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## Efficient Prediction of Stochastic Eye Patterns via Lagrange Interpolation

P. Manfredi, I. S. Stievano, and F.G. Canavero

This letter addresses the statistical assessment of system performance via the efficient estimation of eye diagram parameters under process variability. Statistical information pertaining to eye parameters is reconstructed by interpolation of a reduced set of simulations. The proposed strategy shows a remarkable efficiency improvement with respect to classical blind and brute-force sampling-based methods.

**Introduction.** The increasing speed of electronic devices and the continuous shrinking of their physical size are stressing the impact of process variations on the system performance. The reliability of data links is typically assessed via the prediction of eye patterns and their features [1], like width and opening. However, such simulations typically assume deterministic properties for the physical link. Nowadays, the accuracy of such predictions on signal integrity is unavoidably related to the capability of accounting for possible sources of variations introduced, e.g., by the manufacturing process [2], which may further degrade the link performance. In this framework, traditional repeated-run analyses such as Monte Carlo, where a multitude of different scenarios is deterministically simulated to collect statistical information, are hardly feasible. In fact, for the reconstruction of eye patterns, long sequences of pseudorandom bit streams must be considered at the driving side in order to suitably account for noise, nonlinear effects and jitter. Therefore, the cost of a single simulation for a deterministic configuration of the physical variables is already rather high. In order to overcome this limitation, in this letter we propose an efficient strategy to quickly characterize eye parameters from a statistical standpoint in a link subject to process variability. The technique is based on the Lagrange interpolation [2,3] of a synthetic set of responses, simulated for a suitable choice of points in the space of the uncertain variables.

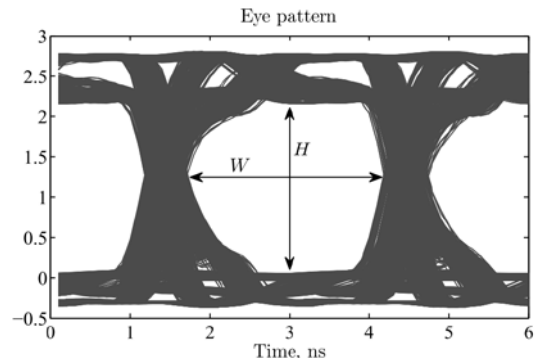
**Lagrange interpolation.** The underlying idea of Lagrange interpolation is to sample the stochastic parameters at a synthetic set of few and cleverly-chosen *collocation* points and to reconstruct the continuous behaviour in the whole random space by interpolation. Given a random output variable of interest  $y$ , depending on multiple random input parameters  $\xi = [\xi_1, \dots, \xi_n]$ , such an interpolation can be expressed as

$$y(\xi) \approx \sum_{k=0}^K y_k(\xi^{(k)}) \Phi_k(\xi) \quad (1)$$

where  $y_k$  are *deterministic* responses, computed for predefined samples  $\xi^{(k)}$  of the random variable  $\xi$ , and  $\{\Phi_k\}$  are multivariate Legendre polynomials. As such, (1) turns out to be a mere analytical (polynomial) expression allowing a fast extraction of statistical information. It is worthwhile noting that  $y$  in (1) can be any output parameter of interest, ranging from a full transient response to a synthetic eye parameter. The collocation points consist of a grid of  $K + 1 = (p + 1)^n$  nodes, where  $p$  is the sampling step per dimension, which can be increased to improve accuracy, while  $n$  is the number of random parameters. Sparse grids can be used to mitigate the increment in the number of points when  $n$  and/or  $p$  is large [4]. The multivariate Legendre polynomials are built as products of univariate ones, which are in turn defined as

$$\Phi_i(\xi) = \prod_{\substack{0 \leq j \leq p \\ j \neq i}} \frac{\xi - \xi_j}{\xi_i - \xi_j} \quad (2)$$

so that  $\Phi_i(\xi_j) = \delta_{ij}$  (Kronecker's delta). Several rules can be outlined for the selection of the grid points. We suggest here to use the zeros of the  $(p+1)$ th-order polynomial orthogonal to the distribution of  $\xi$  (e.g., Hermite or Legendre polynomials for Gaussian and uniform variability, respectively) [3]. This choice already turned out to be suitable in other applications [5]. As a result, statistical information can be fast extracted from (1) instead of running a large number of blind, iterative random simulations.



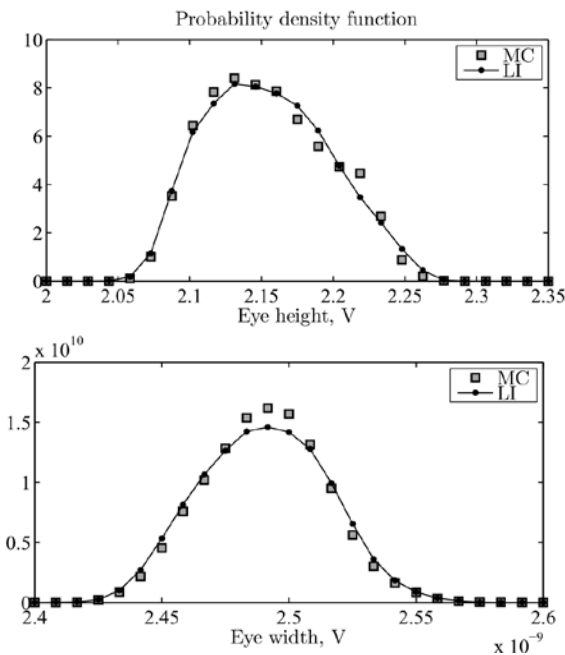
**Fig. 1** Example of eye diagram describing the performance of a realistic PCB link. The eye height ( $H$ ) and width ( $W$ ) are also indicated.

