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IN-CYLINDER PRESSURE-BASED DIRECT TECHNIQUES AND TIME FREQUENCY ANALYSIS FOR COMBUSTION DIAGNOSTICS IN IC ENGINES.

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ABSTRACT

In-cylinder pressure measurement and analysis has historically been a key tool for off-line combustion diagnosis in internal combustion engines, but online applications for real-time condition monitoring and combustion management have recently become popular. The present investigation presents and compares different low computing-cost in-cylinder pressure based methods for the analyses of the main features of combustion, that is, the start of combustion, the end of combustion and the crankshaft angle that responds to half of the overall burned mass. The instantaneous pressure in the combustion chamber has been used as an input datum for the described analytical procedures and it has been measured by means of a standard piezoelectric transducer.

Traditional pressure-based techniques have been shown to be able to predict the burned mass fraction time history more accurately in spark ignition engines than in diesel engines. The most suitable pressure-based techniques for both spark ignition and compression ignition engines have been chosen on the basis of the available experimental data. Time-frequency analysis has also been applied to the analysis of diesel combustion, which is richer in events than spark ignited combustion. Time frequency algorithms for the calculation of the mean instantaneous frequency are computationally efficient, allow the main events of the diesel combustion to be identified and provide the greatest benefits in the presence of multiple injection events. These algorithms can be optimized and applied to onboard diagnostics tools designed for real control, but can also be used as an advanced validation tool for refined combustion models.

The presented results on the pressure-based techniques, including a time frequency analysis, have been compared with the numerical outcomes from previously developed two- and three- zone thermodynamic combustion models.

Keywords: combustion; in-cylinder pressure; time frequency analysis.

Highlights:

- Direct pressure-based techniques have been applied successfully to spark-ignition engines.

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- The burned mass fraction of pressure-based techniques has been compared with that of 2- and 3-zone combustion models.
- The time frequency analysis has been employed to simulate complex diesel combustion events.

1. INTRODUCTION

Since the invention of internal combustion (IC) engines, the measured time history of the in-cylinder pressure has been the principal diagnostic tool for experimenters. The following parameters can be determined through cylinder pressure analysis: peak pressure and maximum pressure gradient, indicated mean effective pressure, pumping mean effective pressure, burn duration, the shape of heat release, and regularity of combustion. The accuracy of the results on one hand depends on the accuracy of each component of the measuring chain, and on the other hand on the correct processing of the pressure signal. [1]. One of the main features of pressure analysis was its simplicity when the available computational power was low. A heat release model that still maintains simplicity is described in [2], even though this model is also capable of considering important effects, such as heat transfer, crevice flow and fuel injection. In recent years, pressure measurement, by means of highly performing transducers, has still represented one of the most important key parameters for detailed engine studies and diagnosis. The demand for prompt engine control requires the real-time analysis of pressure signals for different purposes. A new method that is able to resolve dynamic knock intensity related to each individual engine cycle, is reported in [3]. The in-cylinder pressure can also be used to obtain a refined closed-loop function in order to control the low temperature combustion mode in Euro 6-compliant diesel engines under steady-state and transient conditions [4]. Furthermore, the instantaneous in-cylinder pressure signal, during the compression stroke, can be used as an alternative to the air mass flow measurement in either naturally aspirated spark ignition or turbocharged diesel engines. This could avoid the need to install an air mass flow sensor, or could provide an improved exhaust gas recirculation (EGR) estimate, if the proposed method is used together with an air mass flow sensor [5]. However, digital filtering must be applied if high quality results are required: the selection of the proper cut-off frequency of a low-pass filter to remove noise related to measurement is mandatory to preserve only physically meaningful information for a reliable heat release analysis on a cycle basis [6]. In order to reduce the computational power for real time-applications, an algorithm that uses the difference pressure instead of the in-cylinder pressure for the calculation of IMEP and MFB50 was developed in [7].

The in-cylinder pressure time history is universally measured using piezoelectric pressure transducers, due to their highly dynamic behavior. However, a piezoelectric sensor only provides the pressure variation with respect to a reference measurement, that is, only a relative value can be measured. The relative pressure has to be referenced to a known value by means of a pegging procedure in order to obtain an absolute value [8]. An accurate pegging method consists of the
installation of a fast-response piezoresistive pressure transducer that is capable of measuring the absolute pressure in the intake manifold or runner. The cylinder pressure, measured by the piezoelectric transducer at the induction process bottom dead center (BDC), is set equal to the absolute pressure level at the intake manifold, which is calculated as a time average value of the piezoresistive transducer trace over a certain interval. Furthermore, a direct and reliable measurement of the compression top dead center (TDC) position and of the thermodynamic loss angle are necessary to obtain an accurate phasing of the pressure signal with the crank angle [9] and thus to achieve meaningful thermodynamic results from the pressure measurement. In fact, as far as the imep evaluation is concerned, an erroneous adjustment of the TDC reference position has been recognized to be a major error source: an uncertainty of 1° CA is expected to provide a change of up to 10% in the calculated value of the imep from the pressure-volume indicated diagram [10]. The effects that crank angle phasing errors can have on the heat release rate and mass fraction burnt have not been clearly defined and quantified [11]. Finally, the measurement noise should be removed from the pressure signal by means of a numerical treatment, which is usually applied after the data have been averaged over several consecutive cycles. Several different methods can be used for this numerical treatment, including fast Fourier transform filters, windowing filters and smoothing splines [6]. A derivative pressure signal coming directly from the pressure transducer is sometimes used without a pressure amplifier [12]. Particular attention has to be paid to filtering and smoothing operations because the pressure fluctuations, which are softened by the filters or altered by the interpolating splines, can contain important information for the detection of combustion [13]. A properly processed pressure signal can be used to provide an evaluation of the main combustion features, either by means of the direct elaboration of the pressure measurement or by means of an indirect heat release analysis, which applies the first law of thermodynamics to the chamber contents on the basis of the in-cylinder pressure time history [14]. Both of these approaches allow the mass fraction of the fuel that has burnt ($x_b$) to be estimated as a function of time, but the former assumes that $x_b$ is related to the pressures of the fired and unfired cycles rather than to the fuel-released chemical energy [15].

The most important events related to combustion are the start of combustion (SOC), end of combustion (EOC) and the angular position at which half of the fuel has burnt (MFB50 is the crankshaft angle that corresponds to $x_b=0.5$). Combustion duration can easily be expressed as the difference between the EOC and SOC angles, but sometimes an angular distance between 10% and 90% of $x_b$ is used as an indicator of the combustion period.

One of the first applications of the measured pressure signal used to perform the direct diagnosis of combustion was illustrated by Marvin in [16]. Ideal compression and expansion phases were considered as simple polytropic evolutions, and the crankshaft angles at which the actual pressure starts to differ from the ideal compression and the straight expansion lines in a double logarithmic pressure-volume plane were considered as the SOC and EOC angles, respectively [17].
These can also be evaluated considering the inflection points in the diagrams of the specific heat ratio during the closed part of the cycle [18].

Marvin’s graphical method was analytically developed and experimentally validated by Rassweiler and Withrow (RW) in [19]. Their analysis pointed out that the cylinder pressure pattern also depends on the changes in the volume of the combustion chamber, because of the piston motion, on the heat transfer to the chamber walls and on the mass leakages through the piston crevices, in addition to the influence exerted by combustion. The effects of volume change, heat transfer, and mass loss [2] were isolated and analytical formulas were proposed to calculate the \( SOC \), the \( EOC \) and the burned mass fraction [20]. A refined alternative procedure to that of Rassweiler and Withrow has been developed by McCuiston, Lavoie and Kaufmann (MLK) [21] on the basis of the pressure and the in-cylinder volume time histories. This procedure works efficiently for gasoline combustion and hypothesizes that the mass of the burned gas is proportional to the \( pV\gamma \) quantity, where \( \gamma \) is the constant pressure-to-constant volume specific heat ratio. The \( SOC \), \( EOC \) and the \( x_b \) curve can easily be determined.

Indirect heat release analyses were developed later than direct pressure-based techniques and they stemmed from the application of the first law of thermodynamics. They have been used to develop ordinary differential equation models of combustion that allow the evaluation of the heat release rate (HRR), from which the burned mass fraction can then be worked out. Single-zone models define the thermodynamic state of the cylinder charge, in terms of average properties of a pure phase [2], but do not distinguish between burned and unburned gases. In simpler models, heat transfer and crevices are neglected and an apparent HRR, also referred to as net HRR, is calculated [22] as a function of the measured pressures and volumes. Considering a heat transfer model [23], and also taking the crevice volume into account [2], the gross HRR due to the chemical energy released by the fuel is obtained, and a more accurate \( x_b \) time distribution can be evaluated.

