Abstract—The evaluation of the cycle life of batteries is an essential task in the assessment of the reliability and cost of battery-operated devices. Several compact cycle life models have been proposed in the literature, that exhibit a general trade-off between generality and accuracy. Some models are based on a compact equation derived from experimental data and try to extract a general relationship between cycle life and the relevant parameters (mostly the depth of discharge), but suffer from poor accuracy. At the other extreme, more accurate models, based on incorporating the aging effect into an equivalent circuit, tend to be focused on a specific device and are seldom applicable to another battery.

In this work we propose an equation-based model that tries to overcome the accuracy limits of previous similar models. The model parameters are obtained by fitting the curve based on information reported in datasheets, and can be adapted (with different accuracy levels) to the amount of available information.

We applied the model to various commercial batteries for which full information on their cycle life is available. Results show an average estimation error, in terms of the number of cycles, generally smaller than 10%, which is consistent with the typical tolerance provided in various datasheets (e.g., [16]).

Index Terms—Battery cycle life, Modeling.

I. INTRODUCTION

Secondary (i.e., rechargeable) batteries have become an essential component in many applications like mobile telecommunications, aerospace, renewable energy applications, and electric vehicles. The possibility of an early verification of these systems, including the exchange of energy between energy storage devices and the other components, requires accurate and efficient battery models.

Several models, exhibiting tradeoffs between generality and accuracy, have been proposed in the literature. In electronic design, the most common approach in battery modeling consists of the definition of a generic model template in terms of an equivalent electric circuit (e.g., [1], [2]), which is then populated either using data obtained from direct measurements on actual devices or by extrapolation of battery characteristics available from datasheets (e.g., [3]). These kinds of models are typically generated for a specific battery chemistry and show a high degree of accuracy. This accuracy may significantly degrade if these models are applied to different battery chemistries.

In other contexts (e.g., automotive, aerospace, smart grids), conversely, designers often rely on simpler mathematical macromodels, such as Peukert’s law [4], as a quick estimator for the sizing of the battery sub-system or for preliminary what-if analysis. Such macromodels try to infer a general relationship between the battery runtime and the most relevant parameters, like the Depth of Discharge (DOD) of a battery. This relationship can then be applied to different kinds of batteries with different chemistries, but the estimation results have, in general, a very low degree of accuracy. This is mainly due to the fact that these models tend to primarily focus on a single charge/discharge cycle of a battery, and are not meant to provide information about the “lifetime” of a battery, i.e., its decrease in performance due to long-term inter-cycle effects, such as the fading of the total capacity due to aging or to repeated cycling.

The literature provides several studies on these effects, proposing mathematical models that are based on the electrochemical properties or the physics of the batteries and are therefore strongly bound to specific battery materials and chemistry (e.g., [5]–[9]). Although some other aging models, such as those proposed in [10]–[15], are empirically characterized onto a pre-defined equation template, they are still derived by measurements and therefore are not general enough to support different battery chemistries.

In this work, the aim is to derive an aging model that shares the basic features of a Peukert-like equation, that is, (i) analytical, but that can be empirically populated, and (ii) general enough to support different battery chemistries. Specifically, we propose a mathematical model for estimating the number of cycles with respect to the related capacity fade of batteries.

The accuracy of the approach proposed is demonstrated by applying this model to various commercial batteries of different chemistries, for which the manufacturers provide information on the long-term effects in their datasheets. Results show an average estimation error, in the number of cycles, generally within 10%, which is consistent with the typical tolerance provided in various datasheets (e.g., [16]).

The paper is organized as follows. Section II reports related works on battery modeling, while section III describes the proposed mathematical model for estimating the number of cycles of batteries. Section IV reports the experimental results, while Section V draws some conclusions.
II. BACKGROUND AND MOTIVATIONS

A. Battery Aging Issues

A battery, during its lifetime, deteriorates due to irreversible physical and chemical changes that mostly occur during its usage. The main visible effect of such deterioration is that the battery capacity appears to decrease with multiple charge/discharge cycles. This capacity fading occurs not only during cycling, but also under storage (i.e., long-term battery storage).

