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***Abstract*—This work presents a novel approach for passivity enforcement of state-space macromodels, based on nonsmooth localization algorithms applied to a convex formulation of the passivity constraints. The main advantages of proposed scheme are guaranteed optimality and limited required computational resources. Compared to convex formulations based on a direct implementation of Bounded Real Lemma we are able to reduce both the memory and time requirements by orders of magnitude.**

I. Introduction

Macromodeling techniques have become ubiquitous in modern Computer Aided Design (CAD) flows. Such methods allow the identification of compact dynamical models of complex interconnect structures. These models match the broadband dynamics of the underlying physical structures and are typically available in state-space or pole-residue representation. These two forms can be readily synthesized into SPICE-compatible netlists, thus allowing efficient Signal and Power Integrity analyses using standard circuit solvers.

Macromodels are quite often obtained by a rational curve fitting process applied to scattering responses computed at frequency samples. For this task, the Vector Fitting scheme [1] and its various derivatives [2] prove to be quite successful. These methods have been demonstrated [3] to scale very favorably with system complexity, both in terms of model order and number of interface ports. Unfortunately, such methods are not able to produce guaranteed passive models. Therefore, common design flows involve a second post-processing step, in which an initial state-space model is processed by a passivity checking and enforcement algorithm [3]. We recall that passivity is a fundamental property that any model of a physically passive interconnect structure should have, in order to guarantee global stability in transient simulations [4].

The problem of passivity enforcement is much more challenging than rational fitting. Passivity conditions, which can be cast in various different forms [3], involve a coupling between all transfer matrix elements, or equivalently all individual elements of the state-space matrices. Therefore, the numerical complexity of passivity enforcement schemes is significantly larger than the complexity of state of the art rational fitting schemes. Moreover, since passivity is more restrictive than stability, there is a potential loss of accuracy that needs to be carefully controlled. These issues triggered significant research efforts in the last two decades.

We can roughly divide passivity enforcement methods in two classes: optimal and suboptimal schemes. Optimal methods provide, within a given parameterization form, a single passive model that is closest in some norm to the original data. In this class we include methods based on Positive or Bounded Real Lemma passivity constraints. Methods such as [5], [6] optimize poles and residues simultaneously in a single step, where as methods such as [7]–[9] optimize the poles and residues separately in two steps. The optimality of these methods, which is guaranteed by their convex formulation, comes at the price of possibly large computational requirements, both in terms of runtime and memory.

Suboptimal passivity enforcement schemes are instead based on approximate, linearized, or local forms of passivity constraints. In this class we include methods based on Hamiltonian perturbation and residue or state-space perturbation via localized constraints at specific frequencies [10]–[14]. These methods are much faster, but only approximate. They are not able to guarantee the best possible accuracy in the final passive models, and they require generally iterations, posing the problem of convergence due to lack of convexity.

In this paper, we present an algorithm that tries to combine the advantages of both approaches. We apply a localization scheme, namely the ellipsoid algorithm [15], to a convex nonsmooth formulation of the passivity constraints [9]. Due to convexity, we are able to prove that the optimal passive model will be obtained. Furthermore, the implementation of ellipsiod algorithm requires a reduced memory footprint, and the number of iterations (which will always converge to the optimum) is shown to be moderate. Therefore, the proposed approach is competitive with respect to both optimal methods (it produces the same result), and suboptimal methods (in terms of efficiency). These features are confirmed through application to various examples.

II. Background

A. Passivity Enforcement via State Space Perturbation

A linear dynamical state space model for a system with p ports is given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t). \end{aligned} \tag{1}$$

Here $x(t) \in \mathbb{R}^n$ describes the state variables, $u(t), y(t) \in \mathbb{R}^p$ describe the input and output vectors respectively. Matrices

$A \in \mathbb{R}^{n,n}, B \in \mathbb{R}^{n,p}, C \in \mathbb{R}^{p,n}$ and $D \in \mathbb{R}^{p,p}$ are obtained via rational fitting [1]. The transfer matrix corresponding to the state space system (1) is given by

$$H(s) = C(sI - A)^{-1}B + D. \tag{2}$$

System (1) and (2) is assumed to be stable. In scattering representation a stable system (2) is *passive* (or dissipative) if and only if its $\mathcal{H}_\infty$ norm is unitary bounded, i.e.

