in order to ensure a gap of about 0.01
190 m between the top edge of the weir and the blades, as illustrated in Fig. 5, in
191 order to avoid any contact between the wheel and the weir. The weir 0.18 m
192 high was installed 0.12 m upstream of the water entry point (E in Fig. 5), and
193 the weir 0.28 m high was installed 0.17 m upstream of the entry point. The flow
194 rate was then set by the pump and the rotational speed of the wheel regulated
195 by the brake.

196 In this case, the weir was a vertical wall, thus its downstream profile did not
197 fit the circular shape of the wheel (the circular path of the blade tip during its
198 rotation). This led to volumetric losses (see Fig. 5): a portion of water flows
199 from the buckets toward the space V . Anyway, the water which is initially lost
200 from the bucket is not definitively lost, since it re-enters into the buckets (the
201 losses are not so high, since a portion of the volume V is filled of water). In
202 order to contain the volumetric losses (consider that the higher the weir, the
203 more distant it has to be installed from the wheel), the height of the weir should
204 be $< 1 \div 1.5$ times the depth of the buckets (the external distance between two
205 blades). This recommendation justifies the investigated heights; considering the
206 diameter of the wheel of 2.12 m and 32 blades, the depth of the buckets is about
207 0.2 m.

208 A total of 42 and 36 experiments for the configurations with the weir of
 209 height $h_s = 0.18$ m and $h_s = 0.28$ m, respectively, were carried out. When the
 210 weir was in operation, the water flow did not accelerate passing under the sluice
 211 gate and it entered into the wheel from higher elevation and at lower velocity.
 212 Therefore, considering a certain flow rate, the torque contribution of the water
 213 weight increases with the weir, whereas the torque contribution due to the kinetic
 214 energy of the flow reduces.

215 **4. Results and discussion**

216 *4.1. Experimental results and discussion*

217 Scope of the present section is to compare the performance of the breastshot
 218 water wheel in the two inflow configurations. During experiments, the flow rate
 219 was firstly imposed. When the sluice gate was in operation, the entry flow velo-
 220 city was regulated acting on the opening of the sluice gate and the rotational
 221 speed of the wheel regulated by means of the brake. Instead, when the weir
 222 was in operation, only the flow rate and the wheel rotational speed needed to
 223 be regulated. Consider a representative case with $Q = 0.05$ m³/s, $h_u = 0.5$
 224 m, $h_d = 0.1$ m (hence $H_{gr} = 0.72$ m and $P_{gr} = 354$ W), $N = 1$ rad/s, and
 225 $\eta = 0.7$ (hence $P_{exp} = 248$ W). The accuracy of the head difference estimation is
 226 $\delta H_{gr} = 0.0078$ m, $\delta P_{gr} = 7.35$ W for the power input, $\delta P_{exp} = 6$ W for the power
 227 output and $\delta \eta = 0.031$ for the efficiency. These values are lower if compared to
 228 their respective measured quantities, hence they can be considered acceptable.

229 Figure 6 depicts some efficiency curves for selected flow rates. In the weir
 230 configuration the efficiency trend is quite constant, while in the sluice gate con-
 231 figuration the efficiency increases up to a maximum, and then it decreases (the

232 maximum power output occurs in correspondence of the maximum efficiency).
233 This difference can be explained in this way. The kinetic energy of the flow enter-
234 ing into the wheel is lower when the weir is installed with respect to the kinetic
235 energy of the flow when the sluice gate is installed (the flow accelerates passing
236 under the sluice gate). Hence the contribution of the kinetic energy of the flow to
237 the torque (as well as to the efficiency) is lower in the weir configuration. Since
238 the wheel rotational speed affects especially the transfer of kinetic energy from
239 the flow to the wheel (i.e. the relative flow velocity and the impact power losses),
240 it is reasonable that the efficiency trend is less affected by the wheel velocity
241 when the weir is installed.

242 For each flow rate, the height of the weir affects the efficiency not significantly,
243 while the opening of the sluice gate is very important. The lower the opening,
244 thus the higher the water velocity to the wheel, the lower the efficiency. The
245 efficiency reduction with the lowering of the sluice gate is worsened with the
246 increase of the flow rate, since the water velocity also increases with the flow
247 rate. High water velocities generate significant power losses both during the
248 filling process due to the impact, and in the conveying channel [17]. One other
249 aspect that can be observed for the sluice gate is that the higher the flow rate,
250 the higher the optimal rotational speed (the speed at maximum efficiency). This
251 occurs for the following motivation. As it will be illustrated in section 4.2, the
252 optimal rotational speed is proportional to the square root of the head difference.
253 The higher the flow rate, the higher the upstream water depth, thus the higher
254 the required rotational speed for the optimal efficiency. This can also be justified
255 by the fact that the higher the upstream water depth, the faster the flow velocity
256 to the wheel, thus the higher the value that the rotational speed can assume to

257 optimize the impact conditions.

