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The use of fractional calculus to model the experimental creep-recovery behavior of modified bituminous binders / Sapora, A.G., Cornetti, P., Carpinteri, A., Baglieri, O., Santagata, E.. - In: MATERIALS AND STRUCTURES. - ISSN 1359-5997. - STAMPA. - 49:(2016), pp. 45-55. [10.1617/s11527-014-0473-6]

*Availability:*

This version is available at: 11583/2607955 since: 2020-04-28T17:05:07Z

*Publisher:*

Springer

*Published*

DOI:10.1617/s11527-014-0473-6

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# The use of fractional calculus to model the experimental creep-recovery behavior of modified bituminous binders

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## Abstract

Fitting of experimental creep-recovery curves obtained from rheological tests carried out on viscoelastic materials can reveal difficulties when handled by classical rheological theories, which generally provide exponential-type functions requiring a great number of parameters to be determined. Fractional calculus may represent a natural framework to develop more synthetic and efficient methods, overcoming most of the problems encountered with the classical approach. In this paper, a fractional model consisting of a dashpot in series with a springpot is proposed to describe the viscoelastic behavior of modified bituminous binders used in road pavements. The purpose is twofold: i) to evaluate the adequacy of a rheological model involving a limited number of parameters, each of which with a precise physical meaning; ii) to improve the accuracy of other fractional approaches proposed in literature. The investigation was validated by means of experimental data gathered from shear creep-recovery tests carried out at various temperatures on two modified bituminous binders containing a styrene-butadiene-styrene polymer and crumb rubber from end-of-life tires.

*Keywords:* Viscoelasticity, creep-recovery tests, bituminous binders, fractional calculus.

## 1. Introduction

The property of a body, when deformed, to exhibit both an elastic and a viscous behavior as a result of the simultaneous storage and dissipation of mechanical energy is known as viscoelasticity. In other terms, viscoelastic materials are those for which the stress-strain constitutive relationship depends upon time. Indeed, this framework represents the research field where fractional calculus, i.e. the mathematical branch dealing with derivatives and integrals of arbitrary orders (Samko 1993, Podlubny 1999), has been mostly applied. The reason is related to the fact that many viscoelastic materials show a creep/relaxation behavior of the power-law type (Nutting 1921,1943, Hilfer 2000, Mainardi 1994, Maugin 2006, Di Paola et al. 2013), while classical rheological models used to describe such a response generally rely on exponential-type functions. As a consequence, a huge number of elements (and thus of parameters) have to be taken into account in order to approximate a power-law expression, giving rise to several drawbacks: i) it is not always possible to provide a clear mechanical meaning to all coefficients; ii) the numerical procedure employed to fit the parameters is not trivial, these being subjected to several physical constraints (Tschoegl 1989, Sorvari and Malinen 2007). On the contrary, power-law expressions are naturally generated by assuming a constitutive law of the fractional type, i.e. involving non-integer order derivatives of stress and/or strain (Metzler et al. 1995). Moreover, it should be pointed out that fractional operators have a simple definition in terms of Laplace transforms, so it is not difficult to obtain the relaxation modulus starting from creep compliance (and viceversa), being these functions strictly related in the Laplace domain (e.g. (Mainardi 2010)).

A detailed list of references on fractional applications to viscoelasticity can be found in (Carpinteri and Mainardi 1998, Mainardi 2010, Machado et al. 2011), while significant contributions which are relevant in the context of this paper are the following: the pioneering work by Caputo and Mainardi (1971) on dissipation in anelastic solids; the application to problems in polymer physics and rheology discussed in (Hilfer 2000); the studies on concrete structures carried out in (Barpi and Valente 2003, 2004); the applications to bioengineering reported in (Magin 2006); the approach proposed in (Meral et al. 2010) to simulate the surface wave response of soft tissue-like materials; the analytical investigation of wave propagation in a viscoelastic rod of finite length carried out in (Atanackovic et al. 2011, 2013); the analysis of the behavior of different elastomers (Di Paola et al. 2011); the finite element (FE) implementation presented in (Muller et al. 2013).