Many researchers have studied aging issues in batteries and have devised various types of models [5]–[7]. The key feature of these models is that they mostly relate capacity fading to battery “usage”. The most significant factors that determine battery aging are the following:

- **Temperature.** As with other typical reliability mechanisms, aging increases at higher temperatures (typically according to an Arrhenius-type equation).
- **Depth-of-Discharge (DOD).** The DOD is the % of battery capacity that has been discharged before starting a new charge phase. A DOD of 100% implies that a battery is fully discharged before re-charging it. Aging is increased by deeper discharge cycles (i.e., higher DOD values)
- **Discharge current.** It is usually measured in C-rate, a current normalized to the one necessary to discharge the nominal battery capacity in one hour. Aging is increased by higher discharge currents.
- **Number of cycles (N).** Aging obviously worsens as the number of charge/discharge cycles increases.

B. Battery Aging Models

Concerning cycle life estimation, numerous researchers have proposed analytical models capturing the main aging mechanisms and capacity fading based on the electrochemical properties of the batteries and even including full-physics based models (e.g., [8] for lithium-ion batteries).

In [9] the authors proposed a model to calculate the usable number of cycles $N$ of a battery based on the following equation:

\[ N = N_1 \cdot e^{\alpha \cdot (1 - DOD')} \]  

where $DOD'$ is the normalized depth of discharge ($0 \leq DOD' \leq 1$), $\alpha$ is a characteristic constant of the battery and $N_1$ is the number of cycles at $DOD' = 1$. This model is empirically characterized for lead-acid, nickel-cadmium (Ni-Cd) and nickel-metal hydride (Ni-MH) batteries, whose cycle-life vs. DOD curve has an exponential shape. It is not, however, suitable for many lithium-based cells, whose cycle-life vs. DOD curve sometimes exhibits a more linear behavior (e.g., for $LiFePO_4$ cells).

A slightly different relationship between cycle-life and DOD was introduced in [10]:

\[ N = N_{0.8} \cdot DOD' \cdot e^{\alpha \cdot (1 - DOD')} \]  

where $N_{0.8}$ is the cycle life at $DOD = 80\%$, while $\alpha$ is a constant whose value is, respectively, 3 and 2.25 for lead-acid and nickel metal hydride (Ni-MH) tested battery packs (nowadays, Ni-MH batteries have mostly been substituted by lithium-ion batteries as they have higher energy density).

Thaller [11] has defined another relationship for battery cycle life after considering excess capacity $F$, with respect to the rated capacity, and a penalty factor due to the DOD, by including the $P$ parameter, as reported in (3), which gives this mathematical prediction model for a general battery:

\[ N = \frac{1 + F - DOD'}{A \cdot (1 + P \cdot DOD') \cdot DOD'} \]  

In our work $F$ is always considered equal to 0, so that each analysis is performed after starting from the rated capacity of any commercial cell or cell string. The product $A \cdot DOD'$ represents the irreversible capacity loss in each cycle. Values of the parameter $A$ were originally declared to be in the range $0.000 \div 0.002$ [11].

These previous models estimate the cycle life of a battery, always after considering a fixed irreversible capacity fading (e.g., $20\%$, that is, when the total maximum available capacity reaches $80\%$ of the nominal one).

In [12] the authors introduce a complex cycle life model consisting of different equations, one for each stress factor considered, i.e., $C$-rate, $T$ and $DOD$. Despite its high accuracy, the model derivation requires extensive empirical measurements and the model itself lacks the compactness and the generality of a Peukert-like equation.