258 Figure 7 shows the maximum experimental power output versus the flow rate.
259 For the cases with the sluice gate, the power output increases with the reduction
260 in the sluice gate opening, due to the higher entry flow velocity. When the weir
261 is in operation, the power output increases with the height of the weir, due to the
262 increase in the elevation of the water entry point, thus in the potential energy of
263 water. The maximum power output for the weir configuration is usually higher
264 at a certain flow rate. This occurs because when the weir is installed, the water
265 weight begins to push the blades from higher elevations, although the torque due
266 to the kinetic energy of the flow is lower, with respect to what happens with
267 the sluice gate. Since the weir leads to higher power output, this means that
268 the increase in the torque due to the water weight is more important than the
269 decrease in the kinetic contribution. However, at flow rates bigger than $0.08 \text{ m}^3/\text{s}$
270 the trend of the power output for sluice gate openings lower than 0.075 m seems
271 to overcome the trend for the cases with the weir.

272 Figure 8a depicts the efficiency versus the flow rate. The first general result
273 that can be seen is the difference between the efficiency trends of the wheel in
274 the two geometric inflow configurations. Considering the cases with the sluice
275 gate, the efficiency increases up to a maximum value. Then, for $a > 0.10 \text{ m}$ the
276 maximum value is also almost constant at 75%; the range of constant efficiency
277 is included between $Q = 0.05$ to $Q = 0.08 \div 0.09 \text{ m}^3/\text{s}$. This range corresponds to
278 $(0.56 \div 0.6) \cdot Q_{max}$ and Q_{max} , where Q_{max} is the maximum flow rate in the range
279 of constant efficiency for each geometric inflow configuration. For sluice gate
280 openings $\leq 0.10 \text{ m}$ there is not a constant efficiency range, and Q_{max} corresponds
281 to the flow rate at the maximum efficiency. The efficiency starts to decrease from

282 $Q = 0.05 \div 0.06 \text{ m}^3/\text{s}$. The smaller the sluice gate opening, the lower Q_{max} .
 283 This is justified by the fact that at a certain flow rate, the smaller the opening
 284 of the sluice gate, the higher the upstream water depth and the water velocity
 285 to the wheel, thus the more significant the power losses upstream of the wheel.
 286 Therefore, the smaller the sluice gate opening, the lower the allowable flow rate
 287 in order to avoid excessive flow velocity to the wheel and power losses.

288 Instead, considering the inflow weirs, the efficiency trend is increasing and
 289 more regular (Fig.8a). This result says that the optimal flow rate is higher in the
 290 weir configuration (due to the geometric limitations of the experimental channel,
 291 it was not possible to investigate higher flow rates). Hence the configuration with
 292 the weir allows to exploit efficiently larger flow volumes, i.e. flow rates $Q > 0.08$
 293 m^3/s in the present case. The efficiency of the plant equipped with the weir
 294 is also higher at very low discharges. The maximum efficiency at flow rates of
 295 $Q = 0.02 \text{ m}^3/\text{s}$ improves from $\eta = 0.25 \div 0.30$ with the sluice gate to $\eta = 0.45$
 296 using the weir. This occurs because at very low flow rates the contribution of
 297 the kinetic energy is negligible; therefore, it is more convenient to use a weir in
 298 order to enhance the water elevation, instead of exploiting the kinetic energy by
 299 reducing the sluice gate opening.

300 The previous considerations show that the efficiency with the sluice gate
 301 exhibits a stronger dependence from the flow rate. This result is confirmed in
 302 [19], where a breastshot water wheel equipped with an inflow weir has been
 303 investigated: its efficiency was constant already from flow rates of $0.2 \cdot Q_{max}$,
 304 while the present breastshot wheel with the sluice gate has constant efficiency in
 305 the range $(0.56 \div 0.6) \cdot Q_{max}$ and Q_{max} .

306 In Fig.8b it can be observed that in the range $P_{gr} = 150 \div 400 \text{ W}$ the ef-

307 ficiency with the weir is lower, probably due to the volumetric losses occurring
 308 downstream of the weir, as explained in section 3.2, while the efficiency is higher
 309 for $P_{gr} > 400$ W. The efficiency at high power inputs ($P_{gr} > 400$ W) decreases
 310 with the reduction of the sluice gate opening. This occurs because such situation
 311 corresponds to high flow rates and upstream water depths h_u , leading to high
 312 flow velocities downstream of the sluice gate, and, as a consequence, significant
 313 power losses in the impact against the blades and in the headrace, due to friction
 314 and turbulence [17]. Hence the use of the weir becomes more advisable than the
 315 sluice gate in these conditions.

316 A comparison between the two inflow weirs shows that the power output of
 317 the highest weir is generally higher than the power output of the shortest weir
 318 (Fig. 7), whereas the efficiencies are similar (Fig. 8).