Only few studies focused on bituminous materials used for road pavements. The first attempt, to the authors' best knowledge, is due to Oeser et al. (2008), who employed the fractional Burger element to describe creep-recovery curves of bituminous mixtures. Both the cases of a constant fractional order  $\alpha$  and of an exponent  $\alpha$  varying with strain (i.e., with time) were discussed. The constitutive

law was implemented through a non-linear computational model in the spirit of the finite element method. More recently, Celauro et al. (2012) proposed the simple two-coefficient fractional dashpot (also known as Scott-Blair element or springpot) to analytically obtain loading and unloading curves which were compared to experimental data derived from creep-recovery tests performed on bituminous mixtures. Although promising, this last study presents significant points of weakness. First of all, discrepancies with experimental data were observed for tests involving long loading times and additional parameters were hence introduced, thus leading to a four-parameter model. From a physical point of view, this choice may be questionable. Secondly, according to the springpot element (and, more in general, to all “pure” fractional models as observed in (Oeser et al. 2008)), recovery strains tend to zero for infinite times, whereas a permanent viscous strain is experimentally observed in the case of bituminous materials.

With the purpose of overcoming these drawbacks and of giving an original contribution to research on bituminous binders, a three-parameter model consisting of a springpot in series with a dashpot was considered in the investigation described in this paper (see also (Di Mino et al. in press)). The model was applied to creep-recovery test results obtained at different temperatures for two modified bituminous binders. The paper is organized as follows: classical rheological models are presented in Section 2, while basic fractional approaches are discussed in Section 3; Section 4 includes a description of the performed experimental investigation, an illustration of the procedure adopted to fit model parameters, the comparison between test data and theoretical **results**; finally, Section 5 contains the main conclusions of the study.

## 2. Classical rheological models

Elastic behavior is generally modeled by a one-dimensional Hookean spring, whose stress-strain ( $\sigma$ - $\varepsilon$ ) constitutive relationship, as a function of time  $t$ , is written as:

$$\sigma(t) = E\varepsilon(t) \tag{1}$$

being  $E$  the spring stiffness.

Fluid viscous response is described by a dashpot element, whose constitutive law is:

$$\sigma(t) = \eta \frac{d\varepsilon(t)}{dt} \tag{2}$$

where  $\eta$  is dynamic viscosity.

Viscoelastic materials exhibit both elastic and viscous behaviors, which are then modeled by properly combining springs and dashpots.

In the first part of the analysis presented in this paper, the viscoelastic constitutive equation is expressed by using a creep-based formulation of the following type:

$$\varepsilon(t) = D(t)\sigma_0 \quad (3)$$

being  $\sigma_0$  the applied stress and  $D(t)$  the creep compliance function, which for the spring is simply  $D=1/E$  and for the dashpot takes the form  $D(t)=t/\eta$ .

### 2.1 Maxwell and Kelvin-Voigt models

The Maxwell model consists of a spring and a dashpot in series (Fig. 1a). The two elements are subjected to the same stress, while total strain is given by the sum of the contributions of the spring and of the dashpot. Thus, it is not difficult to obtain the constitutive relationship by differentiating Eq. (1) and by adding the result to Eq. (2):

$$\sigma(t) + \frac{\eta}{E} \frac{d\sigma(t)}{dt} = \eta \frac{d\varepsilon(t)}{dt} \quad (4)$$

The Kelvin-Voigt model is obtained by combining in parallel a spring and a dashpot (Fig. 1b). In this case the two elements are subjected to the same strain, whereas total stress is the sum of two terms associated to the component models. The constitutive law can be achieved by adding Eq. (1) to Eq. (2), leading to:

$$\sigma(t) = E\varepsilon(t) + \eta \frac{d\varepsilon(t)}{dt} \quad (5)$$

In the case of the Maxwell model, creep compliance  $D(t)$  is:

$$D(t) = \frac{1}{E} + \frac{t}{\eta}, \quad (6)$$

while for the Kelvin-Voigt model it is represented by an exponential-decaying function of the following type:

$$D(t) = \frac{1}{E} [1 - \exp(-t/\bar{t})] \quad (7)$$

where  $\bar{t} = \eta/E$  is known as retardation time.

