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ABSTRACT

Aim of this work is to propose a stochastic description of the leakage between two aquifers separated by a semi-permeable layer with low hydraulic conductivity. The source of uncertainty here considered is the random fluctuation of the phreatic surface of surficial aquifer, originated from random rainfall events. The study focuses on an area surrounding a pumping well penetrating the deep aquifer and impacting its piezometric level, where infiltration from the surficial aquifer can be more harmful. Closed form expressions for the leakage between the surficial and the deep aquifer are used to obtain the long-term probability distribution of leakage flow rate, assuming the shallow phreatic surface dynamics modeled with a Poisson-driven stochastic process. A sensitivity analysis is performed to verify the variability of the probability distribution of leakage within the range of feasible parameter values, then the stochastic model is applied to three field cases where time series of the piezometric levels of the phreatic aquifer are available. Results show that the induced variability of the discharge flowing between aquifers is remarkable and that in general it cannot be neglected despite the low hydraulic conductivity of the semi-permeable layer. The proposed probabilistic model is a useful tool for evaluating the risk associated to contaminant transport into deep aquifers and its fate in relation to groundwater withdrawals.

Keywords: groundwater, leakage, recharge, pumping well, Poisson noise, probability distribution

1. INTRODUCTION

Groundwater resource is of great importance for human activities and life: from agricultural uses and industrial processes to drinking requirements. The most precious groundwater is that of deep aquifers, because usually it is less contaminated and, in fact, most of the water supply systems pump water from confined aquifers.

In nature, many deep aquifers are not perfectly separated from the overlying aquifer because the layer dividing them is not completely impermeable and/or because of the presence of abandoned wells and boreholes that may allow for some leakage. In these circumstances, leakage occurs from an aquifer to the other, upward or downward, according to the piezometric levels of the aquifers. When a confined aquifer receives some recharge infiltrating from the overlying phreatic aquifer, it is referred to as a semi-confined aquifer and quantification of such infiltration is important for management purposes, in order to evaluate both the effects on the hydraulic head and velocity fields of the underlying aquifer and, if a contaminant is released, the mass of solute leaching from the surface to the deep aquifer.

Leaving aside the infiltration caused by abandoned wells and improperly plugged boreholes (e.g. Avci, 1994; Lacombe et al., 1995; Nordbotten et al., 2004) and limiting the analysis to the leakage through the semi-permeable layer, the infiltration flow depends on the difference between piezometric levels of the unconfined and semi-confined aquifers and the thickness and hydraulic conductivity of the aquitard (or semi-permeable layer). These quantities may be regarded as deterministic or as space-time random factors when taking into account their natural variability. For instance, the random nature of the hydrogeological parameters has been accounted for in the past to investigate the effects of leakage on the flow field of the semi-confined aquifer. Examples include: Li and Graham (1998, 1999) who dealt with the stochastic analysis of solute transport in heterogeneous semi-confined aquifers subject to spatiotemporal random recharge; Butera and Tanda (1999) who, following the linear theory of Dagan (1984), developed a stochastic approach to describe the effect of a uniform recharge on semi-confined aquifers with heterogeneous transmissivity, with the aim of characterizing the velocity field and the transport processes. Lu e Zhang (2002) investigated through spectral method the non-stationarity features of heterogeneous semi-confined aquifers. More recently Yeh and Chang (2009) applied the nonstationary spectral approach to quantify the variability of hydraulic head and velocities in deep heterogeneous aquifers subject to spatial and periodic recharge, as influenced by the water table fluctuations in the overlying phreatic aquifer. In addition, attention has been paid to the development of methods for computing pumping-induced recharge: Butler and Tsou (2003) outlined an approach for calculating the volume of pumping-induced leakage for both infinite and bounded homogeneous aquifers, assuming a constant head in the upper aquifer. Zhan and Bian (2006) derived closed-form analytical solutions of the steady state and leakage volumes within a given circular area for both constant-rate and constant-drawdown pumping wells, in homogeneous aquifers.

To our knowledge, less attention has been paid to the characterization of leakage as driven by fluctuations of the water table in the phreatic aquifer. When these fluctuations are induced by random rainfall events, the shallow water table dynamics becomes itself stochastic and new (probabilistic) frameworks are necessary to describe the manifold interactions and feedbacks between groundwater, climate, soil and vegetation (e.g., Rodriguez-Iturbe et al., 2007).

Some probabilistic frameworks used in environmental sciences and ecohydrology are detailed by Ridolfi et al. (2011) considering different characteristics of the random forcing (Poisson noise, Gaussian white noise, dichotomous noise, etc.). An example of a probabilistic soil water balance driven by rainfall events is found in Rodriguez-Iturbe et al. (1999), while the soil water balance in the presence of an aquifer has been considered, for example, by Salvucci and Entekhabi (1994), Bierkens (1998), Laio et al. (2009), Tamea et al. (2010). None of such contributions considers the effect of stochasticity on deep aquifers, nor the presence of subsurface geological structures, such as soil layers having different hydraulic conductivity. The goal of the present work is to propose a probabilistic framework describing the infiltration between aquifers, separated by a semi-permeable layer, as driven by random fluctuations of the shallow water table induced by rainfall events.

The proposed model assumes some simplifying hypothesis about the pedology and hydrogeology of the site and about the dynamics of the shallow water table. It then uses a simple and known groundwater solution to describe the flow leaching through the semi-permeable layer in an area surrounding a well, which withdraws water from the deep aquifer and impacts the vertical flow by modifying its piezometric surface. After reconstructing the probability distribution of the phreatic surface, the model returns a probability density function of the flow rate infiltrating over the considered area, under the assumption that infiltration velocities are low enough that the influence of leakage on the piezometric surfaces of the two aquifers is negligible.

The probabilistic description of leakage is of large interest for several reasons: (i) it helps understanding the relevance of shallow water table fluctuations in the presence of a pumping well withdrawing water from deep aquifers; (ii) it allows one to quantify the leakage flow variability as a function of rainfall variability, enabling e.g. quantitative analysis of the effect of some climate changes; and finally, (iii) it gives a tool for evaluating the risk of pollution of deep aquifers or, in general, the fate of transport associated to leakage. Given the relevance of these aspects, the present paper aims at proposing a useful tool for probabilistic assessments in subsurface hydrology.

The paper is so organized: the problem is initially presented and the infiltration flow rate quantified deterministically, next the probabilistic description of the piezometric fluctuations of the phreatic aquifer and of infiltration follows, finally three case studies are presented and discussed.

