

The Wiener-Hopf method in electromagnetics

Original

The Wiener-Hopf method in electromagnetics / Daniele, Vito; Zich, Rodolfo. - STAMPA. - (2014).

Availability:

This version is available at: 11583/2608557 since:

Publisher:

Scitech Publishing

Published

DOI:

Terms of use:

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

(Article begins on next page)



The Wiener-Hopf Method in Electromagnetics

Vito G. Daniele and Rodolfo S. Zich

MARIO BOELLA SERIES ON ELECTROMAGNETISM IN INFORMATION & COMMUNICATION

The Wiener-Hopf Method in Electromagnetics

Mario Boella Series on Electromagnetism in Information and Communication

Piergiorgio L. E. Uslenghi, PhD – Series Editor

The Mario Boella series offers textbooks and monographs in all areas of radio science, with a special emphasis on the applications of electromagnetism to information and communication technologies. The series is scientifically and financially sponsored by the Istituto Superiore Mario Boella affiliated with the Politecnico di Torino, Italy, and is scientifically co-sponsored by the International Union of Radio Science (URSI). It is named to honor the memory of Professor Mario Boella of the Politecnico di Torino, who was a pioneer in the development of electronics and telecommunications in Italy for half a century, and a vice president of URSI from 1966 to 1969.

Published Titles in the Series

Introduction to Wave Phenomena

by Akira Hirose and Karl Lonngren

Scattering of Waves by Wedges and Cones with Impedance Boundary Conditions

by Mikhail Lyalinov and Ning Yan Zhu

Complex Space Source Theory of Spatially Localized Electromagnetic Waves

by S. R. Seshadri

The Wiener-Hopf Method in Electromagnetics

by Vito Daniele and Rodolfo Zich

Forthcoming Titles

Higher Order Numerical Solution Techniques in Electromagnetics

by Roberto Graglia and Andrew Peterson (2015)

Slotted Waveguide Array Antennas

by Sembiam Rengarajan and Lars Josefsson (2015)

The Wiener-Hopf Method in Electromagnetics

ISMB Series

Rodolfo Zich

Instituto Superiore de Mario Boella

Vito Daniele

Polytechnic of Torino



theiet.org



Published by SciTech Publishing, an imprint of the IET.
www.scitechpub.com
www.theiet.org

Copyright © 2014 by SciTech Publishing, Edison, NJ. All rights reserved.

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 646-8600, or on the web at copyright.com. Requests to the Publisher for permission should be addressed to The Institution of Engineering and Technology, Michael Faraday House, Six Hills Way, Stevenage, Herts, SG1 2AY, United Kingdom.

While the author and publisher believe that the information and guidance given in this work are correct, all parties must rely upon their own skill and judgement when making use of them. Neither the author nor publisher assumes any liability to anyone for any loss or damage caused by any error or omission in the work, whether such an error or omission is the result of negligence or any other cause. Any and all such liability is disclaimed.

10 9 8 7 6 5 4 3 2 1

ISBN 978-1-61353-001-6 (hardback)
ISBN 978-1-61353-031-3 (PDF)

Typeset in India by MPS Limited
Printed in the US by Integrated Books International
Printed in the UK by CPI Group (UK) Ltd, Croydon

Contents

Preface	xiii
Foreword	xvii
PART 1 Mathematical Aspects	1
1 Forms of Wiener-Hopf equations	3
1.1 The basic Wiener-Hopf equation	3
1.1.1 An electromagnetic example: The half-plane problem	5
1.2 Modified W-H equations (MWHE)	7
1.2.1 Longitudinally modified W-H equations	7
1.2.2 Transversely modified W-H equations	9
1.2.3 The incomplete Wiener-Hopf equations	10
1.3 Generalized W-H equations	12
1.3.1 An electromagnetic example: The PEC wedge problem	12
1.3.2 An electromagnetic example: The dielectric wedge problem	13
1.4 The Hilbert-Riemann problem	14
1.5 Reduction of W-H equations to the classical form	14
1.5.1 Reduction of the transversely modified W-H equations to CWHE	14
1.5.2 Reduction of the longitudinally modified W-H equations to CWHE	15
1.5.3 The Hilbert-Riemann equations	16
1.5.4 Generalized Wiener-Hopf equations	16
1.6 From Wiener-Hopf equations to Fredholm integral equations in the spectral domain	17
1.7 Fundamental literature	19