## 2.2 Generalized models

Since both models introduced in the previous section have only two parameters, their capability of representing the behavior of complex materials is limited. However, they can be combined in several ways, thus leading to more versatile and useful generalized models (the reader is referred to (Tschoegl 1989) for a complete description).

Typical examples of such models are: (i) the four-parameter Burger model (Fig. 2a), which is obtained by arranging in series a Maxwell element with a Kelvin-Voigt element; (ii) the  $2n$ -parameter generalized Kelvin-Voigt model (Fig. 2b), achieved by considering  $n$  Kelvin-Voigt elements in series with an isolated spring. In this last case, the creep compliance function is:

$$D(t) = \frac{1}{E_0} + \sum_{k=1}^n \frac{1}{E_k} [1 - \exp(-t/\bar{t}_k)] \quad (8)$$

Eq. (8) can be easily written in the form of a Prony series. Indeed, there are several problems related to such a model. Firstly, the physical meaning of all the elements is not always clear. Secondly, fitting an experimental curve by a Prony series is not a trivial task since several restrictions must be imposed (e.g. coefficients have to be positive (Bradshaw and Brinson 1997)). The resulting fitting algorithm consists of a constrained least squares problem and sophisticated numerical methods must be implemented (e.g. Tschoegl 1989, Sorvari and Malinen 2007).

## 3. Fractional models

Fractional generalization of the models presented in Section 2.1 can be obtained by replacing first-order derivatives with derivatives of order  $\alpha \in (0,1)$  (Mainardi 2010). According to Caputo's definition, the  $\alpha$ -derivative of a generic function  $f$  can be expressed in the form:

$$\frac{d^\alpha f(x)}{dx^\alpha} = \frac{1}{\Gamma(1-\alpha)} \int_0^x \frac{f'(y)}{(x-y)^\alpha} dy \quad \alpha \in (0,1) \quad (9)$$

where  $\Gamma$  is the Euler-Gamma function.

According to Eq. (9), function  $f$  is obtained for  $\alpha = 0$ , while the first order classical derivative corresponds to  $\alpha = 1$  (being  $\Gamma(0)=\infty$  and  $\Gamma(1)=1$ ).

It should be noticed that different definitions of fractional derivative have been proposed in literature (Podlubny 1999). For systems at rest at  $t = 0$ , Eq. (9) coincides with that provided by Riemann and Liouville, thus proving that the same notation is coherent with both definitions with no loss of generality. A different notation was adopted in (Schiessel et al. 1995, Hillfer 2000), where fractional elements were obtained by substituting both springs and dashpots with springpots.

### 3.1 Scott-Blair element

Replacement of the first-order derivative with the derivative of order  $\alpha \in (0,1)$  is equivalent, from a practical point of view, to the substitution of a dashpot with a fractional dashpot (or *springpot*) of order  $\alpha$ , which represents the simplest fractional element (also known as Scott-Blair element) (Fig. 3a). The corresponding constitutive equation is ( $\alpha \in (0,1)$ ):

$$\sigma(t) = b_1 \frac{d^\alpha \varepsilon(t)}{dt^\alpha} \quad (10)$$

Eq. (10) coincides with Eq. (1) (i.e., the spring case) for  $\alpha = 0$ , while it leads to Eq. (2) (i.e., the dashpot case) for  $\alpha = 1$ . As  $\alpha$  varies, the physical meaning of parameter  $b_1$  obviously changes (as well as its physical dimensions), passing from a stiffness ( $\alpha = 0$ ) to a viscosity ( $\alpha = 1$ ).

When a creep-based formulation is used, for the Scott-Blair element creep compliance  $D(t)$  assumes the following power-law form:

$$D(t) = \frac{t^\alpha}{b_1 \Gamma(1+\alpha)} \quad (11)$$

### 3.2 Double Scott-Blair element

If two springpots in series are considered, a double Scott-Blair element is originated (Fig. 3b). The constitutive relationship can be expressed as ( $\alpha, \beta \in (0,1)$ ):

$$\frac{d\varepsilon(t)}{dt} = \frac{1}{b_1} \frac{d^{1-\alpha} \sigma(t)}{dt^{1-\alpha}} + \frac{1}{b_2} \frac{d^{1-\beta} \sigma(t)}{dt^{1-\beta}} \quad (12)$$

whereas the creep function  $D(t)$  is:

$$D(t) = \frac{t^\alpha}{b_1 \Gamma(1+\alpha)} + \frac{t^\beta}{b_2 \Gamma(1+\beta)} \quad (13)$$

