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# Detecting mode-shape discontinuities without differentiation – examining a Gaussian process approach

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**Abstract.** Detecting damage by inspection of mode-shape curvature is an enticing approach which is hindered by the requirement to differentiate the inferred mode-shape. Inaccuracies in the inferred mode-shapes are compounded by the numerical differentiation process; since these small inaccuracies are caused by noise in the data, the method is untenable for most real situations. This publication proposes a new method for detecting discontinuities in the smoothness of the function, without directly calculating the curvature i.e. without differentiation. We present this methodology and examine its performance on a finite element simulation of a cracked beam under random excitation. In order to demonstrate the advantages of the approach, increasing amounts of noise are added to the simulation data, and the benefits of the method with respect to simple curvature calculation is demonstrated.

The method is based upon Gaussian Process Regression, a technique usually used for pattern recognition and closely related to neural network approaches. We develop a unique covariance function, which allows for a non-smooth point. Simple optimisation of this point (by complete enumeration) is effective in detecting the damage location. We discuss extensions of the technique (to e.g. multiple damage locations) as well as pointing out some potential pitfalls.

## 1. Background

In recent years there has been a significant effort to employ vibration-based inspection methods for identifying various forms of structural damage; for example, by judiciously assessing the eigen-parameters of a damaged structure, it should be feasible to detect, locate, and even quantify the extent of damage [1].

A survey on the use of natural frequency changes on for damage detection is presented in [2] where it is concluded that the shift in natural frequencies has some important practical limitations. Instead the mode-shapes have been found to be better damage indicators due to the spatial information with respect to location of damage that they can provide. Although the displacement mode-shape is generally not sufficiently sensitive to weak damage [3], recently new methods based on spatial wavelet analysis have emerged as a potential instrument to overcome this problem thanks to their high-resolution: Indeed, although local changes in the mode-shapes due to cracks or defects usually are not obvious, it may be possible to identify singularities in the signal by applying the wavelet transform to the mode-shape. [4]-[5].

As an alternative to the mode-shapes, the modal curvatures have been recognized to be potentially higher quality damage indicators due to their superior capability in localising minor

damage. In particular, in correspondence with the damage, the curvature exhibits a peak that can be detected relatively easily by subtracting the curvature of a undamaged beam from that of the damaged structure, as shown by Pandey et al. [6] the first to introduce the method; however two main drawbacks to this method exist:

- the use of the second-order central difference approximation of Laplace operator to evaluate the modal curvature is very sensitive to noise, amplifying greatly the errors in the mode-shape which may originally have been relatively small [7];
- a good estimation of modal curvature requires high spatial resolution of the measured mode-shape [8].

The latter drawback can be effectively overcome with the use of advanced measurement instrumentation, such as a scanning laser vibrometer (SLV) [9], [10], capable of acquiring data in a very large number of observation points. However, the first drawback relative to the numerical differentiation of mode-shapes still remains a crucial limitation in the applicability of modal curvature in structural damage identification.

## 2. Damage identification methodology

Our damage identification methodology comprises two parts: first, a modal decomposition of the captured data is performed and the mode-shapes calculated; subsequently, Gaussian Process Regression (GPR) is applied to the mode-shapes with a special covariance function designed to identify damage. The marginal likelihood of the GPR can then easily be analysed to compare damaged and un-damaged models, and to identify damage location. At no point in our methodology need the mode-shapes be differentiated.

### 2.1. POD background

Proper orthogonal decomposition (POD) is a method for analysing multiple degree of freedom data. Its application to structural damage identification was proposed [11], and further studies were conducted in e.g. [12]. In previous work, the authors have shown the POD is equivalent to Principal Component Analysis (PCA), and also that the resulting decomposition is a *rotation* of the modal analysis (MA) decomposition [13].

The POD methodology can be outlined as follows:

- (i) An excitation force is applied to the structure, and the response is recorded by an array of sensors.
- (ii) The correlation matrix  $R$  of the data is formed.
- (iii) An eigenvalue decomposition of  $R$  is performed: the eigenvalues are known in the literature as ‘Proper Orthogonal Values’ (POV) and the eigenvectors as ‘Proper Orthogonal Modes’ (POM).
- (iv) The POMs are organised by order of decreasing POV.
- (v) Usually, only the first few POMs are considered (in some cases, only the first one). In this publication, we have considered the first two POMS.