2	The exact solution of Wiener-Hopf equations	21
2.1	Introduction	21
2.2	Additive decomposition	22
2.3	Multiplicative decomposition or factorization	23
2.4	Solution of the W-H equation	24
2.4.1	Solution of the nonhomogeneous equation	24
2.4.2	Remote source	27
2.5	Unbounded plus and minus unknowns	29
2.6	Factorized matrices as solutions of the homogeneous Wiener-Hopf problem	29
2.7	Nonstandard factorizations	31
2.8	Extension of the W-H technique to the GWHE	34
2.9	Important mappings for dealing with W-H equations	35
2.9.1	The $\chi = \sqrt{\tau_o^2 - \alpha^2}$ mapping	35
2.9.2	The $\alpha = -\tau_o \cos w$ mapping	36
3	Functions decomposition and factorization	45
3.1	Decomposition	45
3.1.1	Example 1	47
3.1.2	Decomposition of an even function	51
3.1.3	Numerical decomposition	51
3.1.4	Example 1 revisited	53
3.1.5	The case of meromorphic functions	54
3.1.6	Decomposition using rational approximants of the function	55
3.2	Factorization	57
3.2.1	General formula for the scalar case	57
3.2.2	Example 2	57
3.2.3	Example 3	58
3.2.4	Factorization of meromorphic functions	58
3.2.5	Example 4	60
3.2.6	Factorization of kernels involving continuous and discrete spectrum	63
3.3	Decomposition equations in the w – plane	66
3.3.1	Evaluation of the plus functions	66
3.3.2	Evaluation of the minus functions	69
3.3.4	Use of difference equation for function decomposition	73
3.3.5	The W-H equation as difference equation	73

4	Exact matrix factorization	75
4.1	Introduction	75
4.2	Some possibilities to reduce the order of the kernel matrices	76
4.3	Factorization of triangular matrices	78
4.4	Factorization of rational matrices	80
4.4.1	Introduction	80
4.4.2	Matching of the singularities	81
4.4.3	The factorization in the framework of the Fredholm equations	85
4.5	Techniques for solving the factorization problem	86
4.5.1	The logarithmic decomposition	86
4.6	The factorization problem and the functional analysis	92
4.6.1	The iterative method	92
4.6.2	The Fredholm determinant method	93
4.6.3	Factorization of meromorphic matrix kernels with an infinite number of poles	94
4.7	A class of matrices amenable to explicit factorization: matrices having rational eigenvectors	95
4.8	Factorization of a 2×2 matrix	96
4.8.1	The Hurd method	96
4.8.2	The off-diagonal form	98
4.8.3	Reduction of matrices commuting with polynomial matrices to the Daniele matrices	99
4.8.4	Explicit factorization of Daniele matrices	101
4.8.5	The elimination of the offensive behavior for matrices having the Daniele form	104
4.8.6	A relatively simple case	106
4.8.7	The $\sqrt{a(\alpha)/b(\alpha)}$ rational function of α case	108
4.9	The factorization of matrices commuting with rational matrices	110
4.9.1	Introduction	110
4.9.2	Matrix of order two commuting with polynomial matrices	111
4.9.3	Explicit expression of $\psi_i(\alpha)$ in the general case	113
4.9.4	Asymptotic behavior of the logarithmic representation of $-l(\alpha)P^{-1}(\alpha) + 1$	117
4.9.5	Asymptotic behavior of the decomposed $\psi_{i\pm}(\alpha)$	118
4.9.6	A procedure to eliminate the exponential behavior	120
4.9.7	On the reduction of the order of the system	124
4.9.8	The nonlinear equations as a Jacobi inversion problem	125
4.9.9	Weakly factorization of a matrix commuting with a polynomial matrix	127

5	Approximate solution: The Fredholm factorization	129
5.1	The integral equations in the $\alpha -$ plane	129
5.1.1	Introduction	129
5.1.2	Source pole α_o with positive imaginary part	130
5.1.3	Analytical validation of a particular W-H equation	131
5.1.4	A property of the integral in the Fredholm equation	132
5.1.5	Numerical solution of the Fredholm equations	134
5.1.6	Analytic continuation outside the integration line	141
5.2	The integral equations in the $w -$ plane	143
5.3	Additional considerations on the Fredholm equations	146
5.3.1	Presence of poles of the kernel in the warped region	146
5.3.2	The Fredholm factorization for particular matrices	147
5.3.3	The Fredholm equation relevant to a modified kernel	147
6	Approximate solutions: Some particular techniques	149
6.1	The Jones method for solving modified W-H equations	149
6.1.1	Introduction	149
6.1.2	Longitudinal modified W-H equation	149
6.1.3	Transversal modified W-H equation	152
6.2	The Fredholm factorization for particular matrices	153
6.3	Rational approximation of the kernel	161
6.3.1	Pade approximants	161
6.3.2	An interpolation approximant method	163
6.4	Moment method	167
6.4.1	Introduction	167
6.4.2	Stationary properties of the solutions with the moment method	169
6.4.3	An electromagnetic example: the impedance of a wire antenna in free space	173
6.5	Comments on the approximate methods for solving W-H equations	175
PART 2	Applications	177
7	The half-plane problem	179
7.1	Wiener-Hopf solution of discontinuity problems in plane-stratified regions	179
7.2	Spectral transmission line in homogeneous isotropic regions	180
7.2.1	Circuitual considerations	181