319 4.2. *Practical applications and discussion*

320 As discussed in the previous section, the paper has showed that the optimal
 321 hydraulic conditions where the inflow weir and the sluice gate should operate are
 322 different. In particular, the weir works better in extreme conditions, that is for
 323 low and high flow rates and power inputs. Therefore, the combination and the
 324 regulation of the sluice gate and the weir can be considered a suitable method to
 325 optimize the working conditions and the efficiency of breastshot water wheels.

326 The regulation of the sluice gate opening can be also a way to control the
 327 operational speed of the wheel, when the flow rate is not constant, in order to
 328 guarantee always the optimal operative conditions for the constant speed of oper-
 329 ation. When the flow rate changes, also the optimal speed of the wheel changes,
 330 since the optimal rotational speed depends on the flow rate. In order to shed

light on this, Fig.9 depicts the rotational speed at the maximum efficiency versus
 the sluice gate opening at different flow rates, for the tested wheel. Therefore,
 at a fixed sluice gate opening and flow rate, the wheel rotational speed required
 to obtain the maximum efficiency is determined. For each flow rate, the trend is
 nearly linear: the lower sluice gate openings (a) (thus the higher the flow velocity
 to the wheel), the higher the wheel rotational speed required to obtain the maximum
 efficiency (as discussed for Fig.6). As a consequence, when it is desirable
 that the wheel operates at a constant rotational speed also with variable flow
 rates, using a graph similar to Fig.9, it is possible to determine for each flow rate
 the sluice gate opening which guarantees that the same rotational speed remains
 optimal. Furthermore, since the graph is drawn using the maximum efficiency
 data, the optimal operative conditions are also guaranteed. The control of the
 sluice gate can cooperate with the weir, to be used at very low and big flow
 rates. Otherwise, if the variable speed of operation is allowed, and the geometric
 configuration is fixed (fixed sluice gate opening in this case), a control system
 must be able to change the wheel rotational speed, depending on the flow rate.
 However, a variable speed of operation requires a costly rectifier/control/inverter
 system, and expensive gearboxes [9]. Therefore, since the constant speed is more
 advisable, the sluice gate opening can be regulated to change the hydraulic configuration
 and to guarantee that the rotational speed (which is constant) continue
 to be optimal also at variable flow rates.

Some results are now discussed as a function of dimensionless parameters,
 making the performance results generically applicable. First of all, in a practical
 application the diameter can be chosen in order to obtain a geometrically similar
 water wheel, hence ensuring the same value of $r_p = H_{g,p}/D_p = 0.165$, where with

the subscript p we refer to the variables in the practical case (the wheel at full scale). It is then possible to determine the optimal inflow configuration, wheel rotational speed and width, as explained in the following lines.

The optimal inflow configuration can be estimated by Fig. 10, which shows the maximum efficiency versus the normalized power input. The normalized power input is defined as follows [18]:

$$P_{gr}^* = \frac{P_{gr} \cdot H_g^4}{\rho Q^3} \quad (10)$$

whose error (considering the representative case, to which corresponds $P_{gr}^* = 42.5$) can be estimated as:

$$\delta P_{gr}^* = \frac{H_g^4}{\rho} \delta \left[\frac{P_{gr}}{Q^3} \right] \rightarrow \frac{H_g^4}{\rho} \frac{P_{gr}}{Q^3} \left(\frac{\delta P_{gr}}{P_{gr}} + 3 \frac{\delta Q}{Q} \cdot Q^3 \right) = 0.88 \quad (11)$$

Figure 10 can be used to determine the optimal inflow condition as a function of the normalized power input (in order to ensure the maximum efficiency). The normalized power input depends on the full scale geometric head difference and on the operative flow rate. Since the power input depends on the inflow configuration, which is not known yet, an iterative process has to be adopted. In Fig.10, the dimensions of the inflow configurations are scaled to the geometric head difference H_g (which is 0.35 m in our case), obtaining the normalized sluice gate opening a^* and weir height h_s^* . The graph can also be used as useful generalized tool to estimate the wheel efficiency as a function of the hydraulic conditions. In Fig.10, the lower the flow rate, the higher the dimensionless power input for each configuration. Observing the trends starting from the highest power inputs, hence for increasing flow rates, the trends with the sluice gate initially increase,

376 reaching a maximum, which occurs at the optimal P_{gr}^* , thus the optimal flow
 377 rate. After the maximum, the trends decrease considerably for sluice gate open-
 378 ings $a \leq 0.10$ m or $a^* \leq 0.286$. The smaller the sluice gate opening the higher the
 379 optimal dimensionless power input at the maximum efficiency, thus the lower the
 380 optimal flow rate (as discussed in the description of Fig.8). Instead, the trends
 381 of the weir do not exhibit a maximum, because the experimental channel did not
 382 allow to explore higher flow rates. For $P_{gr}^* > 70$ it is more advisable to use the
 383 weir. For $P_{gr}^* < 70$ the efficiency trends of the weir and of the sluice gate are
 384 practically coincident, except when the efficiency trends for $a^* \leq 0.286$ decrease
 385 at normalized power input lower than the optimal.