While considering limit cases, the model described by Eqs. (12-13) yields:

- A single spring (Eq. (1)) with stiffness  $b_1 b_2 / (b_1 + b_2)$  for  $\alpha = \beta = 0$ .
- A single dashpot (Eq. (2)) with viscosity  $b_1 b_2 / (b_1 + b_2)$  for  $\alpha = \beta = 1$ .
- The classical Maxwell model (Eq. (4)) for  $\alpha = 1, \beta = 0$  v  $\alpha = 0, \beta = 1$
- A single springpot for  $\alpha = \beta \neq 0, 1$  described by the two constants  $b_1 b_2 / (b_1 + b_2), \alpha$ .

It can be observed that for  $\beta = 1$  the second springpot becomes a dashpot (Fig. 3c): this three-parameter model is adopted in Section (4.2) to describe creep-recovery curves obtained from experimental tests.

Interested readers can refer to (Schiessel et al. 1995, Mainardi 2010) for a deeper insight on more complex fractional viscoelastic models. In general it is observed that substitution of a dashpot with a springpot involves the introduction of an additional parameter in the analysis. For example, fractionalization of the abovementioned Burger model (Fig. 2a) involves a total of six parameters.

#### 4. Proposed model and experimental validation

As already observed in Section 1, many experimental tests on viscoelastic materials reveal a power-law behavior under creep loading. From the point of view of classical rheology, this implies the implementation of the generalized Kelvin-Voigt model with all the related drawbacks discussed in Section 2.2. Fractional models discussed in Section 3 are much easier to handle, since power-law creep compliance derives directly from the constitutive law (Eqs. (10)-(13)). The book by Mainardi (2010) describes in detail all the fractional rheological approaches presented so far in different research fields.

Modelling is certainly more challenging when considering the experimental behavior of viscoelastic materials subjected to both loading and unloading conditions. This is the case of the so called creep-recovery tests, which consist in a first phase, in which a given load is applied to a specimen for a predefined time interval, followed by a second phase, in which the load is removed. During such tests, evolution of strain as a function of time is monitored, thus leading to the direct assessment of its reversible and irreversible components.

Creep-recovery tests are used in the laboratory performance-related characterization of bituminous binders and mixtures in order to evaluate their anti-rutting potential (Levenberg 2009, Merusi and Giuliani 2011, Santagata et al. 2013, Bai et al. 2014). Such a task is of premium importance since rutting, which results from the accumulation of permanent deformation produced by repeated axle loads, represents one of the main distress types affecting road pavements. Presence of ruts on pavement surfaces leads to a low level of comfort and safety, thus contributing to the decrease of overall road serviceability.

The single springpot element (Fig. 3.1) was employed in (Celauro et al. 2012) to model data from creep-recovery compression tests carried out on bituminous mixtures in different temperature conditions. The two parameters  $(b_1, \alpha)$  were obtained from fitting which was performed by using the Mathematica<sup>®</sup> software. However, it was observed that for long loading times fitted recovery strains deviate significantly from recorded data and that implementation of the single springpot element is not adequate to capture the presence of permanent strains. This limitation is clearly described by the following equation, derived from Eq. (11):

$$\varepsilon(t) = \frac{[t^\alpha U(t) - (t-t^*)^\alpha U(t-t^*)]}{b_1 \Gamma(1+\alpha)} \sigma_0 \quad (14)$$

which shows that for such a model strain tends to zero for times  $t$  tending to infinite. In Eq. (14) the function  $U(t)$  represents the unit step function, while  $t^*$  is the starting unloading instant.

. Thus, an approach based on two different groups of parameters, one for the loading phase and one for the unloading phase, was proposed. This choice, which leads to a four-coefficient model, is arbitrary and lacks a precise physical meaning.