Another analytical method for battery life prediction is based on the *Amp-hour–throughput*, i.e., the total energy supplied by the battery during its life [13], also called “charge life”. The charge life $\Gamma_R$ in Amp-hours (Ah) is defined as:

\[ \Gamma_R = L_R \cdot DOD' \cdot C_R \]  

where $C_R$ is the rated capacity in Ah at a rated discharge current $I_R$, and $L_R$ is the maximum number of cycles referring to a given normalized depth of discharge $DOD'$ and a discharge current $I_R$. In the model presented in [14], the authors proposed calculating an equivalent $Ah$ weighted-throughput parameter. They claimed the C-rate effect on aging is negligible in Li-ion cells, since hybrid electric vehicles exhibit relatively large C-rates ($\pm 4 C$). This assumption cannot however be generalized for other applications.

The model proposed in [15] adopted this approach to estimate the cycling capacity fade through a modified definition of the Arrhenius equation, characterized by a square root time dependence.

C. Motivations of the Work

In spite of the various differences, all the above models are built by extracting parameter values through measurements on the batteries under test. Although the generated models are typically very accurate, this approach is quite time-consuming (especially when multiple cycles are involved) and requires expensive laboratory instrumentation.
For this reason, methods that rely only on available data (e.g., datasheets) to derive the capacity fade in batteries using analytical models (e.g., [17]) have been reported in the literature in recent years. Clearly, the accuracy of these models depends on the amount of available information reported into battery datasheets.

Before starting to describe the analytical model for battery capacity fade that we propose in this paper, an important consideration must be drawn: if the battery datasheet provides a cycle vs. DOD curve, as the one depicted in Figure 1, one can evaluate the battery cycle life without resorting to any estimation model.

Unfortunately, this kind of information is seldomly present in datasheets. Hence, in these cases, the battery cycle life can be extracted either by means of empirical characterizations, as explained in the previous Section, or by an analytical model able to exploit available information to derive the battery capacity fade.

Many manufacturers provide information about capacity fade in the form of a Capacity vs. Number of cycles curve (e.g., [16]) as also depicted in Figure 2. From these plots, it is no simple matter to perform the battery cycle life evaluation, since the data about the number of cycles are available for a given number of DODs only and, furthermore, sometimes they might even show an uncertainty that may range from 8 to 10%, or even higher.

In this work we propose an analytical model for capacity fade that, using only datasheet information, allows the cycle life of a battery of any chemistry or type to be estimated, while having a simple mathematical form similar to Peukert’s law.

III. MODELING METHODOLOGY

A. Model Definition

The model proposed in this work somehow mimicks the shape of Peukert’s law, as expressed by (5), which models the non-linear dependency between capacity and the discharge current:

\[ t = \frac{C}{I^k} \]  

where \( C \) is the capacity of the battery, \( I \) is the discharge current, and \( t \) is the time for totally discharging the battery; \( k \) is the Peukert’s coefficient; typical values of \( k \) depend on the battery chemistry and the manufacturing process and they typically range from 1.1 to 1.3.

As a matter of fact, the curves describing the capacity vs. number of cycles exhibit a similar non-linear relationship.

Our objective is therefore to derive a model expressing battery cycle life in a compact mathematical form similar to Peukert’s law, and describing the general non-linear relationship between the capacity fade and the DOD.

In the case of capacity fade, the non-linearity concerns both the number of cycles \( N \) as well as the DOD, and the actual relationship among these quantities depends also on the value of the target capacity degradation (i.e., the behavior for a 20% capacity fade will be different from that for a 30% capacity fade). In order to model this non-linearity we need to define a new parameter that characterizes the battery performance during the cycling.

The proposed mathematical model is shown in (6); it allows to estimate the number of charging-discharging cycles \( N \) for a given battery based on four main parameters.

\[ N = L \cdot \frac{C_{\text{fade}}}{DOD^h} \]  

- \( L \) (called the linear factor) is the parameter that characterizes the battery performance over cycling,
- \( C_{\text{fade}} \) is the percentage of capacity loss,
- \( DOD \) is the depth of discharge expressed as a percentage (0, . . . , 100);
- \( h \) is the coefficient that models the nonlinear relationship between \( N \) and \( DOD \) for a certain \( C_{\text{fade}} \).