386 Once the inflow configuration is determined, by Tab.1 the optimal rotational
 387 speed can be estimated. Table 1 shows the optimal normalized tangential speeds
 388 u^* of the wheel at the highest efficiency for each inflow case and flow rate ($u =$
 389 NR).

$$u^* = \frac{N \cdot R}{\sqrt{2gH_{gr}}} \quad (12)$$

390 whose error (for the same previous representative case to which corresponds
 391 $u^* = 0.266$) can be estimated as:

$$\delta u^* = \frac{R}{\sqrt{2g}} \frac{N}{\sqrt{H_{gr}}} \left(\frac{\delta N}{N} + \frac{1}{2} \frac{\delta H_{gr}}{H_{gr}} \right) = 0.0014 \quad (13)$$

392 The wheel tangential speed was normalized to the term $\sqrt{2gH_{gr}}$, in order
 393 to make the results both applicable in a general case, as also done in [15] for
 394 *Zuppinger* and *Sagebien* water wheels, and to relate the rotational speed to the
 395 hydraulic conditions.

396 Considering optimal flow rates $Q > 0.03 \text{ m}^3/\text{s}$ for the cases with the sluice
 397 gate, the normalized tangential speeds are approximately included in the range
 398 $u^* = 0.3 \div 0.4$ (which is a quite limited range). For a certain flow rate, these
 399 values are almost constant at different sluice gate openings; instead they slightly
 400 increase with the flow rate (at a constant sluice gate opening). This means that
 401 the optimal tangential speed is mainly affected by the square root of the head
 402 difference. Instead $u^* = 0.16 \div 0.4$ for the cases with the weir and the height of
 403 the weir affects noticeably u^* (remember that from Fig.6 the efficiency was not
 404 strongly affected by the wheel velocity, thus the efficiency at the optimal speed is
 405 not so higher than the efficiency at different wheel speeds). The calculated ranges
 406 are in agreement with those found for *Zuppinger* and *Sagebien* water wheels [15],
 407 which are between 0.2 and 0.4.

408 Table 2 reports the filling ratio of the buckets at the highest efficiency for
 409 each inflow case and flow rate. The filling ratio is defined as the ratio of the
 410 water volume inside the bucket to the bucket volume, which is delimited by two
 411 blades and the channel's bed. The optimal filling ratio is included in the range
 412 $0.3 \div 0.45$ for the sluice gate and $0.27 \div 0.6$ for the weir. In a practical application,
 413 the table can be used to determine the width of the wheel, which should ensure
 414 that the optimal filling ratio is respected. In order to use these tables, the actual
 415 flow rate and inflow type have to be scaled in Froude similarity to the conditions
 416 investigated in this work. Finally, the wheel speed and width can be adjusted
 417 to ensure a water depth in the buckets higher than the tailrace water depth, to
 418 avoid adverse hydrostatic forces.

419 5. Conclusions and future work

420 Although water wheels were forgotten in the Twentieth century, nowadays
421 they can represent interesting hydraulic machines in very low head sites. Water
422 wheels are efficient, cheap and exhibit low environmental impacts. In particular,
423 breastshot water wheels are used in sites with abundant flow rates, such as in
424 irrigation and mill channels, with heads generally less than 4 m. Some of them
425 are located in old water mills, thus their restoration can also contribute to the
426 preservation of the natural and cultural heritage. We can also claim that water
427 wheels are seeing a revival, but the engineering information is not completed in
428 detail, thus further work is needed.

429 In this paper, experimental results are reported to illustrate how the efficiency
430 of a breastshot water wheel changes under different hydraulic and geometric
431 configurations. Two different inflow configurations are investigated: the former
432 has a sluice gate upstream of the wheel (whose opening could be regulated), the
433 second a weir. Two weirs of different heights were installed upstream of the wheel
434 and investigated.

435 The maximum efficiency for sluice gate openings > 0.075 m was $\eta = 0.75$,
436 which was quite constant in the range between $0.05 < Q < 0.08$ m³/s for $a > 0.10$
437 m, while the efficiency with the weir is increasing, suggesting that the wheel is
438 able to exploit larger water volumes. The weirs improve the efficiency of the
439 wheel at very low discharges ($Q < 0.03$ m³/s), and they give also appreciable
440 effects at high power input ($P_{gr} > 400$ W).

441 The optimal normalized tangential speeds are included in the range $u^* =$
442 $0.3 \div 0.4$ and $u^* = 0.16 \div 0.4$ for the cases with the sluice gate and the weir,

443 respectively. The optimal filling ratio is approximately included in the range
444 $0.3 \div 0.5$. These ranges can be considered optimal operative conditions for similar
445 breastshot water wheels.