7.2.2	Jump of voltage or current in a section where it is present a discontinuity	182
7.2.3	Jump of voltage or current in a section where a concentrated source is present	182
7.3	Wiener-Hopf equations in the Laplace domain	183
7.4	The PEC half-plane problem	185
7.4.1	E-polarization case	185
7.4.2	Far-field contribution	188
7.5	Skew incidence	191
7.6	Diffraction by an impedance half plane	197
7.6.1	Deduction of W-H equations in diffraction problems by impenetrable half-planes	197
7.6.2	Presence of isotropic impedances Z_a and Z_b	200
7.7	The general problem of factorization	203
7.7.1	The case of symmetric half-plane	205
7.7.2	The case of opposite diagonal impedances $Z_b = -Z_a$	206
7.8	The jump or penetrable half-plane problem	206
7.9	Full-plane junction at skew incidence	207
7.10	Diffraction by an half plane immersed in arbitrary linear medium	208
7.10.1	Transverse equation in an indefinite medium	208
7.10.2	Field equations in the Fourier domain	210
7.10.3	The W-H equation for a PEC or a PMC half-plane immersed in a homogeneous linear arbitrary medium	216
7.11	The half-plane immersed in an arbitrary planar stratified medium	220
8	Planar discontinuities in stratified media	223
8.1	The planar waveguide problem	223
8.1.1	The E-polarization case	223
8.1.2	Source constituted by plane wave	225
8.1.3	Source constituted by an incident mode	227
8.1.4	The skew plane wave case	228
8.2	The reversed half-planes problem	230
8.2.1	The E-polarization case	230
8.2.2	Qualitative characteristics of the solution	231
8.2.3	Numerical evaluation of the electromagnetic field	232
8.2.4	Numerical solution of the W-H equations	233
8.2.5	Source constituted by a skew plane wave	237

8.3	The three half-planes problem	244
8.3.1	The E-polarization case (normal incidence case)	244
8.3.2	The skew incidence case	247
8.4	Arrays of parallel wire antennas in stratified media	248
8.4.1	The single antenna case	248
8.4.2	The W-H equations of an array of wire antennas	250
8.4.3	Spectral theory of transmission lines constituted by bundles of wires	254
8.5	Spectral theory of microstrip and coplanar transmission lines	254
8.5.1	Coplanar line with two strips	254
8.5.2	The shielded microstrip transmission line	260
8.6	General W-H formulation of planar discontinuity problems in arbitrary stratified media	261
8.6.1	Formal solution with the factorization method	263
8.6.2	The method of stationary phase for multiple integrals	267
8.6.3	The circular aperture	268
8.6.4	The quarter plane problem	272
9	Wiener-Hopf analysis of waveguide discontinuities	279
9.1	Marcuvitz-Schwinger formalism	279
9.1.1	Example 1	280
9.1.2	Example 2	283
9.2	Bifurcation in a rectangular waveguide	285
9.3	The junction of two waveguides	287
9.4	A general discontinuity problem in a rectangular waveguide	289
9.5	Radiation from truncated circular waveguides	292
9.6	Discontinuities in circular waveguides	297
10	Further applications of the W-H technique	301
10.1	The step problem	301
10.1.1	Deduction of the transverse modified W-H equations (E-polarization case)	301
10.1.2	Solution of the equations	303
10.2	The strip problem	303
10.2.1	Some longitudinally modified W-H geometries	304
10.3	The hole problem	304
10.4	The wall problem	305
10.5	The semi-infinite duct with a flange	307