2. PROBLEM DESCRIPTION

Let us consider the case of two homogeneous, isotropic and indefinite aquifers separated by a horizontal semi-permeable soil layer which allows for some water infiltration between the two strata. The surficial aquifer (identified by subscript "1") is characterized by a horizontal phreatic surface laying at a positive depth from the soil surface, ; such depth may change in time due to the fluctuations induced by rainfall events and it has a maximum depth indicated by . The deep aquifer (identified by subscript "2") has a piezometric surface which, in the absence of perturbations, lies at a fixed (positive) depth, , from the soil surface. In general such depth is different from the surficial phreatic surface thus generating a flow through the semi-permeable layer, from one aquifer to the other.

A fully penetrating well of radius withdraws water from the semi-confined aquifer, lowering the piezometric surface a depth which varies with the distance, , from the center of the well (Figure 1). The drawdown in the piezometric head due to the well, , never lowers the piezometric surface below the top of the deep aquifer and it becomes negligible at a distance identifying the radius of influence of the well, while the rate of water pumped out from the well, , is assumed to be constant in time.

The semi-permeable layer separating the two aquifers allows for the water flowing either downward or upward, depending on the relative position of the two piezometric surfaces. According to the Darcy's law, the overall flow rate through the semi-permeable layer is a function of the characteristics of the layer and of the piezometric head difference in the two aquifers. Focusing on the vertical flow occurring within the radius of influence of the well and assuming the domain to be axial-symmetric without other wells or disturbances nearby, the discharge can be quantified by

$$— \qquad (1)$$

where is the hydraulic conductivity of the semi-permeable layer and its thickness, both assumed to be uniform in space. If the piezometric surface of the deep aquifer has a constant gradient, the following analysis is still valid, provided that is taken as the average piezometric depth in the area of radius .

It should be noted that the hydraulic conductivity of the semi-permeable layer is here taken small enough to prevent the phreatic surface (of surficial aquifer) from being influenced by the well and to avoid that leakage significantly modifies the pressure regime of the deep aquifer. The decoupling of the dynamical systems of shallow and deep aquifers is necessary for the application of the leakage model (end of Section 2.1) and the development of the stochastic framework presented in Section 3.1.

2.1 COMPONENTS OF THE INFILTRATING FLOW

Three contributions to the infiltrating flow can be identified by separating three different components of the piezometric head difference in (1).

Assuming that fluctuations of the phreatic surface occur as stochastic upward displacements from the deepest position (see Section 3), the first component of the piezometric head difference in (1) is given by the vertical distance between the (time-varying) position of the phreatic surface and its maximum depth, i.e. . Such difference is, by construction, always positive and the phreatic surface is here assumed to increase simultaneously in the area of influence of the well, thus the contribution is uniform in space.

The second component of the piezometric head expresses the head difference between the two aquifers, in the absence of phreatic-surface fluctuations and of a drawdown imposed by the well. This component is given by the vertical distance between the maximum depth of the phreatic surface and the undisturbed position of the piezometric surface in the deep aquifer, i.e. . Such difference is constant in time and uniform in space, and it can take positive or negative values according to the relative position of the piezometric head in the two aquifers. In case of a constant piezometric gradient in the deep aquifer, such difference would not be uniform, but using the spatial average of the resulting flow rate will be equal to the one obtained in uniform conditions.

The third component of the piezometric head difference expresses the role of the pumping well on the vertical flow between the aquifers and it is given by the space-varying drawdown induced by the well in the piezometric surface of deep aquifer, i.e. . By construction, the head difference is positive and it is considered to be constant in time when the well works continuously and the aquifer pressure and velocity field have reached the steady state.

The piezometric drawdown induced, at steady state, by a well having a constant extraction rate and fully penetrating the confined aquifer can be quantified by the Thiem solution (e.g., Batu, 1998). Under the hypothesis of a Darcian horizontal flow in a homogeneous and isotropic aquifer, and a finite area of influence of the well of radius , the polar-coordinate expression of the drawdown reads

$$— \quad – \qquad (2)$$

with being the generic distance from the well and being the transmissivity of the confined aquifer.

If the well had not to be fully penetrating in the deep aquifer, an alternative well function should be used and a different formulation of the steady-state drawdown would be reached. However, one may consider that for practical purposes the difference between the extreme cases of no well penetration and full penetration is limited to a restricted fraction of the radius of influence of the well (Batu, 1998), thus equation (2) is considered a valid approximation.

In order to evaluate the use of Thiem equation in the problem considered (i.e., in a semi-confined aquifer), additional considerations have to be made about the modifications induced by leakage on the piezometric surface of the deep aquifer. To this purpose, a comparison (not shown) has been carried out between the Thiem solution (2) and the steady state solution for a leaky semi-confined aquifer with a withdrawing well having a finite radius of influence (e.g., Batu, 1998). Considering the set of parameter values used in the reminder of the paper (see Section 4) and thanks to the high leakage factor ( ), the two solutions in terms of drawdown are undistinguishable. On one hand, this confirms that leakage and the phreatic surface do not affect the piezometric surface of the deep aquifer as required by the stochastic framework; on the other hand, this suggests that under the conditions of high leakage factor Thiem's solution is a good compromise between simplicity and adequacy.

In preliminary tests, Thiem solution (2) has also been compared to the steady state solution for a leaky semi-confined aquifer with a withdrawing well in an infinite domain; using parameter values of Section 4, the difference is limited to fractions of a meter (maximum: 0.4 m at the Thiem's domain edge). This finding allows the Thiem solution to be employed here without much concern about the value of the radius of influence of the well, considering also that spatial influence of a well can rarely be assumed to be actually infinite, due to the presence of hydrogeological discontinuities.

2.2 THE OVERALL INFILTRATING FLOW

Specifying the three components of the piezometric head difference in (1) and solving each part of the integral accordingly, the three contributions to the infiltrating flow rate read as follows:

— , (3)

which represents the contribution given by the surficial water table fluctuations and it is always positive and directly proportional to both the time-varying piezometric head difference and the area considered;

— , (4)

which is a sort of *base flow* quantifying the infiltration between aquifers in the absence of surficial fluctuations and of the well; the flow rate is directly proportional to both the area considered and the piezometric head difference, being either positive or negative according to the sign of the difference in ;

— — — (5)

which is the infiltration driven by the water extraction in the well and it is positive as long as the well is used to withdraw (and not to inject) water.