The correlation proposed by Woschni [24], and validated on a huge number of experimental tests, is widely adopted for the simulation of the heat transfer coefficient, but some constant parameters of this correlation have to be tailored on to the basis of the considered engine layout and working conditions [25]. The combustion efficiency measured from exhaust emissions can be used to calibrate these constant parameters, and a physically-consistent final predicted value of \( x_b \) [17] can thus be obtained from the combustion model.

Two-zone models, in which one zone represents the unburned mixture upstream of the flame, and the other simulates the burned mixture downstream from the flame [25], are more refined and complex than single zone models. The grade of complexity of the model is further increased in the multi-zone approach [26], in which the burned gases are simulated by means of different zones, even though this modeling strategy can also be considered reasonable when an enhanced prediction of engine-out emissions is required. \( SOC \) is generally easier to determine than \( EOC \), because the latter is less repetitive, due to the inherent difficulty in its localization and the arbitrariness of its practical definition [13]. The energy
conservation equation of the unburned gas mass can be considered for SOC evaluation in the context of the HRR analysis.

Since changes in the sensible internal energy of the charge during the compression phase are only due to work and heat exchanges with the combustion chamber walls, SOC can ideally be evaluated as the instant at which the energy conservation mass of the unburned gas fails to be verified [27]. For a practical and effective evaluation, a threshold can be set on the value of the $x_b$, e.g. 1%. However, the necessity of larger threshold values in the SOC identification process has already been pointed out [28], due to uncertainties that are related to pressure and volume measurements and to the large dispersion of this parameter if ensemble-averaged pressure data are considered. EOC can be evaluated in the HRR analysis as the crank angle at which $x_b$ reaches a certain threshold (e.g. 0.99) for single-zone models [29], or as the crank angle at which the theoretical $x_b$ reaches the experimental combustion efficiency measured from emissions for two- and multi-zone models. Otherwise, since HRR expresses the rate at which the fuel releases its chemical energy, a correspondence can be set between EOC and the crank angle at which the HRR falls below a minimum threshold, e.g. 3% of the maximum HRR.

Since accurate heat release analyses are rather complex and time consuming, the direct utilization of the in-cylinder pressure is often the preferred option in order to obtain fast, although approximate, results with the possibility of real-time calculation. Some pressure-based direct approaches, for instance, the polytropic volume method [30], have the aim of maintaining the simplicity and the small computational efforts of the RW method, but have been developed to overcome its inherent limitations [31]. Another example is that of the Pressure-Ratio Management (PRM) method, developed for naturally aspirated spark-ignition engines, which involves the calculation of the ratio of the fired pressure to the corresponding motored in-cylinder pressure at each crank angle [32]. This methodology, combined with optical analysis, has also been applied to combustion investigation in [33].

The simple PRM approach has been modified to obtain the Pressure Departure Ratio (PDR) algorithm [32], which is more suitable for diesel propulsion systems. In the PDR model, the ratio of the fired to the motored pressure is corrected by means of two constants, which should be calibrated for each engine setup. The PDR algorithm is able to predict $MFB50$ with adequate accuracy and is suitable for real-time applications, without any preliminary treatment for the pressure data, unlike apparent HRR analyses, in which the derivative term of the pressure can be affected negatively by high measurement noise, and moving average techniques should be applied to the original signal in order to remove spurious high frequency oscillations from the HRR trace [34]. Other pressure-based direct methods for the real-time calculation of the main combustion parameters are founded on the pressure-difference apparent heat release analysis [7], in which the difference between the actual pressure and motored pressure (the latter is calculated assuming a polytropic law for the compression phase) is used instead of the in-cylinder pressure, since the heat released by the motored pressure is nil [7]. The advantage of these procedures is that the pressure differences are not affected by pegging uncertainties.
A simple pressure-based direct procedure, which is founded on the MLK technique, relates the EOC to the angle at which quantity \(pV^{1.15}\) reaches a maximum value [11]. The calculation is started 10 CA ATDC and continues up to 10 CA, before exhaust valve opening by steps of 1 CA. EOC is set equal to the crank angle that corresponds to the maximum of the \(pV^{1.15}\) plus 10\(^\circ\)CA. A low value of the polytropic index is chosen for the expansion phase (the normally adopted values are around 1.3, which are significantly higher than 1.15) to ensure reliable results, even in the presence of large pressure uncertainties which can make the detection of the maximum point difficult. The addition of 10 CA to the angle that corresponds to the maximum of \(pV^{1.15}\) allows the low value of the polytropic index to be partly compensated and guarantees complete combustion [11].

The investigation of the derivative of the pressure signal with respect to time has also been proposed for combustion detection. A fourth-order finite difference scheme can be applied to the pressure signal in order to reduce the noise measurement effects in the calculus of the pressure derivative [12]. A sudden increase in the \(\frac{dp}{dt}\) signal appears at the combustion development stage and this can be used to determine the SOC, while EOC can be detected on the basis of the attenuation of the oscillations in the pressure derivative time history [13]. Time-frequency analysis of the in-cylinder pressure derivative has recently been applied to detect combustion [13]. This signal processing technique was introduced into the engine field to detect knock and block vibration and for the diagnosis of the injection process [35]. The fundamental idea behind time-frequency analysis is to be able to understand and describe how the spectral content of a signal changes in time, while classical signal analysis usually dealt with time and frequency separately [36]. In general, time-frequency analysis is approached using the spectrogram, which is based on the short-time Fourier transform (STFT) concept [37], but many other methods have been proposed and applied [38]. Qualitative criteria of the signal frequency should allow the existence of combustion to be detected, while time information can serve for combustion localization.

In the present work, a comparison has been made between the most popular pressure-based direct methods and single, two-zone and three-zone combustion models; reference has been made to different working conditions for both spark-ignition and compression-ignition IC engines. The time frequency analysis and the spectrogram theory have also been applied to both the in-cylinder pressure signal and the difference between fired and motored pressure in diesel engines. The mean instantaneous frequency [39] has been calculated from a home-made tool on the basis of the spectrogram, and this signal allows detailed information on the development and evolution of diesel combustion to be extracted.

### 2. PRESSURE-BASED DIRECT METHODS FOR COMBUSTION SIMULATION.

Marvin’s original work referred to gasoline engines [17]. A typical \(p-V\) diagram of a naturally-aspirated spark-ignition engine \((V=2000\text{ cm}^3)\) applied to a passenger car [40] is reported in Fig. 1a (\(\text{bmep}=790\text{ kPa}, n=3300\text{ rpm}\)) and a portion of this diagram is plotted on logarithmic coordinates in Fig. 1b to illustrate the procedure. The dashed line shows actual
in-cylinder pressures measured by a piezoelectric pressure transducer, whereas the ideal Otto combustion is represented by means of a vertical segment. The compression and the expansion phases are represented as polytropic evolutions, according to the $pV^{m}=\text{const}$ law, and they become straight lines plotted with a solid line in the log$p$ - log$V$ diagram. Polytropic exponents equal to $m_{c} \approx 1.34$ and $m_{e} \approx 1.27$ have been considered for the closer-to-TDC portion of the compression and expansion phases, respectively, in line with values in [18]. In [33], the following values are proposed: $m_{c} = 1.35$ and $m_{e} = 1.25$. The crankshaft angles at which the actual pressure starts to differ from the ideal compression and straight expansion lines can be identified as the SOC and EOC angles, respectively: SOC $\approx 348°$ CA and EOC $\approx 414°$ CA in Fig. 1b. The combustion duration should be estimated as $\Theta_{\text{comb}}(EOC-SOC)$ and results to be around $66°$ CA for the considered example. Furthermore, Marvin assumed that the pressure rise above the straight compression line was proportional to the mass of burned fuel. Therefore, $x_{b}$ at a certain point $C$ (Fig. 1b) is given roughly by the ratio of length $C'A$ to length $AB$. Point $C'$ is the projection, on the top dead center line, of point $C$, which belongs to the actual pressure curve. This projection occurs by means of the straight line $C'C$, which is parallel to the polytropic compression straight line when $C$ is located between SOC and TDC ($\theta_{\text{TDC}} = 360°$ CA) and is parallel to the polytropic expansion straight line when $C$ is situated between $TDC$ and $EOC$.

