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Wiener-Hopf Solution for an Unaligned PEC Wedge over a Dielectric Substrate

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Abstract **– This paper deals with the problem of evaluating the electromagnetic field of a perfect electrical conducting (PEC) wedge over dielectric substrate. In this paper the directions of the two faces of the wedge are arbitrary. We formulate the problem in terms of generalized Wiener-Hopf equations (GWHE) and we propose a possible method of solution based on the reduction of the GWHE to Fredholm integral equations of second kind**

1 INTRODUCTION

In this paper we consider the problem constituted by the evaluation of the electromagnetic field in the physical structure shown in Fig.1.

The faces of the perfect electrical conducting (PEC) wedge are defined by $\varphi=\Phi_a$ (face a) and $\varphi=-\pi-\Phi_b$ (face b). The half-space y>0 is free space and constituted by two angular regions: region 1 $0\leq\varphi\leq\Phi_a$ and region 3 $-\pi-\Phi_b\leq\varphi\leq-\pi$. The half-space y<0 (region 2) is constituted by a homogenous dielectric infinite layer with permittivity ε_r at a distance d from the edge of the wedge.

A plane wave with direction $\varphi=\varphi_o$ ($0\leq\varphi_o\leq\Phi_a$) is incident on a PEC wedge located in region 1. For the sake of simplicity the direction of the incident plane wave is taken at normal incidence on the wedge; the skew incidence case is a possible extension.

Preliminary studies on this topic have been carried out in [1] where the structure was simplified and only GTD coefficients were explored: the PEC wedge was with face b parallel to the dielectric layer and the angular region 1 was obtuse. The unaligned PEC wedge problem is formulated in [2].

The literature shows apparently few works on this problem. However the problem considered in this paper is close to several topics of great interest that have been studied by many authors: for instance the diffraction by a buried body.

Particular cases of a wedge immersed in a stratified medium were studied in [3]-[4] by using the Uniform Theory of Diffraction (UTD). However the application of this method is limited to edges not close to the stratified regions.

Moreover waves in layered media are studied in depth in [5],[18].

A lot of effort has been done in numerical method. In particular [6] investigates integral equations formulations for current induced by a known excitation on a conducting cylinder/strip located near (at least in contact) to the planar interface between two semi-infinite homogeneous half-spaces of different electromagnetic properties.

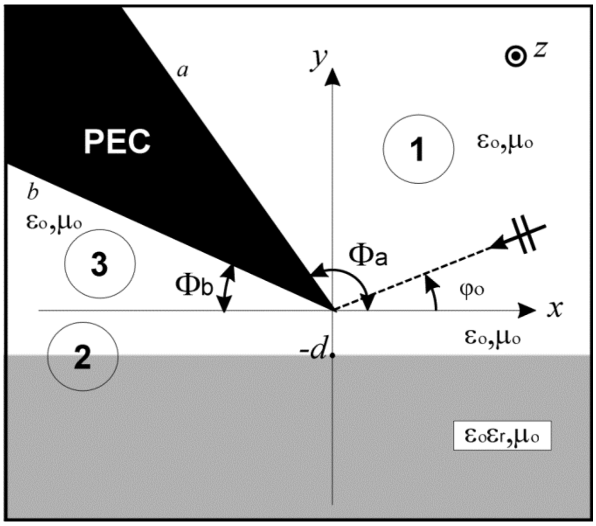

Figure 1: The physical structure of the problem.

Reference [7] presents a compact representation of dyadic Green's functions for plane-stratified media and mixed-potential integral equations for arbitrarily shaped, conducting or penetrable objects embedded in the multi-layered medium. These papers were source of numerous works on scattering by buried perfectly conducting structure

We recall that the use of Finite methods should be combined by suitable singular basis functions capable to model the singularity of the physical quantities [8].

