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Abstract

This paper provides and compares two alternative solutions for the simulation of cables and interconnects with the inclusion of the effects of parameter uncertainties, namely the Polynomial Chaos (PC) method and the Response Surface Modeling (RSM). The problem formulation applies to the telegraphers equations with stochastic coefficients. According to PC, the solution requires an expansion of the unknown parameters in terms of orthogonal polynomials of random variables. On the contrary, RSM is based on a least-square polynomial fitting of the system response. The proposed methods offer accuracy and improved efficiency in computing the parameter variability effects on system responses with respect to the conventional Monte Carlo approach. These approaches are validated by means of the application to the stochastic analysis of a commercial multiconductor flat cable. This analysis allows us to highlight the respective advantages and disadvantages of the presented methods.

1. Introduction

The constant and rapid pace of technological innovation today has produced increasingly complex electronic devices to such an extent that they are often physically separated into several sub-devices and then connected together. The integrity of signals propagating on interconnections is then a fundamental point for the smooth functioning of the overall system. Cable bundles represent one of the most common means by which modern electronic systems and subsystems are interconnected. A large variety of examples exists, ranging from transportation vehicles (cars, aircrafts, ships) to Information Technology equipment and to industrial plants. The electromagnetic interaction among closely spaced individual wires induces disturbances in all other adjacent circuits. This crosstalk can cause functional degradation of the circuits at the ends of the cable. The magnitude of the electromagnetic interference varies significantly as a function of a number of factors including the wires geometries.

The sensitivity of crosstalk to random wires position in the cable has led to several probabilistic models for the crosstalk according to the frequency ranges. Instead of using brute-force Monte Carlo (MC) method, some alternative solutions based on the derivation of pseudo-analytical expressions for the statistical parameters of the responses of distributed systems have been proposed so far [1]. However, their principal limitation is related to their scarce flexibility and restriction to the particular structures and output variables for which they have been derived. Possible complementary methods based on the optimal selection of the subset of model parameters in the whole design space have also been proposed [2]. However, these methods become inefficient when applied to the analysis of complex realistic structures, since they require a large set of simulations with different samples of the random parameters.

Recently, an effective solution based on the so-called polynomial chaos (PC) has been proposed to overcome the previous limitations. This methodology is based on the representation of the stochastic solution of a dynamical circuit in terms of orthogonal polynomials. For a comprehensive and formal discussion of PC theory, the reader is referred to [3] and references therein. This technique has been successfully applied to several problems in different domains, including the extension of the classical circuit analysis tools, like the modified nodal analysis (MNA), to the prediction of the stochastic behavior of circuits [4]. However, so far, the application has been mainly focused on the gaussian variability of model parameters and limited to dynamical circuits consisting only of lumped elements. The authors of this contribution have recently proposed an extension of PC theory to distributed structures described by transmission-line equations [5], also in presence of uniform random variables.

The main drawback of PC is related to the reduction of its efficiency when the number of random variables increases. A possible solution consists in performing prelimary tests to identify the most influential variables to be included in the model. Yet, an approach based on the Response Surface Modeling (RSM) is also possible and presented in this paper as an alternative to PC, followed by a comparison between the two methods. The RSM is based on the fitting of a system response using polynomial terms, whose coefficient are computed in a least-square sense starting from a reduced set of samples. In order to be validated and compared, the advocated techniques are applied to the stochastic analysis of a commercial multiconductor flex-cable used for the communication between PCB cards.

2. Variability via Polynomial Chaos

The idea underlying the PC technique is the spectral expansion of a stochastic function (intended as a given function of a random variable) in terms of a truncated series of orthogonal polynomials. Within this framework, a function H, that in our specific application will be the expression of the parameters and the resulting frequency-domain response of an interconnect described as a transmission line, can be approximated by means of the following truncated series

$$H(\xi) = \sum_{k=0}^{P} H_k \cdot \phi_k(\xi), \tag{1}$$

where $\{\phi_k\}$ are suitable orthogonal polynomials expressed in terms of the random variable ξ. The above expression is defined by the class of the orthogonal bases, by the number of terms P (limited to the range $2 \div 5$ for practical applications) and by the expansion coefficients H_k. The choice of the orthogonal basis relies on the distribution of the random variables being considered. The tolerances given in product documentation and datasheets are usually expressed in terms of minimum, maximum and typical values. Since the actual distribution is generally unknown, a reasonable assumption is to consider the parameters as random variables with uniform distribution between the minimum and maximum values. Readers are referred to [5] for additional details on the application of PC to transmission lines.