The burned mass fraction curve, determined according to the Marvin procedure ($x_{b,M}$), is reported in Fig. 2 on the basis of the data given in Fig. 1b and is compared with the $x_{b}$ distribution obtained using a two-zone combustion diagnostic model (the $RW$ method, to which $\Delta p_{\text{comb},RW}$ and $x_{b,RW}$ in Fig. 2 refer, will be discussed in detail later on in this section). The agreement between the Marvin distribution and the theoretical curve of the two-zone model is acceptable, with a difference of $\Delta x_{b} \approx 2.5\%$ between the two curves at $\theta = 400°$ CA. The most interesting point of the burned mass fraction profile is its half value ($x_{b} \approx 0.5$): Fig. 2 shows that the difference is about $1.5°$ CA between the $MFB50$ values estimated by means of the two approaches. None of the abovementioned discrepancies between Marvin’s method and the two-zone model can be regarded as being only due to the inaccuracy of the Marvin estimation of the burned mass fraction, since the $x_{b}$ curve of the two-zone model is also affected by some uncertainties, which are primarily due to the calibration procedure of the coefficients of the wall heat transfer model. In general, the less the difference between exponents $m_{c}$ and $m_{e}$, the better the performance of the Marvin method in the calculation of the $x_{b}$ curve. Marvin’s graphical procedure for the evaluation of $x_{b}$ cannot be extended easily to diesel engines because the Sabathé cycle, which is the general reference for compression ignition engines in passenger cars, also involves a combustion phase at constant pressure.
The RW method represents a sort of analytical version of Marvin’s graphical procedure. It is based on the hypothesis that the changes in the pressure, due to piston motion and the charge-to-wall heat transfer, can be represented by polytropic processes. In this method, the pressure variation during any crank angle increment $\Delta \theta = \theta_i - \theta_{i-1}$ consists of two parts: a pressure rise due to combustion ($\Delta p_{\text{comb}, \text{RW}}$) and a pressure contribution due to volume change and heat exchanges. The term $\Delta p_{\text{comb}, \text{RW}}$ can be evaluated as the difference between the actual and polytropic pressures by means of the following formulas, in which the pressure values are measured and the volumes are calculated [19]:

$$
\begin{aligned}
\Delta p_{\text{comb}, \text{RW}}(\theta) &= p(\theta) - p_{i-1}\left[\frac{V_{i-1}}{V(\theta)}\right]^{\frac{n_i}{n_i-1}} & \text{if } \theta \leq \theta_{TDC} \\
\Delta p_{\text{comb}, \text{RW}}(\theta) &= p(\theta) - p_{i-1}\left[\frac{V_{i-1}}{V(\theta)}\right]^{\frac{n_i}{n_i-1}} & \text{if } \theta \geq \theta_{TDC}
\end{aligned}
$$

Figure 2. $x_b$ with the Marvin and RW procedures
(gasoline engine, $bme=790$ kPa, $n=3300$ rpm).

Figure 1a. Indicated $p$-$V$ diagram
(gasoline engine, $bme=790$ kPa, $n=3300$ rpm)

Figure 1b. Enlargement of the indicated diagram in the log$p$-log$V$ plane ($bme=790$ kPa, $n=3300$ rpm).
The SOC (lower than $\theta_{TDC}$) and the EOC (higher than $\theta_{TDC}$) are the maximum and the minimum crankshaft angles, respectively, for which $\Delta p_{\text{comb, RW}}$ becomes virtually nil. The fitting of the $m_c$ and $m_e$ polytropic indexes should be carried out by means of a least square technique performed over crank angle intervals of $20^\circ \div 50^\circ \text{CA}$ just before the start and after the end of combustion, respectively. In particular, $m_c$ can be worked out over an angular interval located just before the high voltage-to-spark instant, whereas $m_e$ can be calculated over an angular window that starts just after the $(\theta_{\text{max}}+10^\circ)$ angle, where $\theta_{\text{max}}$ provides the maximum value of quantity $pV^m_c$, this being in line with the procedure outlined in [11].

If reference is made to the data in Fig. 1a ($m_e=1.27$), one obtains $\theta_{\text{max}}+10^\circ \approx 430^\circ \text{CA}$, that is, a higher angle than the EOC value estimated by means of the Marvin procedure (cf. Fig. 1b).

Figure 2 reports the $\Delta p_{\text{comb, RW}}$ versus $\theta$ distribution (solid line) that has been calculated by means of Eq. (1) on the basis of the experimental pressure data shown in Fig. 1a. It results that $\text{SOC} \approx 346^\circ \text{CA}$ and $\text{EOC} \approx 418^\circ \text{CA}$, and these values are very similar to the outcomes of the Marvin graphical procedure.

In the case of the gasoline engine, the elemental combustion heat ($\delta Q_{\text{hrr}}$) released during the $\Delta \theta$ interval can roughly be modeled as a heat adsorbed during an isochoric evolution, according to the following expression:

$$\delta Q_{\text{hrr}} = M c_v \Delta T_{\text{comb, RW}}$$  \hspace{1cm} (2)

where $M$ is the constant mixture mass during combustion. The temperature increase $\Delta T_{\text{comb}}$ can be calculated as

$$\Delta T_{\text{comb, RW}} = \frac{\Delta p_{\text{comb, RW}} V_{TDC}}{M R}$$  \hspace{1cm} (3)

By substituting Eq. (3) in Eq. (2) and taking into account Eq. (1), one obtains:

$$\delta Q_{\text{hrr}} \approx \frac{V_{TDC}}{\gamma - 1} \left( p(\theta_i) - p_{n-1} \left[ V_{n-1}/V(\theta_i) \right]^m \right)$$  \hspace{1cm} (4)

where $m$ can either be $m_c$ or $m_e$ and $\gamma = c_p/c_v$. Parameter $\gamma$ can be considered as a constant property or expressed as a function of the temperature. Quantity $\delta Q_{\text{hrr}}$ does not include the heat exchanged with the walls, since it has already been included in the $m$ polytropic exponent. In other words, $\delta Q_{\text{hrr}}$ represents the gross heat release, while the apparent or net heat release ($\bar{\delta}Q_{\text{hrr}}$) can be obtained by replacing polytropic index $m$ with factor $\gamma$ in Eq. (4). Furthermore, heat $\bar{\delta}Q_{\text{hrr}}$ results to be proportional to $\Delta p_{\text{comb, RW}}$, in agreement with Marvin’s hypothesis. Therefore, the RW algorithm establishes $x_b$ at the end of the $i$-th interval of amplitude $\Delta \theta$ ($\theta = i \Delta \theta$) as [19].
\[ x_{b,RW}(\theta_i) = \frac{m_b(\theta_i)}{m_{b,\theta_0}} = \frac{\sum_{k=0}^{N} \Delta p_{comb,RW}(\theta_k)}{\sum_{k=0}^{N} \Delta p_{comb,RW}(\theta_k)} \]  

(5)

where \( m_b(\theta) \) denotes the mass of fuel that has already burned at crank angle \( \theta \), \( i=0 \) indicates the SOC and \( N = \Theta_{comb}/\Delta \theta \) designates the EOC. As can be seen in Fig. 2, the burned mass fraction obtained by means of the RW technique is virtually coincident with the curve achieved by means of the Marvin graphical method. It is worth pointing out that if a constant polytropic index equal to \( 1/2(m_c+m_e) \) was considered for both the compression and expansion phases, as different experimenters have done [8], the pattern of the \( x_{b,RW} \) curve in Fig. 2 would not change appreciably. In spite of the evident approximating nature of the RW procedure, its accuracy can be considered acceptable and it is still in fact widely used, even for diesel engines, due to its relative simplicity and satisfactory computational efficiency. The criticism inherent to both the Marvin and RW methods concerns the choice of the proper angle intervals over which the polytropic exponents are fitted, and the determination of these intervals affects the \( x_b \) curve to a great extent.

McCinston, Lavoie and Kauffmann proposed a relation of the same type as the following one to approximate the \( x_b \) curve, on condition that the volume of the burning mixture does not vary significantly during the combustion process:

\[ x_{b,MLK} = \frac{p V^m - p_{SOC} V_{SOC}^m}{p_{EOC} V_{EOC}^m - p_{SOC} V_{SOC}^m} \]  

(6)

where \( p \) and \( V \) represent the pressure and volume at a certain crank shaft angle \( \theta \) and \( p_{SOC}, p_{EOC}, V_{SOC}, \) and \( V_{EOC} \) designate the pressure values and volumes at SOC and EOC. Unlike Eq. (5), the MLK formula also evaluates the volume variations. Eq. (6) can be obtained from the energy equation, provided adequate simplifications are introduced. If crevice volume and blow-by effects are neglected, the heat release equation can be written as

\[ \delta q_{hot} = dx_b H_i = c_v dT + pdv - \delta q_{ht} \]  

(7)

where \( v \) is the specific volume of the mixture, \( H_i \) is the lower heating value of the fuel and \( \delta q_{hot} = cdT \) represents the heat transfer with the walls, \( c = c_v(m-k)/(m-1) \) being the polytropic specific heat for the wall heat exchange. After some arrangements and bearing in mind that \( pv = RT (R = c_p - c_v) \), Eq. (7) becomes

\[ dx_b H_i = \frac{m}{m-1} pdv + \frac{1}{m-1} vdp \]  

(8)
If the heat exchanged with the walls is overlooked ($\delta q_{ht} \approx 0$), $m$ can be replaced by $\gamma$ in Eq. (6) and one obtains the original MLK formula: the resulting heat release thus becomes equal to the apparent heat release rate ($\delta q_{ahrr}$). Apparent heat release rates are usually 10-20% lower than gross heat release rates [20].