446 Therefore, both the correct design of the elevation of the weir and the opening
447 of the sluice gate can be considered a suitable method to optimize the working
448 conditions and the efficiency of breastshot water wheels.

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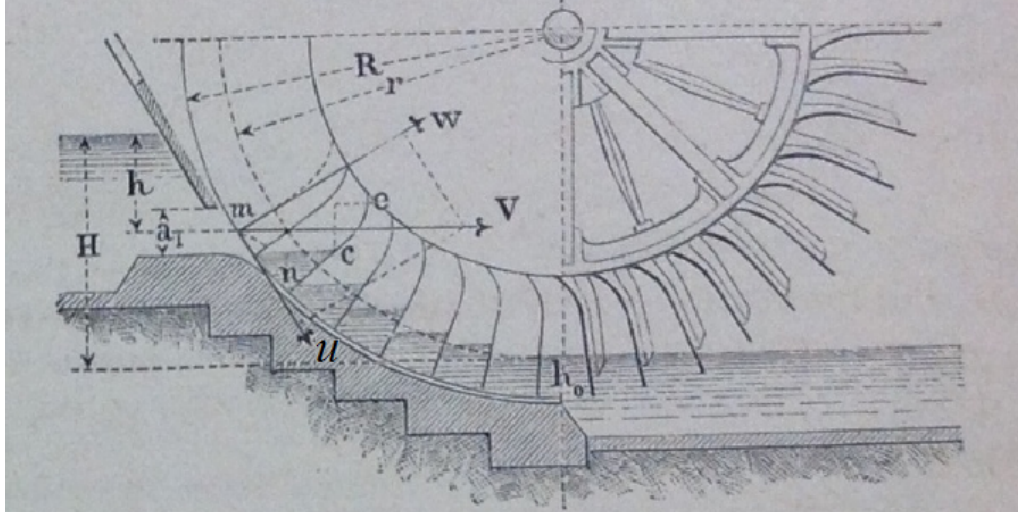


Fig. 1. Classical configuration of a fast breastshot wheel equipped with a sluice gate of opening a (Garuffa, 1897 [7]). The relative flow velocity $\vec{w} = \vec{v} - \vec{u}$ is oriented as the blades in the impact point, where \vec{v} is the absolute flow velocity and \vec{u} is the tangential velocity of the wheel.

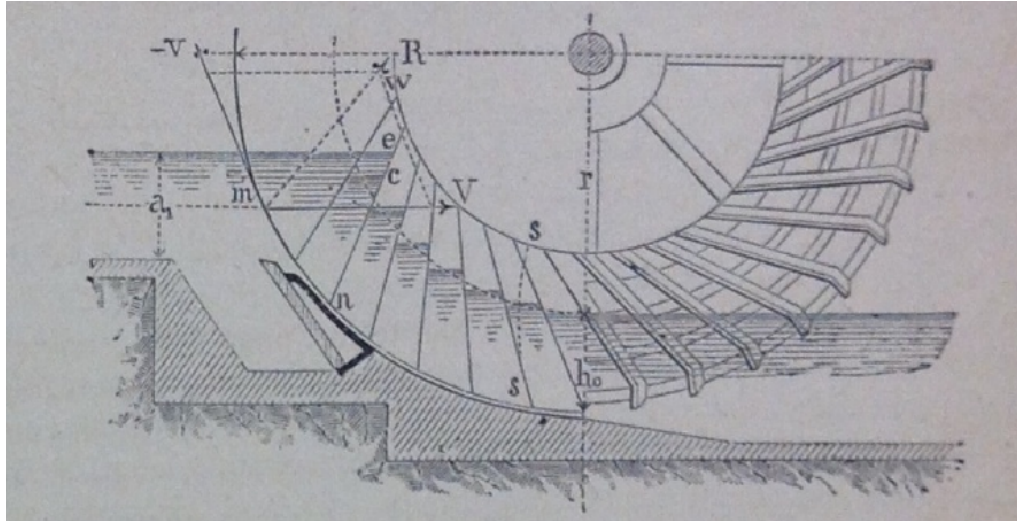


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Fig. 3. The installed wheel in 2.12 m in diameter and 32 blades. At the shaft axis the gearbox and the generator are visible.