The proposed formulation of the problems is based on the use of generalized Wiener-Hopf equations (GWHE) whose introduction an development starts on [9] and subsequence works for example [10]-[13]

Following the method proposed in [1]

1. first we formulate the entire problem with coupled generalized Wiener-Hopf equations *(*GWHE),
2. second we reduce them using Fredholm factorization [14],
3. in order to numerically obtain [15] estimates of the spectra,
4. to asymptotically evaluate the electromagnetic field.

This procedure has been effectively used in previous works in particular in [16],[17] where the diffraction by impenetrable and penetrable wedges have been studied.

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In this work, the notation and the terminology of [1] are applied

2 FORMULATION AND SOLUTION

With reference to Fig.1, we consider time harmonic electromagnetic fields with a time dependence specified by the factor $e^{j\omega t}$ which is omitted. Cartesian coordinates *(x,y,z)* as well as polar coordinates (ρ,φ,z) are used. The incident field is constituted by the E-polarized plane wave with longitudinal component

$$E_z^i = E_o e^{jk\rho\cos(\varphi-\varphi_o)} \quad (1)$$

where $k=\omega\sqrt{\mu_o\varepsilon_o}$ is the free space propagation constant, φ_o the azimuthal angle of incidence.
The following Laplace transforms (2) and (3) in the η-plane assume a fundamental role.

$$\begin{cases} V_+(\eta,\varphi)=\int_0^\infty E_z(\rho,\varphi)e^{j\eta\rho}d\rho \\ I_+(\eta,\varphi)=\int_0^\infty H_\rho(\rho,\varphi)e^{j\eta\rho}d\rho \end{cases}, \quad (y>0, 0\le\varphi\le\Phi_a) \quad (2)$$

$$\begin{cases} v_\eta(y)=\int_{-\infty}^\infty E_z(x,y)e^{j\eta x}dx \\ i_\eta(y)=\int_{-\infty}^\infty H_x(x,y)e^{j\eta x}dx \end{cases}, \quad (y<0) \quad (3)$$

In particular we define the spectral unknowns: $V_+(\eta)=V_+(\eta,0)$, $I_+(\eta)=I_+(\eta,0)$, $V_{\pi+}(\eta)=V_+(\eta,\pm\pi)$, $I_{\pi+}(\eta)=I_+(\eta,\pm\pi)$, $V_-(\eta)=V_{\pi+}(-\eta)$, $I_-(\eta)=-I_{\pi+}(-\eta)$. From here on, all these quantities will be called the spectra.
The axial spectra are the spectral unknowns (2) and (3) evaluated along $\varphi=0$ and $\varphi=-\pi$ directions.
The system of GWHE is constituted by three equations and are given in terms of the axial spectra and the spectra on the face a and b.
The first equation of this system relates the spectral quantities at section $y=0$

$$Y(\eta)v_\eta(0)=-i_\eta(0) \quad (4)$$

where $v_\eta(0)=V_{\pi+}(-\eta)+V_+(\eta)$, $Y(\eta)$, is obtained from circuital consideration of wave propagation in layered media [18]

$$Y(\eta)=\frac{Y_d(\eta)\cos(\xi(\eta)d)+jY_c(\eta)\sin(\xi(\eta)d)}{Y_c(\eta)\cos(\xi(\eta)d)+jY_d(\eta)\sin(\xi(\eta)d)}Y_c(\eta) \quad (5)$$

where $Y_c(\eta)=\xi(\eta)/kZ_o$, $\xi(\eta)=\sqrt{k^2-\eta^2}$, ($\xi(0)=k$), $Y_d(\eta)=\sqrt{\varepsilon_r k^2-\eta^2}/(kZ_o)$, k is the free space propagation constant, Z_o the free space impedance.
Using the definition of the axial spectra, (4) is rewritten as

$$Y(\eta)(V_{\pi+}(-\eta)+V_+(\eta))-I_{\pi+}(-\eta)+I_+(\eta)=0 \quad (6)$$

For the angular region 1 with perfect conducting boundary on face a (the voltage spectrum is vanishing on the face a) we obtain the following equation [10]:

$$Y_c(\eta)V_+(\eta)-I_+(\eta)=-I_+(-m_a,\Phi_a)=-I_{a+}(-m_a) \quad (7)$$

where $m_a=-\eta\cos\Phi_a+\xi\sin\Phi_a$.
A similar equation is obtained for region 3

$$Y_c(\eta)V_{\pi+}(\eta)+I_{\pi+}(\eta)=-I_+(-m_b,\pi-\Phi_b)=I_{b+}(-m_b) \quad (8)$$

where $m_b=-\eta\cos\Phi_b+\xi\sin\Phi_b$.
Since no general closed form solution of the system of GWHEs (6), (7) and (8) is available, we reduce the problem to a system of Fredholm integral equations (FIEs). The procedure to reduce the GWHE to Fredholm equations is discussed in several papers, for instance [14].
This procedure consists in the use of Cauchy integration and in the extraction of offending singularities derived from geometrical optics fields.
Moreover, while reducing the problem to FIEs, we enforce that the kernels are not singular and possibly compact to get better convergence.
The FIEs can be analytical manipulated to get a system of coupled FIEs in the unknowns $V_+(\eta)$ and $V_{\pi+}(\eta)$ that can be solved by numerical quadrature [15], obtaining approximate axial spectra which is valid only in particular subdomains.
Alternatively to the Fredholm formulation, the GWHE can be reduced to difference equations in the w-plane by introducing the mapping $\eta=-k\cos w$, useful to obtain analytical continuation for the approximate axial spectra.
At the present time the FIE formulation in the η plane gives limited precision for angular region 1 with acute aperture angles ($\Phi_a<\pi/2$). This phenomenon was not observed for acute angular regions in GWHE formulation in $\overline{w}$ plane ($\overline{w}=w\pi/\Phi_a$), see [19].
Using the equations reported for example in [1], [17]

$$\hat{V}_{\varphi d}(w)=\frac{Z_o(\hat{I}_+(w-\varphi)-\hat{I}_+(w+\varphi))+\hat{V}_d(w-\varphi)+\hat{V}_d(w+\varphi)}{2} \quad (9)$$

$$I_{\varphi+}(w)=\frac{Z_o(\hat{I}_+(w-\varphi)+\hat{I}_+(w+\varphi))+\hat{V}_d(w-\varphi)-\hat{V}_d(w+\varphi)}{2} \quad (10)$$

where $\hat{V}_{\varphi d}(w)=\sin(w)V_+(-k\cos(w),\varphi)$ and $\hat{I}_{\varphi+}(w)=I_+(-k\cos(w),\varphi)$, we obtain the spectra for any direction on the angular regions.

By using the inverse Laplace transform (11)

$$E_z(\rho,\varphi)=\frac{k}{2\pi}\int_{\lambda(B_r)}\hat{V}_{\varphi d}(w,\varphi)e^{jk\rho\cos w}dw \quad (11)$$

where $\lambda(B_r)$ is the mapping of the Bromwich B_r contour of the η-plane into the w-plane and by using asymptotic techniques (SDP) we obtain estimates of far field in terms of geometrical optics field (GO) and diffracted field (GTD). Using the Uniform Theory of Diffraction [20] the far field is estimated by removing caustics of GTD.

Following circuital consideration [18] in region 2 the spectra at $y=-d$ is given by

$$v_\eta(-d)=\frac{Y_c(\eta)}{Y_c(\eta)\cos(\xi d)+jY_d(\eta)\sin(\xi d)}v_\eta(0) \quad (12)$$

The application of the inverse Fourier transform (13) and asymptotic techniques (SDP) give estimates of far field in the dielectric layer.

$$E_z(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}v_\eta(y)e^{-j\eta x}d\eta \quad (13)$$

The complete procedure and numerical validation will be presented at the conference and proposed in [21].

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