Eq. (8) can be rewritten as follows [21]

$$dx_b = \frac{1}{H_i} \frac{d \left( pv^m \right)}{(m-1)V^{m-1}} = \frac{d \left( \frac{V^m}{V_{SOC}^m} \right)}{(m-1)P_{SOC}V_{SOC} \left( \frac{V}{V_{SOC}} \right)^{m-1}}$$  \hspace{1cm} (9)

where the instantaneous volume $V$ of the combustion chamber and instantaneous pressure have been normalized to their values at SOC and quantity $y$ is defined by $y = (V/V_{SOC})(P/P_{SOC})^{1/m}$. Eq. (9) can easily be integrated and, for constant volume combustion, the following expression can then be obtained ($P_{SOC}V_{SOC} = MRT_{SOC}$):

$$y^m(\theta) = 1 + \frac{m-1}{\gamma - 1} \frac{H_i}{c_r T_{SOC}} x_b$$  \hspace{1cm} (10)

Eq. (10) provides a linear expression for $y^m(\theta)$ as a function of $x_b$, and this expression can also be formulated as [21]

$$y^m = 1 + \left[ y^m(\theta_{EOC}) - 1 \right] x_b$$  \hspace{1cm} (11)

where $y^m(\theta_{EOC})$ corresponds to $x_b = 1$. Eq. (11) can be solved, with respect to $x_b$, in order to obtain the approximate relation for $x_b$ given by Eq. (6). In short, the MLK method can be considered a rough application of the heat release analysis and is based on the observation that the heat released by the fuel to the charge in the period from SOC to EOC is proportional to $pv^m$ and it can be normalized to the value expressed by Eq. (12) in order to obtain the $x_b,_{MLK}$ curve. Although Eq. (6) only holds for isochoric combustion, the inaccuracies in the determination of the angles at which combustion begins and ends are small when the volume does not vary significantly during combustion. Therefore, the simplified MLK procedure can be applied to production gasoline engines, but is not suitable for diesel engines in which combustion occurs during a significant change in the $V$ volume.

Figure 3 plots quantity $pv^m$, divided by a reference value, as a function of $\theta$, for the pressure and volume data reported in Fig. 1a ($m = m_r \approx 1.27$ since most of the combustion develops in the $\theta > \theta_{TDC}$ range for the considered case).
The $pV_m/(pV_m)_{ref}$ versus $\theta$ trace is approximately constant in the compression phase, increases during combustion ($SOC \approx 346^\circ CA$) and is again nearly constant for the remainder of the cycle. This is consistent with the results given in [41], where $pV^\gamma$ is nearly constant during the approximately isentropic processes, while it is not constant during a period of energy release. A maximum point of $pV_m$ is observable in Fig. 3 at $\theta \approx 424^\circ CA$ and in the MLK method this angle corresponds to EOC, this being in line with other results concerning the meaning of the maximum value of quantity $pV_1^{1.15}$ [11]. The $x_b$ time histories predicted by means of the MLK technique and the two-zone combustion model have also been reported in Fig. 3. As can be inferred, the $x_b,MLK$ curve is virtually coincident with the two-zone model burned mass fraction distribution: the difference between the MFB50 points of the two curves is less than 0.2$^\circ CA$.

The PRM algorithm [32] involves the calculation of the ratio between the fired pressure and the corresponding motored cylinder pressure ($p_{mot}$) at each crank angle. The curve $p_{mot}$ versus $\theta$ can be measured experimentally, in the absence of combustion, when the engine is run by an electrical machine, or can be approximated by means of a polytropic evolution (the latter solution is mandatory if the engine is supercharged or turbocharged).

The pressure ratio ($PR$) is defined as [33]

$$ PR = \frac{p(\theta)}{p_{mot}(\theta)} \quad (13) $$

This parameter has a unit value before combustion and becomes higher than one during combustion, reaching a maximum value at the end of combustion. The increase in $PR$, with respect to the unit value, is referred to as the modified pressure ratio:

$$ MPR(\theta) = \frac{p(\theta)}{p_{mot}(\theta)} - 1 \quad (14) $$
The maximum value of PMR is reached when PR is at a maximum and is called the final PR value (FPR). It usually ranges from 2.8 to 4.0 and typically occurs at around 55° CA ATDC for spark-ignition engines [42]. FPR reaches a maximum value for stoichiometric mixtures and decreases as the excess of air, EGR or residual gas are increased; therefore, it can also be useful as an indicator of the charge dilution of the combustion system. The MPR, once it has been normalized to FPR, provides the pressure ratio management procedure for the estimation of \( x_b \) [32]:

\[
x_{b,PRM} = \frac{MPR(\theta)}{FPR}
\]  

(15)

Figure 4a plots the \( p(\theta) \) and \( p_{mot}(\theta) \) signals for the same engine layout and working condition to which Figs. 1-3 refer.

The PR ratio is reported in Fig. 4b as a function of \( \theta \) and its maximum value occurs at \( \theta \approx 430° \) CA, that is, around 70° ATDC. Furthermore, Fig. 4b shows that the \( x_{b,PRM} \) curve is almost coincident with the two-zone model \( x_b \) distribution.

The direct application of the PMR technique to diesel engines can result in a burned mass fraction curve that can differ from the actual cumulative heat release trace, which is calculated by means of a combustion model. In fact, FPR occurs towards the exhaust valve opening, due to the much higher compression ratios of the diesel engine, and the PRM estimation therefore departs from the actual diesel combustion characteristics, even for single fuel injection schedules.

Furthermore, diesel combustion can consist of discrete heat release events because of multiple injection strategies and this represents another main difference from spark ignition engines.

\[ PDR(\theta) = \frac{p(\theta) + C_1}{p_{mot}(\theta) + C_2} - 1 \]  

(16)
where $C_1$ is the fired pressure characterization coefficient and $C_2$ is the motored pressure characterization coefficient [32]. These empirical coefficients are constant for a given engine configuration. The PDR has an almost zero value before combustion and rises to a maximum value ($PDR_{\text{max}}$) which corresponds to EOC. An estimate of the mass fraction burnt is obtained by normalizing PDR to its maximum value [32]:

$$x_{b,PDR} = \frac{PDR(\theta)}{PDR_{\text{max}}}$$ (17)

The $C_2$ coefficient in Eq. (16) is adjusted so that the $x_{b,PDR}$ trace matches the results of a heat release model as closely as possible: small changes in the $C_2$ value cause the curve to pivot around a point. The $C_1$ constant is then selected to shift the pivotal point as close as possible to the actual MFB50, because this ensures that small deviations at the extreme ends of the curve will have minimal effects on the prediction of MFB50.

Figures 5 and 6 compare the performance of the RW, MLK and PRM methods when applied to the previously considered gasoline engine at $bme_p=440$ kPa, $n=2000$ rpm and at $bme_p=620$ kPa, $n=2570$ rpm, respectively. The $x_b$ curves, which have been worked out by means of a calibrated two-zone gasoline combustion model, are used as references to evaluate the different pressure-based direct techniques. Furthermore, Fig. 7 shows a comparison of the same abovementioned methods for a spark-ignited CNG supercharged engine ($iV=1242$ cm$^3$) at $bme_p=800$ kPa and $n=3000$ rpm. The reference heat release curve has been calculated with a two-zone combustion diagnostic model of the considered CNG engine [43].