Once the POMs have been extracted from the data, we apply a Gaussian Process Regression (GPR) technique as detailed below. GPR contributes the crux of our methodology: similar methods could equally be applied to mode-shapes recovered by modal analysis or any other similar technique.

## 2.2. GPR background

Gaussian Process regression was proposed some time ago by [14], and was popularised in the machine learning community by authors such as [15] and [16].

The premise is simple: assume that the values of a function are jointly Gaussian-distributed. Let  $f(\mathbf{x})$  be the value of some unknown function at the points  $\mathbf{x}$ , and  $f(\mathbf{x}^*)$  be the value of the same function at the points where we would like to regress (interpolate). The probability function of a vector containing all of these points is:

$$p\left(\begin{bmatrix} f(\mathbf{x}) \\ f(\mathbf{x}^*) \end{bmatrix}\right) = \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} K_{x,x} & K_{x,x^*} \\ K_{x^*,x} & K_{x^*,x^*} \end{bmatrix}\right) \quad (1)$$

The common convention of a zero-mean prior has been used for simplicity. Predictions for the function  $f$  at unknown points  $\mathbf{x}^*$  can be easily made by marginalising the joint Gaussian - this can be done in closed form (see [15]).

$$p(f(\mathbf{x}^*)|f(\mathbf{x})) = \mathcal{N}(K_{x^*,x}K_{x,x}^{-1}f(\mathbf{x}), K_{x^*,x}K_{x,x}^{-1}K_{x,x^*}) \quad (2)$$

Note that in the context of this publication, interpolation is not the goal: rather we wish to find a Gaussian Process which is good at (in a probabilistic sense) interpolating the mode-shapes.

The crux of GP regression lies in the *covariance function*: the values of the positive-definite covariance matrices  $K$  are related to the argument of the function,  $\mathbf{x}$  by some covariance function  $k$ :

$$K_{x,x}[i, j] = k(x_i, x_j) \quad (3)$$

There are a large number of covariance functions available. In this publication, we will make use of the polynomial covariance function:

$$k_{poly}(x_i, x_j) = \alpha(1 + x_i x_j)^N + \delta_{i,j}\beta \quad (4)$$

and the radial-basis-function (RBF) covariance function:

$$k_{rbf}(x_i, x_j) = \alpha \exp\{-\gamma(x_i - x_j)^2\} + \delta_{i,j}\beta \quad (5)$$

where the parameters  $\alpha, \gamma, N$  and  $\beta$  control the amplitude and length-scale (roughness) or polynomial degree of the function  $f(x)$ , and  $\beta$  again controls the noise. The parameters can be easily optimised via the *marginal likelihood*. This can be calculated in closed form:

$$\ln p(f(\mathbf{x})|\mathbf{x}, \theta) = -0.5 \ln |K_{x,x}| - 0.5 f(\mathbf{x})^\top K_{x,x}^{-1} f(\mathbf{x}) - 0.5 N \ln(2\pi) \quad (6)$$

It is common practise to optimise the parameters using a gradient based optimiser. After optimisation, the marginal likelihood is the key object in detecting damage in our methodology. In the next section, we shall propose a covariance function which models a discontinuity in the function  $f$ : it is through comparing the likelihood of GPR with this covariance function and GPR with a smooth covariance function that damage will be identified.

## 2.3. A discontinuous covariance function

In order to model data which has some discontinuity, we introduce to the covariance function a new parameter  $x_d$ , representing the position of the discontinuity. Thus the covariance function becomes

$$k_d(x_i, x_j) = \begin{cases} k_{poly}(x_i, x_j) + k_{rbf}(x_i, x_j) & x_i < x_d, x_j < x_d \\ k_{poly}(x_i, x_j) + k_{rbf}(x_i, x_j) & x_i > x_d, x_j > x_d \\ k_{poly}(x_i, x_j) & \text{otherwise} \end{cases} \quad (7)$$

The above formulation states that if  $x_i$  and  $x_j$  are both smaller or both larger than  $x_d$ , then the covariance function is a sum of the RBF and polynomial covariance functions. If  $x_i$  and  $x_j$  are on ‘either side’ of the point  $x_d$ , then the covariance function consists of only the polynomial terms. In all cases, the noise term  $\beta$  is shared.