The similarity with Peukert’s law is evident. \( N \), considered as an inter-cycle “lifetime” parameter, is obtained as the ratio of capacity and a weighted metric of discharge current (\( DOD^h \)). There are however two relevant differences: (1) factor \( L \) is used to scale the “lifetime” across multiple cycles, and (2) \( h \) is not constant, but depends on \( C_{\text{fade}} \). This makes our approach more general with respect to previous models and allows one to adapt it to the available manufacturer’s data.

Equation 6 defines a generic model template, which is empirically populated based on the extraction of data from typical capacity fade vs. \# of cycles plots, as described in the next section.

B. Extraction of Model Parameters

Besides the “physical” quantities (\( C_{\text{fade}} \) and \( DOD \)) the model includes two other scale parameters, i.e., the linear factor \( L \) and the binding coefficient \( h \), which have to be determined by fitting empirical data derived from available information (e.g., datasheet).

The actual parameter identification depends on the amount of available information. As discussed in Section II, our model is meaningful if the battery under analysis only provides information in the form of two or more curves in the (capacity, number of cycles) plane, each corresponding to a different DOD.

Let us assume that there are \( M \) such curves available in a datasheet or in a measured set of data. Obviously the larger
together with the relative errors (array \( \text{Err} \)) determine the model (Lines 27-28), which is used as an index in corresponding to the smallest error is selected as single we have no insight of possible values of largest value of of the datasheet. As a rule of thumb, it is usually near to the is evaluated. The value of computed using the model equation (6) (Line 16), and the erro r is evaluated by Algorithm 1.

At the end of the iteration over \( L \), the value of \( L \) corresponding to the smallest error is selected as single \( L_{\text{opt}} \) for the model (Lines 27-28), which is used as an index in \( h \) to determine \( h_{\text{opt}} \) for each \( C_{\text{fade}} \).

Since we need to determine two parameters from the curve(s) \( (h \) and \( L \)), and given the limited number of samples points to be considered, it is feasible to derive them from an exhaustive exploration for all \( C_{\text{fade}} \) and \( \text{DOD} \) points, as the values of \( h \) and \( L \) that minimize the maximum error with respect to the curves. However, an exploration requires a feasible range for these two parameters, which is not easy to determine because they are only weakly linked to “physical” quantities. Of the two, \( L \) is the one with some physical interpretation since it can be regarded as a correction factor of the number of cycles \( N \). Therefore, we can assume that \( L \) ranges between 1 and a value \( L_{\text{max}} \), determined by inspection of the datasheet. As a rule of thumb, it is usually near to the largest value of \( N \) reported in the datasheet curves. Conversely, we have no insight of possible values of \( h \). For this reason, we implement the search as a two-phase process, as described by Algorithm 1.

The search is organized into two main iterations over \( L \). In the first one (Lines 1-7), for all values of \( C_{\text{fade}} \) (assumed to be discretized into \( P \) values) and of the \( M \) \( \text{DOD} \) values it computes the resulting value of \( h \) using (7), which is simply a re-arrangement of (6) expressing \( h \) instead of \( N \), and determines thus a feasible range \( \mathcal{H} = [h_{\text{min}}, h_{\text{max}}] \) for \( h \).

\[
h = \frac{\log(L \cdot C_{\text{fade}})}{\log(\text{DOD})} \tag{7}
\]

Now that we have a feasible range for \( h \), in the second iteration (Lines 10-26), we determine the optimal values of \( h \) and \( L \), as follows. In the outer loop over \( L \) (Line 10), the optimal value of \( h \) is calculated first; for each value of \( h \) (using some discretization step), \( C_{\text{fade}} \) and \( \text{DOD} \) \( N \) is computed using the model equation (6) (Line 16), and the error between this value and the one extracted from the datasheet is evaluated. The value of \( h \) that yields the least average error is stored as the best for a given value of \( L \) into an array \( h \), together with the relative errors (array \( \text{Err} \), Lines 22-23).