![Figure 5. Comparison of the procedures pertaining to $x_b$ (gasoline engine, $bme_p=440$ kPa, $n=2000$ rpm).](image1)

![Figure 6. Comparison of the procedures pertaining to $x_b$ (gasoline engine $bme_p=620$ kPa, $n=2570$ rpm).](image2)
The MLK algorithm generally gives results that are virtually coincident with those of two-zone models, although the PMR method also guarantees very satisfactory performance. The MLK and the PRM generally give very similar results because Eqs. (6) and (15) represent the same physical law: in fact, if one divides the numerator and denominator of the right hand member of Eq. (6) by $m_{SOCSOCVp}$, the quantity $m_{SOCSOC VVp}$ represents the polytropic pressure (motored pressure) and, as a consequence, Eq. (15) is obtained. The only slight difference between the two techniques can reside in the evaluation of the EOC, which coincides with the maximum of $p/p_{mot}$ for the PMR procedure and with the maximum of $pV^m$ for the MLK procedure: these two maxima can occur at different crankshaft angles.

Finally, the data in Figs. 8-10 refer to a twin-stage turbocharged diesel engine ($iV=2000$ cm$^3$) [44], fuelled with distinct injection schedules at different bmep and $n$ conditions. The crankshaft angle based $x_b$ traces derived from the application of single-zone and three-zone diagnostic tools for the simulation of diesel combustion. In the calculus of $x_b,PDR$, the calibrated values of $C_1$ and $C_2$ were set equal to 1.1 and 1.2, respectively, for the turbocharged engine setup tested in the current investigation. These values were selected on the basis of preliminary tests aimed at optimizing the PDR method response. As can generally be inferred from Figs. 8-10, the prediction capability of the considered pressure-based techniques becomes worse when passing from spark ignition to compression ignition engines. In particular, the accuracy of the pressure based techniques in the prediction of SOC, EOC and the $x_b$ trace deteriorates in diesel engines because the combustion evolution becomes more complex, also because of the multiple injection events, and requires more sophisticated approaches. The PDR method generally guarantees the best approximation of the $x_b$ distributions obtained by means of the single zone and three-zone models; both the SOC and the EOC are evaluated with satisfactory accuracy for the single (Fig. 8) and the double injection (Fig. 9) strategies. The RW technique gives a better performance than the PMR method, which is confirmed to be inadequate for diesel engines, even in the presence of single injection events.
3. TIME FREQUENCY ANALYSIS

The Fourier transform is suitable for describing stationary phenomena in the frequency domain, but it does not allow transient phenomena that undergo a time evolution to be analyzed. In fact, Fourier transform coefficients represent the time integral of the product between the considered signal and a complex sinusoidal wave, which is determined perfectly in terms of frequency and phase, but is not localized in the time domain [38]. In other words, the FFT points out the contribution of the different harmonic terms to the signal, but the time instants at which each frequency component is relevant are not clarified. Instead, time frequency analysis is used to evaluate the changes, with respect to time, in the frequency spectrum of a transient signal $f(t)$. A large number of fast Fourier transforms are realized over different consecutive short time intervals and each FFT is referred to the mean instant of the short time interval over which the FFT has been performed. In fact, the unsteady signal is assumed to have stationary behavior in each short time interval and a local Fourier spectrum is therefore calculated over this short interval.

From a mathematical point of view, a windowing operation of signal $f(t)$ is realized in the time-frequency analysis by multiplying $f(t)$ by a mobile window function $h(t-\tau)$, where $\tau$ is a variable parameter that represents the centre of the window support. The Short Time Fourier Transform (STFT) is defined in the following way:

$$\hat{F}_l(\omega, \tau) = \int_{-\infty}^{\infty} f(t) h(t-\tau)e^{-j\omega t} dt$$ (18)

The position of the window in Eq. (18) can be shifted with respect to time by varying $\tau$, and different Fourier spectra can then be obtained. Integrating Eq. (18) with respect to $\tau$, one obtains

$$\hat{F}(\omega) = \int_{-\infty}^{\infty} \hat{F}_l(\omega, \tau) d\tau = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} f(t) h(t-\tau) dt \right] e^{-j\omega t} dt$$ (19)
Function $\hat{F}(\omega, \tau)$ represents a local Fourier transform, and the summation of all of the STFT over $\tau$ values ranging from $-\infty$ to $+\infty$ is equal to $\hat{F}(\omega)$. Since $\hat{F}(\omega)$ is the Fourier transform of a time convolution, it is given by the product of the Fourier transform of the signal, namely $F(\omega)$, and the Fourier transform of the window, namely $H(\omega)$:

$$\hat{F}(\omega) = F(\omega) H(\omega) \quad (20)$$

When the STFT is applied to the analysis of the time variations of a physical signal, the $\tau$ parameter in Eq. (18) is varied by finite steps of amplitude $\Delta \tau$. The higher the overlap period of two consecutive positions $\tau_i$ and $\tau_{i+1} = \tau_i + \Delta \tau$ of window $h$ in the time domain, the more gradual the variation of the Fourier spectrum with respect to time. The extension of the time support $\Delta T_h$ of the window, i.e., the time interval in which the $h(t)$ window is higher than zero, can be related to the sampling circular frequency ($\omega_s$) and to the number of sampling points within the window support ($N$) by the relation $\Delta T_h = 2\pi N/\omega_s$. The overlap period is equal to $\Delta t_{ov} = s \Delta T_h = 2\pi n/\omega_s$, where $n$ is the number of samplings in the overlap period $\Delta t_{ov}$ and $s = n/N$ is the overlap factor. As a result, the temporal resolution of the SFST can be worked out as follows:

$$\Delta \tau = \Delta T_h - \Delta t_{ov} = \frac{2\pi}{\omega_s} N \left(1 - s\right) \quad (21)$$

Both a reduction in $N$ and an increase in $s$ at fixed sampling frequency $\omega_s$ induces an improvement in the time resolution, that is, a diminution in $\Delta \tau$. The minimum value of $\Delta \tau$ is $2\pi/N$ and is reached for $s = N/(N-1)$.

A window function can be characterized in terms of its centre and its semiamplitude. The centre ($t_0$) and the semiamplitude ($\Delta t$) of a window function $h$ are defined as

$$t_0 = \frac{1}{\| h \|_2} \int_{-\infty}^{\infty} t |h(t)|^2 \, dt \quad \Delta t = \frac{1}{\| h \|_2} \left[ \int_{-\infty}^{\infty} (t - t_0)^2 |h(t)|^2 \, dt \right]^{1/2} \quad (22)$$

where the norm $\| h \|_2$ represents the energy of the $h$ function, which is given by

$$\| h \|_2 = \int_{-\infty}^{\infty} |h(t)|^2 \, dt \quad (23)$$

The definitions of centre and semiamplitude extend naturally to the frequency domain for the $H(\omega)$ function:

$$\omega_0 = \frac{1}{\| H \|_2} \int_{-\infty}^{\infty} \omega |H(\omega)|^2 \, d\omega \quad \Delta \omega = \frac{1}{\| H \|_2} \left[ \int_{-\infty}^{\infty} \left(\omega - \omega_0\right)^2 |H(\omega)|^2 \, d\omega \right]^{1/2} \quad (24)$$

where $\| H \|_2 = \int_{-\infty}^{\infty} |H(\omega)|^2 \, d\omega$. 
The selection of the optimum $h$ window function is a central theme in the time frequency theory. Different shapes of window functions have been analyzed and tested by specialists in the field. Eq. (20) shows that the window modifies the frequency content of signal $f(t)$, which is given by $F(\omega)$. A rectangular shaped window ($h_R$, cf. Fig. 11) is a simple solution, but it is able to alter the spectrum of the original signal to a great extent. As an example, if $f(t)$ is a sinusoid with circular frequency $\omega = \omega_0$, the signal, which is windowed by means of a $h_R$ function with its centre at $\omega_0$, features a frequency spectrum that can be represented as a band centered on $\omega_0$ ($H_R$ is reported in Fig. 11) rather than as a single line at $\omega_0$. Furthermore, the step transitions that occur in $h_R$ generate high-frequency harmonic components in the $\tilde{F}(\omega)$ spectrum. Other window functions have been proposed to partially solve these drawbacks of the rectangular window. One popular window function is the Hann filter (cf. $h_H$ in Fig. 11), which is defined as follows (the here considered window has the center at point $n=(N-1)/2$):

$$h_H(n) = \frac{1}{2} \left[ 1 - \cos \left( \frac{2\pi n}{N-1} \right) \right] \quad 0 \leq n \leq N - 1$$

(25)

The $h_H$ function goes to zero at $n=0$ and $n=N-1$, without any type of discontinuity, and therefore does not introduce any spurious high-frequency components in the frequency content of signal $f$. The Fourier spectrum of $h_R$, that is, $H_R$, has been illustrated in Fig. 11: a central lobe exists in the Fourier spectrum of the windowed function, with a higher extension than the corresponding one in the Fourier spectrum of the $h_R$ windowed signal, but the number and the maximum values of the lateral lobes is reduced compared to the $h_R$ case.