$$||H||_{\mathcal{H}_\infty} = \sup_{\omega \in \mathbb{R}} \sigma_1(H(j\omega)) \leq 1, \tag{3}$$

where σ_1 denotes the largest singular value [16]. If (3) does not hold, passivity is enforced by introducing a perturbation in the model. As described in [3], a common choice is to perturb the state-space C matrix with the assumption that $||D||_2 = \sigma_1(D) \leq 1$, which is a necessary condition for (3) to hold. Hence, a *perturbed* system is defined as

$$H(\Delta C, s) = (C + \Delta C)(sI - A)^{-1}B + D \tag{4}$$

where the perturbation matrix $\Delta C \in \mathbb{R}^{p,n}$ is unknown. Given an initial non-passive system (2) our goal is to enforce passivity by introducing a *minimal* perturbation. This problem can be formulated as

$$\underset{\Delta C}{\text{minimize}} \ ||\Delta C||_F \quad \text{s.t.} \ \ ||H(\Delta C)||_{\mathcal{H}_\infty} \leq 1. \tag{5}$$

Here the minimal perturbation condition is expressed in terms of the Frobenius norm of ΔC. Other weighted norms can be used with trivial extensions, e.g., it is shown in [10] that using the controllability Gramian of the system as a matrix weight yields a solution that provides minimal impulse response perturbation in the $\mathcal{L}^2$ (energy) norm.

Let $vec(\cdot)$ define an operator that stacks up the columns of a matrix to form a vector. To simplify the notation, we define $\delta_C = vec(\Delta C) \in \mathbb{R}^q$ ($q = np$) and rewrite (5) as

$$\underset{\delta_C}{\text{minimize}} \ f(\delta_C) \quad \text{s.t.} \ \ h(\delta_C) \leq 1, \tag{6}$$

where $f(\delta_C) = ||\delta_C||_2 = ||\Delta C||_F$ and $h(\delta_C) = ||H(\Delta C)||_{\mathcal{H}_\infty}$. The optimization problem (6) is convex [17]. Additionally, (6) is feasible since one can always find a feasible point, namely $\Delta C = -C$. Hence, problem (6) has a unique globally optimal solution. Moreover, as described in [9], $h(\delta_C)$ is continuous but non-smooth. In this work we propose using a localization method to solve (6).

B. Localization Methods

Localization methods, such as the ellipsoid algorithm [15], are the optimization techniques where an initial set containing the global minimum becomes smaller at each iteration, thus bracketing the solution more and more tightly as the iterations progress. These methods are memory efficient and can handle nonsmooth problems, such as (6).

The main idea behind the ellipsoid algorithm is to define an initial ellipsoid, χ_0, that is guaranteed to contain the global minimum. The initial point $\delta_C^{(0)}$ is defined to be at the center of this ellipsoid. The ellipsoid is then modified iteratively as $\chi_0 \rightarrow \chi_1 \rightarrow \cdots \rightarrow \chi_\nu \rightarrow \ldots$, according to the feasibility of its center $\delta_C^{(\nu)}$.

Suppose that the current center $\delta_C^{(\nu)}$ is *infeasible*. Therefore, the value of $h(\delta_C^{(\nu)}) > 1$ has to be decreased to find a feasible point. The algorithm defines a hyperplane orthogonal to a subgradient $g \in \partial h$ of the constraint function. The half-space

$$\mathcal{S}_\nu = \{z | \ g^T(z - \delta_C^{(\nu)}) \leq 0\} \tag{7}$$

is the one where the constraint function decreases and hence intersects with the feasible region. The algorithm constructs an ellipsoid $\chi_{\nu+1}$ that includes $\chi_\nu \cap \mathcal{S}_\nu$, and such that $vol(\chi_{\nu+1}) < vol(\chi_\nu)$, where $vol(\cdot)$ denotes the volume its the argument.

Suppose instead that the center $\delta_C^{(\nu)}$ of the ellipsoid χ_ν is *feasible*. The algorithm then defines $g \in \nabla f$ as the gradient of the objective function, which decreases in the half-space (7), which in turn contains the global minimum δ_C^*. The next ellipsoid $\chi_{\nu+1}$ is constructed as in the previous case. This process is continued until the size of the updated ellipsoid is small enough such that all of its interior points fall with in an ϵ-neighborhood of the global minimum δ_C^*. Note that localization methods are not descent algorithms. Hence, they keep track of the best feasible solution attained through all iterations. The update steps of ellipsoid algorithm, given in [15], [18], are analytical, involving only matrix vector products. The ellipsoid algorithm is efficient in terms of memory, but it may take several iterations to converge.

III. Passivity Enforcement via Localization Methods

In this paper, we employ the ellipsoid algorithm of Sec. II-B to solve the convex non-smooth problem (6).

A. Initial Set

One of the main challenges in using the ellipsoid algorithm is to define an initial set χ_0 that is guaranteed to contain the global minimum. This initial set needs to be as small as possible, because for larger initial sets the algorithm may take more iterations to converge. We define χ_0 in the form of a hypersphere with radius R. We also compute an upper and a lower bound on R, which help us to pick a value of R that is appropriate (see [19] for a detailed derivation).