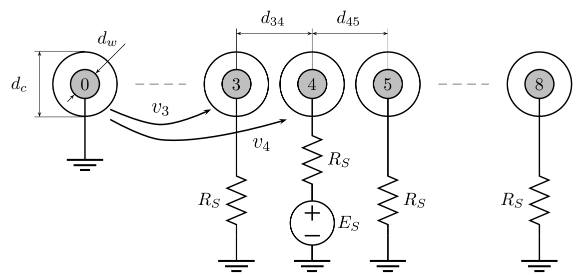
3. Variability via Response Surface Modeling

Although PC provides an accurate stochastic model, even at high frequencies, the amount of time taken by the overhead and by the solution of the augmented system rapidly grows with the number of polynomial terms. Hence, the indiscriminate inclusion of any possible random variable in the PC model may be critical for this method and should be avoided. The variables should be carefully chosen among the most influential instead. Nonetheless, an alternative and effective method for the inclusion of a higher number of random variables exists and it is provided by the RSM.

The Response Surface Model [6] is a polynomial function which approximates the input/output behaviour of a complex system; the model is a non-linear equation constructed by fitting observed responses and inputs via a least-square fitting technique and it is used to predict the system output in response to arbitrary combinations of input variables. In our specific application, the second-order bivariate RSM of the magnitude of a transfer function $H(j\omega)$, analogous to (1), is composed of 6 terms and takes the following form:

$$\left|H\left(j\omega\right)\right|_{\mathrm{dB}} = \left|H_0\left(j\omega\right)\right|_{\mathrm{dB}} + \beta_1\left(\omega\right)\xi_1 + \beta_2\left(\omega\right)\xi_2 + \beta_{11}\left(\omega\right)\xi_1^2 + \beta_{22}\left(\omega\right)\xi_2^2 + \beta_{12}\left(\omega\right)\xi_1\xi_2, \tag{2}$$

where β_0 is set equal to the nominal transfer function $\left|H_0\left(j\omega\right)\right|_{\mathrm{dB}}$ without any effect of parameter variability. The remaining five terms have to be estimated through a least-square fitting technique based on the solutions of the transmission-line equation computed for the values of the input variables specified by the sampling plan. Extension to arbitrary multivariate models is straightforward. It is worth noting that the system response and therefore model coefficients are frequency-dependent, hence a least square problem has to be solved for each frequency point. The



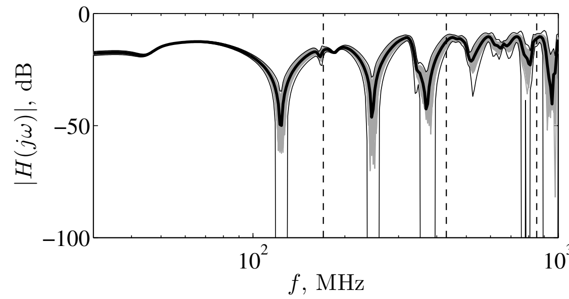


Figure 1: Application test structure: 80 cm long commercial flex cable. $R_S = 50\,\Omega$, $d_w = 15$ mils, $d_c = 35$ mils. Distance between adjacent wires $d_{ij} = 50$ mils.

Figure 2: Bode plots (magnitude) of the near-end crosstalk transfer function $H(j\omega)$. Solid black thick line: deterministic response; gray lines: a sample of responses obtained by means of the MC method.

choice of normalized random variables with support $[-1, 1]$ as inputs and of the magnitude of the transfer function in dB as output reduces the variation of the fit coefficients, thus avoiding numerical instabilities in the model. However, a Response Surface Model for the estimation of the linear magnitude or phase may be created as well.

Once their coefficients are determined, both PC and RSM represent analytical functions of the random variables, that can be used to compute the PDF of $|H(j\omega)|_{\mathrm{dB}}$ through standard techniques [7].

4. Validation

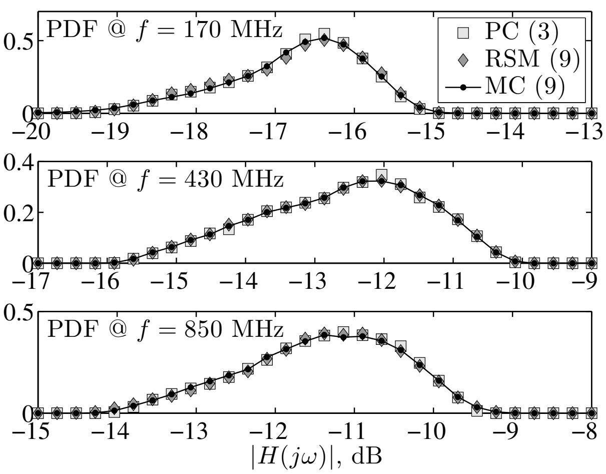


Figure 3: Comparison of the probability density functions of $|H(j\omega)|$ computed with PC (third-order expansion with 3 variables), RSM (second-order model with 9 variables) and MC (40000 runs, involving the same variables of the Response Surface Model).