At the end of the iteration over \( L \), the value of \( L \) corresponding to the smallest error is selected as single \( L_{\text{opt}} \) for the model (Lines 27-28), which is used as an index in \( h \) to determine \( h_{\text{opt}} \) for each \( C_{\text{fade}} \).

**Algorithm 1** Search for the best value of \( L \)

1. for all \( L \in [1, L_{\text{max}}] \) do
2. \hspace{1em} for all \( C_{\text{fade}} = 1 \ldots P \) do
3. \hspace{2.5em} for all \( \text{DOD} = 1 \ldots M \) do
4. \hspace{4em} Compute \( h \) by (7)
5. \hspace{2.5em} end for
6. \hspace{1em} end for
7. \hspace{1em} end for
8. \hspace{1em} \( \mathcal{H} \leftarrow [h_{\text{min}}, h_{\text{max}}] \)
9. \hspace{1em} \( \text{MinMaxErr} \leftarrow \infty \)
10. \hspace{1em} for all \( L = 1 \ldots L_{\text{max}} \) do
11. \hspace{2.5em} \( \text{MaxErr} \leftarrow 0 \)
12. \hspace{2.5em} for all \( h \in \mathcal{H} \) do
13. \hspace{4em} \( \text{TotErr} \leftarrow 0 \), \( \text{MinAvgErr} \leftarrow \infty \).
14. \hspace{4em} for all \( C_{\text{fade}} = 1 \ldots P \) do
15. \hspace{6em} for all \( \text{DOD} = 1 \ldots M \) do
16. \hspace{8em} Calculate \( N \) using (6) and compute the absolute error \( E \)
17. \hspace{8em} \( \text{TotErr} \leftarrow \text{TotErr} + E \)
18. \hspace{6em} end for
19. \hspace{4em} end for
20. \hspace{4em} \( \text{AvgErr} \leftarrow \text{TotErr} / (P \cdot M) \)
21. \hspace{4em} if \( \text{AvgErr} < \text{MinAvgErr} \) then
22. \hspace{6em} \( \mathcal{H}[L] \leftarrow h \)
23. \hspace{6em} \( \text{Err}[L] \leftarrow \text{AvgErr} \)
24. \hspace{6em} end if
25. \hspace{4em} end for
26. \hspace{4em} end for
27. \hspace{4em} \( L_{\text{opt}} \leftarrow \text{argmin}(\text{Err}) \)
28. \hspace{4em} \( h_{\text{opt}} \leftarrow \mathcal{H}[L_{\text{opt}}] \)

IV. MODEL VALIDATION

The validation of the proposed model is performed after considering batteries of various chemistries produced by different manufacturers. Although the type of aging data differs from one datasheet to another, we have collected the available information and translated it into the tabular format described in Section III; using these data, we ran the search algorithm to populate the model for each battery under analysis.

A. VRLA Batteries

We start our evaluation from Valve Regulated Lead Acid (VRLA) batteries, which have a more evident nonlinear aging behavior with respect to other chemistries. Moreover, datasheets for most VRLA batteries include more detailed information on aging, typically in the form of the plot of Capacity vs. Number of cycles (e.g., Figure 2).

Table I reports the extracted manufacturer data and the resulting model parameters for the AGM-VRLA XTV1272 battery by CSB. The first three columns represent the data extracted from the datasheet [16] for three different \( C_{\text{fade}} \) points, namely 10, 20, and 40%. The last four columns report the parameters obtained by the search algorithm, the resulting number of cycles \( N_{m} \) from the model, and the estimation error. After comparing \( N_{m} \) against the cycle life extracted from the
Although the error is not negligible, it is worth emphasizing that the datasheet for this battery reports a possible range of the number of cycles rather than a single curve, to indicate the intrinsic uncertainty of the estimation. The spread of the values actually increases for increasing DODs. From the datasheet, we found that the possible variation of the cycle life (measured as the difference between the minimum or maximum value with respect to the average) might even be up to 10, 11, and 16% for \( C_{fad persistence} = 10, 20, \) and 40%, respectively.