In order to overcome these drawbacks, a new model consisting of a springpot in series with a dashpot (Fig. 3c) is proposed in this paper to approximate the whole creep-recovery behavior. As observed in Section 3.2, such a model represents a particular case of the double Scott-Blair element (Fig. 3b) obtained by setting  $\beta = 1$ . The choice of adding a dashpot in series to a springpot is justified by the need of obtaining a non-zero permanent strain at the end of recovery. Starting from Eq. (14) and setting  $\beta = 1$ , the constitutive law becomes:

$$\varepsilon(t) = \left\{ \left[ \frac{t^\alpha}{b_1 \Gamma(1+\alpha)} + \frac{t}{b_2} \right] U(t) - \left[ \frac{(t-t^*)^\alpha}{b_1 \Gamma(1+\alpha)} + \frac{(t-t^*)}{b_2} \right] U(t-t^*) \right\} \sigma_0 \quad (15)$$

In order to validate the model, several shear creep-recovery tests were carried out at various temperatures on two different modified bituminous binders. The test procedure is described in Section 4.1, while the comparison between experimental data and theoretical results is discussed in Section 4.2.

It should be mentioned that the adopted fractional approach lies in a framework in which non-linear effects are not taken into account or discussed. Nevertheless, since these aspects could be relevant for future applications (Masad et al. 2008, Di Mino et al. in press), it may be useful for the reader to briefly synthesize the most important studies developed in this field so far. A detailed reference list can be found in the work by Schapery (2000).

Schapery's non-linear viscoelastic theory (Schapery 1969, Lou and Schapery 1971) was derived by using principles of irreversible thermodynamics. In the case of uniaxial loading, if strain is assumed as the independent state variable, the theory leads to a single integral expression, where non-linear stress-dependent terms are related to Gibb's free energy and can be calibrated by specifying a creep compliance function (Lou and Schapery 1971). Furthermore, for low stresses, strain reduces to a power-law function of time, which is linear with respect to applied stress. As already outlined, this can be achieved also by assuming a constitutive law of fractional type.

More recently, Lai and Bakker (1995) successfully applied the theory to uniaxial loading of a semi-crystalline polymer, high-density polyethylene. Total strain was assumed to be decomposed into its recoverable visco-elastic and irrecoverable plastic portions, the former being represented by Schapery's thermodynamic theory, the latter being accumulated during loading history. This viscoplastic strain formulation, as well as some work of others on various polymers and composites, were later discussed by Schapery (1997), who then generalized his theory to properly take damage and viscoplastic effects into account (Schapery 1999). Starting from the models presented above, many applications were developed in different research fields (Uzan, et al. 1986; Uzan, 1996; Park et al. 1996; Provenzano et al. 2002; Ye et al. 2010).

Eventually, even in the framework of fractional calculus, non-linear analyses have been discussed both in (Oeser et al. 2008) and (Di Mino et al. in press).

#### *4.1 Materials and test procedure*

Materials used in the experimental investigation included a polymer-modified binder (PMB) and an asphalt rubber (AR). Both binders were evaluated in short-term aged conditions after being subjected to the Rolling Thin Film Oven Test (RTFOT) in accordance with AASHTO T240-2009.

PMB was originated from a base bitumen by adding a high percentage of styrene-butadiene-styrene (SBS) according to the undisclosed processing scheme adopted by the plant which provided the

material. Modification of bitumen by means of SBS polymers may provide a significant enhancement of its viscoelastic properties (King et al. 1986; Collins et al. 1991). For this reason, the use of SBS polymer-modified bituminous binders is suitable for heavy-duty pavements subjected to high traffic volumes with a significant percentage of heavy and slow-moving vehicles.

AR was a commercially available product containing approximately 18% crumb rubber (by weight of total binder) derived from the grinding of end-of-life tires. The use of crumb rubber in road paving applications has been known since the 60's (McDonald 1981) and represents an attractive design option since it allows to solve a serious waste management problem and can simultaneously lead to a significant improvement of pavement performance. In particular, due to their enhanced stiffness and elasticity, crumb rubber modified binders are expected to reduce the accumulation of permanent deformation, thus contributing to the prevention of rut formation (Lee et al. 2008, Hsu et al. 2011, Moreno et al. 2013).