The parameter  $x_d$  can be optimised using the marginal likelihood in a similar way to the other parameters, though a gradient based approach is impossible: in this work we present the (log) marginal likelihood as a function of  $x_d$ : where a peak occurs, damage is implicated.

### 3. Case study: application to a cracked beam

In order to establish the validity of the damage location method a case study was performed involving a cracked cantilevered beam that behaves non-linearly [17]. An advantage of the method is that, since an a priori model is not required, it can be applied indifferently to linear and non-linear structures.

In the simulation it was supposed that the damage affected just the stiffness matrix of the element containing the crack and not the global mass and the damping matrices  $\mathbf{M}$  and  $\mathbf{C}$ . Undamaged sections of the beam were modelled by Euler-type finite elements with two nodes and two degrees-of-freedom (transverse displacement and rotation) at each node. For the section with the crack the finite element proposed in reference [18] has been used.

In order to model accurately the non-linear behaviour of the beam it is necessary to determine the precise moment that the beam changes state, ie. when the crack opens or closes. In the results presented, it has been assumed that the change between fully-open and fully-closed takes place instantaneously, giving rise to a bilinear-type stiffness non-linearity.

When the crack closes and its interfaces are completely in contact with each other, the dynamic response can be determined directly using the global stiffness matrix of the uncracked beam  $\mathbf{K}_u$ . However, when the crack opens, the stiffness matrix of the cracked element must be introduced in replacement at the appropriate rows and columns of the global stiffness matrix  $\mathbf{K}_d$ .

In the numerical simulation, the change of state is imposed in terms of the beam curvature at the cracked section: the crack is assumed to be open if the curvature is in the positive sense, otherwise it is closed. Under the action of the excitation force, alternate crack opening and closing causes the equations of motion of the cracked beam to be non-linear:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F} \quad (8)$$

where,

$$\mathbf{K} = \mathbf{K}_u - \delta\Delta\mathbf{K} \quad (9)$$

and by denoting the changes in the global stiffness matrix due to the crack:

$$\Delta\mathbf{K} = \mathbf{K}_u - \mathbf{K}_d \quad (10)$$

with:

$$\delta = \begin{cases} 1 & ; \text{ when the crack is open} \\ 0 & ; \text{ when the crack is closed} \end{cases}$$

For the numerical simulations presented, the non-linear equations of motion for the cracked beam rewritten in an incremental form have been solved with an implicit time integration scheme and modified Newton iteration according to Bathe [19].

The cantilever steel beam, with length of  $0.7m$  and cross-section of  $20 \times 20mm^2$ , was subdivided in 50 elements. For the beam material, a Young’s modulus of  $2.06e11N/m^2$ , density of  $7.85kg/m^3$  and damping ratio of 2% was assumed.

In order to generate data for the application of the damage detection procedure presented, a conventional random test was simulated using the finite element model of the beam and applying

at its free end a transverse force with a random amplitude (for this purpose, the MatLab function `randn` was used). In the simulations performed, transverse accelerations and displacements were calculated in 50 points equally spaced along the beam, choosing a timestep equal to the inverse of the 40th natural frequency of beam structure (approximately 150KHz).

An assortment of damage scenarios was examined: the crack depth  $a$  was varied from 0 to  $0.4h$  (where  $h$  is the depth of the beam) in steps of  $0.1h$ , and the crack was considered at locations corresponding to  $0.2L$  (where  $L$  is the length of the beam) and  $0.4L$ . Detecting discontinuities is a trivial task when dealing with simulated data: it is the noise which occurs in real-world testing which provides a significant challenge. To this end, we added artificial noise to our data at various signal-to-noise ratios (SNR), again utilising the `randn()` function. We considered SNRs of 50,100,500 and 1000, as well as examining the data with no noise at all (SNR= $\infty$ ).

### 3.1. Results and discussion

Figure 1 shows the results of our method applied to the data with a fracture simulated at a position 20% of the length of the beam. Each sub-plot shows the marginal likelihood for our model as a function of  $x_d$ , the specified discontinuity position. Each column of plots has the same crack length (as indicated by at the top of each column), and each row has the same SNR, as indicated to the left.