The sum of the three contributions above expresses the total discharge passing through the semi-permeable layer. Considering the typical dimensions of a well radius (m) as compared with the range of the well influence radius (m) (*Hamill and Bell*, 1992), one can simplify with respect to thus reducing the final expression of the infiltration discharge to

— —— . (6)

Such expression sets the basis of the evaluation and comparison of natural infiltration components versus well-driven infiltration within the area of influence of the well itself. Notice that the sign of is positive when the flow goes from the surficial to the deep aquifer, but it may also be negative when the term is negative and so large to turn negative the hole parenthesis in (6). In fact, while in recharge areas the surficial aquifer has higher piezometric head and the natural flow between the two aquifers occurs downwards, in the drainage or floodplain areas the deep aquifer can have higher piezometric head than the surficial aquifer, thus imposing a natural upward flow through the semi-permeable layer separating them.

3. THE PROBABILISTIC APPROACH

The phreatic surface of the unconfined aquifer may fluctuate (increase) as a reaction to water infiltration generated by random rainfall events. Although phreatic surfaces at large depths from the soil surface, or in soils with particular hydrogeologic characteristics, may have negligible rainfall-induced fluctuations, shallow phreatic surfaces react in a more rapid and intense way to rainfalls. In such case, phases of rising phreatic surface, induced by rainfall, are shorter and have larger temporal gradients than descending phases, when the phreatic surface slowly moves towards deeper positions. A temporal scale exists at which the dynamics show a typical jump-decay structure with risings occurring instantaneously, as in Figure 2, and for the cases considered in the present analysis (Section 4) this scale is about one or few days. The jump-decay structure can then be modeled with a Poisson-driven stochastic process such as those described by *Ridolfi et al.* (2011).

The generic position of the phreatic surface (see Figure 2) can be written as where the variable name is used in the following instead of for the sake of clarity. The values of are measured with respect to the lowest position at which the undisturbed phreatic surface tends in the absence of rainfall and which is assumed to be imposed by external conditions. Assuming the decay phases to be exponential in time, the phreatic surface dynamics can be described by

— (7)

where is the decay coefficient and is the random forcing having the form of a marked Poisson noise, with constant rate λ (reciprocal of the mean interarrival time) and exponentially distributed jumps with mean value α. The probability density function of , , describing the long-term stochastic dynamics (7) reads (e.g., *Ridolfi et al.*, 2011)

—— (8)

where is the Euler complete Gamma function. The above probability distribution, which is a Gamma distribution, has mean equal to — and variance equal to —.

The probability distribution (8) is superiorly unbounded, but the phreatic surface dynamics has a discontinuity at the soil surface (see, e.g., *Tamea et al.*, 2010) thus represents a physical upper bound for . In case that the cumulative probability for is non negligible, the bounded probability distribution should be used instead,

(9)

with being the upper incomplete Gamma function; this distribution expresses the probability associated to conditional on having a below-ground phreatic surface, i.e. on assuming that an above-ground position, if reached, is immediately left because of surface runoff and evapotranspiration.

The assumption of exponential decays in the phreatic surface dynamics (7) has a twofold justification. The first one is that shallow groundwater can be assumed to behave like a simple linear reservoir, where storage is proportional to discharge, and the latter is represented by the groundwater flow towards a surface water body or deep aquifers. In such case, after cessation of a rainfall event, the storage drops according to an exponential law (e.g. *Bras*, 1990, chapter 9.6) and assuming a linear dependence between storage and the phreatic surface position, the decay will correspondingly be exponential. The second justification is given by the analytical modeling of temporal evolution of the piezometric head following a local rise of the phreatic surface induced by rainfall. Considering a horizontal radial homogeneous flow field in polar coordinates, a piezometric head increase from the rest condition evolves in time as a sum of exponential functions (e.g., *Carslaw and Jaeger*, 1959, page 198), both in case of a constant initial condition over the domain, or any generic function going to zero at the boundary. Such property of the solution confirms the functional correctness of the exponential-decay assumption in (7); in fact, although different behavior of the solution are possible (even non-monotonic), there is always a time after which the decrease becomes typically exponential. Furthermore, the exponential-decay assumption and, in general, the appropriateness of the phreatic surface dynamical model (7) will find a strong support in the data analysis, as detailed in the following Section 4.1.

As a remark, the probabilistic framework here proposed is based on the assumption that phreatic surface position does not depend on the deep aquifer piezometric levels nor on leakage, since such inter-dependence would invalidate the first-order stochastic model in (7) and call for more complex models which are unlikely to be solved for an impulsive forcing such as Poisson noise. Limiting the application of the model to cases with semi-permeable layer of very low hydraulic conductivity (K), which limits the infiltration velocities and minimizes the pressure transduction, allows us to ensure the independence of the piezometric surfaces of the two aquifers, thus enabling the stochastic modeling of the problem.

3.1 PROBABILISTIC DESCRIPTION OF THE INFILTRATION FLOW

Due to the probabilistic nature of the phreatic surface fluctuations, also the flow rate infiltrating from one aquifer to the other, that is , is a probabilistic variable. Given the probability distribution of , and the monotonic relationship between the two variables (equation (6)), one can express the probability density function of the discharge through the semi-permeable layer, , as a derived distribution of , i.e.

(10)

where is the inverse function of (6). Then, by using equation (8), the long-term probability distribution of infiltration discharge becomes

(11)

where the two parameters and , are the flow rate per unit head difference and the flow rate through the semi-permeable layer in the absence of phreatic surface fluctuations, respectively. Notice that represents the minimum discharge between the aquifers in the presence of the well and that two contributions can be identified: the base flow rate and the well-driven flow rate . The probability distribution (11) has a mean value equal to and a variance equal to . In case that truncated distribution (9) is used, the probability distribution of results superiorly bounded at , corresponding to a phreatic surface coincident with the soil surface.

The probabilistic model proposed is relatively parsimonious in describing a complex problem with few variables, but it is still built on six groups of parameters describing: the phreatic surface dynamics ( ), the piezometric difference between aquifers ( ), the semi-permeable layer ( ), and the well ( ), plus the discharge . The number of functional dependencies of the probabilistic solution can be decreased by using the non-dimensional parameters obtained with the dimensional analysis (e.g., Barenblatt, 1987). Given the kinematic nature of the problem, one can chose two independent parameters, for example and , and use only these five non-dimensional groups, reducing the functional dependencies of two units,

(12)

to describe the probability distribution of the non-dimensional flow rate, i.e.