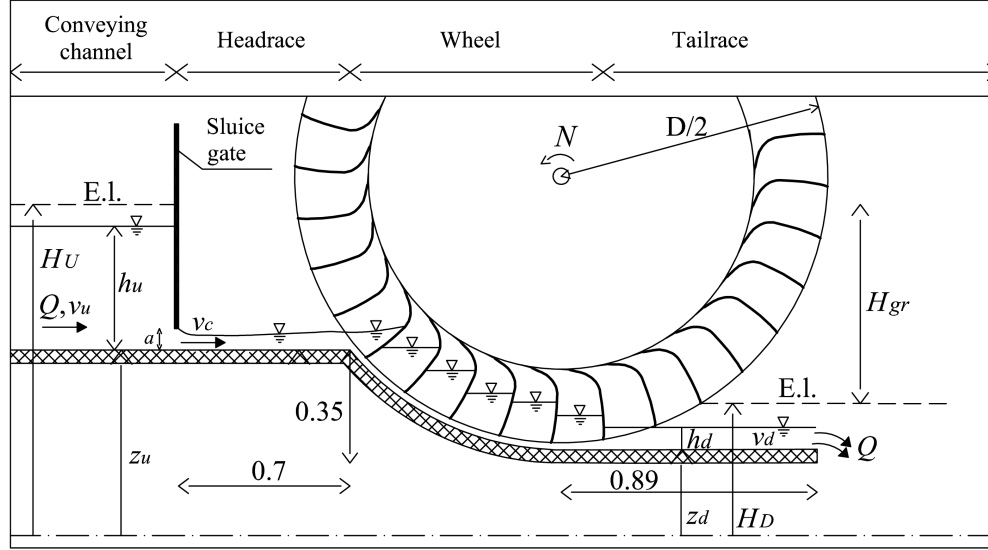


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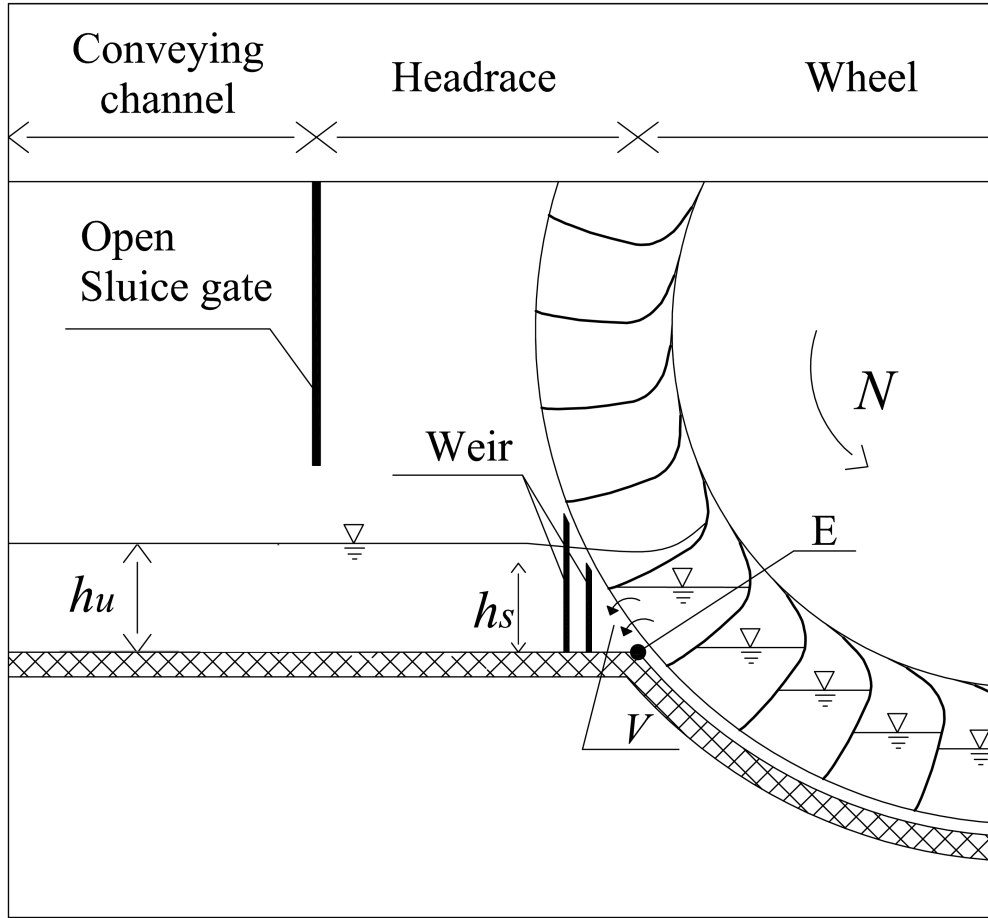