10.6	Presence of dielectrics	308
10.7	A problem involving a dielectric slab	310
10.8	Some problems involving dielectric slabs	313
10.8.1	Semi-infinite dielectric guides	314
10.8.2	The junction of two semi-infinite dielectric slab guides	314
10.8.3	Some problems solved in the literature	314
10.9	Some problems involving periodic structures	315
10.9.1	Diffraction by an infinite array of equally spaced half-planes immersed in free space	315
10.9.2	Other problems solved in the literature	317
10.10	Diffraction by infinite strips	318
10.10.1	Solution of the key problem	319
10.10.2	Boundary conditions	321
10.10.3	Solution of the W-H equation	321
10.11	Presence of an inductive iris in rectangular waveguides	323
10.12	Presence of a capacitive iris in rectangular waveguides	324
10.13	Problems involving semi-infinite periodic structures	324
10.14	Problems involving impedance surfaces	325
10.15	Some problems involving cones	326
10.16	Diffraction by a PEC wedge by an incident plane wave at skew incidence	330
10.17	Diffraction by a right PEC wedge immersed in a stratified medium	334
10.18	Diffraction by a right isorefractive wedge	337
10.18.1	Solution of the W-H equations	342
10.18.2	Matrix factorization of $g_e(\alpha)$	345
10.18.3	Near field behavior	347
10.19	Diffraction by an arbitrary dielectric wedge	349
References		351
Index		361

Preface

In 1931, Wiener and Hopf (1931) invented a powerful technique for solving an integral equation of a special type. By introducing the Laplace transform of the unknown, the integral equation was rephrased in terms of a functional equation in a suitably defined complex space. The solution method of the latter is very ingenious indeed. It is based on a sophisticated procedure exploiting some properties of the analytic functions and it stands as one of the most important mathematical inventions for obtaining analytical solutions of very difficult problems.

In electromagnetic geometries, a fundamental approach due to Jones (1952a) applies the Laplace transforms directly to the partial differential equations, and the complex variable functional equations are obtained directly without having to formulate an integral equation. Jones's approach has been adopted systematically by Noble (1988) in his book on the Wiener-Hopf technique. Noble's work presents many applications of the Wiener-Hopf technique in a systematic way and is fundamental for readers interested in this powerful method. Unfortunately, this book was written many years ago (the first edition was in 1958); in the meantime, many scientists have devoted efforts to studying the Wiener-Hopf technique and have achieved important developments.

The main purpose of this book is to provide students and scientists of diffraction phenomena with a comprehensive treatment of the Wiener-Hopf technique, including its latest developments. In particular, these developments illustrate the wide range of possible applications of this method. In practice, it is now possible to solve all canonical diffraction problems involving geometrical discontinuities using the Wiener-Hopf technique, which has definitively established it as the most general and powerful analytical method for this purpose.

A great number of problems can be effectively approached using the W-H technique (Fig. 1). Shown in the figure are geometrical structures that can be considered equivalent to a (uniform or nonuniform) waveguide in which semi-infinite geometrical discontinuities have been introduced. These discontinuities may be also modified in the transversal or longitudinal direction of the waveguide, thus augmenting considerably the number of possible problems that can be effectively studied by this technique. It must be observed that most of these problems are very important and that often there are no alternative approaches available for solving them efficiently, even numerically. Some general remarks about the W-H techniques are necessary before delving into specific problems in detail.

First of all, no W-H problem is simple to study. For instance, for a given electromagnetic problem that perhaps may be formulated in terms of W-H equations, it could be