. (13)

Indicative range of values for the model parameters are indicated in Table 1 and corresponding minimum and maximum values of the non-dimensional groups, are shown in Table 2; such values represent the range considered in the sensitivity analysis of the probability distribution of which has been performed by varying one non-dimensional group at a time.

Results of the sensitivity analysis are shown in Figure 3, where darker (lighter) curves indicate smaller (larger) values of the non-dimensional groups. It is clear that the first two groups -upper panels- do not modify the stochastic nature of the process since distributions are simply translated along the horizontal axis with a mere change in the mean value of the distribution. On the contrary, the third and fourth groups -lower panels- involve parameters of the stochastic process, thus their variation leads to a change in the mean, variance and shape of the distribution, although remaining always positively skewed and not modifying the lower limit ( ). Notice that infiltration flow rate, , in the diagrams can be calculated by after specifying the values of necessary parameters.

4. DATA ANALYSIS AND RESULTS

The proposed approach is based on a number of simplifying assumptions (soil homogeneity and isotropy, independent phreatic surfaces, jump-decay dynamics of surficial aquifer, etc…), which are necessary for describing the complexity of phenomena with simple, yet informative, models. However some of these assumptions, such as those concerning the phreatic surface dynamics, may find their basis on the analysis of groundwater level measurements. We consider here three time series measured in continuous-monitoring piezometers located in the northwest of Italy, in the towns of Biella (a), Frassineto Po (b), and Basaluzzo (c) (Figure 4).

4.1 SHALLOW WATER TABLE DYNAMICS

Daily time series of phreatic surface depth, measured from the soil surface, are available from year 2004 (a-b) and 2006 (c) to 2013. In the data analysis, is set at the deepest recorded position of the phreatic surface and is used to recover the time series of fluctuations above such value. In order to avoid the effect of seasonal oscillations and recover the jump-decay structure of the underlying process, time series are de-seasonalized in the mean and in the variance according to the following expression:

$$ \frac{\quad}{\quad} \quad , \qquad (14) $$

where is the generic element of the measured series (day of the year), and are the mean and standard deviation of the day of the year, smoothed out with a 30-days moving average, and and are the mean and standard deviation of the whole series. De-seasonalized time series, , thus have the mean and standard deviation of the original series and are shown in Figure 5, while time series parameters are indicated in Table 3.

Parameters describing the stochastic properties ( ) of each series are derived from an analysis of jump magnitudes (difference between two subsequent values) and interarrival times between two jumps (in number of days). In order to ignore random disturbances in the time series, only jumps of magnitude larger than 20% of the standard deviation and time intervals longer than 2 days are considered in the analysis; thresholds are arbitrary but necessary to filter out the noisy groundwater signal. The effect of filtering on the parameter estimation is indicated in Table 3, where original unfiltered parameters and are also shown in parenthesis.

Decays of the time series are approximated with an exponential function, whose parameter ( ) has been evaluated with two approaches. The first estimate of is obtained with a least square linear interpolation of the logarithm of the longest decay in the time series; this approach is valuable if such decay is sufficiently long and representative of the general decay dynamics. As a counterpart, a different and more comprehensive evaluation is followed for the second estimate of , which involves all points belonging to decays, with the graphic construction depicted in Figure 6. Logarithms of decays are first sorted by decreasing initial value and the first –highest– curve is placed on a graphics, starting from . Then each curve, from the second to the last one, is added to the graph and translated along the time axis such that the initial point of the curve is found at the time of the closest value already on the graph. Least square linear interpolation is then performed over all points within this graphic construction and the resulting angular coefficient is the parameter wanted. Also for the analysis of decays, only time intervals greater than 2 days are considered, in order to avoid the effect of noisy oscillations. Table 3 shows the two parameter estimate (the one related to the longest curve in parenthesis) and confirms that the difference between them is minimal.

The cumulative frequency distribution of the deseasonalized time series at the three sites is finally compared with the probability distribution of the Poisson process (8) using the parameter values specified on Table 3, outside the parenthesis. The comparison is shown in the right panels of Figure 5 and the very good matching in all cases proves the appropriateness of the probabilistic model (7) for describing the surficial phreatic surface dynamics.

4.2 STOCHASTIC INFILTRATION

After testing the probabilistic model for the phreatic surface, some other parameters are required to apply the result of Section 3.1 and to obtain the probability distribution of leakage flow rate between the surficial and deep aquifers over the well area of influence.

At the sites considered, geological and pedological information are derived from reports of the local Environmental Agency and regional authorities (e.g., Regione Piemonte, 2010); the radial influence of wells in deep aquifers, , is taken from *Hamill and Bell* (1992) for a medium-grain-size sand, thus for the generic type of soil at the tree sites; transmissivity of deep aquifer is assumed based on the site characteristics, and the flow rate withdrawn from the well is given a realistic value based on experience (Table 4). All such values are taken equal at the three sites, while the uniform time-invariant piezometric head difference, , is different among the sites. Values are assumed to be positive for wells located in recharge areas (site 'a' and to a lesser extent, site 'c') and negative for wells located in the drainage area close to the Po river floodplain (site 'b'); they are taken at the minimal edge of the range of head differences that can be found in the area, according to the recharge/drainage maps of the region (Regione Piemonte, 2010).

It is worth noting that most parameters values are taken at the safest side of the range of possible values (low-permeability thick layer and low head difference between aquifers), which should provide a low estimate of the discharge infiltrating from one aquifer to the other and hence a lower impact of the stochasticity on . Results of the probabilistic model (8) are shown in Figure 7 and Table 5, the latter reporting the contributions given by the well ( ), by the time-invariant head difference between aquifers (, or base flow rate) and by the phreatic surface fluctuations ( ) through the mean and 90% quantile values.

By looking at Table 5, one can briefly comment on the contributions at the three sites. At site (a), the infiltrating flow rate between the aquifers is dominated by and, to a lesser extent, by the shallow water table fluctuations. At site (b), the base flow rate is negative, thus directed upward, and contribution given by the phreatic surface fluctuations is comparable in magnitude but opposite in sign. At site (c) the base flow rate is small but positive (downward), whereas the infiltrating flow between the aquifers is dominated by the shallow water table fluctuations. The well contribution to the overall infiltration, , is equal at all sites and it is small compared to the other contributions. The sites, thus, show marked differences with respect to leakage phenomena, despite the hydro-geological similitude (equal parameters in Table 4, apart from).