**Application example:** In order to show the strength of the proposed approach, we consider a single point-to-point PCB interconnect, linking two ICs, one on a card and one on board, via two sections of microstrip transmission lines interposed by a connector represented by a lumped LC equivalent model ( $L = 2$  nH,  $C = 2$  nF). The microstrip traces are characterised by a nominal width of 150  $\mu\text{m}$  and a copper thickness of 30  $\mu\text{m}$ , whilst the board substrate has a thickness of 100  $\mu\text{m}$ , a relative permittivity of 4.1, and a loss tangent of 0.02. The link is driven by a Texas Instruments transceiver (model name SN74ALVCH16973, power supply voltage  $V_{DD} = 2.5$  V), whose HSPICE transistor-level description is available from the vendor. Finally, the input ports of the receiving IC are modelled by means of a 10-pF capacitor and clamp diodes.

Without loss of generality, we consider an equivalent circuit model for the driver in place of its transistor-level description. This is in order to make the repeated-run analysis feasible, thus providing reference results for comparison. Different approaches can be used for generating IC models. One standard solution is to use the Input/Output Buffer Information Specification (IBIS). Recently, other approaches complementing the IBIS and providing good accuracy for recent device technologies have been proposed (see, e.g., [6]). In this example, we use the modelling methodology presented in [6], where the model parameters are computed via a well-established procedure from port voltages and current responses to a predefined set of stimuli. The demonstrated accuracy of such models in the prediction of eye diagrams with respect to transistor-level simulations is on the order of 2%.

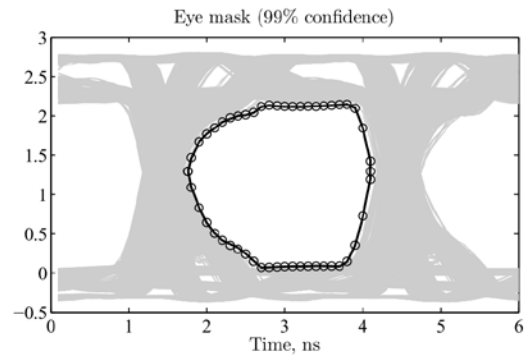
In our simulation, the link variability is provided by the strength of the driver (10% relative standard deviation), the width of the PCB microstrips (5% deviation), and the relative permittivity of their dielectric substrate (5% deviation), all modelled as three independent Gaussian random parameters. This choice is of practical interest since it allows to model typical fluctuations occurring in the output currents of different manufactured IC buffers, as well as the randomness introduced by the etching process and the impurities in substrate materials. Fig. 1 illustrates the eye diagram at the receiver side of the described link, computed via HSPICE simulations and by considering a 1000-bit data sequence with a bit time of 3 ns and a Gaussian jitter with 0.15-ns standard deviation, as well as a limited set of realisations of the random link parameters. In presence of random link variations, the eye opening  $H$  and width  $W$  become random themselves and require therefore a statistical characterisation.

For instance, Fig. 2 shows the probability distribution of the eye height and width resulting from the variability of the link properties. The curve labelled as “MC” refers to the result obtained with a 10000-run Monte Carlo analysis (a large number of simulations is necessary for an accurate prediction of distribution functions) performed with the available feature in HSPICE, which took about 16 hours. We recall here that a faster model was used for the driver in place of the transistor level in order to speed-up this analysis. The line labelled as “LI” is instead obtained by interpolating a limited collection of 64 eye data, whose simulation only took 5 min and 47 s. An excellent accuracy is established, and a remarkable speed-up of 160× is achieved. Based on the information in Fig. 2, it is possible to estimate that, for the considered link, with a probability of 99% the eye height will be larger than 2.0652 V (the repeated-run analysis predicts a value of 2.0666 V, with an error which is well below the accuracy achieved using such a large number of Monte Carlo samples). Analogously, the predicted minimum eye width with a 99% confidence level is 2.4321 ps (compared to 2.4337 ps obtained with the repeated-run analysis).



**Fig. 2** Probability density functions of the eye height (top panel) and width (bottom panel) resulting from the random nature of the physical link. The squares denote the information obtained via 10000 Monte Carlo simulations, whilst the dots indicate the prediction provided by the proposed technique.

Following a similar reasoning, it is possible to estimate an eye opening profile with a confidence level of 99%. This is shown in Fig. 3, where the result of the proposed technique (solid line) is compared with that of the repeated-run analysis (circles), exhibiting again very good agreement. Briefly speaking, the information in Fig. 3 provides the following valuable information to the designer: there is only a 1% chance that the stochastic link response will lie inside the displayed mask.



**Fig. 3** Eye diagram of Fig. 1, with the inclusion of the profile of the eye opening computed with a confidence level of 99%. Solid line: prediction via the proposed technique; circles: profile obtained via Monte Carlo simulations.

**Conclusions.** In this letter, we propose an efficient strategy for the statistical characterisation of eye diagram parameters in electrical links affected by variability of their physical parameters. Instead of running repeated simulations for large sets of uncertain parameters, a limited set of simulation outcomes is suitably interpolated by means of Lagrange polynomials. Such simulations are performed based on a clever grid of nodes in the random space. The proposed strategy exhibits excellent accuracy and remarkable speed-up with respect to traditional blind sampling-based methods.

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