Examine for a moment the top row of figure 1. In this case, the method has been applied to the data with no noise added, for various crack depths. In each case where the crack depth is not zero, we observe a spike in the log marginal likelihood of our model at a position of approximately 20% of the beam length: the model is detecting the damage here. We confirm similar results when the model is applied to similar data with the crack positions at 40% of the beam length (Figure 3).

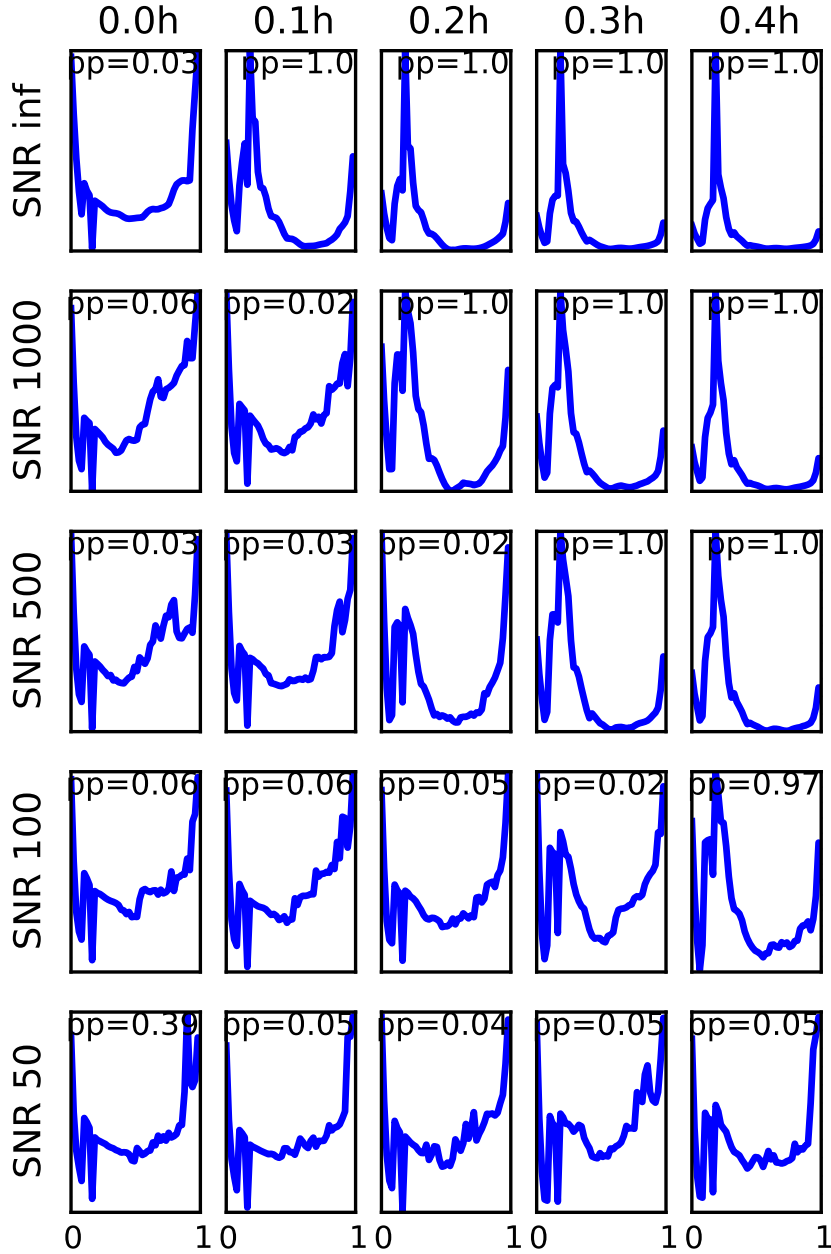
It is interesting to note that the ‘tails’ of the plot rise (towards  $x_d = 0, 1$ ). Indeed, for the case where no damage is present, the tails rise much higher than the remainder of the plot. The explanation for this is simple: when  $x_d$  moves toward the ends of the beam, the majority of the data are being modelled as smooth, with no discontinuity. When  $x_d$  reaches the end of the beam, *all* of the data is modelled as smooth. For the case with no noise, it is natural that a totally smooth model should be the best. This observation gives rise to a simple method for calculating the posterior probability of damage, by model comparison.