As a proof of the capabilities of the proposed techniques, the analysis of the test structure depicted in Fig. 1 is presented. The structure represents a .050" High Flex Life Cable (28 AWG Standard, PVC) in a 9-wire configuration. Figure 1 collects both the key parameters defining the geometry of the wires as well as the information on the two-terminal circuit elements connected at the near-end of the cable. The cable length is 80 cm and the far-end terminations are defined by identical RC parallel elements ($R = 10\,\mathrm{k}\Omega$, $C = 10\,\mathrm{pF}$) connecting the wires #1,...,#8 to the reference wire #0.

Fig. 2 shows the response variability of $H(j\omega) = V_3(j\omega)/E_S$, defining the near-end crosstalk between two adjacent wires in a bundle of many wires. The following tolerance limits, available from the official datasheet of the cable, have been used for the bundle parameters: separation between wires $d_{ij} \in [48, 52]$ mils; overall radius of each wire including the dielectric coating $r_{c,i} \in [16, 19]$ mils; permittivity of the PVC dielectric coating $\varepsilon_r \in [2.9, 4.1]$.

In order to reduce the number of random variables included in the PC model, a reasonable choice is to assume that only the separations between the generator and the two adjacent wires are effective on crosstalk, as well as the permittivity. Therefore, the variability is considered to be provided by the relative permittivity ε_r of the coating and the separations d_{34} and d_{45} between the active and its immediately adjacents lines. These quantities are assumed to behave as independent uniform random variables lying in the aforementionated ranges. All the other parameters are considered to be equal to their nominal values.

Furthermore, a RSM of $|H(j\omega)|_{\mathrm{dB}}$ is built considering 9 random variables as inputs, in order to include the variability of each wire-to-wire separation d_{ij}, as well as of the relative dielectric constant. The resulting polynomial function needs 55 terms, whose fit coefficients are estimated from the evaluation of 250 samples.

A quantitative prediction of the crosstalk variability can be appreciated in Fig. 3, comparing the PDF of $|H(j\omega)|$ computed for different frequencies with the distributions obtained via the analytical PC and RSM expressions. The frequencies selected for this comparison correspond to the dashed lines shown in Fig. 2. The good agreement between the curves allows us to conclude the following: (i) both the approches provide accurate results in reproducing the tails and the large variability of non uniform shapes of the reference distributions; (ii) RSM is indeed capable of handling a larger number of variables w.r.t. PC and (iii) the limited set of variables included in the PC model represented a smart choice.

5. Conclusions

This paper presents two alternative methods enabling to compute quantitative information on the sensitivity to parameters uncertainties of complex distributed interconnects described by multiconductor transmission-line equations.

PC is based on the expansion of the voltage and current variables into a sum of a limited number of orthogonal polynomials. It is shown that it provides very high accuracy when compared to conventional solutions like Monte Carlo in the evaluation of statistical parameters, even at high frequencies. Moreover, PC allows to build a stand-alone (augmented) model describing an interconnect affected by parameters variability. This model can be reused when simulating different test conditions, such as different loads and line lengths. However, it suffers from a loss of computational efficiency when the number of included random variables is raised.

RSM represents an alternative solution to overcome the previous limitation and it is based on a polynomial fitting of the desired output variables in a least-square sense. Yet, the model is limited to the specific conditions for which it is computed, and it needs to be re-built whenever the loads or the line length change. Typically, it is less accurate since some interaction terms are neglected to limit the amount of samples required.

Both methods have been applied to the stochastic analysis of a commercial multiconductor flex cable with uncertain parameters described by independent uniform random variables. The PC and RSM computation of the curves of Fig. 3 turns out to be faster by a factor ranging between 30 and 150 with respect to MC computation. This holds even if for fairness we consider the computational overhead required by the generation of the proposed models. This comparison confirms the strength of the proposed methods, that allow to generate accurate predictions of the statistical behaviour of a realistic interconnect with a great efficiency improvement.

6. References

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