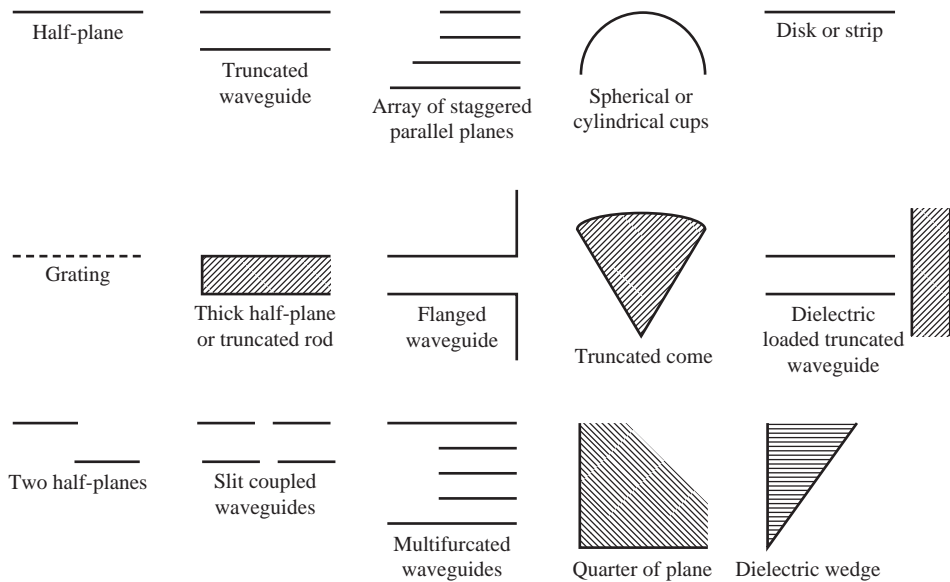


Fig. 1: A few examples of W-H geometries

quite difficult to obtain these equations. In the literature, many problems are formulated in terms of functional equations that, even though equivalent to Wiener-Hopf equations, do not present the so-called standard forms considered in this book. We emphasize that it is important to formulate the problems in terms of standard W-H equations because it provides a uniform methodology to obtain exact or approximate solutions in a systematic way. The key function containing all the information in the standard Wiener-Hopf equations is the W-H kernel. It is generally a matrix \mathbf{G} function of a complex variable α . It follows that the first step of the W-H technique is to find $\mathbf{G}(\alpha)$ for a specific geometry. Sometimes this is a difficult task requiring a profound knowledge both of the formulation of electromagnetic problems and of the underlying physical concepts.

The central problem in solving the standard W-H equations is conceptually very simple: the factorization of the matrix $\mathbf{G}(\alpha)$. This problem constitutes a very beautiful mathematical problem that in the past has become a cult activity for many students. However, even though this problem has been extensively studied in the past, up to now a method to factorize a general $n \times n$ matrix (chapter 4) was not known. Fortunately, several approximate factorization techniques have recently been developed. In particular, the reduction of the factorization problem to the solution of Fredholm integral equations of the second kind constitutes a powerful tool that provides efficiently the approximate factorized matrices of $\mathbf{G}(\alpha)$.

Once the factorization of $\mathbf{G}(\alpha)$ is achieved, new efforts are necessary to extract solutions. In fact, even if formal solutions may be obtained, a long and difficult elaboration is always required to make them effective from the physics and engineering points of view.

The W-H technique involves complex and cumbersome algebraic manipulations. Nowadays these manipulations do not constitute a serious impediment because powerful algebraic manipulator codes are readily available. In particular, all the results in this book were obtained by intensive use of the computing software MATHEMATICA.

Concerning the overall philosophy of the subject presentation, this book has been written for readers primarily interested in the fundamental concepts and possible applications of the presented method. For this reason, the considered arguments are often only delineated and not discussed in great mathematical depth. The W-H technique requires the knowledge and use of many advanced topics of complex analysis, whose exposition might discourage readers who are interested primarily in application aspects. Of course, the best way to render the mathematical tools appealing is to present them only in as much detail as is required for the specific applications. We tried to follow this principle, but it was sometimes impossible. Therefore, we divided the book into two parts. The first part (chapters 1–6) is devoted to the mathematical aspects of the W-H technique, whereas the second part (chapters 7–10) presents applications that we hope illustrate the beauty, aims, and power of the theory. In particular, in the applications we often emphasized only the first and more difficult step of the W-H technique: the deduction of the matrix kernel $\mathbf{G}(\alpha)$ of the problem. In fact, this is the step that in some sense lacks of a general methodology. It is the intensive presentation of the deduction of $\mathbf{G}(\alpha)$ in different problems that provides the useful tools and the practice needed for solving new problems.

The Wiener-Hopf equations studied in this book are substantially one dimensional. It is possible to introduce multidimensional W-H equations (Meister & Speck, 1979) and generalize the concept of factorization that constitutes the fundamental tool that distinguishes the W-H equations from other integral equations. In particular, two works by Radlow (1961, 1964) attempted to solve two fundamental diffraction problems¹ by factorizing kernels defined in two-dimensional space. In these cases, the factorization method needs function-theoretic tools employing analytical functions with two complex variables. The involved analytical difficulties may easily lead to errors, and as a consequence unfortunately Radlow's solutions are incorrect. To date, the only way to solve multidimensional W-H equations appears to be the use of the moment method. Even though approximate, this kind of solution is very powerful; some examples will be considered in chapter 8.