Equipment used for testing was a Physica MCR 302 Dynamic Shear Rheometer (DSR) from Anton Paar Inc., an air bearing stress-controlled device equipped with a permanent magnet synchronous drive (minimum torque = 0.1  $\mu\text{Nm}$ , torque resolution < 0.1  $\mu\text{Nm}$ ) and an optical incremental encoder for measurement of angular rotation (resolution < 1  $\mu\text{rad}$ ). The 25 mm parallel plates geometry was employed, with a gap between the plates set at 1.0 mm in the case of PMB and 1.5 mm in the case of AR.

Rheological measurements consisted in shear creep-recovery tests carried out at four temperatures (ranging from 58°C to 76°C, with 6°C increments) at a single stress level (equal to 100 Pa). Duration of creep and recovery phases was set in order to allow materials to reach steady-state flow conditions under loading and to recover most of the delayed elastic deformation after load removal. As indicated in Table 1, this approach led to very long creep and recovery times which were however coherent with those adopted in previous investigations carried out on similar highly modified binders (Santagata et al. 2013, 2014). Two replicates were run for each material-temperature combination and average data were considered in the analysis.

#### *4.2 Results and discussion*

The three parameters ( $\alpha, b_1, b_2$ ) of Eq. (15) were determined by means of a numerical algorithm which allows calculation of the coefficients of a nonlinear regression function by using a least squares estimate. Data recorded in the first 1% of the loading history were not considered in the analysis, since they may be affected by inaccuracies due to initial settlements of test specimens under loading and to inertia phenomena of the test setup.

Obtained values of  $(\alpha, b_1, b_2)$  are reported in Table 2. It can be noticed that  $\alpha$  monotonically decreases as a function of increasing temperature  $T$ , i.e., when a more viscous behavior is expected. At first glance, this trend might appear misleading, since for  $\alpha$  tending to zero the springpot approaches the behavior of a spring. However, it must also be observed that  $b_1$  decreases with temperature, indicating a progressive reduction of springpot resistance to shear deformation. In other terms, as a result of the combined effects of  $\alpha$  and  $b_1$  variations, as temperature increases and coherently with expectations, the relative contribution of the springpot to overall deformation becomes smaller than that of the dashpot. As expected, parameter  $b_2$ , representing viscosity of the dashpot, also decreases with temperature.

By comparing results obtained for the two binders, it can be observed that at any given temperature AR exhibits lower values of  $\alpha$ , higher values of  $b_1$  and higher values of  $b_2$  than PMB. Coherently with the composition of the considered materials, this indicates that AR has a more pronounced elastic behavior and a higher stiffness than PMB.

Experimental data were compared with those obtained by using Eq. (15) with fitted parameters (Table 2). Results referring to different temperatures for PMB are reported in Figs. 4-5a,b, while Figures 4-5c,d display the percent relative error  $\delta$  of each case. Indeed, a fairly good agreement is generally found. In the creep phase,  $\delta$  remains always below 10%, with maximum values (nearly 8%) observed at  $T = 58^\circ\text{C}$  and  $76^\circ\text{C}$ , i.e. in the least and most viscous cases, respectively. During recovery,  $\delta$  does not exceed 3%, showing a monotonic trend with time. Similar considerations hold for tests carried out on AR (Figs. 6-7): theoretical results seem to be accurate both during the creep phase (with maximum errors, close to 7%, occurring once again for the extremes cases corresponding to  $T = 58^\circ\text{C}$  and  $76^\circ\text{C}$ ) and during recovery ( $\delta$  being always less than 5%).

In order to verify the capability of the model to predict material response under different test conditions, shear strain values calculated by implementing the proposed model were compared with available experimental data not used in the calibration process. These data refer to creep tests carried out on binder PMB at a lower stress level (20 Pa) and the two extreme temperatures (58 and  $76^\circ\text{C}$ ). Theoretical predictions, obtained by using the parameters given in Table 2, and experimental curves are shown in Fig. 8. Matching appears to be satisfactory in both cases with maximum relative deviations lower than 5%.