The long-term probability distribution of discharge at site (a) (Figure 7) is centered at flow rates larger than the other sites, mainly because the head difference between the two aquifers is large (intense recharge through). Similar maximum flow rates are found at site (c) where, instead, the phreatic surface dynamics has large fluctuations (see variance in Table 3) which impose a large variance to the infiltration discharge; however, at site (c) minimum discharge values lower than (a) are expected due to a lower base flow contribution. The site (b) is the lowest in altitude and it lays in the floodplain, where the head difference between aquifers, , is assumed to be negative; as a result is directed upward, the mean flow rate given by the probabilistic model (in Table 5) is negative, and the probability distribution is mainly found on the negative flow rate axis (Figure 7). Nevertheless, the well withdrawal and the phreatic surface fluctuations may turn the overall flux downward, and in fact the infiltrating discharge exceeded with a 10% probability (i.e.,) is positive and a portion of probability distribution covers positive fluxes. This can also be seen in the lower-right panel of Figure 7, where the cumulative probability distributions are depicted together; the range of possible flow rates in Figure 7 highlights the importance of accounting for stochastic variability in the quantification of infiltration between aquifers.

The relevance of the probabilistic approach is given by the variability of infiltrating flow rate and by the associated probability, as opposed to the trivial result which could be obtained by simply taking the mean phreatic surface depth and using it to compute the infiltrating flow rate. Given the linear relation (6) between the two variables, the trivial approach would result in the mean infiltrating flow rate, without indication of the range of possible values. The real advance of the probabilistic model is then given by the quantification of stochastic variability and associated probability, which may be useful in several applications (such as risk assessments or scenario evaluations) where the probability distribution matters.

Also, it should be noted that ranges of computed leakage at all sites are small, if not negligible, with respect to the flow rate withdrawn from the well ( ); this agrees with the assumption that, in the considered case, infiltration through the semi-permeable layer does not affect the groundwater flow field nor the piezometric surface of the deep aquifer. However in the long term, assuming a constant withdrawal from the well, the flow infiltrating into the deep aquifer goes inevitably into well pumping, provided that no change in storage occurs and that outflow from the area of influence of the well cannot occur with the given piezometric surface. Since the leakage from phreatic aquifer may bring chemicals and pollutants deeper in the soil, the infiltration between aquifers which is captured by the pumping well may pose big problems about the quality of water pumped.

5. CONCLUSIONS

A probabilistic model to quantify water leakage between aquifers, in the presence of a well and a fluctuating phreatic surface, is here proposed. Building on some simplifying yet realistic assumptions, which were verified analytically or basing on data, the model returns the long-term probability distribution associated to infiltrating flow rates in steady conditions (e.g., without changes in well withdrawal).

Results indicate that the stochastic component of infiltration due to random fluctuations of the phreatic surface can be relevant compared to the infiltration induced by the pumping well and it may strongly enhance the overall leakage between the surficial and the deep aquifer. In fact, although low hydraulic conductivity of the semi-permeable layer (m/s) and high values of the leakage factor (m) can induce to neglect the leakage phenomenon in the three case studies proposed, the phreatic surface fluctuations are shown to have a significant impact on the variability of the discharge infiltrating between aquifers, even with a negligible impact of leakage on the piezometric head of the deep aquifer.

Knowing the probability distribution of the infiltrating flow rate, through a given area, gives a complete description of its variability, including the variance, interquantile ranges and symmetry, and provides a full picture of the impact of rainfall randomness. The proposed model is a useful tool, for example, for the assessment of climate change impacts, because a change in the mean rainfall intensity and/or interarrival time would modify the probability distribution of infiltrating flow rate; even when maintaining a constant total average rainfall (thus a constant) the resulting infiltrating flow rate will have an unchanged mean but a different distribution. Other aspects related to water management, such as pumping limitations or pollution concerns, may also benefit from the proposed model in that, although a fair amount of information is already given by deterministic estimates of infiltrating discharges, the random forcing of rainfall, having impulsive nature and high interannual fluctuations, warns against considerations ignoring the variability of the problem and its probabilistic characteristics.

Finally, the probability distribution of infiltrating flow rates when associated to the transfer of contaminants between aquifers, can give useful indications for evaluating the risk of deep aquifer contamination, when appropriate measures considering the whole range of possible flows (and pollution transport) are used (e.g., the operational value of probabilistic predictions in Laio and Tamea, 2007).

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TABLES

α	λ	β	K_L	b_L	Δ_0	R	Q_p	T
[m]	[events/d]	[1/d]	[m/s]	[m]	[m]	[m]	[m^3/s]	[m^2/s]
0.1 - 1	0.02 - 0.08	0.005 - 0.02	10^{-8}- 10^{-7}	1 - 10	1 - 10	100 - 1000	0.01 - 0.1	10^{-3} - 10^{-1}

Table 1: typical range of parameter values

$\Pi_h = \Delta_0 / R$	$\Pi_p = Q_p / (4 \cdot \pi \cdot T \cdot R)$	$\Pi_\alpha = \alpha / R$	$\Pi_n = \lambda / \beta$
$10^{-3} – 10^{-1}$	$8 \cdot 10^{-6} – 8 \cdot 10^{-2}$	$10^{-4} – 10^{-2}$	1 – 16

Table 2: range of values for non-dimensional groups

	$\boldsymbol{h_{1,0}}$ [m]	overbar($\boldsymbol{y}$) [m]	$\boldsymbol{\sigma}^2$ [m^2]	$\boldsymbol{\lambda}$ [1/d]	$\boldsymbol{\alpha}$ [m]	$\boldsymbol{\beta}$ [1/d]
(a) Biella	13.91	2.058	0.595	0.043 (0.068)	0.506 (0.325)	0.013 (0.012)
(b) Frassineto Po	8.43	0.833	0.104	0.068 (0.127)	0.129 (0.071)	0.014 (0.013)
(c) Basaluzzo	24.34	2.711	1.888	0.064 (0.105)	0.638 (0.400)	0.021 (0.020)

Table 3: parameter values of time series of groundwater measurements in the case studies; in parenthesis: λ and α unfiltered parameters, β of the longest decay curve

$\boldsymbol{K_L}$ [m/s]	$\boldsymbol{b_L}$ [m]	**aquifer soil type**	$\boldsymbol{R}$ [m]	$\boldsymbol{T}$ [m^2/s]	$\boldsymbol{B}$ [m]	$\boldsymbol{Q_p}$ [l/s]	$\boldsymbol{\Delta_0}$ [m]
10^{-8}	4	Medium sand	500	$50 \cdot 10^{-3}$	4472	30	+3 (site a) -1 (site b) +1 (site c)

Table 4: parameter values to run the model

	Site (a)	**Site (b)**	**Site (c)**
Q_w	*0.09*	*0.09*	*0.09*
Q_0	*5.89*	*-1.96*	*1.96*
Q_f *(mean - 90% quantile)*	*3.56 - 6.08*	*1.32 - 2.10*	*4.01 - 7.01*

Table 5: contributions to the leakage over the well influence area given by the well (Q_w), by the base flow rate (Q_0) and by the phreatic surface fluctuations (Q_f); values are in l/s.