Hence, the absolute maximum estimation error obtained by the proposed model (i.e., around 12, 11, and 12%, respectively) is comparable with the maximum tolerance given by the manufacturer.

### B. Other Battery Chemistries

Evaluation of other battery chemistries is complicated by the fact that in general only the manufacturers of VRLA batteries provide plots of Capacity vs. Number of cycles, for different DODs. In particular, datasheets usually report only a single Capacity vs. Number of cycles curve referring to a single DOD value for lithium-ion (Li-ion) batteries. The availability of just one DOD reference, however, would yield a model with little practical use in this case, since the calibration for discharge patterns would be different from that used for characterization.

Therefore, in order to have a more meaningful assessment of the accuracy of the proposed model, we only selected those batteries whose datasheets report the Number of cycles vs. DOD characteristic, even just for a single \( C_{fad persistence}^* \) value. In any case, values of DOD below 10% are not used for the derivation of the model because (i) they are not representative of typical battery usage and (ii) they are not statistically representative. It is worth noticing that the number of cycles should approach infinity as DOD \( \rightarrow 0 \); therefore, as DOD gets smaller it would be correct to consider a range of values rather than a precise value. Of course, all the characteristics given by the manufacturers always refer to certain operating and working conditions (e.g., charge/discharge current and temperature), which are usually different from one brand to another. In this work, we do not consider the differences among these conditions, in order to firstly validate the basic proposed model. The parameters and estimation errors for these batteries are reported in Tables II and III, which also report, for a more comprehensive validation, the results of the application of the existing and most meaningful analytical models [9], [11]. As (1) requires the number of cycles at DOD=100% as input parameter, the evaluation of that previous model was not possible for two batteries because this value is not available in their datasheets, as reported in Table II. On the other hand, as the model proposed by [11] is useless for DOD′ = 1 (in this case, \( N \) in (3) would be equal to zero), the analysis was re-performed by considering the maximum DOD=80% as reported in Table III.

In Table II, the largest absolute estimation error of the model occurs for a LiFeMgPO₄ battery, almost 20%, while the maximum mean value is 11.35% for the Alpha® one. However, the total average error of the maximum errors for the 10 batteries in the table is 10.66%. The mean errors are obviously smaller, in general less than 10%, and in one case 11.35%. In general, the proposed model shows robustness and accuracy for different types of electric storage devices. For the Li-ion battery by Saft Evolion the linear factor \( L \) is very high with respect to any other battery. In fact, the linear factor usually depends on the battery properties of cycling, while the range of the \( h \) parameter strictly depends on the linearity of the cycle life with respect to the DOD. The lowest \( h \) coefficient found in the model validation is 0.225627 for the Discover 22-24-700 battery, whereas the highest \( h \) is 2.000414 for the Saft Evolion.

The chart in Figure 3 reports a comparison of the estimation models after applying each of them to the benchmarks. For a comprehensive report, it also includes the main results obtained for the analysis of the model by [10], whose estimation errors are too great to be reported. Furthermore, for the here proposed model, this chart considers the worst case (i.e., data reported in Table II). Although the previous models have

![Fig. 3. Maximum and mean estimation errors given by the models for all the selected benchmarks.](image-url)
V. Conclusion

A novel mathematical model for estimating the number of cycles of a battery with respect to an expected capacity fade, has been proposed. The related equation describes the cycling behavior of batteries of different chemistries, and it demonstrates the possibility to obtain a very fast and also accurate exploration of battery lifespan. The characterization of the long-term effects for a specific battery only requires two parameters: a linear factor $L$ and the exponential $h$ coefficient. Results show an estimation mean error generally within 10%.

Future works will include temperature and current rates in the model, in order to analyze the estimation error after considering different operating and working conditions with respect to the reference cases.

To the number of cycles vs. DOD characteristic, the maximum DOD is 80% for all the analyses.

### References