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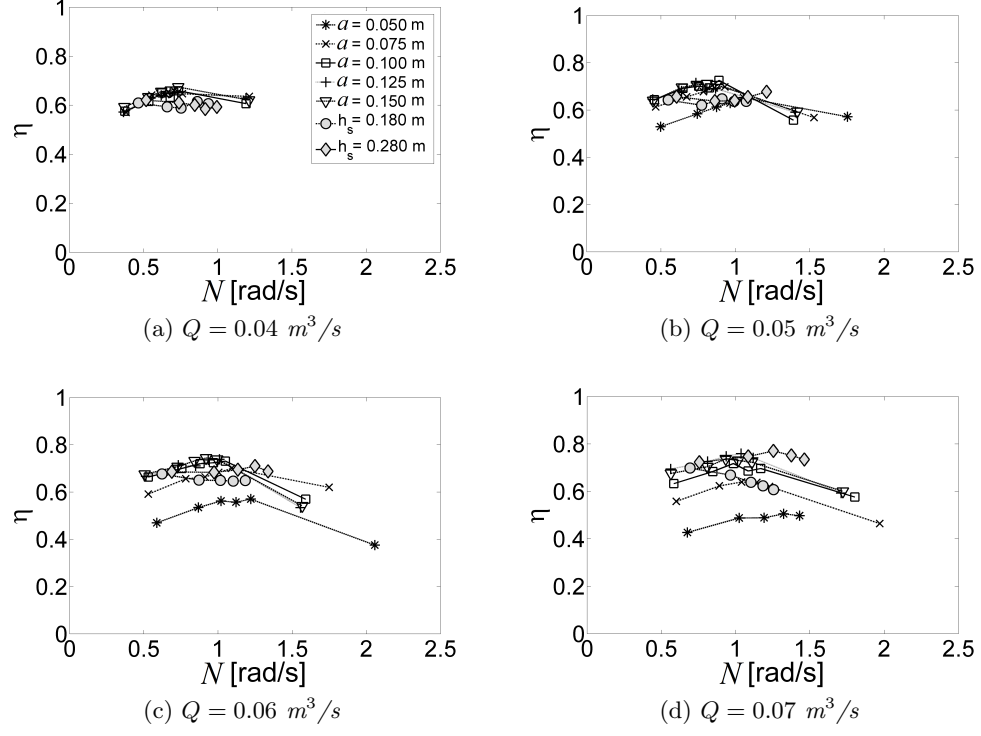


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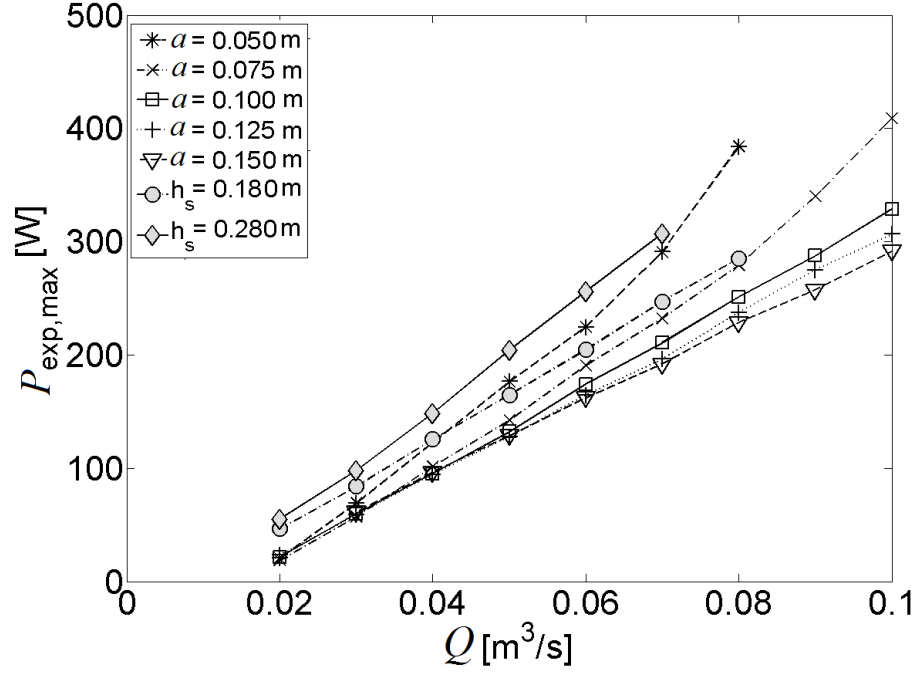


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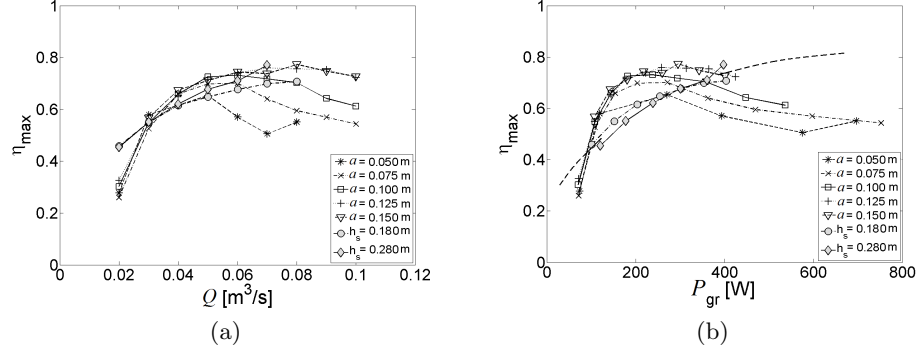