FIGURES

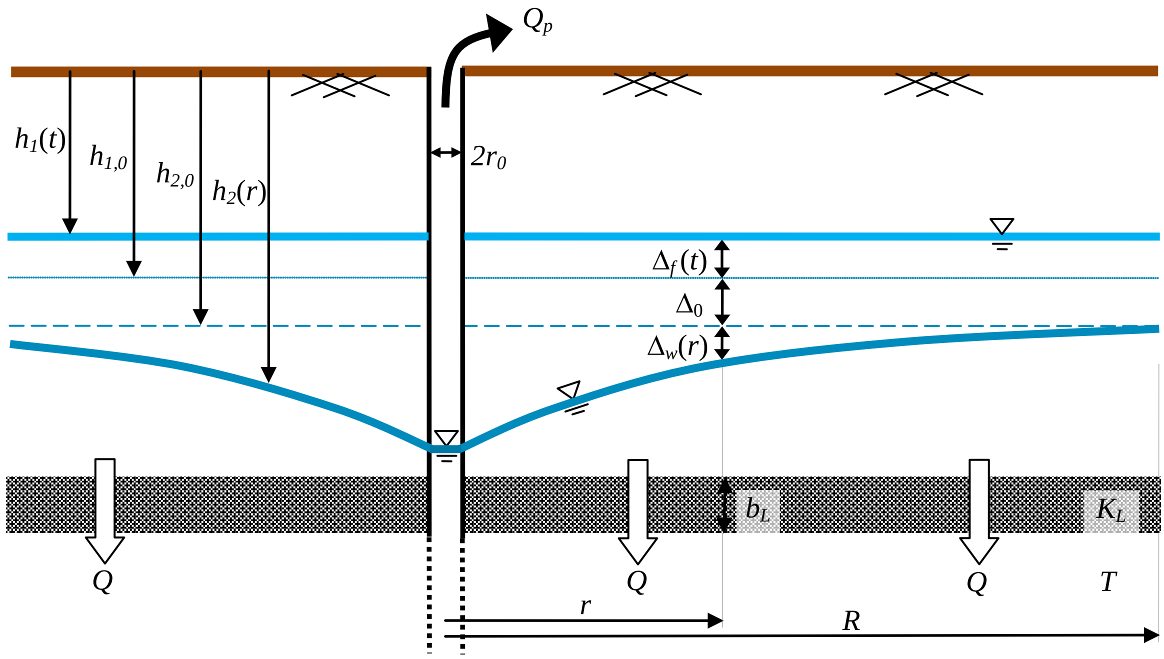

Figure 1: scheme of the two aquifers and the well, with indication of the variables introduced in the problem description.

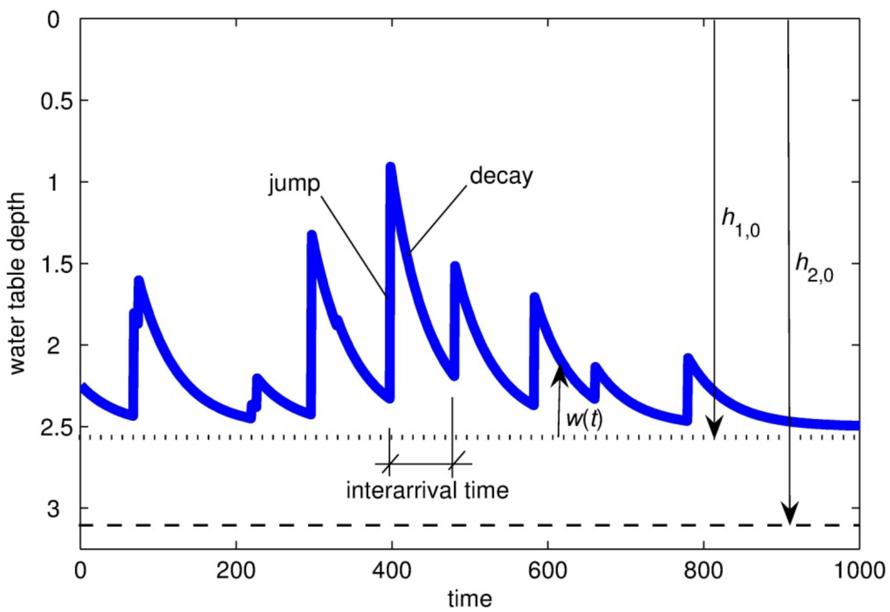

Figure 2: sketch of a jump-decay dynamics.

