

Recent advances in the method of moments to model surface junctions

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# Recent Advances in the Method of Moments to Model Surface Junctions

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**Abstract**— This paper summarizes and disseminates our efforts in developing new techniques to handle singularities in the Method of Moment. In this context singularities are due to mathematical formulation of the problem in terms of integral equations and to the physical properties of the electromagnetic quantities at geometrical/material discontinuities. One of these cases is the surface junction among plates.

## I. INTRODUCTION

The method of moments (MoM) is one of the most widely used numerical techniques for solving scattering and radiation problems. The classical scheme with the traditional Rao-Wilton-Glisson (RWG) basis functions [1] has been partially extended to handle complex structures with metallic plates joined together [2]-[6] that generate surface junctions defined at the connection points among different plates. Frequently the connection among plates involves sharp edge structures whose modeling is not considered in the current literature. Indeed real-life complex targets present surface junctions typically constituted by sharp edge plates as for examples wings, fins, winglet of air vehicles or electronic manufactures (for instance heat-sinks).

This problem involves: 1) the sharpness of the geometry that may excite electromagnetic singularities, 2) the singularity of the Green's function in the mathematical formulation of the problem through integral equations, 3) the connection among plates that creates a new constraint among the physical unknowns.

## II. MODELLING SHARP GEOMETRIES

Recently, the authors of this paper have extended the method of moments (MoM) [7] to handle structures with sharp edges [8]-[13] where the charge and the current densities show singular behaviors [14]-[15].

In [11] additive singular divergence-conforming vector bases are capable to model the electromagnetic singularities. We recall that the additive scheme augments the degree of freedoms for the element connected to sharp edges by introducing edge singular elements and vertex singular

elements with special singular vector functions defined in parent domain. For a complete treatise, please refer to [11].

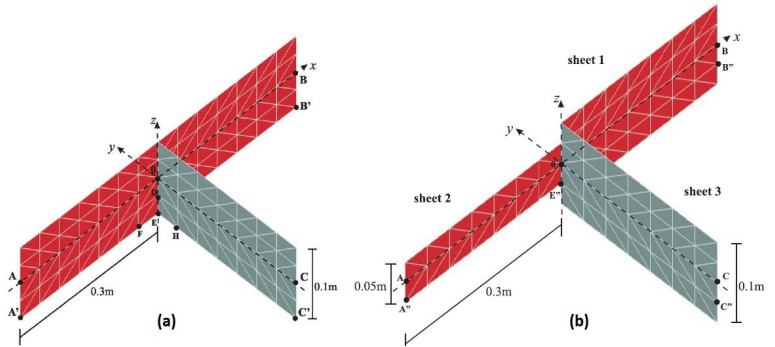


Fig. 1. T-shape surface formed by joining three zero-thickness rectangular plates: (a) three plates with same size, (b) one of the three plates described in (a) is resized (sheet 2).

## III. THE SINGULARITY OF THE GREEN'S FUNCTION

Potential integrals with unbounded singular kernels arise in the moment-method solution of the integral equations of electromagnetism whenever the source and the testing subdomain coincide, that is, in the so-called self-term case. The integrals become nearly singular if the source and testing subdomains are very close to each other, but do not overlap.

Recently, we have developed a new singularity cancellation technique [16] that allows to obtain machine precision results and anticipate the precision of the numerical results in terms of sample points. This technique is based on variable transformations whose Jacobian cancels out the singularity of the integral kernel. The superiority of the cancellation method with respect to the subtraction method is established for example in [16]-[17].

## IV. MODELLING SURFACE JUNCTIONS

The junction problem among plates is two-fold and it can be classified in: (a) sharp edge plates joined to sharp edge plates (plate-to-plate junctions), (b) sharp edge plates joined to or merged in a smooth surface (plate-to-surface junctions). Fig. 1 illustrates these two cases.

As stated in our recent paper [18] both the junctions of Fig. 1 contain corner-points (for instance in Fig. 1(a) point E' and in Fig. 1(b) point E''). One of the major numerical difficulties is in modeling the current density near this kind of point where one cannot resort to any analytical method to predict the current density behavior.

We recall that, on infinitely thin plates, the current density tangent to the edge-profiles could be singular at the edges and it is expected to be probably singular also at the corner-point.

Moreover, to enforce the correct mathematical/physical model we need to guarantee the current continuity along the line of junction (in Fig. 1 the line of junction is the  $z$  axis).

In the neighborhood of the corner-point we need two different approach to model the current density: one for the case of plate-to-plate junctions (Fig. 1(a)) and one for the plate-to-surface junctions (Fig. 1(b)). In particular in the second case the infinite current density running along the edge has to flow out on a surface at the corner-point (in Fig.1(b) the surface is constituted by sheets 1 and 3).

In [18] we propose a complete procedure to handle junctions in presence of sharp edges in surface integral equation method by defining the required basis functions and unknowns. In particular we show how to incorporate higher order interpolatory vector functions and singular vector functions in MoM codes when surface junctions are present exploiting the additive property of the bases. This procedure is based on the application of Kirchhoff's current law in terms of degrees of freedoms (coefficients of basis functions).

In particular, for the region near a corner point, in case of plate-to-surface junctions, we have defined new singular elements that model properly the transition region. In this scheme the edge-aligned strong half-vertex singular elements [18] are used in the element connected to the corner point inside the surface.

These singular elements require high precision integration tools for double singular integrals (singularity of the Green's function and singularity of the basis functions) that are derived through the methods proposed in [16], [19].

Several numerical tests will be presented at the conference to validate our procedure.

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