## 5. Conclusions

A fractional model, consisting of a springpot in series with a dashpot, was used in this study to describe the creep-recovery behavior of two different modified bituminous binders (containing SBS

polymer and crumb rubber, respectively). Theoretical results matched the experimental data gathered from shear creep-recovery tests carried out at different temperatures. In particular, during the creep phase, absolute percent relative errors were generally lower than 8%, while during the recovery phase maximum deviations were comprised in the  $\pm 5\%$  range. For one of the two binders, parameter calibration was successfully verified by considering a different data set, obtained from creep tests carried out at a lower loading level. Based on the findings synthesized above, it can be concluded that the proposed fractional model is superior to others adopted for the analysis of bituminous materials for road pavements. In particular, it seems to be more accurate than the single springpot considered in (Celauro et al. 2012) and easier to be implemented than the six-parameter fractional Burger element (Oeser et al. 2008).

Future developments of the study described in this paper will consider three-dimensional characterization, numerical implementation of the model via a finite element code and generalization of the approach to take into account non-linear effects (Di Mino et al. in press).

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## List of Tables

Temperature [°C]	Shear stress level [Pa]	Creep time [s]	Recovery time [s]
58	100	10800	43200
64	100	1800	21600
70	100	600	10800
76	100	300	7200

Table 1. Test conditions adopted in the experimental investigation.

$T$ [°C]	PMB			AR		
	$\alpha$	$b_1$ [ $s^\alpha \cdot N/m^2$ ]	$b_2$ [ $s \cdot N/m^2$ ]	$\alpha$	$b_1$ [ $s^\alpha \cdot N/m^2$ ]	$b_2$ [ $s \cdot N/m^2$ ]
58	0.611	0.555	9.26	0.461	0.909	15.7
64	0.571	0.312	1.88	0.372	0.476	6.02
70	0.549	0.213	0.758	0.359	0.345	2.47
76	0.502	0.154	0.215	0.341	0.256	1.08

Table 2. Fitted parameters ( $\alpha, b_1, b_2$ ) for creep-recovery tests carried out at different temperatures.

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**Fig. 1.** Maxwell model (a); Kelvin-Voigt model (b)

**Fig. 2.** Burger model (a); generalized Kelvin-Voigt model (b)

**Fig. 3.** Scott-Blair model or springpot (a); double Scott-Blair model (b); proposed model (c)

**Fig. 4.** Creep-recovery tests on binder PMB: a,b) experimental data (dashed line) and theoretical results (solid line) (a: for  $T = 58^{\circ}\text{C}$ ; b: for  $T = 64^{\circ}\text{C}$ ; c,d) Percent relative error (c: for  $T = 58^{\circ}\text{C}$ ; d: for  $T = 64^{\circ}\text{C}$ ).

**Fig. 5.** Creep-recovery tests on binder PMB: a,b) experimental data (dashed line) and theoretical results (solid line) (a: for  $T = 70^{\circ}\text{C}$ ; b: for  $T = 76^{\circ}\text{C}$ ; c,d) Percent relative error (c: for  $T = 70^{\circ}\text{C}$ ; d: for  $T = 76^{\circ}\text{C}$ ).

**Fig. 6.** Creep-recovery tests on binder AR: a,b) experimental data (dashed line) and theoretical results (solid line) (a: for  $T = 58^{\circ}\text{C}$ ; b: for  $T = 64^{\circ}\text{C}$ ; c,d) Percent relative error (c: for  $T = 58^{\circ}\text{C}$ ; d: for  $T = 64^{\circ}\text{C}$ ).

**Fig. 7.** Creep-recovery tests on binder AR: a,b) experimental data (dashed line) and theoretical results (solid line) (a: for  $T = 70^{\circ}\text{C}$ ; b: for  $T = 76^{\circ}\text{C}$ ; c,d) Percent relative error (c: for  $T = 70^{\circ}\text{C}$ ; d: for  $T = 76^{\circ}\text{C}$ ).

**Fig. 8.** Creep tests on binder PMB: comparison between experimental data (dashed line) and predictions (solid line) (a: for  $T = 58^{\circ}\text{C}$ ; b: for  $T = 76^{\circ}\text{C}$ ).