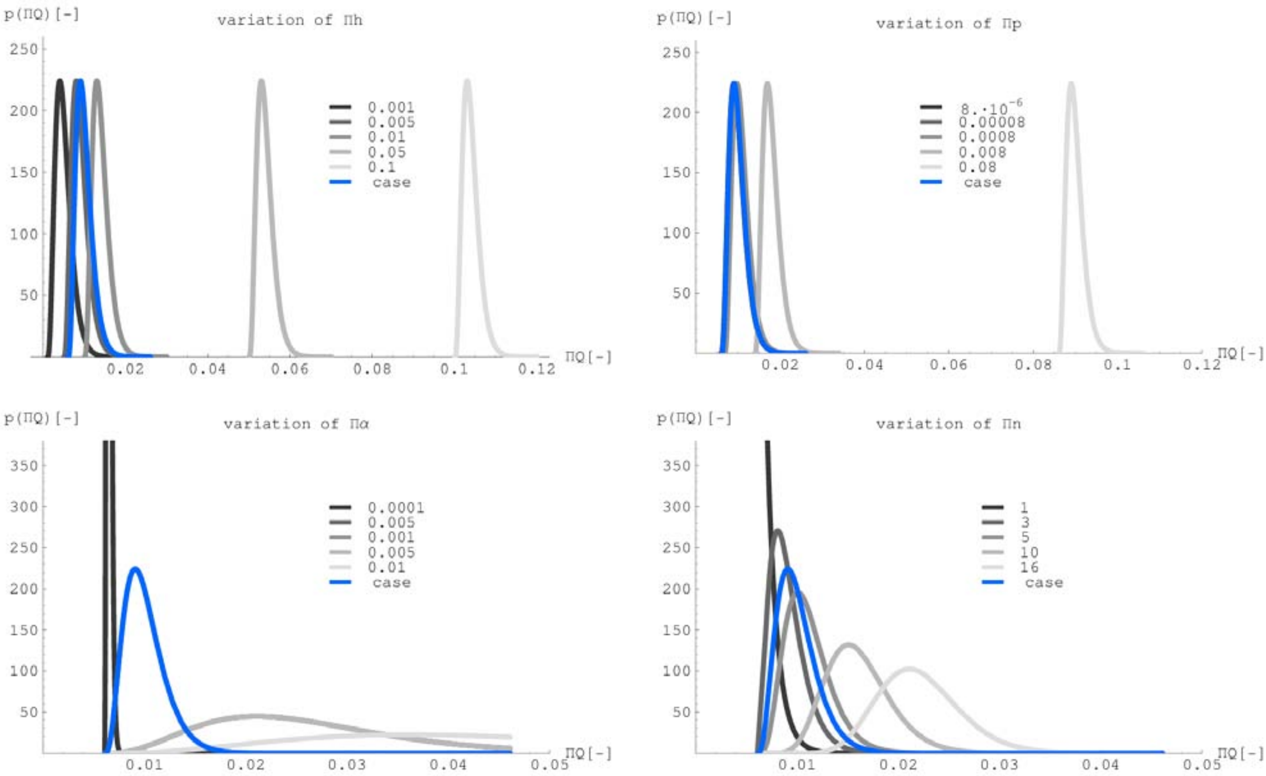

Figure 3: sensitivity analysis of the probability distribution, , of non-dimensional infiltrating flow rate; the blue line indicates a reference case with parameter values given in Table 4 and .

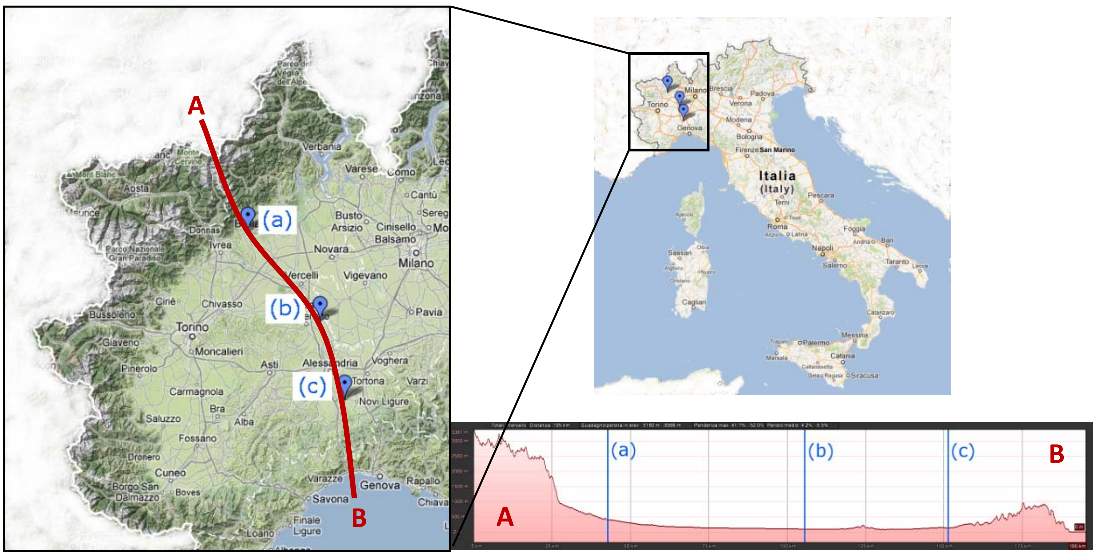

Figure 4: geographical location, in the North-West of Italy, of the three wells considered in the case studies and altimetry profile of the section AB.