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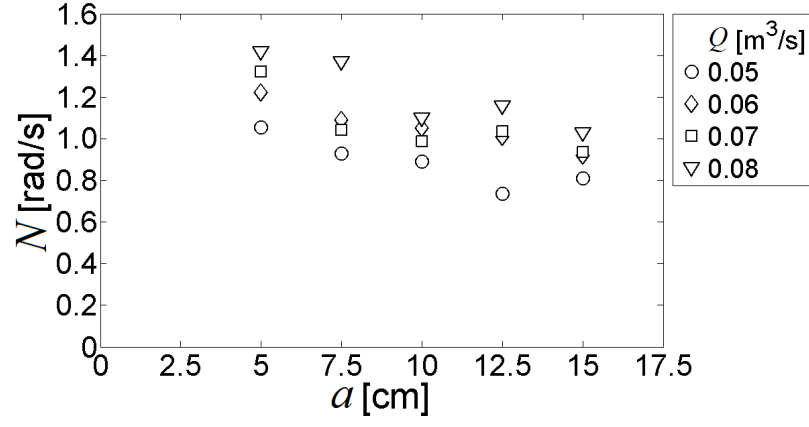


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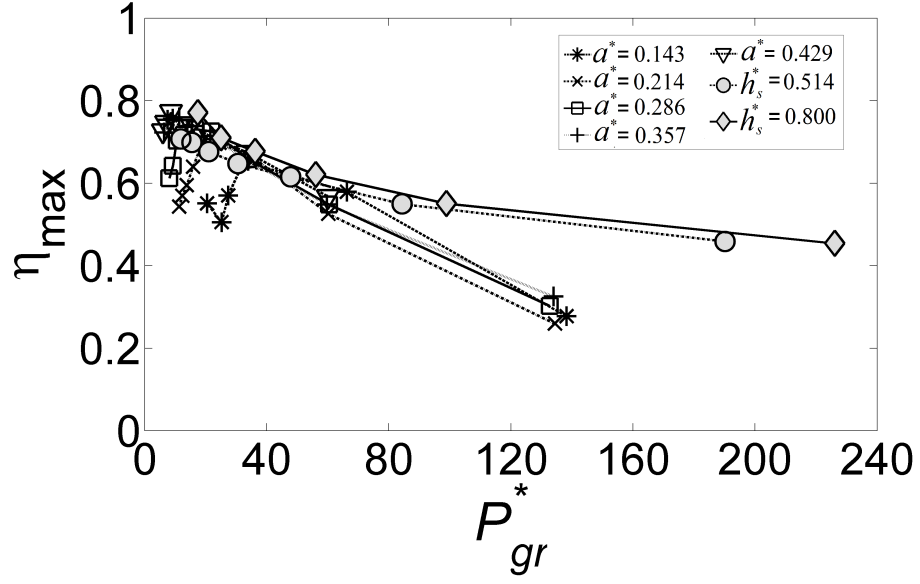


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Table 1. Normalized tangential speed of the wheel at the optimal efficiency for each inflow case and flow rate.

Inflow case (m)	Q (m ³ /s)								
	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
$a=0.05$	0.06	0.21	-	0.34	0.36	0.35	0.36	-	-
$a=0.075$	0.10	0.18	0.27	0.34	0.38	0.34	0.42	0.41	0.34
$a=0.1$	0.10	0.22	0.29	0.35	0.40	0.36	0.39	0.35	0.36
$a=0.125$	0.12	0.19	0.29	0.29	0.39	0.40	0.44	0.37	0.38
$a=0.15$	-	0.22	0.29	0.32	0.36	0.36	0.40	0.37	0.45
$h_s=0.18$	0.099	0.24	0.29	0.30	0.21	0.23	0.25	-	-
$h_s=0.28$	0.13	0.18	0.16	0.37	0.38	0.39	-	-	-

Table 2. Filling ratio at the optimal efficiency for each inflow case and flow rate.

Inflow case (m)	Q (m ³ /s)								
	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
$a=0.05$	0.72	0.31	-	0.28	0.29	0.31	0.33	-	-
$a=0.075$	0.45	0.38	0.33	0.32	0.32	0.39	0.34	0.37	0.47
$a=0.1$	0.46	0.32	0.32	0.33	0.33	0.42	0.43	0.51	0.53
$a=0.125$	0.40	0.37	0.32	0.40	0.35	0.40	0.41	0.53	0.55
$a=0.15$	-	0.32	0.32	0.36	0.38	0.44	0.46	0.55	0.49
$h_s=0.18$	0.40	0.24	0.27	0.32	0.57	0.59	0.62	-	-
$h_s=0.28$	0.29	0.30	0.45	0.24	0.28	0.33	-	-	-