1) An Upper Bound on R (R_{UB}): Since we have defined the objective function in terms of an ℓ^2 norm of the decision variable, the ℓ^2 norm of *any* feasible point δ_C^F will define an upper bound on R. Hence, a hypersphere centered at origin with radius equal to the Euclidean distance of δ_C^F from the origin is guaranteed to contain the global minimum. As described in Section II, one of such feasible points is $\delta_C^F = -vec(C)$, therefore we set $R_{UB} = ||vec(C)||$.

2) Lower Bound on R (R_{LB}): We assume that the initial unperturbed system is not passive, hence $\delta_C^{(0)}$ is infeasible. We define R_{LB} to be the radius of an infeasible hypersphere ϵ_{LB} centered at the initial point $\delta_C^{(0)}$, such that

$$h(\delta_C) > 1 \quad \forall \delta_C \in \epsilon_{LB}. \tag{8}$$

Since $h(\delta_C)$ is convex, we have $\forall \delta_C \in \epsilon_{LB}$

$$h(\delta_C) \geq h(\delta_C^{(0)}) + \partial h(\delta_C^{(0)})^T(\delta_C - \delta_C^{(0)})$$
$$= h(\delta_C^{(0)}) - R_{LB}\sqrt{\partial h(\delta_C^{(0)})^T \partial h(\delta_C^{(0)})}, \quad (9)$$

where ∂h denotes a subgradient [9] of the constraint function. From (9), we note that all points in ϵ_{LB} are infeasible if

$$h(\delta_C) \geq h(\delta_C^{(0)}) - R_{LB}\sqrt{\partial h(\delta_C^{(0)})^T \partial h(\delta_C^{(0)})} > 1. \quad (10)$$

Solving (10) gives us R_{LB} as

$$R_{LB} = \frac{h(\delta_C^{(0)}) - 1}{\sqrt{\partial h(\delta_C^{(0)})^T \partial h(\delta_C^{(0)})}} = \frac{h(\delta_C^{(0)}) - 1}{||\partial h(\delta_C^{(0)})||_2}, \quad (11)$$

so that all the interior points of any hypersphere with radius $R < R_{LB}$ are infeasible. Hence, we must select $R > R_{LB}$ in order to guarantee that the hypersphere with radius R includes the global minimum.

3) Practical Considerations: From earlier discussions, the value of R must satisfy the following inequality

$$R_{LB} < R \leq R_{UB}. \quad (12)$$

In our implementation we select $R = R_{UB}$.

B. Computational Complexity

The main attractive feature of the ellipsoid algorithm is its low memory usage. The ellipsoid algorithm has two major components: a) computing the $\mathcal{H}_\infty$ norm, and b) updating and storing ellipsoid parameters. The corresponding computing requirements are summarized in Table I. The algorithm requires

TABLE I
COST PER ITERATION

Component	memory	time
(a) $\mathcal{H}_\infty$ norm	$\mathcal{O}(n^2)$	$\mathcal{O}(n^3)$
(b) Ellipsoid Parameters	$\mathcal{O}(q^2)$	$\mathcal{O}(q^2)$

only a modest storage of $\mathcal{O}(q^2)$, where q denotes the degrees of freedom ($\delta_C \in \mathbb{R}^q$). Notice that the cost per iteration for the ellipsoid algorithms is dominated by the computation of the $\mathcal{H}_\infty$ norm, which is $\mathcal{O}(n^3)$.

IV. RESULTS

We consider two simple but challenging test cases. Given an initial nonpassive model identified by vector fitting [1], passivity was enforced using the proposed localization approach, an existing optimal passivity enforcement technique [7], and a suboptimal technique [10].

A. A packaging interconnect

The first example is a $p = 16$ port coupled packaging interconnect, for which an original non-passive model ($n = 598$ states) exhibits very large passivity violations over a large bandwidth ($\sigma_1 = 72$) as shown in Fig. 1. The bounds computed on the initial hypersphere's radius are $0.52 < R \leq 1.97$. We selected $R = 2.0$. The ellipsoid algorithm took less than 500 iterations to find an accurate feasible solution, with a total runtime of about 50 minutes. However, note that over 80% (4.5 seconds per calculation) of the time was spent in computing the $\mathcal{H}_\infty$ norm. Here, we have used a standard algorithm to compute the $\mathcal{H}_\infty$ norm [16]. Memory usage was less than 90 MB.