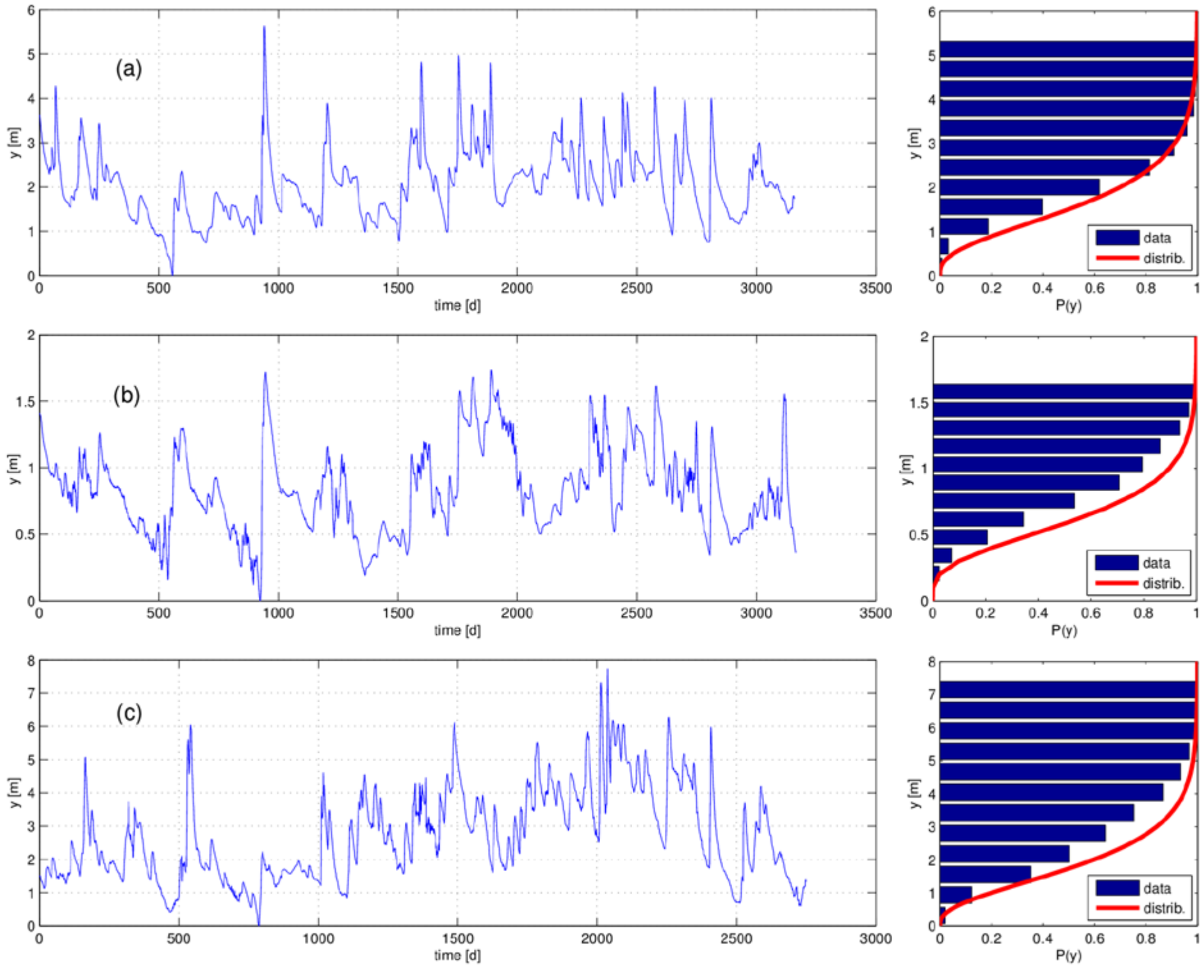

Figure 5: deseasonalized phreatic surface time series at the three sites and comparison with cumulative probability distribution of corresponding Poisson processes (parameter values given in Table 4).

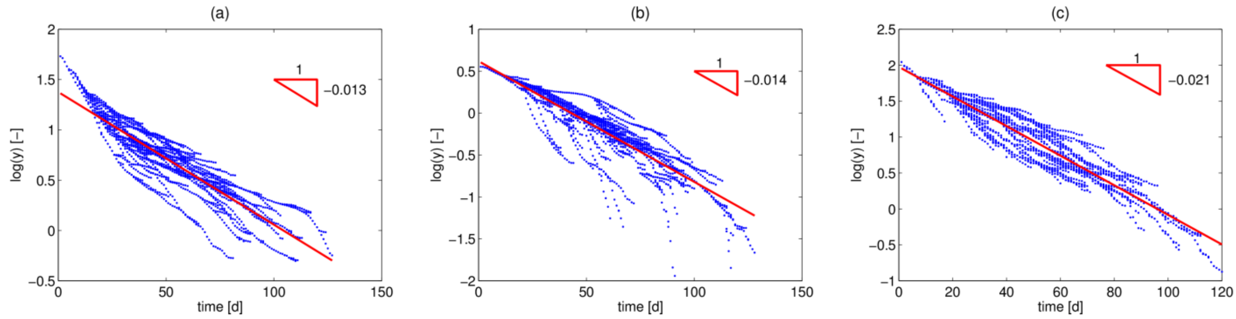

Figure 6: graphic construction of decays and corresponding linear interpolation in the logarithmic plane.

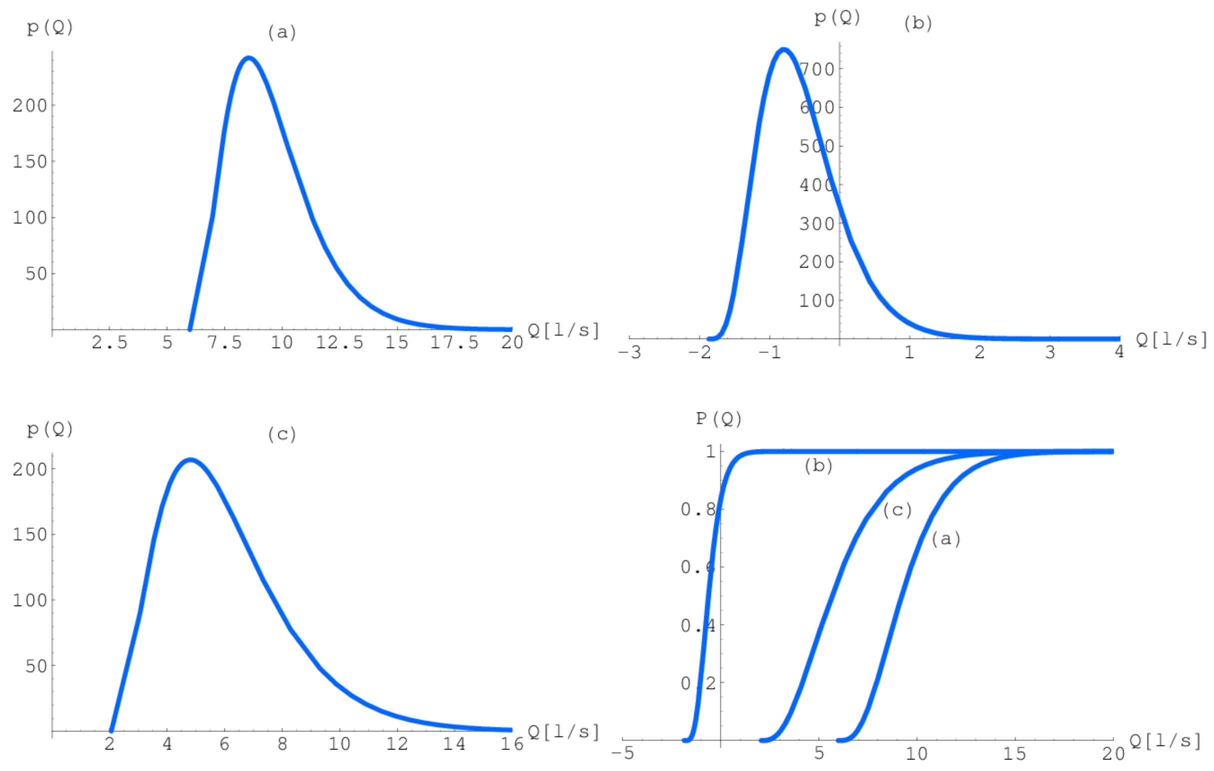

Figure 7: probability density functions of infiltrating flow rate Q at sites 'a' (upper-left), 'b' (upper-right), 'c' (lower-left) and the three cumulative probability distributions (lower-right).