For any window function $h$, the amplitude of $H(\omega)$, i.e. $2\Delta_0$, increases as the amplitude of $h(t)$, i.e. $2\Delta$, reduces. The Heisenberg principle of indetermination governs the relationship between $\Delta_0$ and $\Delta_t$ according to the following expression [37]:

![Figure 11. Different $h$ functions and their Fourier transforms.](image)
The two formulations are equivalent because $\nu = \omega / 2\pi$ and hence $\Delta t = \Delta \omega / 2\pi$. It is impossible, on the basis of these relations, to simultaneously reduce $\Delta \omega$ and $\Delta t$ in order to obtain a high resolution of the unsteady signal $f(t)$ in either the time or the frequency domains. If the frequency resolution is improved, $\Delta \omega$ should be reduced, but this leads to an increase in $\Delta t$ and therefore to a diminution in the capability of locating events in the time domain: phenomena that are closely coupled become difficult to distinguish. In the particular case of the FFT, the window function in Eq. (18) is $h_{FT} = 1$, $\Delta t_{FT} \to \infty$ while $\Delta \omega_{FT} \to 0$ because $H_{FT}$ is the Dirac delta function in the frequency domain: the description is therefore very accurate in the frequency domain, but it is impossible to localize the events with respect to time.

The Gaussian window, i.e., $h_G(n) = \exp\left\{-\frac{1}{2} \left[ \frac{n - (N-1)/2}{\sigma(N-1)/2} \right]^2 \right\}$, is the window function that minimizes the product between $\Delta t$ and $\Delta \omega$. In fact, it can be proved that the Gaussian window is the only function for which $\Delta t \Delta \omega = 1/2$. When $h_G$ is used in Eq. (18), the thus obtained $\tilde{F}(\omega, \tau)$ is named the Gabor transform of the $f$ function.

### 3.1 Spectrogram and mean instantaneous frequency numerical model

The time-frequency distribution related to signal $f(t)$ is a function $P_f(t, \nu)$, which is defined as follows [37]:

$$ E_f = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_f(t, \nu) dtd\nu = \int_{-\infty}^{\infty} \left| f(t) \right|^2 dt $$

where $E$ represents the energy of signal $f(t)$. The $P_f(t, \nu)$ distribution therefore represents an energy density function with respect to both time and frequency. The Parseval identity allows the energy of the signal to be calculated, starting from the knowledge of the Fourier spectrum ($F(\omega)$ or alternatively $F(\nu)$, which has been considered in what follows):

$$ E_f = \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} |F(\nu)|^2 d\nu $$

The marginal energy density functions of signal $f$ are the density functions with respect to either $t$ or $\nu$. These functions are obtained by integrating $P_f(t, \nu)$ with respect to one of the independent variables:

$$ M_f(\nu) = \int_{-\infty}^{\infty} P_f(t, \nu) dt \quad m_f(t) = \int_{-\infty}^{\infty} P_f(t, \nu) d\nu $$

The marginal properties are said to hold when the following relations are satisfied:

$$ |F(\nu)|^2 = \int_{-\infty}^{\infty} P_f(t, \nu) dt \quad \left| f(t) \right|^2 = \int_{-\infty}^{\infty} P_f(t, \nu) d\nu $$
If the marginal properties are satisfied, Eqs. (27) and (28) are automatically verified, but the opposite is generally not true. Therefore, the marginal properties represent a more severe requirement than Eqs. (27) and (28).

The time-frequency distribution can be interpreted as a density probability function and has been used in the simulation code to calculate the mean instantaneous frequency \( \overline{\nu}(t) \) according to the following formula [37]:

\[
\overline{\nu}(t) = \frac{1}{\int_{-\infty}^{\infty} P_j(t, \nu) d\nu} \int_{-\infty}^{\infty} \nu P_j(t, \nu) d\nu
\]  

Eq. (31) gives the baseline harmonic contribution to signal \( f(t) \) at each time instant and is a fundamental relation for many engineering applications. The spectrogram \( P_{sp}(t, \nu) \) is the square modulus of \( \hat{F}_j(\omega, \tau) \) and has been selected as the time-frequency distribution in the developed code:

\[
P_{sp}(t, \nu) = \left[ \int_{-\infty}^{\infty} f(\tau) h(\tau-t) e^{-j2\nu \tau} d\tau \right]^2 = \hat{F}_j(\omega, \tau)^2
\]  

Function \( P_{sp} \) gives the energy density with respect to time and frequency of the \( f(t)h(t-\tau) \) windowed signal. In fact, the following relation holds:

\[
\int_{-\infty}^{\infty} P_{sp}(t, \nu) d\nu = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} f(\tau) h(\tau-t) e^{-j2\nu \tau} d\tau \right] \left[ \int_{-\infty}^{\infty} f^*(\tau') h^*(\tau'-t) e^{j2\nu \tau'} d\tau' \right] d\nu =
\]

\[
= \int_{-\infty}^{\infty} e^{-j2\nu (\tau-\tau')} d\nu \int_{-\infty}^{\infty} \left[ f(\tau) f^*(\tau') h(\tau-t) h^*(\tau'-t) e^{-j2\nu \tau'} \right] d\tau d\tau'
\]  

where \( f' \) and \( h' \) are the complex conjugates of functions \( f \) and \( h \), respectively. Furthermore, since the representation of the Dirac delta function in the frequency domain is generally given by

\[
\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} d\omega = \int_{-\infty}^{\infty} e^{j2\nu \tau} d\nu
\]  

it is possible to obtain:

\[
\int_{-\infty}^{\infty} P_{sp}(t, \nu) d\nu = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ f(\tau) f^*(\tau') h(\tau-t) h^*(\tau'-t) \delta(\tau'-\tau) \right] d\tau d\tau' =
\]

\[
= \int_{-\infty}^{\infty} |f(\tau)|^2 |h(\tau-t)|^2 d\tau \neq |f(t)|^2
\]  

Finally, by developing a symmetric procedure for quantity \( \int_{-\infty}^{\infty} P_{sp}(t, \nu) d\tau \), the following expression can be obtained:

\[
\int_{-\infty}^{\infty} P_{sp}(t, \nu) dt = \int_{-\infty}^{\infty} |F(\nu)|^2 |H(\nu')|^2 d\nu' \neq |F(\nu)|^2
\]
The marginal properties are not verified by Eqs. (36) and (35); they could be satisfied if \( h(t-t)=\delta(t-t) \) and \( H(t-v)=\delta(t-v) \) simultaneously, but, on the basis of the Heisenberg principle, this is not possible. In other words, the windowing operation alters the signal and leads to spurious contributions, due to the square of \( |h| \) and \( |H| \). This is why the integrals of \( P_{sp} \) with respect to \( t \) or \( \nu \) are not equal to \( \|F(v)\|^2 \) and \( \|f(t)\|^2 \), respectively. However, if \( \int_{-\infty}^{\infty} h(t-t) dt = 1 \), integral \( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_{sp}(t,\nu) dtd\nu \) becomes equal to energy \( E \) of signal \( f(t) \), and the spectrogram can therefore be confirmed to be a feasible choice for the \( P_{sp}(t,\nu) \) function in Eq. (31) because it does not alter the energy of the signal, although the marginal properties are not satisfied.

Eqs. (18), (21)-(25), (31)-(32), together with energy consistency condition \( \int_{-\infty}^{\infty} \|h_t(t-t)\|^2 dt = 1 \), which makes Eqs. (27) and (28) hold, have been implemented in the developed Matlab numerical tool to perform the combustion time frequency analyses. In this code, \( f(t) \) can be either the in-cylinder pressure signal \( p(t) \) or the pressure difference basis of a preliminary campaign of STFT tests on \( p(t) \): \( N=50, s=0.96, w=120 \text{ kHz} \) and, from Eq. (21), \( \Delta \tau=170 \mu s \).