In this book we consider only time harmonic fields with a time dependence specified by the factor $e^{j\omega t}$ (electrical engineering notations), which is omitted throughout, and where the imaginary unit is indicated with j . Conversely, in applied mathematics the factor $e^{j\omega t}$ is usually replaced by $e^{-i\omega t}$. This means that in the natural domain the change $j \Rightarrow -i$ transforms the engineering notation into applied mathematics notation (and vice versa). However, in the spectral domain, usually the same notations are used in both engineering and applied mathematics. In fact, regarding for example the Fourier transforms, the following definitions are the most frequently used in the literature:

$$F_e(\alpha) = \int_{-\infty}^{\infty} f_e(x) e^{j\alpha x} dx, \quad F_a(\alpha) = \int_{-\infty}^{\infty} f_a(x) e^{i\alpha x} dx$$

where the subscript e means engineering and the subscript a means applied mathematics. Consequently, in the spectral domain on the real axis we have

$$F_a(\alpha) = F_e(-\alpha)$$

and j is replaced by $-i$ (and vice versa).

¹ The diffraction problems studied by Radlow are the diffraction by a quarter-plane and the diffraction by a right-angle dielectric wedge.

For example, let us consider in the natural domain the propagation factor that is defined in electrical engineering notation by

$$f_e(x) = e^{-jkx}$$

with the propagation constant k defined by

$$k = \beta - ja, \quad a \geq 0$$

The same propagation factor in applied mathematics notation is written

$$f_a(x) = e^{ik_a x}$$

with $k_a = \beta + ia$.

In the Laplace domain, on the real axis, we have

$$F_e(\alpha) = \int_0^{\infty} f_e(x) e^{-j\alpha x} dx = \frac{j}{\alpha - k}$$

which in applied mathematics notations is written

$$F_e(\alpha) = \frac{j}{\alpha - k} \Rightarrow F_a(\alpha) = \frac{-i}{-\alpha - k_a} = \frac{i}{\alpha + k_a}$$

Analytic continuations define the previous functions in the whole complex plane α . This means that the Laplace Transforms are defined for every value of α by

$$F_e(\alpha) = \frac{j}{\alpha - k}, \quad F_a(\alpha) = \frac{i}{\alpha + k_a}$$

In the following we will define plus $F_+(\alpha)$ and minus $F_-(\alpha)$ (section 1.1). Notice that a plus (or minus) function in the electrical engineering notation is also a plus (or minus) function in the applied mathematics notation. The only difference between the two is given by the location of the singularities. For example, $F_e(\alpha)$ and $F_a(\alpha)$ are plus functions both with engineering and applied mathematics notation. However, $F_e(\alpha) = \frac{j}{\alpha - k}$ has a singularity at $\alpha = k = \beta - ja$, whereas $F_a(\alpha) = \frac{i}{\alpha + k_a}$ has it at $\alpha = -k_a = -\beta - ia$. The notation and definitions presented in this preface will be used throughout the book.

In the 80 years since the seminal 1931 paper by Wiener and Hopf, an enormous amount of work has been performed using their powerful function-theoretic method and its further extensions. It would not be possible to reproduce all that work in detail within a single volume. Therefore, we simply report many results without proof, referring the interested reader to the bibliographical sources for additional details. Similarly, we list many applications of the method to electromagnetic boundary-value problems, often just providing the results without the detailed derivations that readers may find in the original publications.

Foreword

The Mario Boella series offers textbooks and monographs in all areas of radio science, with a special emphasis on the applications of electromagnetism to information and communication technologies. The series is scientifically and financially sponsored by the Istituto Superiore Mario Boella affiliated with the Politecnico di Torino, Italy, and is scientifically cosponsored by the International Union of Radio Science (URSI). It is named to honor the memory of Professor Mario Boella of the Politecnico di Torino, who was a pioneer in the development of electronics and telecommunications in Italy for half a century and was vice president of URSI from 1966 to 1969.

This advanced research monograph is devoted to the Wiener-Hopf technique, a function-theoretic method that has found applications in a variety of fields, most notably in analytical studies of diffraction and scattering of waves. It contains a compendium of the research work of Professor Vito G. Daniele of the Politecnico di Torino, who is a foremost international authority on the Wiener-Hopf method. Professor Daniele has teamed with his colleague and coauthor, Professor Rodolfo S. Zich, past rector of the Politecnico di Torino and current president of the Istituto Superiore Mario Boella, in writing this monograph.

It is hoped that this work will be well received by scientists, engineers, and applied mathematicians and will serve as a benchmark reference in the field of theoretical electromagnetism for the foreseeable future.

Piergiorgio L. E. Uslenghi
Series Editor
Chicago, January 2014