As well as the log marginal likelihood, each plot in Figures 1 and 3 also contains an indication of the posterior probability of fracture (pp, shown numerically on each plot). This has been obtained by comparing the our discontinuous model with a smooth model, obtained simply by fixing  $x_d = 0$ . We assumed a prior probability of fracture  $p(f) = 0.5$ , and numerically marginalising  $x_d$ , obtained a posterior probability of fracture by Bayes’ rule:

$$\begin{aligned}
 p(\mathcal{D}|f) &= \int_{x_d} p(\mathcal{D}|x_d, \theta) dx_d \\
 p(\mathcal{D}|\bar{f}) &= p(\mathcal{D}|x_d = 0, \theta) \\
 p(f|\mathcal{D}) &= \frac{p(\mathcal{D}|f)p(f)}{p(\mathcal{D}|f)p(f) + (1 - p(f))p(\mathcal{D}|\bar{f})}
 \end{aligned} \tag{11}$$

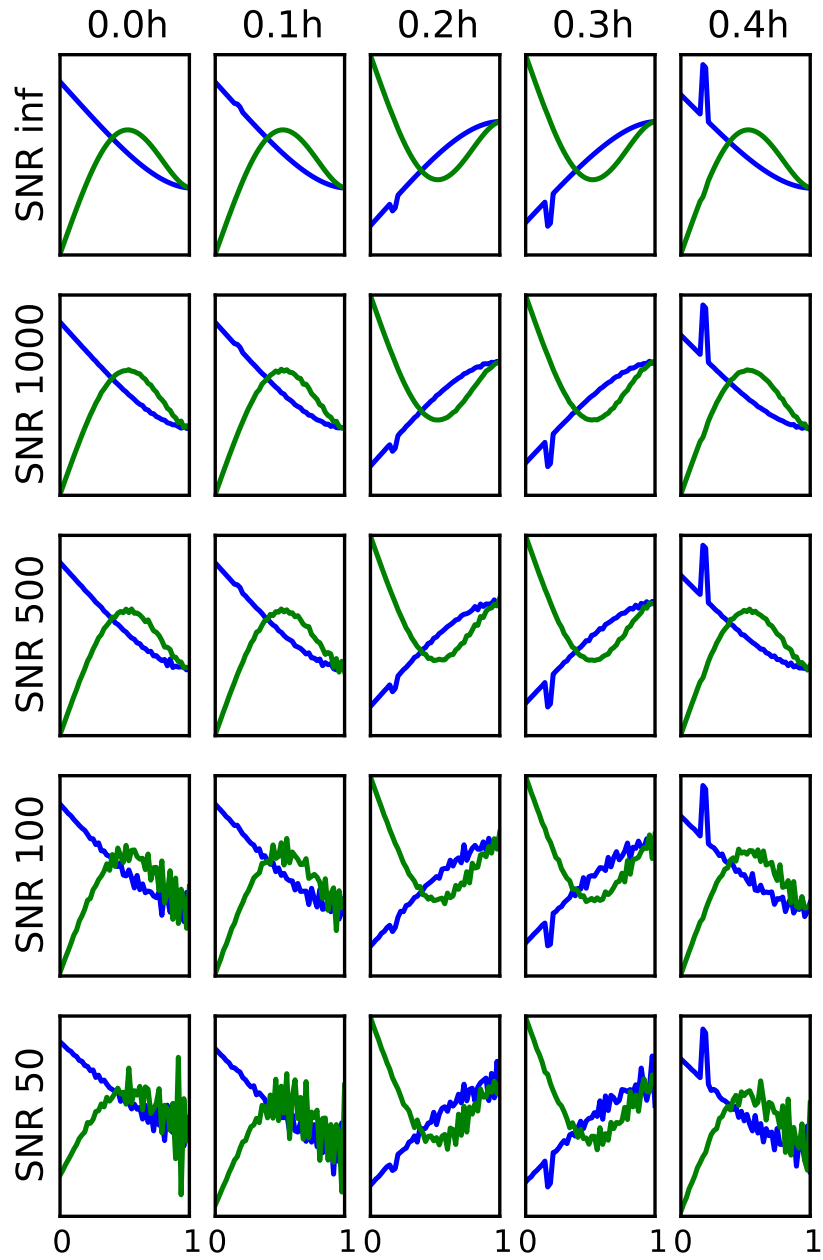
In order to compare our proposed methodology with a simple existing procedure, we provide Figures 2 and 4, which show the curvature (by numerical differentiation) of the POMs (to which our method applies GPR). These figures are formatted similarly to Figures 1 and 3, in that identical crack depths appear in the columns, and identical SNRs appear in the rows.