### 3.2 Diesel combustion characterization by means of time-frequency analysis.

Figures 12, 14, 16 and 17 plot the injected mass flow-rate \( (G_{inj}) \), the cylinder pressure (in the upper part), the \( HRR \), the \( \tau_b \) and the mean instantaneous frequency crankshaft angle-based distributions (in the bottom part) at different engine working conditions for the same diesel engine layout to which Figs. 8-10 refer. Besides, Figs. 13 and 15 report the 3D graphs of spectrogram \( P_{sp} \), normalized to its maximum value, for two distinct engine working conditions; \( P_{sp} \) has been calculated by means of Eq. (32), in which \( f=p, h=h_H \) and \( t \) is replaced by \( \theta \). The plotted \( G_{inj} \) data were measured at the hydraulic rig, in the absence of combustion, under the same injector working conditions as those of the engine tests, whereas the cylinder pressure time histories were measured in the engine at the dynamometer cell. The \( HRR \) and the \( \tau_b \) plotted traces have been calculated by means of a three-zone diesel combustion model on the basis of the measured \( p \) and \( G_{inj} \) histories. It is worth observing that the measured \( G_{inj} \) can differ significantly from the actual injected flow-rate time history measured at the dynamometer cell, especially for small injection events, because the temperature levels of the fuel are much higher at the dynamometer cell than at the hydraulic test rig. Finally, the mean instantaneous frequency has been evaluated through Eq. (31), in which function \( f(t) \) has been replaced by \( p(\theta) \). The reported diagrams allow the pattern of the \( \tau(\theta) \) signal to be interpreted on the basis of the other physical variable distributions according to a learning procedure.

Figures 12 and 13 refer to a pilot-main injection schedule (two injection pulses are present in the \( G_{inj} \) trace) performed at \( bmep=500 \text{ kPa} \) and \( n=2000 \text{ rpm} \). Fig. 13 shows that \( P_{sp} \) is only higher than zero in the 320°-450° CA \( \theta \) range. Furthermore,
the spectrogram indicates that the frequency content of the considered combustion event is lower than 4 kHz. In Fig. 12, the first maximum point ($P_1$), which occurs along the $\tau(\theta)$ distribution, is caused by the injection of fuel into the combustion chamber. The subsequent decrease in $\tau$ can be ascribed to fuel evaporation and the SOC is located in correspondence to the minimum $P_2$ point. The possible discrepancies between the phasing of $P_2$ and the beginning of the $HRR$ trace can be ascribed to the inaccuracy of the combustion model when simulating the start of combustion. The time frequency analysis can therefore also be applied to validate and possibly improve the prediction capability of combustion diagnostic tools. The increase in $\tau$, which is detected after $P_2$, is a consequence of the combustion development. A first, more pronounced peak, related to the premixed combustion phase of the pilot injection, can be detected ($P_3$) and this is followed by a second one ($P_4$), which is due to a smaller, late combustion event related to the pilot injected fuel. The intensity of the $P_4$ peak is also reduced because the piston has already started its downstroke after TDC; if this phenomenon is taken into account, the late combustion event of the pilot injected fuel seems to be underestimated in the $HRR$ trace obtained when the model is used.
The diminution in the $\nu$ values, which occurs within the $365-368^\circ\text{CA}$ range, is induced by the evaporation of the main injected fuel and by the combustion chamber expansion due to the piston downstroke. The $\text{SOC}$ of the main injected fuel is placed at about $\theta \approx 368^\circ \text{CA}$, and a minimum point ($P_5$) can be observed in the $\nu$ curve. The consequent $\nu$ increase is characterized by the presence of two phases: a first phase (up to $375^\circ \text{CA}$), in which the frequency grows at a higher rate, and a second phase (from $375^\circ \text{CA}$ to $P_6$), in which the slope of $\nu$ with respect to $\theta$ becomes lower. It can be stated, on the basis of the $\text{HRR}$ trace, that the former phase is related to the premixed combustion of the main injected fuel and the latter phase refers to its diffusive combustion regime. The $\text{EOC}$ is evaluated as the time instant at which the $\nu$ curve matches a horizontal line, which is represented by a dashed horizontal line in Figs 14-17. The $\text{EOC}$ prediction of the time frequency analysis is physically consistent with the pattern of the $\chi_b$ and $\text{HRR}$ curves at that angle.

Figs 14 and 15 refer to $\text{bmeq}=800$ kPa and $n=2500$ rpm under a pilot-main-after triple injection schedule (cf. the $G_{\text{inj}}$ trace). In Fig. 14, the $\text{SOC}$ of the pilot injection occurs at $P_1$, which represents the minimum just before the first distinct
maximum; the increase in the mean instantaneous frequency, due to pilot combustion, is soon overbalanced by both the
cylinder volume expansion after TDC and by the evaporation of the main injected fuel; a maximum point of $\tau$ therefore
occurs in $P_2$. The following minimum $\tau$ point, i.e. $P_3$, indicates the beginning of the main combustion after the
evaporation of part of the fuel injected in the main pulse; the $P_4$ peak is related to the premixed combustion part of the
main injected fuel and also to a possible residual pilot combustion, whereas $P_5$ refers to the diffusive combustion of the
main injected fuel and corresponds to a slope variation in the HRR trace. Furthermore, the after-injection causes peak $P_6$
in the $\tau$ distribution (also perceivable in the HRR trace) and, finally, the $\tau$ curve becomes almost horizontal at $\theta \approx 430^\circ CA$,
where the HRR trace is practically null. Fig. 15 reports the 3D spectrogram that corresponds to the data in Fig. 14: two
bumps can be noticed in the $P_{sp}(\nu, t)$ graph, as in Fig. 13, even though the number of injection shots is different (three
shots in Fig. 15 and two shots in Fig. 13). In both cases, the earlier peak corresponds to the piston at TDC (it has been
verified that this peak is also present in the spectrogram of the motored pressure), while the other bump is phased with
the main combustion HRR peak.

Figure 16 reports a pilot-pilot-main injection case at $bmep=200$ kPa and $n=1500$ rpm. The closest-to-main pilot injection
is referred to as pilot 1, whereas the more distant pilot injection is referred to as pilot 2. Since the timing of the pilot 2
injection occurs early during the piston compression stroke, the pilot 2 injected fuel exhibits a two-stage ignition: the
HRR dashed curve becomes different from zero at $\theta \approx 350^\circ CA$, but the sharp increase in HRR occurs later, at $\theta \approx 356^\circ CA$.
Correspondingly, in the curve of the mean instantaneous frequency, $P_2$ and $P_3$ are the SOC and the maximum $\tau$ point
related to the cool flames, whereas $P_4$ and $P_5$ (related to the first peak in the HRR trace) are analogous to $P_2$ and $P_3$ for
the hot flames. The pilot 1 SOC is phased with $P_6$, while $P_7$ represents the maximum $\tau$ point due to pilot 1 combustion
and is related to the second peak in the HRR trace. The SOC of the main injected fuel occurs at $P_6$, and the development
of the main combustion leads to $P_9$, which is related to the third and maximum peak in HRR. The end of combustion
occurs at $\theta \approx 420^\circ CA$, an angle at which both HRR and $x_b$ are practically constant.

Finally, Fig. 17 refers to a pilot-pilot-main-after quadruple injection event ($bmep=500$ kPa and $n=1500$ rpm). Even though
the overall combustion evolution is complex and rich in events, the mean instantaneous frequency distribution allows the
main features of the different combustion events to be identified. The SOC of the pilot 2, pilot 1 and main injected fuel
quantities are phased with $P_1$, $P_3$ and $P_6$, respectively, and related to the local minima of the $\tau$ curve. The local maximum
$P_2$ refers to pilot 2 combustion, while $P_3$ is related to both the pilot 2 and pilot 1 injected fuel masses (part of the pilot 2
injected fuel burns together with the pilot 1 injected fuel) and $P_5$ concerns a late combustion event of the pilot 1 injected
fuel. The local maxima $P_7$ and $P_8$ of the $\tau$-$\theta$ distribution correspond, respectively, to a premixed phase of the main
combustion and to a diffusive phase of both the main combustion and after combustion. The after combustion is responsible for the weak decrease in $\tau$ that occurs after $P_6$, and the EOC is located close to $\theta=400^\circ$CA.