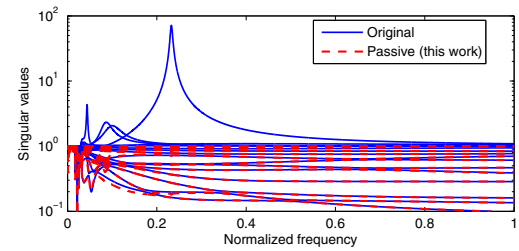

Fig. 1. Packaging interconnect: singular values of original non-passive (solid blue) and proposed passive (dashed red) models.

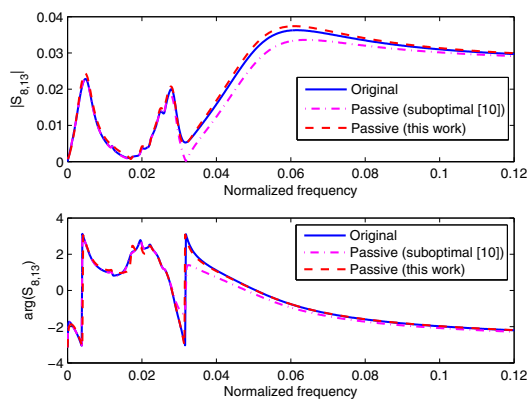

Fig. 2. Packaging interconnect: original non-passive and perturbed passive model responses ($S_{8,13}$)

Figure 2 demonstrates the accuracy of our passive model even in the presence of large passivity violations. Compared with the suboptimal technique [10], the proposed scheme is able to achieve passivity with a smaller perturbation amount, hence increased accuracy. Application of the alternative optimal technique [7] resulted in a problem with 217365 equations and 772865 variables. The required memory, as reported by Matlab was over 40 GB, and the problem could not be set up because the solver ran out of memory.

B. A SAW Filter

For this 3-port SAW filter, an initial stable but non passive model with 144 states was computed using vector fitting [1] from measured data. The original non-passive model exhibited large passivity violations at DC ($\sigma_1 = 25.83$) and around a normalized frequency of 0.156 ($\sigma_1 = 6.51$), as shown in Fig. 3. The bounds computed on the initial hypersphere's radius are $0.06 < R \leq 1.0$. We selected $R = 1.0$. The ellipsoid algorithm converged in less than 300 iterations to an accurate feasible solution in 30 seconds. However, note that over 82% of the time was spent in computing the $\mathcal{H}_\infty$ norm. Memory usage was less than 1 MB.

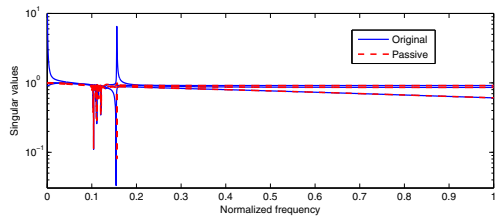

Fig. 3. SAW filter: singular values of original non-passive (solid blue) and proposed passive (dashed red) models.

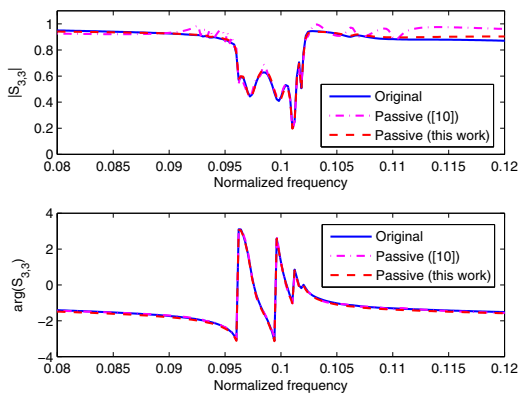

Fig. 4. SAW filter: original non-passive and perturbed passive model responses ($S_{3,3}$)

Figure 4 demonstrates the accuracy of our passive model even in the presence of large passivity violations. Also in this case, as expected, the proposed model is more accurate than with suboptimal techniques such as [10]. An optimally accurate model could also be obtained with the algorithm [7], which resulted in a problem with 14250 equations with 46156 variables. The required memory, as reported by matlab was over 2 GB, and the corresponding runtime was about 2.5 hours. We conclude that proposed approach is able to compute an optimal passive macromodel with a fraction of memory and orders of magnitude speed-up with respect to [7].

V. Conclusions

We have applied a localization method (the ellipsoid algorithm) to solve the problem of passivity enforcement for linear dynamical models. The main advantages are guaranteed optimality of the computed passive models, thanks to a convex formulation, and moderate computing requirements when compared to alternative optimal techniques. We further remark that within proposed framework, global optimality can be traded for efficiency by stopping iterations at any time, as soon as the achieved model is passive and meets sufficient accuracy requirements.

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