We note several improvements of our method over the curvature method. First, our method produces a clear numerical indication of the detection of a crack, the sensitivity of which can

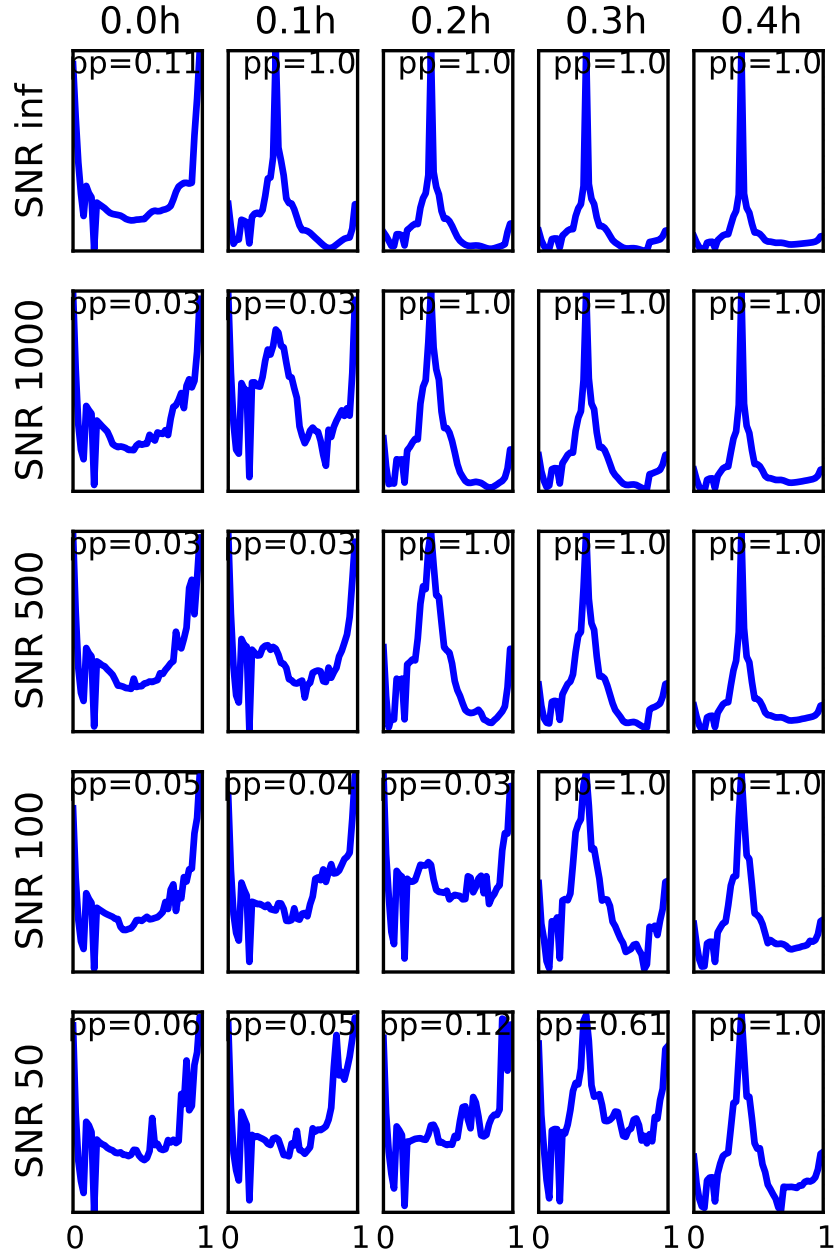


**Figure 1.** Log-marginal likelihood of the model as a function of the parameter  $x_d$ , for various crack depths located at 20% of the length of the beam.