The time frequency analysis can also be applied to the difference, $\Delta p_{\text{comb}}$, between $p(\theta)$ and motored pressure $p_{\text{mot}}(\theta)$ instead of to the $p(\theta)$ signal. This is justified by the fact that $\Delta p_{\text{comb}}(\theta)$ roughly represents the $p(\theta)$ portion, which should be more closely related to the combustion events, whereas $p_{\text{mot}}(\theta)$ is closely related to the chamber volume variation that is due to piston motion. Figs. 18 and 19 report $\nu$ calculated on the basis of $\Delta p_{\text{comb}}$ ($f:=\Delta p_{\text{comb}}$ in spectrogram $P_{\text{sp}}$) for the same engine working conditions as Figs. 14 and 17, respectively. The main advantages of the time frequency analysis applied to $\Delta p_{\text{comb}}$ are the amplification of the peaks in the $\nu$ distribution and a general reduced uncertainty in the determination of EOC, compared to the $p$ analysis. In fact, the $\theta$ value at which the $\nu$ curve matches the horizontal line pattern at the end of combustion results to be more clearly identified for the $\Delta p_{\text{comb}}$ case than for the $p$ case. On the other hand, the knowledge of the motored pressure time history is required for the calculus of $\Delta p_{\text{comb}}(\theta)$. However, the determination of SOC is always easier, and virtually corresponds to the crank angle at which $p(\theta)$ and $p_{\text{mot}}(\theta)$ start to differ, whereas the two pressure traces will never result in the same values when combustion extinguishes because $p(\theta)$ will always remain higher. In addition, the suitable $p_{\text{mot}}(\theta)$ signal cannot be experimentally measured in a turbocharged engine. In fact, because of the difference in intake manifold pressure between firing and motoring tests, the $\Delta p_{\text{comb}}$ signal, obtained as the difference between the experimental measurements of $p(\theta)$ and $p_{\text{mot}}(\theta)$, would be higher than zero before SOC. Therefore, in the case of turbocharged engines, the proper motored trace, which is useful for the $\Delta p_{\text{comb}}$ calculus, has to be calculated starting from a firing cycle and considering a polytropic evolution from an angular position where combustion has not yet started. Finally, by comparing Fig 14 with Fig. 18 and Fig 17, with Fig 19, it can be observed that the trends of the mean instantaneous frequency distributions pertaining to $p(\theta)$ and $\Delta p_{\text{comb}}$ are really quite similar.

Figure 16. Time frequency analysis of $p$
(diesel engine, $bmeq=200$ kPa, $n=1500$ rpm).

Figure 17. Time frequency analysis of $p$
(diesel engine, $bmeq=500$ kPa, $n=$
4. CONCLUSIONS.

The most popular in-cylinder pressure-based direct techniques have been analyzed mathematically, and the hypotheses on which each technique is founded have been pointed out. Different typologies of engines, i.e., gasoline, methane and diesel engines, and different $bmep$ and $n$ working conditions have been considered in order to perform a significant comparison of the tested pressure-based direct methods. The predictions obtained using these methods have been compared with the numerical outcomes of two-zone and three-zone combustion models of spark-ignition and compression-ignition engines, respectively.

The start and end of combustion, for spark ignition engines fuelled with either gasoline or methane, are generally identified with satisfactory precision by the pressure-based methods. The simulation of the burned mass fraction time history is also accurate: the best performance has been achieved with the $MLK$ procedure, which accurately reproduces the $xb$ traces obtained with the two-zone combustion model, but the $PMR$ technique also provides very satisfactory results.

In particular, the $PMR$ and $MLK$ procedures have been proved to be based on the same physical law and this justifies the very similar performance that is obtained when these methods are used.

As far as diesel combustion is concerned, the accuracy of the pressure-based direct techniques in the prediction of $SOC$, $EOC$ and the $xb$ trace generally deteriorates, compared to spark-ignition engines, and the results can become worse as the number of injections increases, because the combustion evolution is more complex, and more sophisticated approaches are required. The only acceptable technique has proved to be the $PDR$ method: both the $SOC$ and the $EOC$ can be evaluated.
with adequate accuracy for single and double injection strategies. On the other hand, an accurate prediction of the burned 
mass time history with pressure-based direct techniques seems to be impracticable for diesel engines.
The time frequency analysis of the in-cylinder pressure, in spite of not being able to provide a complete \( x_b \) curve, 
represents a useful means of investigation for diesel engines because it can offer a great deal of information on the 
combustion evolution as well as on the initial and final instants of the overall process. A home-made numerical tool has 
been developed to perform STFT tests on the in-cylinder pressure. The mean instantaneous frequency versus crankshaft 
angle distribution, which is obtained from the spectrogram of the in-cylinder pressure, allows the \( SOC \), the fuel 
evaporation phase and the combustion stage pertaining to each injection shot to be identified clearly. Furthermore, the 
ignition delays of both cool and hot flames as well as the timings of both premixed and diffusive combustion can be 
localized accurately during the piston compression and expansion strokes.
The application of the time frequency analysis to the pressure difference between the in-cylinder pressure and the motored 
pressure leads to better results than in the case of the in-cylinder pressure signal. In particular, it leads to amplified peaks 
in the \( \nu \) distribution and to less uncertainty in the determination of \( EOC \) than in the case of the in-cylinder pressure.

5. NOMENCLATURE.

\begin{itemize}
\item \textit{bmep} \hspace{1cm} \text{brake mean effective pressure}
\item \textit{BDC} \hspace{1cm} \text{bottom dead center}
\item \textit{c} \hspace{1cm} \text{constant specific heat of the polytropic evolution}
\item \textit{c_v} \hspace{1cm} \text{specific heat at constant volume}
\item \textit{dp/dt} \hspace{1cm} \text{pressure derivative with respect to time}
\item \textit{E} \hspace{1cm} \text{energy of the signal}
\item \textit{EOC} \hspace{1cm} \text{end of combustion}
\item \textit{f(t)} \hspace{1cm} \text{time function}
\item \textit{F(\omega)} \hspace{1cm} \text{Fourier transform of function } f
\item \textit{FFT} \hspace{1cm} \text{fast Fourier transform}
\item \textit{\hat{F}_f(\omega, \tau)} \hspace{1cm} \text{short time Fourier transform}
\item \textit{\hat{F}(\omega)} \hspace{1cm} \text{integral of the short time Fourier transform with respect to time}
\item \textit{FPR} \hspace{1cm} \text{final pressure ratio}
\item \textit{G_{inj}} \hspace{1cm} \text{injected mass flow-rate}
\item \textit{h(t)} \hspace{1cm} \text{window function}
\end{itemize}
Fourier transform of the window function

indicated mean effective pressure

engine displacement

parameter that defines the position of the window function in the time domain

lower heating value

heat release rate

internal combustion engines

marginal function in the time domain; polytropic exponent

mass of burned fuel

total mass of burned fuel

exponents of the polytropic compression and expansion phases

mass of the burning mixture; marginal function in the frequency domain

angle at which 50% of the combustion mixture has burned

McCuis-ton, Lavoie and Kaufmann (procedure)

modified pressure ratio

engine speed; number of points in the overlap zone during the shift of the window function

number of intervals in the combustion process; number of points of the window function

time-frequency distribution function

noteworthy points along the mean instantaneous frequency time distribution

spectrogram

in-cylinder instantaneous pressure

pressure departure ratio (algorithm)

pressure ratio

pressure ratio management (procedure)

elastic constant of the gas

Rassweiler and Withrow (procedure)

start of combustion

short time Fourier transform

time

center of the window function in the time domain

top dead center
<table>
<thead>
<tr>
<th>No.</th>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>639</td>
<td>$v$</td>
<td>specific volume</td>
</tr>
<tr>
<td>640</td>
<td>$V$</td>
<td>instantaneous volume of the combustion chamber</td>
</tr>
<tr>
<td>641</td>
<td>$x_b$</td>
<td>burned gas mass fraction</td>
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<tr>
<td>642</td>
<td>$y$</td>
<td>dimensionless quantity in the MLK algorithm</td>
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<td>643</td>
<td>$\gamma$</td>
<td>ratio between the principal specific heats</td>
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<td>644</td>
<td>$\delta(t)$</td>
<td>Dirac delta function</td>
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<tr>
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<td>$\delta Q$</td>
<td>heat per unit mass</td>
</tr>
<tr>
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<td>amplitude of the window function in the time domain</td>
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<td>$\Delta \nu$</td>
<td>amplitude of the window function in the frequency domain</td>
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<td>$\Delta \omega$</td>
<td>amplitude of the window function in the circular frequency domain</td>
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<td>650</td>
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<tr>
<td>651</td>
<td>$\Delta T$</td>
<td>temperature difference</td>
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<td>652</td>
<td>$\Delta T_h$</td>
<td>time support of the window function</td>
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<tr>
<td>653</td>
<td>$\Delta t_{ov}$</td>
<td>overlap time interval between consecutive window function positions</td>
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<td>$\Delta \tau$</td>
<td>time resolution of the short time Fourier transform</td>
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<td>crankshaft angle increment</td>
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<td>$\Theta$</td>
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<td>660</td>
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<td>$\omega_0$</td>
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<tr>
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<td>$\omega_s$</td>
<td>sampling circular frequency</td>
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**Subscripts**

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<td>$ahrr$</td>
<td>apparent heat release rate</td>
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<td>666</td>
<td>$comb$</td>
<td>combustion</td>
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<td>667</td>
<td>$EOC$</td>
<td>at end of combustion</td>
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<td>668</td>
<td>$f$</td>
<td>relative to function $f$</td>
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6. REFERENCES.