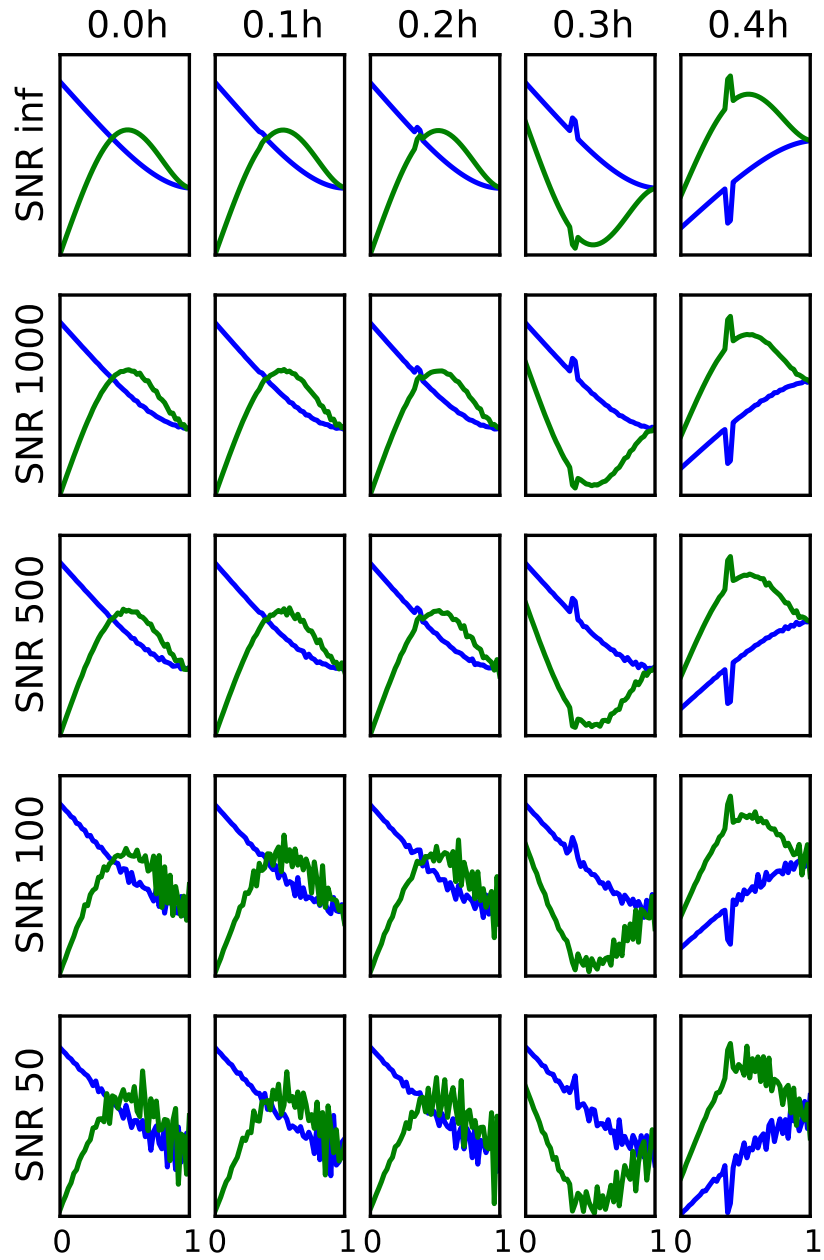
be adjusted by selecting  $p(f)$  in equation 11. Further, whilst both methods are susceptible to noise, ours would appear to be *less susceptible*.



**Figure 2.** POM curvature (first two POMs shown) for damage cases with a crack located at 20% of the beam length, cf. Figure 1



**Figure 3.** Log-marginal likelihood of the model as a function of the parameter  $x_d$ , for various crack depths located at 40% of the length of the beam.



**Figure 4.** POM curvature (first two POMs shown) for damage cases with a crack located at 40% of the beam length, cf. Figure 3

#### 4. Scope for improvement and future work

In this publication, we have introduced a damage detection methodology based upon GPR of POMs. Comparisons with a simple curvature method reveal that there may be some advantages of this method. We hope that through future work, the method can show a greater sensitivity to damage in the case of noisy data. This could be achieved through several methods: the method could be incorporated into a fully probabilistic model of the vibration data; the method could be applied to mode-shapes, instead of POMs; a more sensitive discontinuous covariance structure could be formulated.

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